Water Resources Engineering and Management

Exercises Lecture 5: Hydropower and flood control

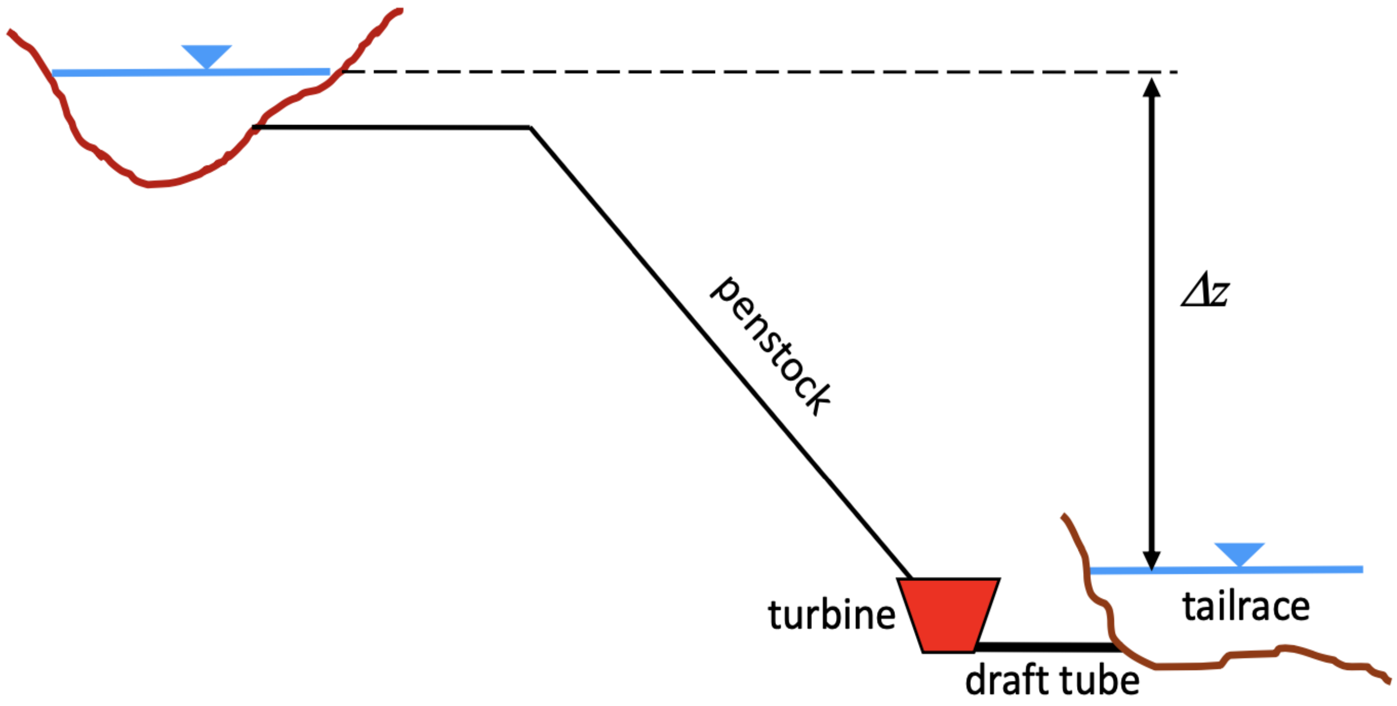
17/03/2025

Exercise 1: Hydropower with storage

A turbine with known operational properties is installed between the two reservoirs sketched in figure, with constant elevation difference between water level. The geometrical and hydraulic characteristic of the penstock are given. The power plant does not work continuously throughout the day.

<u>DATA and ASSUMPTIONS</u>

- Penstock length → $L_P = 250$ m
- Penstock diameter → $D_P = 1$ m
- Draft tube length → $L_D = 30$ m
- Draft tube diameter → $L_D = 2.5$ m
- Penstock and draft tube roughness → $k_s = 0.1$ mm
- Inlet loss coefficient of the penstock → $\xi_{IN} = 0.5$
- Outlet loss coefficient of the draft tube → $\xi_{OUT} = 1$
- Elevation difference → $\Delta z = 100$ m
- Flow rate → $Q = 8\ m^3/s$
- Turbine efficiency → $\eta = 0.8$
- Working duration → $K_t = 6$ hours/day



<u>QUESTIONS</u>

- Determine the power P_T generated by the turbine
- Determine the yearly energy production E_T

Exercise 1. Friction losses in rectilinear pipe - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

