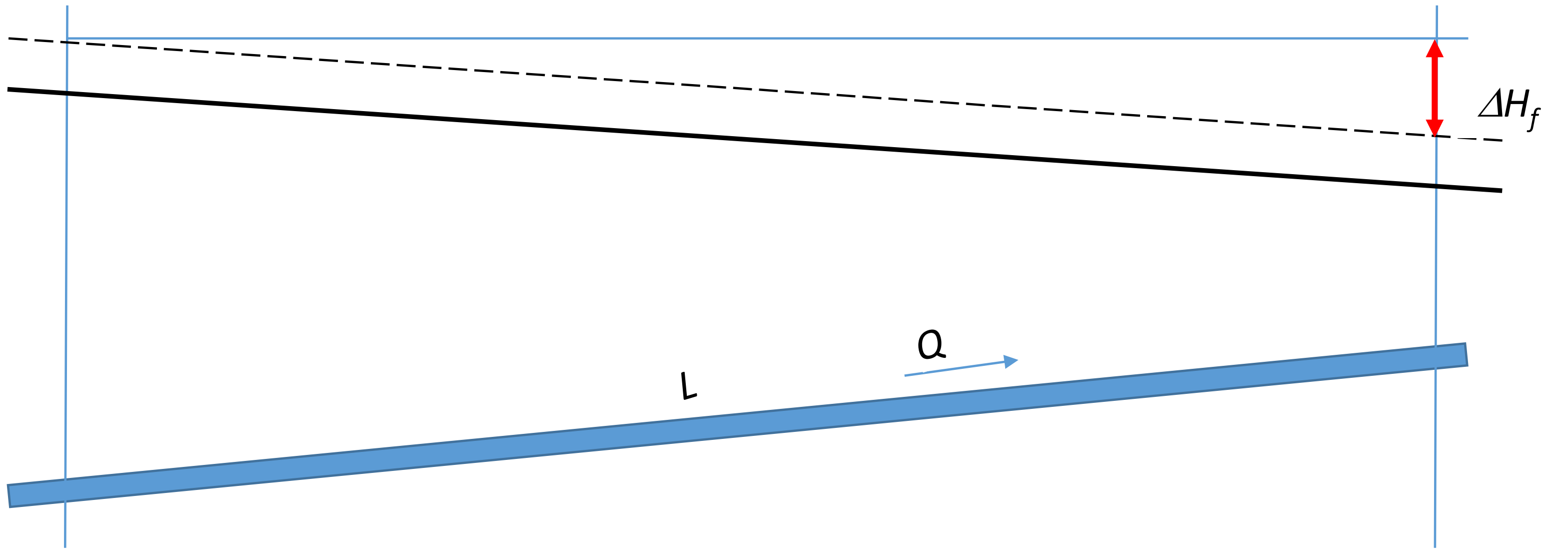
Water Resources Engineering and Management

Exercises Lecture 3: pipe hydraulics, energy and hydraulic grade lines, pipe loops, pumping systems

03/03/2025

Exercise 1. Friction losses in rectilinear pipe

A 20m-long rectilinear pipe is used to convey a flowrate of Q=5 L/s.
The pipe diameter is 6.5 cm and the wall roughness is k_s= 0.26 mm.



<u>QUESTIONS</u>

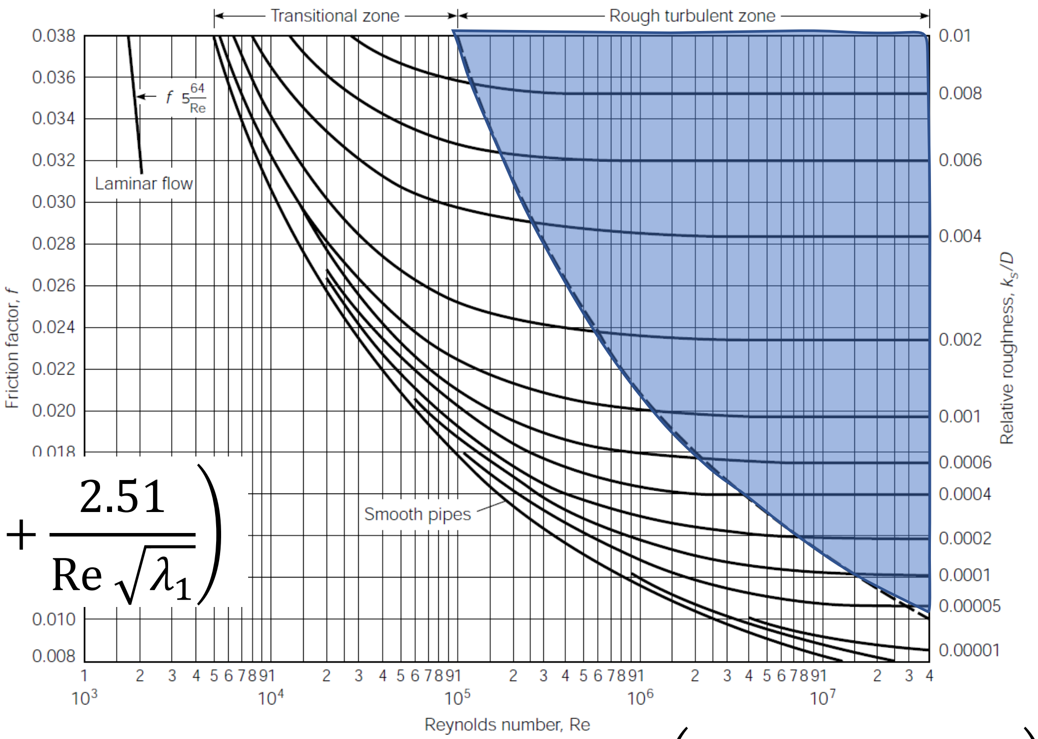
- The energy loss, ΔH_f, at the end of the pipe
- The energy loss, ΔH_{SM}, in the hypothesis of hydraulically smooth pipe

