Water Resources Engineering and Management

Exercises Lecture 4: reservoir design and operation, domestic and agricultural water uses

10/03/2025

Exercise 1: Reservoir design and operation

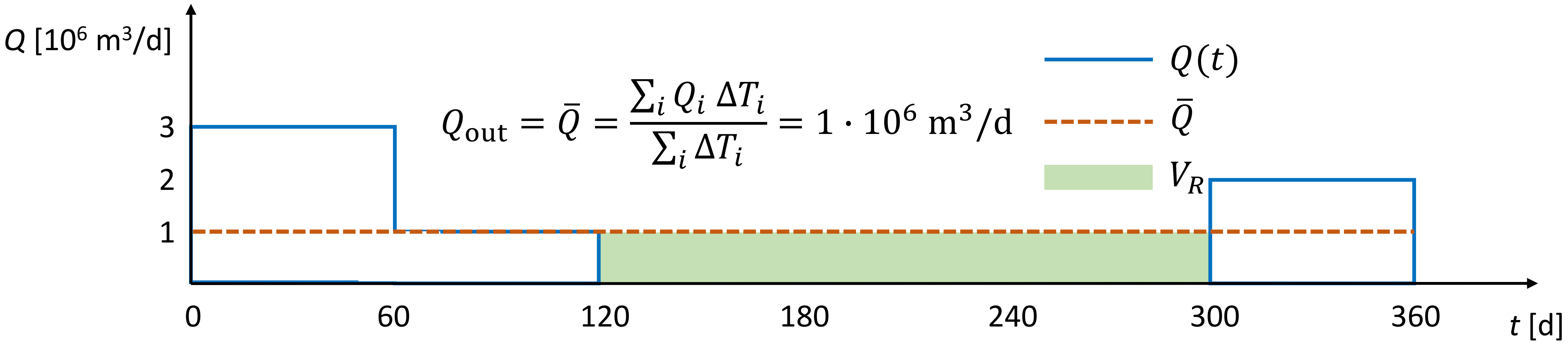
The yearly hydrologic regime of a river can be subdivided in 4 periods, with the duration and the average discharge summarized in the table below.

Period *i*	1	2	3	4
Duration ΔT_i [d]	60	60	180	60
Average flow rate Q_i [m³/d]	$3 \cdot 10^6$	$1 \cdot 10^6$	0	$2 \cdot 10^6$

<u>QUESTIONS</u>

- The reservoir capacity, V_R, needed to withdraw a firm yield (i.e. constant flow rate) equal to the average flow during the year
- The volume of water that must be stored in the reservoir at the beginning of the year, V_0, to obtain an annual steady state functioning of the reservoir
- The best regulation that approximates the average constant discharge supposing that after 20 years the reservoir has lost 50% of its initial volume because of deep sedimentation

Exercise 1: Reservoir design and operation - SOLUTION



$$Q_{\text{out}} = \bar{Q} = \frac{\sum_i Q_i \, \Delta T_i}{\sum_i \Delta T_i} = 1 \cdot 10^6 \text{ m}^3/\text{d}$$

$$V_R = \max[V_{\text{in}} - V_{\text{out}}] - \min[V_{\text{in}} - V_{\text{out}}]$$

Let's define
$$\begin{cases} \Delta T_1 = T_1 \\ \Delta T_1 + \Delta T_2 = T_2 \\ \Delta T_1 + \Delta T_2 + \Delta T_3 = T_3 \\ \Delta T_1 + \Delta T_2 + \Delta T_3 + \Delta T_4 = T_4 \end{cases}$$

$$V_{\text{in}} = \int_t Q_i(t)\, dt = \begin{cases} Q_1 \, t & 0 \le t \le T_1 \\ Q_1 \Delta T_1 + Q_2 (t - T_1) & T_1 < t \le T_2 \\ Q_1 \Delta T_1 + Q_2 \Delta T_2 + Q_3 (t - T_2) & T_2 < t \le T_3 \\ Q_1 \Delta T_1 + Q_2 \Delta T_2 + Q_3 \Delta T_3 + Q_4 (t - T_3) & T_3 < t \le T_4 \end{cases}$$