Energy production $\rightarrow E_T = P_T \, \Delta t$

Timeframe $\rightarrow \Delta t = 1\text{ y} \cdot 365\text{ d/y} \cdot K_T$

Power $\rightarrow P_T = \eta_T \, \gamma \, Q \, \Delta H_T$

Net head at the turbine $\rightarrow \Delta H_T = \Delta z - \Delta H_{TOT}$

Total energy loss $\rightarrow \Delta H_{TOT} = \sum \Delta H_{MAJOR} + \sum \Delta H_{MINOR}$

Major loss for each pipe $i \rightarrow \Delta H_{MAJOR,\,i} = \dfrac{\lambda_i}{D_i} \dfrac{U_i^2}{2\,g} L_i$

Minor loss for each pipe $i \rightarrow \Delta H_{MINOR,\,i} = \xi_i \dfrac{U_i^2}{2\,g}$

Colebrook – White Eq. Iterative sol. (see Ex. L3) $\rightarrow \lambda_i = \left(-2\log_{10}\left(\dfrac{k_{s,i}}{3.71\,D_i} + \dfrac{2.51}{\mathrm{Re}_i\sqrt{\lambda_i}}\right)\right)^{-2}$

Reynolds number $\rightarrow \mathrm{Re}_i = \dfrac{U_i\,D_i}{\nu}$

L_P [m]	L_D [m]	D_P [m]	D_D [m]	k_s [m]	Δz [m]	Q [m3/s]	η [-]	K_{IN} [-]	K_{OUT} [-]	K_t [h/d]
250	30	1	2.5	0.0001	100	8	0.8	0.5	1	6

PIPE	U [m/s]	Re [-]	λ_1 [-]	λ_2 [-]	λ_3 [-]	λ_4 [-]	ΔH_{MAJOR} [m]	ΔH_{MINOR} [m]	ΔH_{TOT} [m]	ΔH_T [m]	P_T [MW]	E_T [MWh]
PENSTOCK	10.186	10185916	0.011974	0.012158	0.012157	0.012157	16.07	2.64	18.87	81.13	5.09	11155
DRAFT TUBE	1.630	4074367	0.010132	0.010920	0.010914	0.010914	0.02	0.14				

Exercise 2: Run-of-river hydropower plant

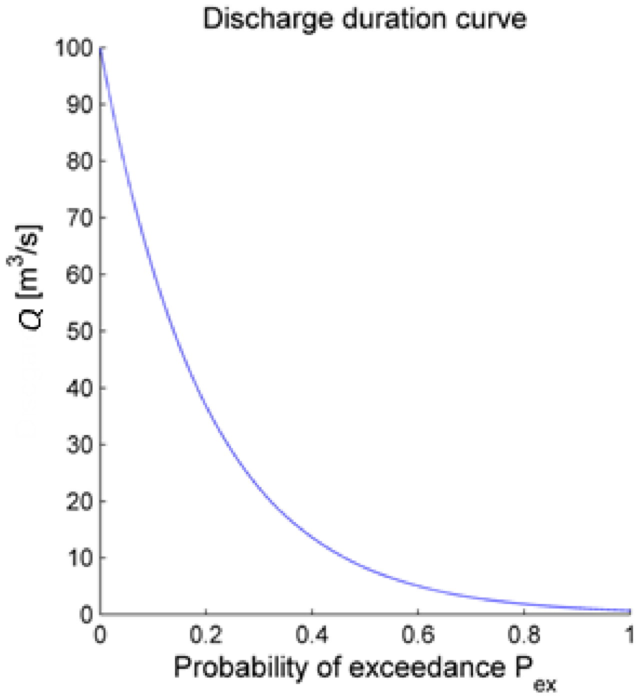
A run-of-river hydropower plant is installed in a river site characterized by a known discharge duration curve in terms of the P_{ex} the exceedance probability (i.e. the fraction of time a certain discharge is equalled or exceeded).
The tailwater rating curve can be approximated as $y[\mathrm{m}] = 0.05\,Q$, where y is the water depth in the tailwater channel.
The water depth in the forebay can be maintained constant for the whole range of discharges considered (0-100 m^3/s).
The hydraulic capacity Q_R of the plant is equal to the discharge that is exceeded 30% of the time ($P_{ex,R}$=0.3).
A horizontal-shaft Kaplan turbine is installed with rated discharge equal to the hydraulic capacity.
The minimum discharge allowable in the turbine is 35% of the rated discharge.
The ratio between the minimum and maximum allowable heads is equal to 0.33.