Exercise 1. Friction losses in rectilinear pipe - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k_s}{3.71\,D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

Colebrook-White equation
To be solved numerically!!



Alternatively, iterative method:

1) Consider "rough turbulent" regime → $\lambda_1 = \left(2\log_{10}\left(\frac{3.71\,D}{k_s}\right)\right)^{-2}$

   Hp) $\mathrm{Re} \longrightarrow \infty$

2) Use λ_1 to use Colebrook-White equation → $\frac{1}{\sqrt{\lambda_2}} = -2\log_{10}\left(\frac{k_s}{3.71\,D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda_1}}\right)$

$$\mathrm{Re} = \frac{U\,D}{\nu}$$ Reynolds number

3) Repeat the previous step until convergence ($\left|\frac{\lambda_{i+1} - \lambda_i}{\lambda_{i+1}}\right| \leq 0.1\%$, usually 2-3 iterations) → $\frac{1}{\sqrt{\lambda_{i+1}}} = -2\log_{10}\left(\frac{k_s}{3.71\,D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda_i}}\right)$

λ_1 = 0.0283931
λ_2 = 0.0295225 $\Delta\lambda_{12}$ = 3.826%
λ_3 = 0.0295014 $\Delta\lambda_{23}$ = -0.0715%
λ_4 = 0.0295018 $\Delta\lambda_{34}$ = 0.0014%

Iterative solution

→ $\lambda = \lambda_4 = 0.0295018$

Numerical solution

$\lambda_{\mathrm{NUM}} = 0.029501798$

$$\Delta H_f = \frac{\lambda}{D}\frac{U^2}{2\,g}L = \frac{\lambda}{D}\frac{16\,Q^2}{\pi^2\,D^4 2g}L = \frac{8\,\lambda}{\pi^2\,g}\frac{Q^2}{D^5}L = 1.05 \text{ m}$$

EPFL

Exercise 1. Friction losses in rectilinear pipe - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

In the case of hydraulically smooth pipe, $k_s = 0$

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

Colebrook-White equation for smooth pipe

Similar iterative procedure, starting from the same λ_1 (rough pipe, real k_s)

$$\lambda_1 = \left(2\log_{10}\left(\frac{3.71\,D}{k_s}\right)\right)^{-2}$$

$\lambda_1 = 0.0283931$
$\lambda_2 = 0.0171511$
$\lambda_3 = 0.0181785$ $\Delta\lambda_{23} = 5.65\%$
$\lambda_4 = 0.0180553$ $\Delta\lambda_{34} = -0.682\%$
$\lambda_5 = 0.0180696$ $\Delta\lambda_{45} = 0.0791\%$

Iterative solution

$$\rightarrow \lambda_{SM} = \lambda_5 = 0.0180696$$

Numerical solution

$$\lambda_{\mathrm{SM,\,NUM}} = 0.0180681$$

$$\Delta H_{SM} = \frac{\lambda_{SM}}{D}\frac{U^2}{2\,g}L = \frac{\lambda_{SM}}{D}\frac{16\,Q^2}{\pi^2\,D^4 2g}L = \frac{8\,\lambda_{SM}}{\pi^2\,g}\frac{Q^2}{D^5}L = 0.64\ \mathrm{m}$$

Exercise 2. Hydraulic diagnostic (verification) problem

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

We consider a conduct constituted by two pipes and with diameter-dependent Darcy coefficients β_1 and β_2.
The pipe system delivers water from a tank with constant water elevation.
The geometry of the system is defined, as shown.
The energy loss coefficient ξ at the restriction is given in Table 1.