$$V_{\text{out}} = \int_t Q_{\text{out}}(t) dt = \bar{Q} \, t$$

Exercise 1: Reservoir design and operation - SOLUTION

$$V_{\text{in}} - V_{\text{out}} = \begin{cases} (Q_1 - \bar{Q})\, t & , \quad 0 \leq t \leq T_1 \\ (Q_1 - \bar{Q})T_1 + (Q_2 - \bar{Q})(t - T_1) & , \quad T_1 < t \leq T_2 \\ (Q_1 - \bar{Q})\Delta T_1 + (Q_2 - \bar{Q})\Delta T_2 + (Q_3 - \bar{Q})(t - T_2) & , \quad T_2 < t \leq T_3 \\ (Q_1 - \bar{Q})\Delta T_1 + (Q_2 - \bar{Q})\Delta T_2 + (Q_3 - \bar{Q})\Delta T_3 + (Q_4 - \bar{Q})(t - T_3) & , \quad T_3 < t \leq T_4 \end{cases}$$

We can now calculate the quantity $V_{\text{in}} - V_{\text{out}}$, for each period, at the beginning or at the end of each period.
Let's do it at the beginning of each period, i.e. for *i*=1, *t*=0; for *i*=2, $t=T_1$; and so on…

$$V_{\text{in}} - V_{\text{out}} = \begin{cases} 0 & , \quad t = 0 \\ (Q_1 - \bar{Q})T_1 & , \quad t = T_1 \\ (Q_1 - \bar{Q})\Delta T_1 + (Q_2 - \bar{Q})\Delta T_2 & , \quad t = T_2 \\ (Q_1 - \bar{Q})\Delta T_1 + (Q_2 - \bar{Q})\Delta T_2 + (Q_3 - \bar{Q})\Delta T_3 & , \quad t = T_3 \end{cases} = 10^6\ \text{m}^3 \cdot \begin{cases} 0 & , \quad t = 0 \\ 120 & , \quad t = T_1 \\ 120 & , \quad t = T_2 \\ -60 & , \quad t = T_3 \end{cases}$$

$$\max[V_{\text{in}} - V_{\text{out}}] = \max \begin{bmatrix} 0 \\ 120 \\ 120 \\ -60 \end{bmatrix} \cdot 10^6\ \text{m}^3 = 120 \cdot 10^6\ \text{m}^3 \qquad \min[V_{\text{in}} - V_{\text{out}}] = \min \begin{bmatrix} 0 \\ 120 \\ 120 \\ -60 \end{bmatrix} \cdot 10^6\ \text{m}^3 = -60 \cdot 10^6\ \text{m}^3$$

$$V_R = \max[V_{\text{in}} - V_{\text{out}}] - \min[V_{\text{in}} - V_{\text{out}}] = \big(120 - (-60)\big) \cdot 10^6\ \text{m}^3 = 180 \cdot 10^6\ \text{m}^3$$