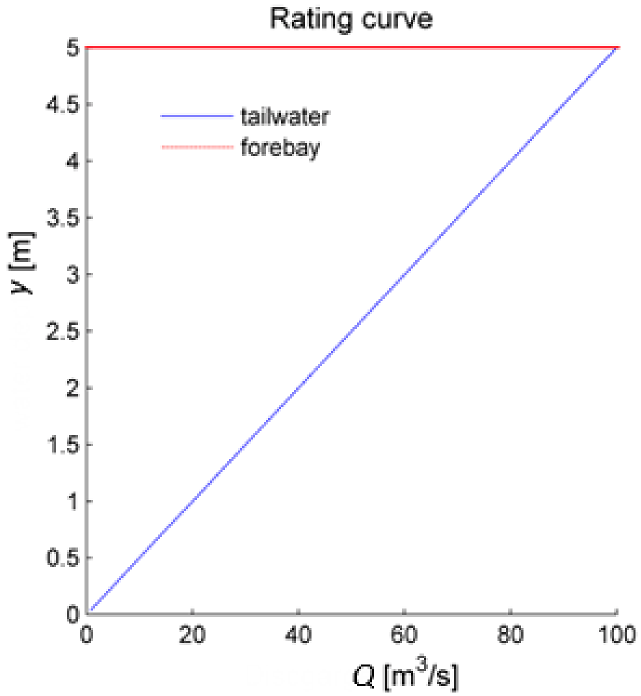
DATA and ASSUMPTIONS

- Flow duration curve → $Q[\mathrm{m}^3/s] = 100\, e^{-5\,P_{\mathrm{ex}}}$
- Flow rating curve → $y = 0.05\, Q$
- Forebay water depth → y_F = 5 m
- Hydraulic capacity → $Q_{\mathrm{R}} = Q\left(P_{\mathrm{ex},R}\right)$
- Minimum flow rate → $Q_{\mathrm{MIN}} = 0.35\, Q_{\mathrm{R}}$
- Turbine efficiency → η = 0.8
- Min-to-max head ratio → K_H = 0.33 m
- Negligible head losses in the power plant

QUESTIONS

○ Determine the expected annual energy production E_T





Exercise 2: Run-of-river hydropower plant - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

Hydraulic capacity $\rightarrow Q_{\mathrm{R}} = 100\, e^{-5 \cdot P_{ex,R}}$

Minimum flow rate $\rightarrow Q_{\mathrm{MIN}} = 0.35\, Q_{\mathrm{R}}$

Net head at turbine $\rightarrow H_{\mathrm{T}}(P_{\mathrm{ex}}) = y_{\mathrm{F}} - y(P_{\mathrm{ex}})$

Maximum net head $\rightarrow \max[H_{\mathrm{T}}] = y_{\mathrm{F}} - y_{\mathrm{MIN}}$

Maximum water depth $\rightarrow y_{\mathrm{MAX}} = y_{\mathrm{F}} - \min[H_{\mathrm{T}}]$

Minimum water depth $\rightarrow y_{\mathrm{MIN}} = 0.05\, Q_{\mathrm{MIN}}$

Net head at turbine $\rightarrow H_{\mathrm{T}}(P_{\mathrm{ex}}) = y_{\mathrm{F}} - 0.05 \cdot 100 e^{-5\, P_{\mathrm{ex}}} = 5(1 - e^{-5\, P_{\mathrm{ex}}})$

Minimum net head $\rightarrow \min[H_{\mathrm{T}}] = K_{\mathrm{H}} \cdot \max[H_{\mathrm{T}}]$

Maximum flow rate $\rightarrow Q_{\mathrm{MAX}} = \dfrac{y_{\mathrm{MAX}}}{0.05}$

Maximum exceedence probability $\rightarrow P_{\mathrm{ex,\,MAX}} = \dfrac{1}{5} \log\left[\dfrac{100}{Q_{\mathrm{MIN}}}\right]$

Minimum exceedence probability $\rightarrow P_{\mathrm{ex,\,MIN}} = \dfrac{1}{5} \log\left[\dfrac{100}{Q_{\mathrm{MAX}}}\right]$