DATA

- Diameter of pipe 1 → D_1= 470 mm
- Diameter of pipe 2 → D_2= 300 mm
- Length of pipe 1 → L_1= 45 m
- Length of pipe 2 → L_2= 25 m
- Water level in the tank → z_1= 10 m
- Elevation at the end of pipe 2 → z_2= 5 m

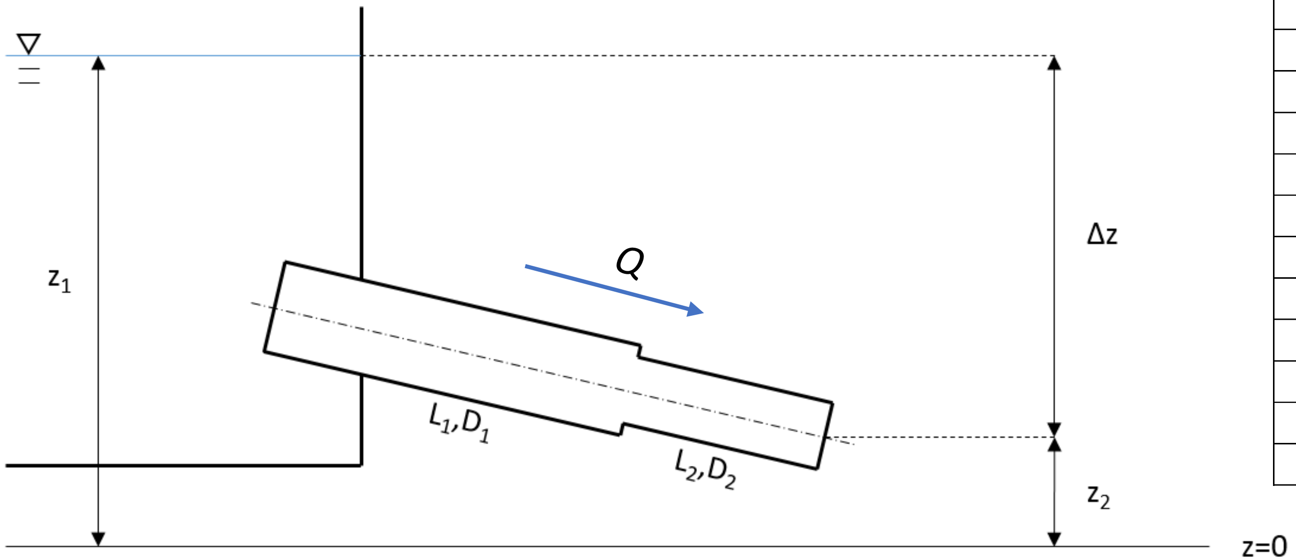


Table 1

D_2/D_1	ξ
0.01	0.5
0.1	0.477
0.2	0.452
0.3	0.425
0.4	0.396
0.5	0.358
0.6	0.31
0.7	0.243
0.8	0.166
0.9	0.086
1	0

Darcy's law for β coefficient

$$\beta = 2\left(0.00164 + \frac{0.000042}{D}\right)$$

Energy loss with β coefficient

$$\Delta H_i = \beta_i \frac{Q_i^2}{D_i^5} L_i$$

QUESTIONS

- The flow rate Q
- Draw the hydraulic grade (piezometric) line and the energy grade line

Exercise 2. Hydraulic diagnostic problem - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