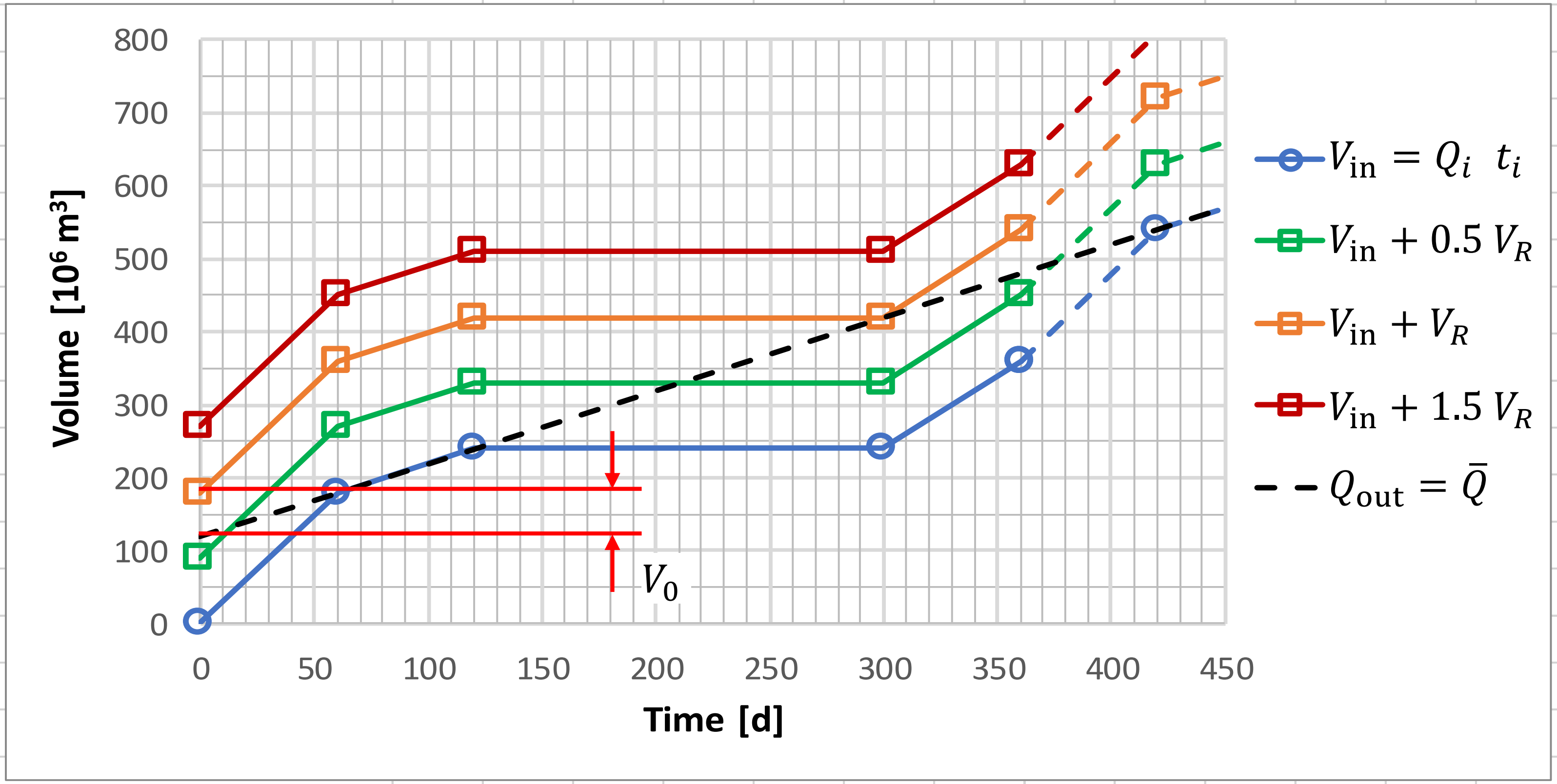
$$V_0 = -\min[V_{\text{in}} - V_{\text{out}}] = -(-60) \cdot 10^6\ \text{m}^3 = 60 \cdot 10^6\ \text{m}^3$$

Exercise 1: Reservoir design and operation - SOLUTION

Q [10^6 m^3/d]	1
0.5 V_R [10^6 m	90
V_R [10^6 m^3]	180
1.5 V_R [10^6 m	270

Period	1	2	3	4		1	2	3	4	1	2	3	4
Duration [d]	60	60	180	60	0	60	60	180	60	60	60	180	60
Time [d]	-360	-300	-240	-60	0	60	120	300	360	420	480	660	720
Average flow rate [10^6 m^3/d]	3	1	0	2		3	1	0	2	3	1	0	2
Flown volume [10^6 m^3]	-360	-180	-120	-120	0	180	240	240	360	540	600	600	720
Flown volume + 0.5 V_R [10^6	-270	-90	-30	-30	90	270	330	330	450	630	690	690	810
Flown volume + V_R [10^6 m^3]	-180	0	60	60	180	360	420	420	540	720	780	780	900
Flown volume + 1.25 V_R [10^6	-90	90	150	150	270	450	510	510	630	810	870	870	990
Volume [10^6 m^3]					60	180	180	0	60				