Potential power $\rightarrow P_{\mathrm{P}}(P_{ex}) = \eta\, \gamma\, Q(P_{ex})\, H_T(P_{ex})$

$$\text{Actual power} \rightarrow P_{\mathrm{A}}(P_{ex}) = \begin{cases} \eta\, \gamma\, Q_R\, H_T(P_{\mathrm{ex}}) & P_{\mathrm{ex,\,MIN}} \leq P_{\mathrm{ex}} \leq P_{ex,R} \\ P_{\mathrm{P}}(P_{ex}) & P_{ex,R} \leq P_{\mathrm{ex}} \leq P_{\mathrm{ex,\,MAX}} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Average power} \rightarrow P_{\mathrm{T}} = \int_0^1 P_A(P_{ex})\, dP_{ex}$$

Exercise 2: Run-of-river hydropower plant - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

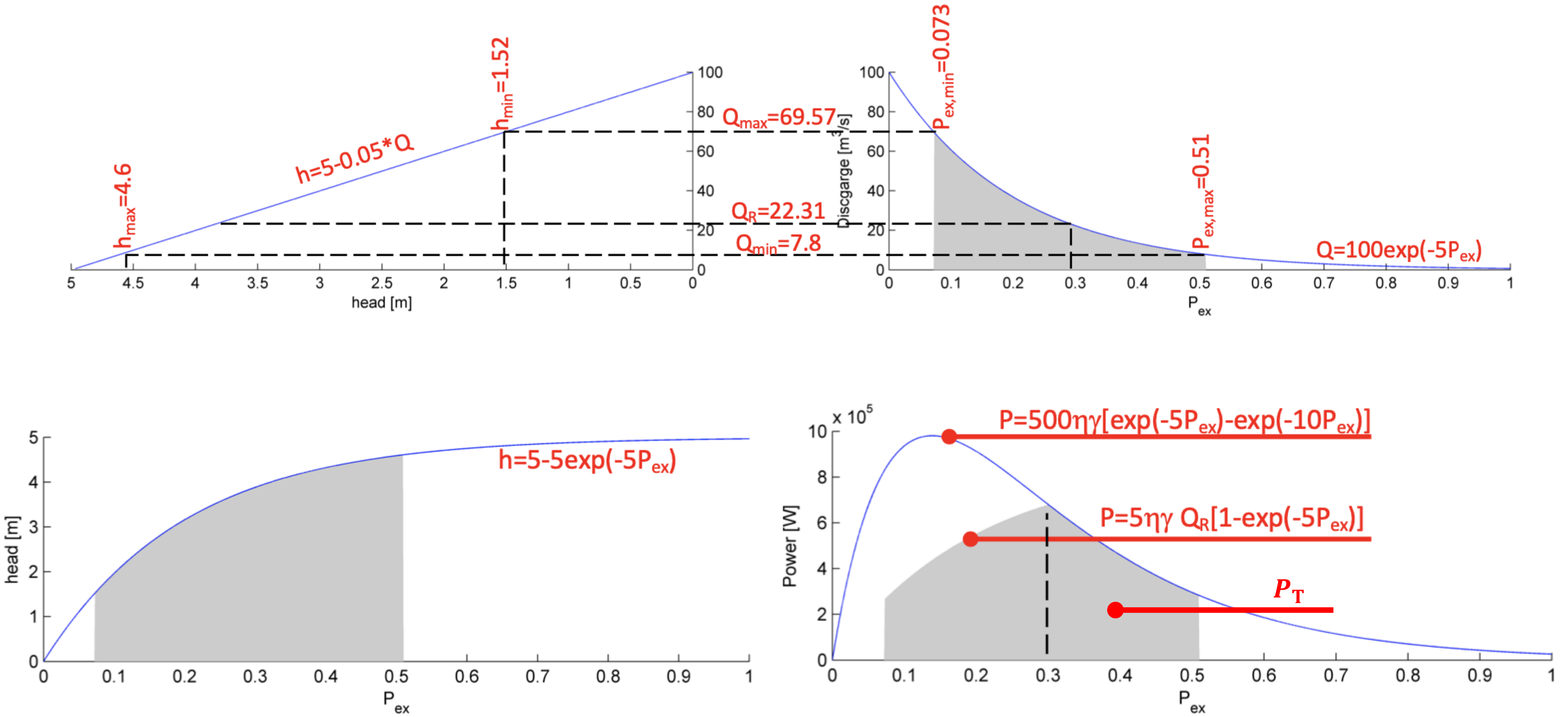
Average power $\rightarrow$ $P_T = \int_0^1 P_A(P_{ex})\, dP_{ex} = \underbrace{\cancel{\int_0^{P_{ex,\,MIN}} P_A(P_{ex})\, dP_{ex}}}_{=0} + \int_{P_{ex,\,MIN}}^{P_{ex,\,R}} P_A(P_{ex})\, dP_{ex} + \int_{P_{ex,\,R}}^{P_{ex,\,MAX}} P_A(P_{ex})\, dP_{ex} + \underbrace{\cancel{\int_{P_{ex,\,MAX}}^{1} P_A(P_{ex})\, dP_{ex}}}_{=0}$