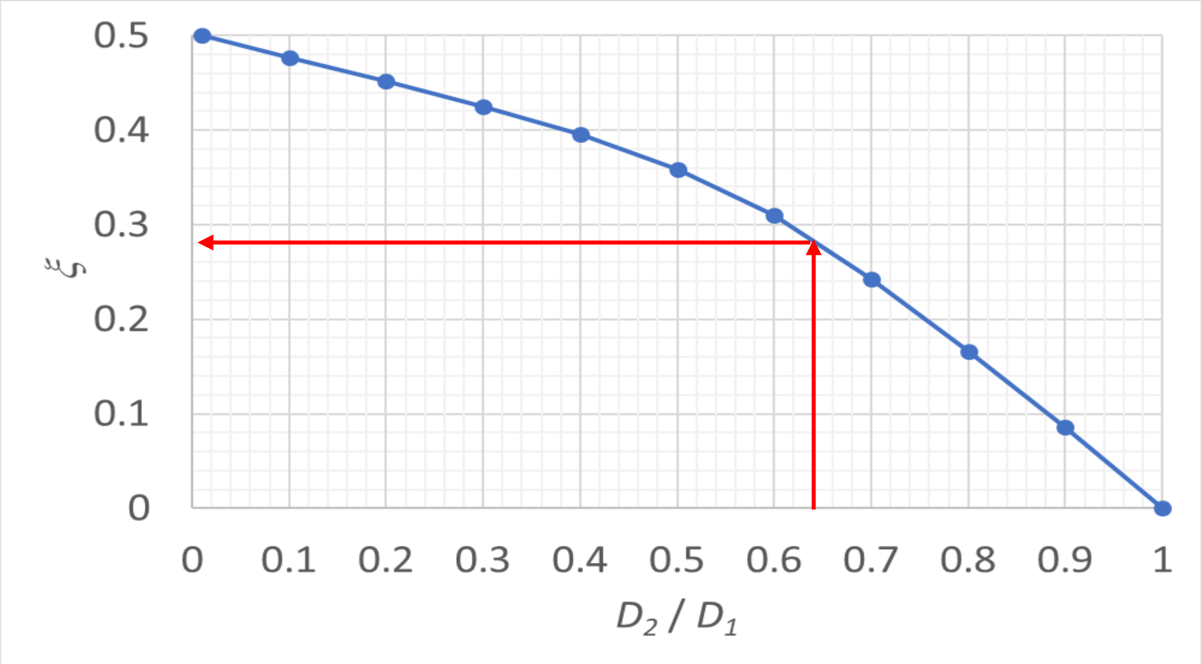
Determination of the minor loss coefficient at the restriction $D_2/D_1 = 0.6383$

By graphical evaluation

$D_2/D_1 \approx 0.64$

Table 1

D_2/D_1	ξ
0.01	0.5
0.1	0.477
0.2	0.452
0.3	0.425
0.4	0.396
0.5	0.358
0.6	0.31
0.7	0.243
0.8	0.166
0.9	0.086
1	0



$\xi \approx 0.28$

By linear interpolation:

$$\xi = \xi\left(\inf\left(\frac{D_2}{D_1}\right)\right) + \frac{\xi\left(\sup\left(\frac{D_2}{D_1}\right)\right) - \xi\left(\inf\left(\frac{D_2}{D_1}\right)\right)}{\sup\left(\frac{D_2}{D_1}\right) - \inf\left(\frac{D_2}{D_1}\right)}\left(\frac{D_2}{D_1} - \inf\left(\frac{D_2}{D_1}\right)\right) = \frac{0.243 - 0.310}{0.7 - 0.6}(0.6383 - 0.6) + 0.31 = 0.284$$

EPFL

Exercise 2. Hydraulic diagnostic problem - SOLUTION

$$z_A + \frac{p_A}{\gamma} + \frac{U_A^2}{2\,g} = z_B + \frac{p_B}{\gamma} + \frac{U_B^2}{2\,g} + \Delta H$$

ENERGY BALANCE

A: water surface in the tank → $p_A = 0$
B: outlet of pipe 2 → $p_B = 0$

A: water surface in the tank → $z_A = z_1$; $p_A = 0$; $Q_A = 0$
B: outlet of pipe 2 → $z_B = z_2$; $p_B = 0$; $Q_B = Q$

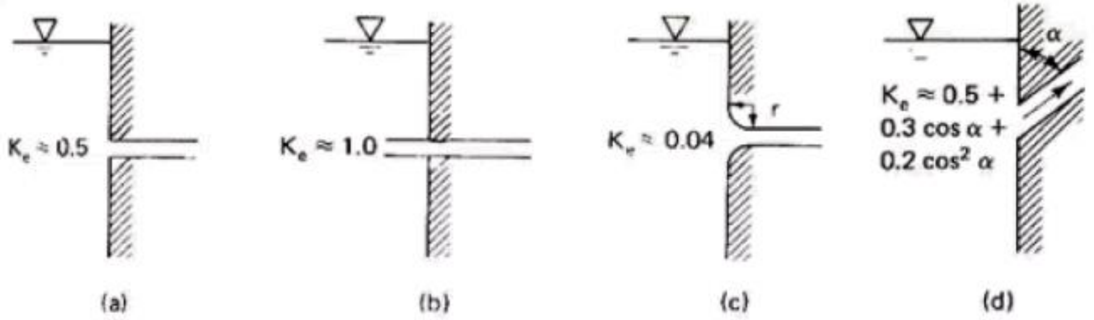
We express all the terms as a function of the flowing discharge, *Q*

1) The major loss in the pipe 1 → $\beta_1 = 2\left(0.00164 + \frac{0.000042}{D_1}\right) = 0.00346$ → $\Delta H_1 = \beta_1 \frac{Q^2}{D_1^5} L_1$