Constant yield		
Time	Volume	Qout
[d]	[10^6 m^3]	[10^6 m^3 / d]
0	120	
60	180	1
420	540	1
450	570	1



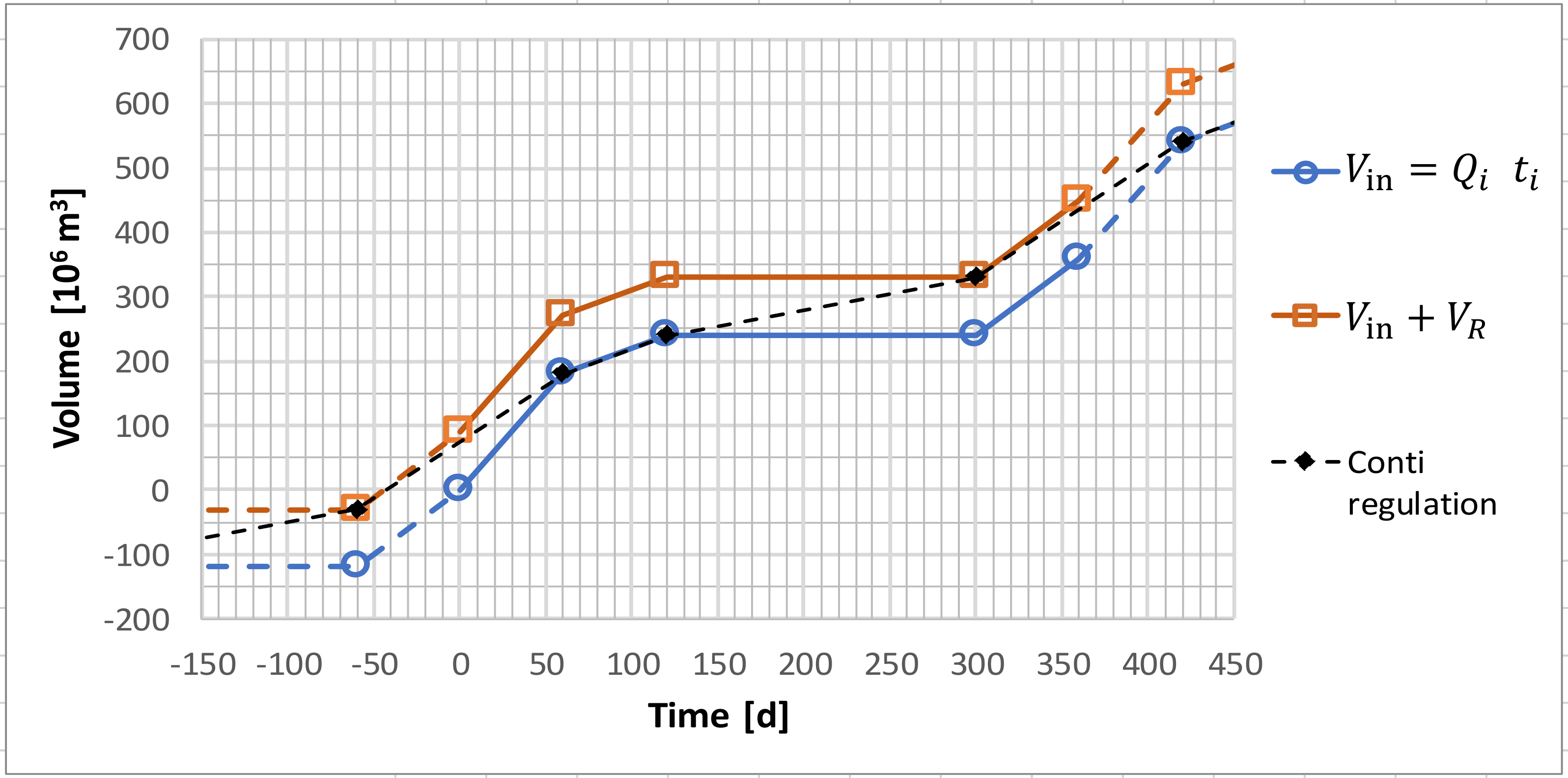
Exercise 1: Reservoir design and operation - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