$$P_T = \eta\, \gamma\, Q_R \int_{P_{ex,\,MIN}}^{P_{ex,\,R}} 5(1 - e^{-5\,P_{ex}})\, dP_{ex} + \eta\, \gamma \int_{P_{ex,\,R}}^{P_{ex,\,MAX}} 500(e^{-5\,P_{ex}} - e^{-10\,P_{ex}})dP_{ex}$$

$$= \eta\, \gamma\, Q_R 5\left(P_{ex}\Big|_{P_{ex,\,MIN}}^{P_{ex,\,R}} + \frac{1}{5} e^{-5\,P_{ex}}\Big|_{P_{ex,\,MIN}}^{P_{ex,\,R}} \right) + \eta\, \gamma\, 500\left(-\frac{1}{5} e^{-5\,P_{ex}}\Big|_{P_{ex,R}}^{P_{ex,\,MAX}} + \frac{1}{10} e^{-10\,P_{ex}}\Big|_{P_{ex,R}}^{P_{ex,\,MAX}} \right) = 212.8\ \mathrm{kW}$$

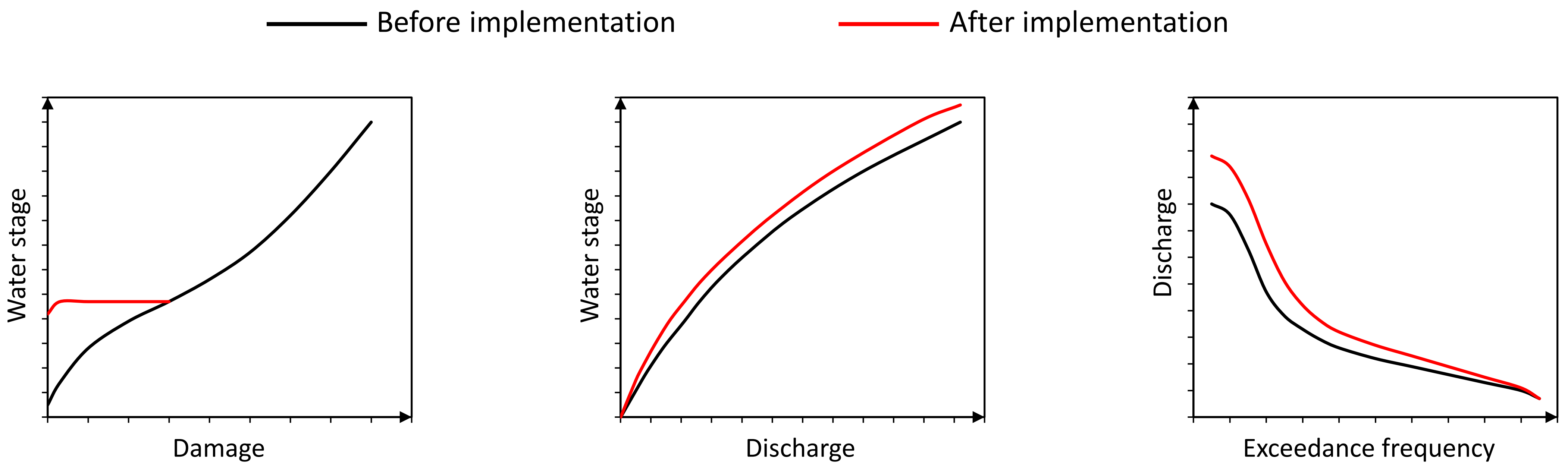
Annual energy production $\rightarrow$ $E_T = P_T \cdot \Delta t = 1.867\ \mathrm{GWh}$

Exercise 2: Run-of-river hydropower plant - SOLUTION



Exercise 3: Flood control

As a consequence of the implementation of a flood control measure, the flood assessment functional relationships are modified as illustrated below.



QUESTIONS

- Describe what type of flood control measure can produce such modifications
- Qualitatively derive the damage-exceedance frequency relationship and the annual expected damage before and after the implementation of the flood control measure

Exercise 3: Flood control - SOLUTION

The areas below the curves correspond to **the annual expected damage** before (gray shading) and after (red pattern) the implementation of the flood control measure