2) The major loss in the pipe 2 → $\beta_2 = 2\left(0.00164 + \frac{0.000042}{D_2}\right) = 0.00356$ → $\Delta H_2 = \beta_2 \frac{Q^2}{D_2^5} L_2$

3) The minor loss at the pipe inlet → $\Delta H_1^c = \frac{U_1^2}{2\,g} = \frac{Q^2}{2\,g\,\Omega_1^2}$

4) The minor loss at the pipe restriction → $\Delta H_{12}^c = \xi \frac{U_2^2}{2\,g} = \xi \frac{Q^2}{2\,g\,\Omega_2^2}$

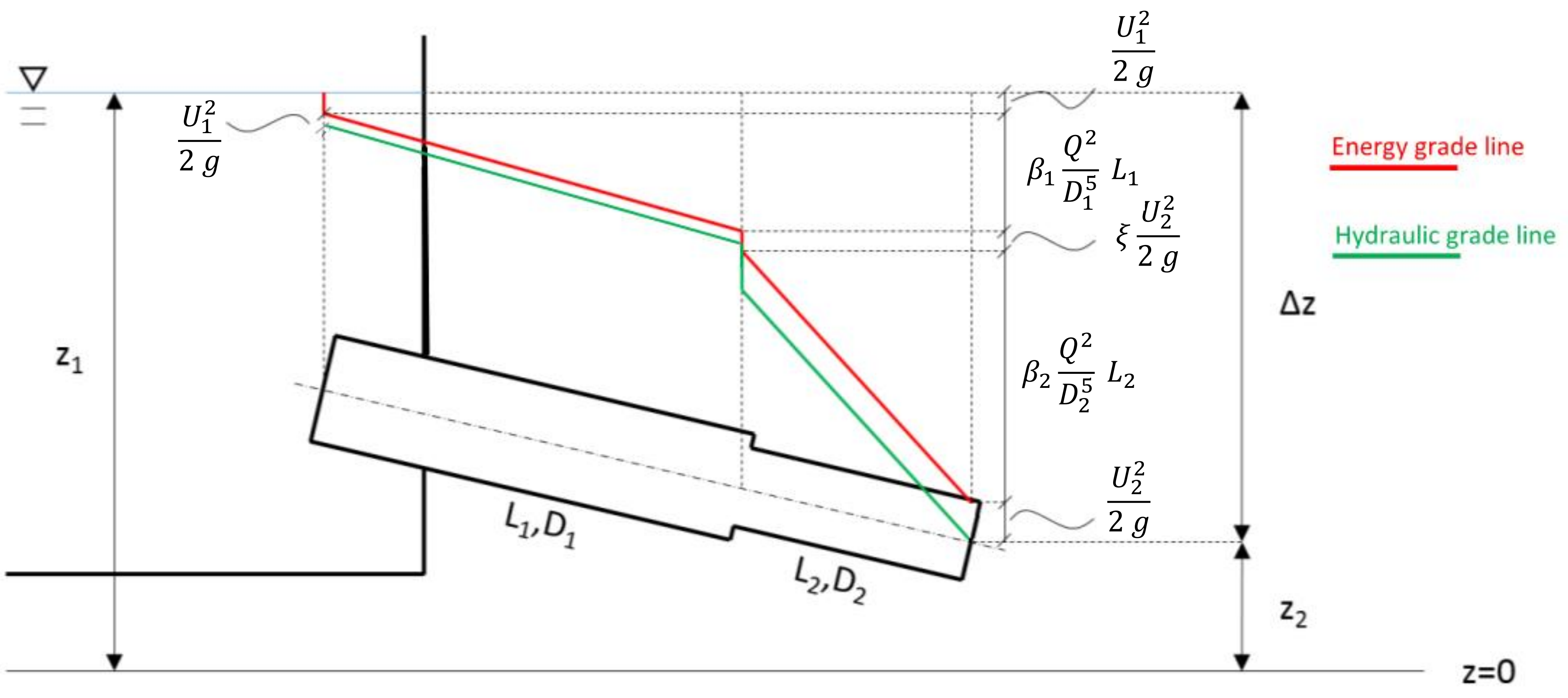


$$\Delta H = \sum_i \Delta H_i = Q^2\left(\beta_1 \frac{L_1}{D_1^5} + \beta_2 \frac{L_2}{D_2^5} + \frac{8}{g\,\pi^2 D_1^4} + \frac{8\,\xi}{g\,\pi^2 D_2^4}\right)$$

ENERGY BALANCE $z_1 = z_2 + \frac{Q^2}{2\,g\,\Omega_2^2} + \Delta H \Rightarrow Q = \sqrt{\frac{z_1 - z_2}{\frac{1}{D_1^4}\left(\frac{8}{g\,\pi^2} + \beta_1 \frac{L_1}{D_1}\right) + \frac{1}{D_2^4}\left(\frac{8}{g\,\pi^2}(1+\xi) + \beta_2 \frac{L_2}{D_2}\right)}} = 0.293\ \text{m}^3/\text{s}$$