Q [10^6 m^3/d]	1
V_{R0} [10^6 m^3]	180
ΔV [%]	-50
$V_{R,t}$ [10^6 m^3]	90

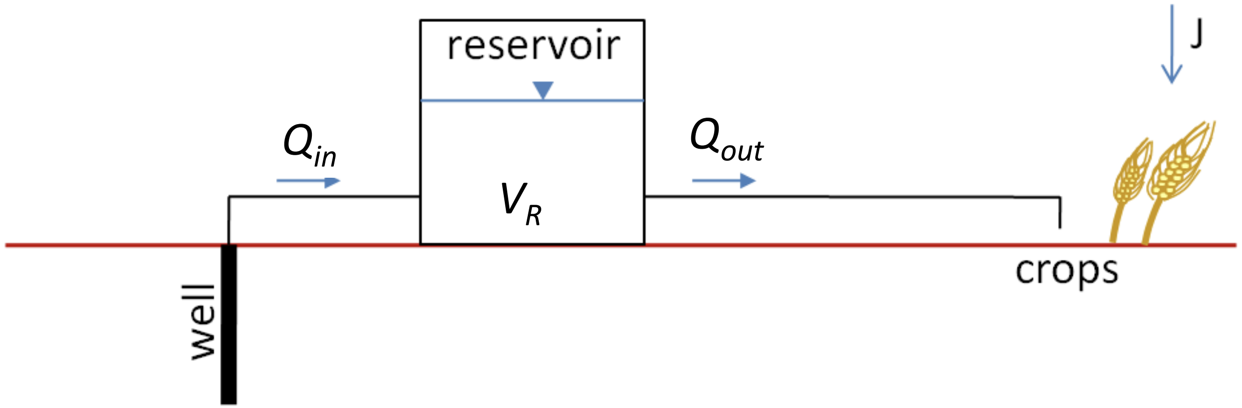
Period	1	2	3	4		1	2	3	4	1	2	3	4
Duration [d]	60	60	180	60	0	60	60	180	60	60	60	180	60
Time [d]	-360	-300	-240	-60	0	60	120	300	360	420	480	660	720
Average flow rate [10^6 m^3/d]	3	1	0	2		3	1	0	2	3	1	0	2
Flown volume [10^6 m^3]	-360	-180	-120	-120	0	180	240	240	360	540	600	600	720
Flown volume + V_R [10^6 m^3]	-270	-90	-30	-30	90	270	330	330	450	630	690	690	810
Volume [10^6 m^3]					60	180	180	0	60				

Conti regulation		
Time	Volume	Qout
[d]	[10^6 m^3]	[10^6 m^3 / d]
-240	-120	
-60	-30	0.5
60	180	1.75
120	240	1
300	330	0.5
420	540	1.75
480	600	1



Exercise 2. Irrigation need and reservoir design

An irrigation system is constituted by a groundwater well which supplies a constant discharge Q_{in} to a reservoir and a pipeline system which connects the water reservoir to a crop area. The yearly hydro-climatic and crop growth regimes of the irrigated area can be subdivided into three periods as illustrated in the table below.



Period	1	2	3
Duration ΔT [d]	100	65	200
Average crop evapotraspiration ET_0 [mm/d]	2.5	1	1.5
Crop factor K_c [-]	1.2	0.5	1
Average rainfall J [mm/h]	0.1	0.1	0.1

DATA and ASSUMPTIONS

- Crop Area → $A = 1\ Km^2$
- Irrigation system efficiency → $\eta = 0.6$
- Effective rainfall to crop → $K_p = 0.5$
- Evaporation from the reservoir can be neglected

QUESTIONS

- The constant flow rate Q_{in} that must be pumped throughout the year to meet the irrigation water demand
- The design storage of the reservoir, V_R
- The initial volume , $V_{R,0}$, that must be stored in the reservoir at the beginning of the year.