Exercise 2. Hydraulic diagnostic problem - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

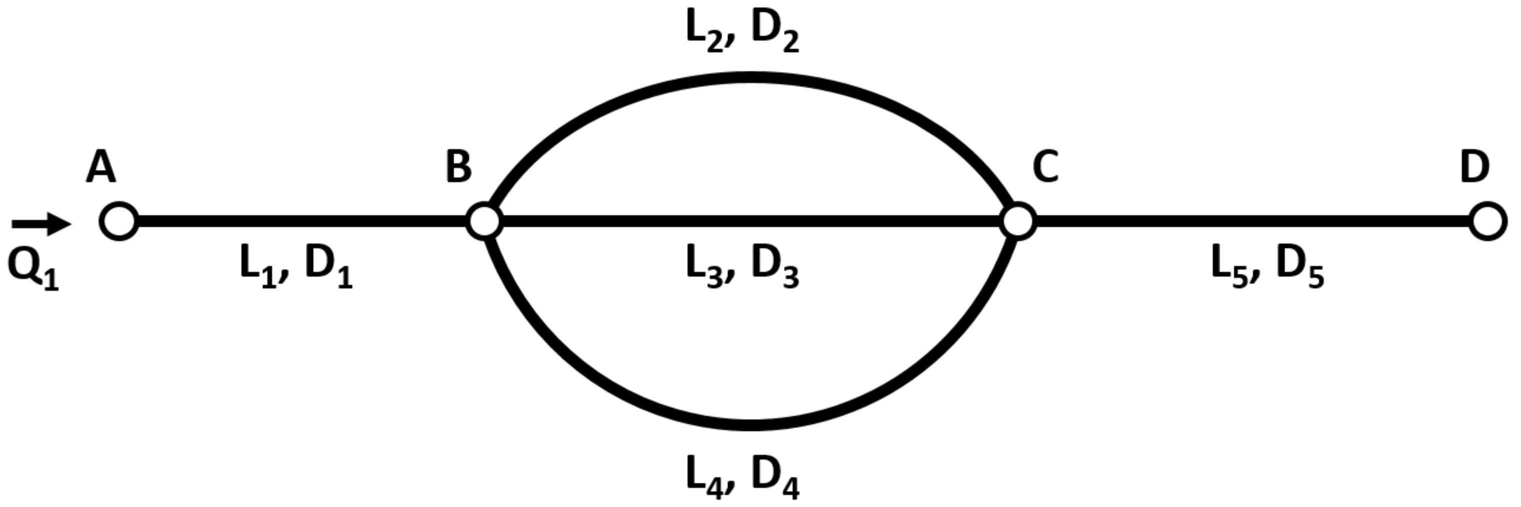


Exercise 3. Pipes in series and parallel

The system of long pipes shown in the figure below is made of new steel pipes.
In this case, the major losses can be computed using the formula: $\Delta H_i = 0.0012\, Q_i^2 D_i^{-5.26}\, L_i$
We can neglect the minor losses at the junctions.
In the future, the three pipes in parallel (2,3 and 4) are going to be replaced by a single pipe of length L_3.

DATA

- Diameter of pipe 1 → D_1= 600 mm
- Diameter of pipe 2 → D_2= 350 mm
- Diameter of pipe 3 → D_3= 300 mm
- Diameter of pipe 4 → D_4= 400 mm
- Diameter of pipe 5 → D_5= 500 mm
- Length of pipe 1 → L_1= 1200 m
- Length of pipe 2 → L_2= 800 m
- Length of pipe 3 → L_3= 700 m
- Length of pipe 4 → L_4= 900 m
- Length of pipe 5 → L_5= 1500 m
- The flow rate in pipe 1 → Q_1= 350 L/s



QUESTIONS

- The total loss ΔH between the points A and D, at the current state
- The diameter of the new pipe, assuming the same total head loss, ΔH, between the points A and D

Exercise 3. Pipes in series and parallel - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

Mass continuity → $Q_1 = Q_2 + Q_3 + Q_4 = Q_5$

For flow in parallel, the head loss is the same → $\Delta H_{BC} = \Delta H_2 = \Delta H_3 = \Delta H_4$

Head loss in pipes 2,3 and 4 → $\Delta H_2 = 0.0012\, Q_2^2 D_2^{-5.26}\, L_2$ $\quad \Delta H_3 = 0.0012\, Q_3^2 D_3^{-5.26}\, L_3$ $\quad \Delta H_4 = 0.0012\, Q_4^2 D_4^{-5.26}\, L_4$

Reverting → $Q_2 = \sqrt{\dfrac{\Delta H_2\, D_2^{5.26}}{0.0012\, L_2}}$ $\quad Q_3 = \sqrt{\dfrac{\Delta H_3\, D_3^{5.26}}{0.0012\, L_3}}$ $\quad Q_4 = \sqrt{\dfrac{\Delta H_4\, D_4^{5.26}}{0.0012\, L_4}}$

And plugging into mass continuity eq. → $Q_1 = Q_2 + Q_3 + Q_4 \;\Rightarrow\; Q_1 = \sqrt{\dfrac{\Delta H_2\, D_2^{5.26}}{0.0012\, L_2}} + \sqrt{\dfrac{\Delta H_3\, D_3^{5.26}}{0.0012\, L_3}} + \sqrt{\dfrac{\Delta H_4\, D_4^{5.26}}{0.0012\, L_4}}$

Considering $\Delta H_2 = \Delta H_3 = \Delta H_4 = \Delta H_{BC}$ and reverting → $\Delta H_{BC} = 0.0012\, Q_1^2 \left(\sqrt{\dfrac{D_2^{5.26}}{L_2}} + \sqrt{\dfrac{D_3^{5.26}}{L_3}} + \sqrt{\dfrac{D_4^{5.26}}{L_4}} \right)^{-2} = 3.16\ \mathrm{m}$

Head loss in pipes 1 and 5 → $\Delta H_1 = 0.0012\, Q_1^2 D_1^{-5.26}\, L_1 = 2.59\ \mathrm{m}$ $\quad \Delta H_5 = 0.0012\, Q_5^2 D_5^{-5.26}\, L_5 = 8.45\ \mathrm{m}$

Total head loss → $\Delta H = \Delta H_1 + \Delta H_{BC} + \Delta H_5 = 14.20\ \mathrm{m}$

Exercise 3. Pipes in series and parallel - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

Mass continuity → $Q_1 = Q_3 = Q_5$

After replacement, the head loss → $\Delta H_{3,\text{future}} = \Delta H_{BC} = 0.0012\, Q_3^2 D_{3,\text{future}}^{-5.26}\, L_3$

Considering mass continuity eq. and reverting → $D_{3,\text{future}} = \left(0.0012 \dfrac{Q_1^2}{\Delta H_{BC}} L_3\right)^{\frac{1}{5.26}} = 0.522\text{ m}$

Exercise 4. Pumping system

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

A pipeline is installed to deliver water from a reservoir to an upper tank, through a pump with known characteristics.
The pipe geometry and the static lift (difference between water levels in the reservoir and in the tank) are given.

DATA

- Diameter of the pipe → D = 0.15 m
- Length of the pipeline → L = 250 m
- Pipe roughness → k_s = 0.3 mm
- Static lift → Δz = 10 m
- Pump characteristics in Table 1

QUESTIONS

- Graphically estimate the pump efficiency η at the functioning point
- Graphically estimate the power requirement P_{pump} at the functioning point
- Comment on the suitability of this pump and pipeline combination, and eventually propose what could be changed (and how) to increase the efficiency at the new functioning point
- Graphically estimate the pump efficiency η* and power P^*_{pump} at the new functioning point

Table 1

Flow rate **Q [m³/s]**	**Pump head** **H_{pump} [m]**	**Efficiency** **η [%]**
0	26.25	0
0.01	24	28
0.02	21.75	51
0.03	19.5	68
0.04	17.5	80
0.05	15	85
0.06	11.75	80
0.07	6.75	64

Exercise 4. Pumping system - SOLUTION

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

CURRENT STATE

D [m]	0.15
L [m]	250
Δz [m]	10
k_s	0.0003

Q [m^3/s]	H_{pump} [m]	η [%]	U [m/s]	Re [-]	λ [-]	λ(CW) [-]	ΔH [m]	$\Delta H + \Delta z$ [m]	P_{hyd} [W]	P_{pump} [W]
0	26.25	0	0	0			0	10	0	0
0.01	24	28	0.566	84883	0.025359	0.025359	0.690	10.69	2354	8409
0.02	21.75	51	1.132	169765	0.024442	0.024442	2.660	12.66	4267	8367
0.03	19.5	68	1.698	254648	0.024112	0.024112	5.903	15.90	5739	8439
0.04	17.5	80	2.264	339531	0.023941	0.023941	10.420	20.42	6867	8584
0.05	15	85	2.829	424413	0.023837	0.023837	16.210	26.21	7358	8656
0.06	11.75	80	3.395	509296	0.023766	0.023766	23.274	33.27	6916	8645
0.07	6.75	64	3.961	594178	0.023715	0.023715	31.611	41.61	4635	7243