Exercise 2. Irrigation need and reservoir design - SOLUTION

For each period *i*, we calculate…

Average crop water need → $ET_{c,\,i} = K_{c,i} \cdot ET_{0,i}$

Irrigation water need → $IWN_i = \max\left[0\,;\,ET_{c,i} - K_p \cdot J_i\right]$

Irrigation water withdrawal → $IWW_i = \dfrac{IWN_i}{\eta}$

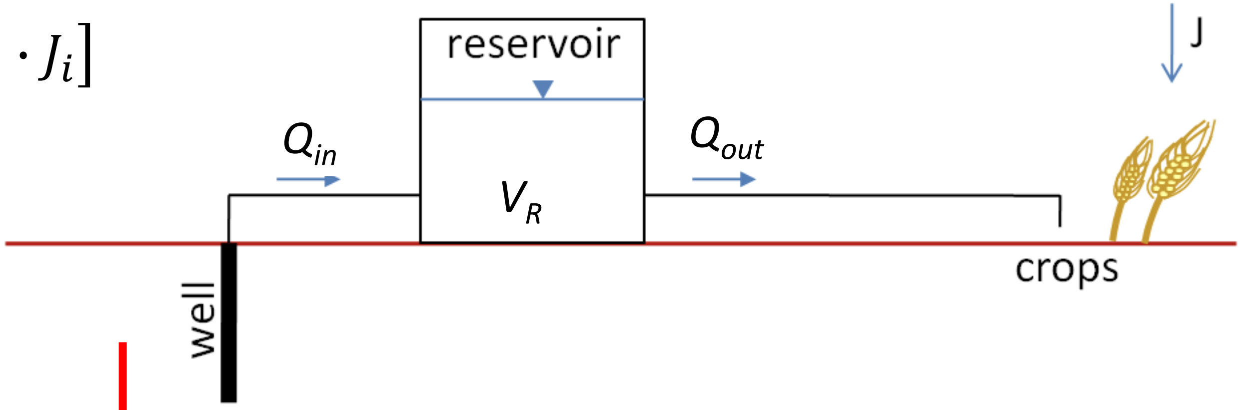
Required irrigation flow rate → $Q_{\text{out},\,i} = IWW_i \cdot A$

Pump flow rate → $Q_{\text{in}} = \dfrac{\sum_i Q_{\text{out},\,i}\,\Delta T_i}{\sum_i \Delta T_i}$

Cumulative irrigation volume → $V_{\text{out},\,i} = \sum_{x=1}^{i} Q_{\text{out},\,x} \cdot \Delta T_x$

Cumulative pumped volume → $V_{\text{in},\,i} = \sum_{x=1}^{i} Q_{\text{in}} \cdot \Delta T_x$

Required reservoir volume → $\Delta V_i = V_{\text{in},\,i} - V_{\text{out},\,i}$



Pump flow rate → $Q_{\text{in}} = \dfrac{\sum_i Q_{\text{out},\,i}\,\Delta T_i}{\sum_i \Delta T_i}$

Reservoir volume → $V_R = \max[\Delta V_i] - \min[\Delta V_i]$

Initially stored water volume → $V_{R,\,0} = -\min[\Delta V_i]$

Exercise 2. Irrigation need and reservoir design - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

A [Km2]	1
η [-]	0.6

Duration ΔT [d]	100	65	200
Average crop evapotraspiration ET_0 [mm/d]	2.5	1	1.5
Crop factor K_c [-]	1.2	0.5	1
Average rainfall J [mm/h]	0.1	0.1	0.1
Average crop water need ET_c [mm/d]	3	0.5	1.5
Irrigation water need, IWN [mm/d]	1.8	0	0.3
Irrigation water withdrawal, IWW [mm/d]	3	0	0.5
Required irrigation flow rate, Q_{out} [m^3/d]	3000	0	500
Pump flow rate, Q_{in} [m^3/d]	1096		
Cumulative irrigation volume, V_{out} [m^3]	300000	300000	400000
Cumulative pumped volume, V_{in} [m^3]	109589	180822	400000
Volume difference , ΔV_i [m^3]	-190411	-119178	0
Reservoir volume, V_R [m^3]	190411		
Initially stored water volume, $V_{R,0}$ [m^3]	190411		