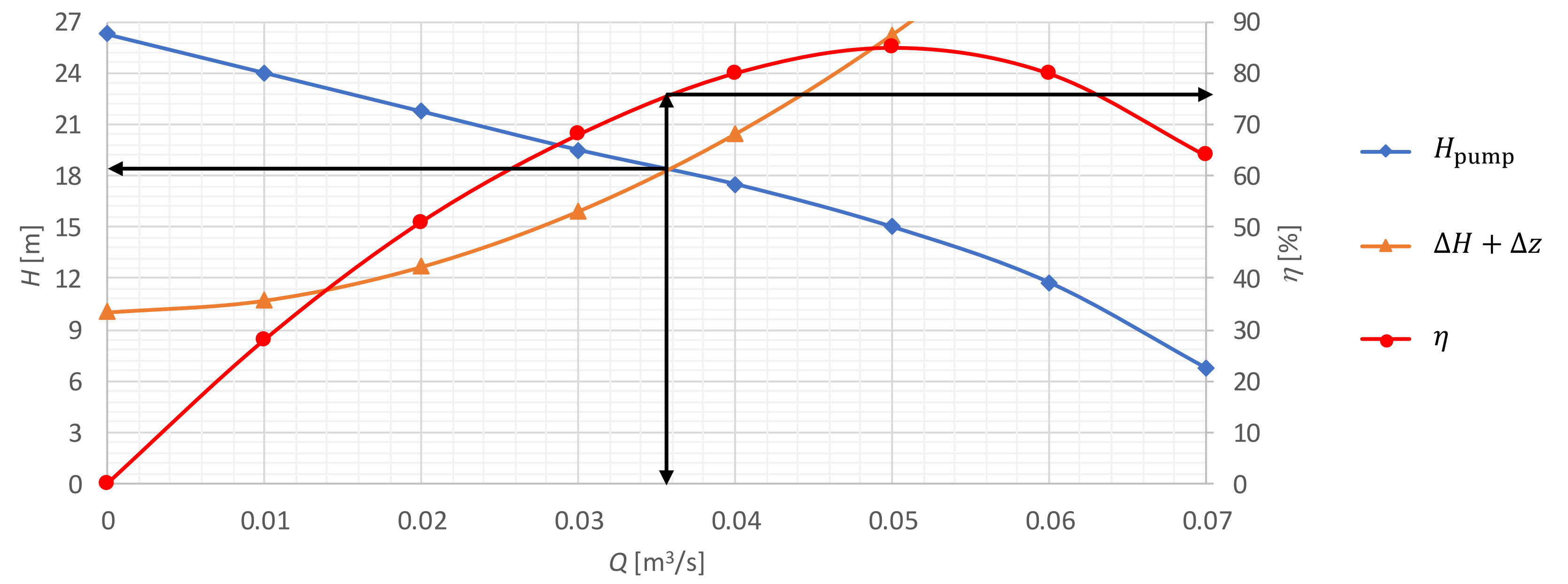
$Q \approx 0.036$ m^3/s

$H \approx 18.6$ m

$\eta \approx 76$ %

$P_{hyd} \approx 6570$ W

$P_{pump} \approx 8640$ W



Exercise 4. Pumping system - SOLUTION

POSSIBLE IMPROVEMENT → Increase pipe diameter to reduce head loss

D [m]	0.2
L [m]	250
Δz [m]	10
k_s	0.0003

Q [m³/s]	H_{pump} [m]	η [%]	U [m/s]	Re [-]	λ [-]	λ(CW) [-]	ΔH [m]	$\Delta H + \Delta z$ [m]	P_{hyd} [W]	P_{pump} [W]
0	26.25	0	0	0			0	10	0	0
0.01	24	28	0.318	63662	0.024667	0.024667	0.159	10.16	2354	8409
0.02	21.75	51	0.637	127324	0.023337	0.023337	0.603	10.60	4267	8367
0.03	19.5	68	0.955	190986	0.022837	0.022837	1.327	11.33	5739	8439
0.04	17.5	80	1.273	254648	0.022572	0.022572	2.331	12.33	6867	8584
0.05	15	85	1.592	318310	0.022409	0.022409	3.616	13.62	7358	8656
0.06	11.75	80	1.910	381972	0.022298	0.022298	5.182	15.18	6916	8645
0.07	6.75	64	2.228	445634	0.022217	0.022217	7.027	17.03	4635	7243

$Q^* \approx 0.0525$ m³/s

$H^* \approx 14.4$ m

$\eta^* \approx 84$ %

$P^*_{hyd} \approx 7420$ W

$P^*_{pump} \approx 8830$ W