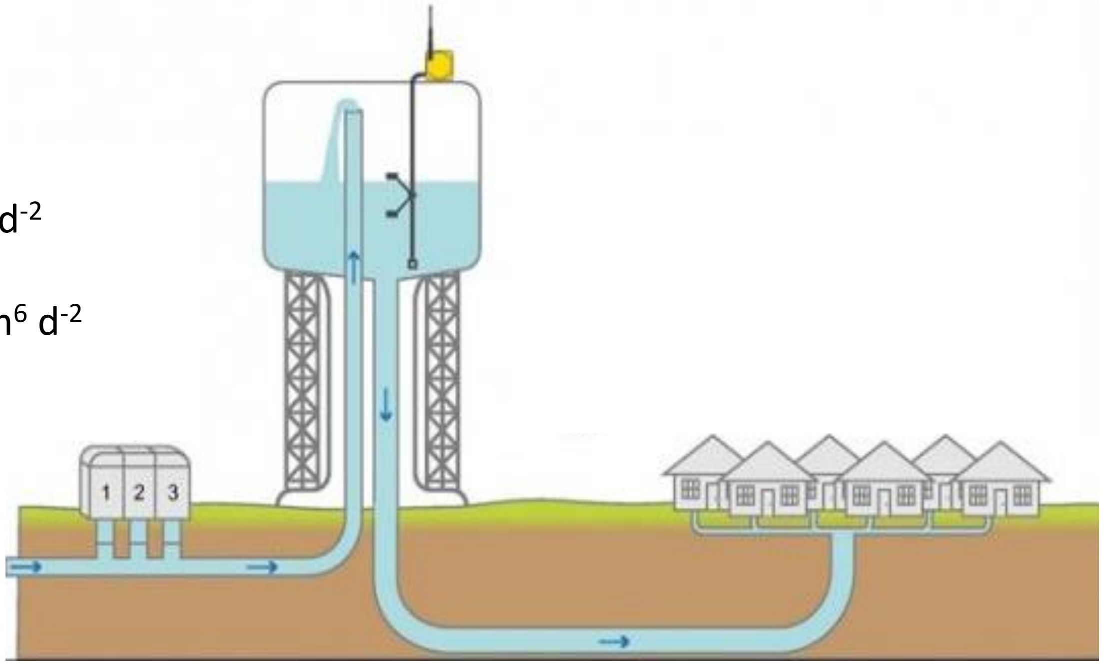
Exercise 3. Probabilistic design of water reservoir

An existing reservoir has been designed to function at a null failure rate for a certain return period.
The corresponding empirical variance of the inflow data corresponding to the assumed return period is known.

The community is considering to increase the size of the reservoir that may work for a longer period.
The characteristic properties of the process behind the reservoir design are known.

DATA and ASSUMPTIONS

- Initial reservoir volume → V_1 = 200 m^3
- Initial return period → T_1 = 50 years
- Empirical variance of inflow data, first design process → S_1^2 = 3320 m^6 d^{-2}
- Future return period → T_2 = 80 years
- Empirical variance of inflow data, future design process → S_2^2 = 3450 m^6 d^{-2}
- Hurst exponent → 0.56
- Failure rate does not change



QUESTIONS

- The reservoir volume, V_2, in the future scenario

Exercise 3. Probabilistic design of water reservoir - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

$$\frac{V_i}{S_i} = K\,T_i^H$$

$$\frac{V_1}{S_1} = K\,T_1^H \quad \Longrightarrow \quad K = \frac{V_1}{S_1\,T_1^H}$$

$$\frac{V_2}{S_2} = K\,T_2^H \quad \Longrightarrow \quad V_2 = V_1\,\frac{S_2}{S_1}\left(\frac{T_2}{T_1}\right)^H = 265\ \mathrm{m}^3$$