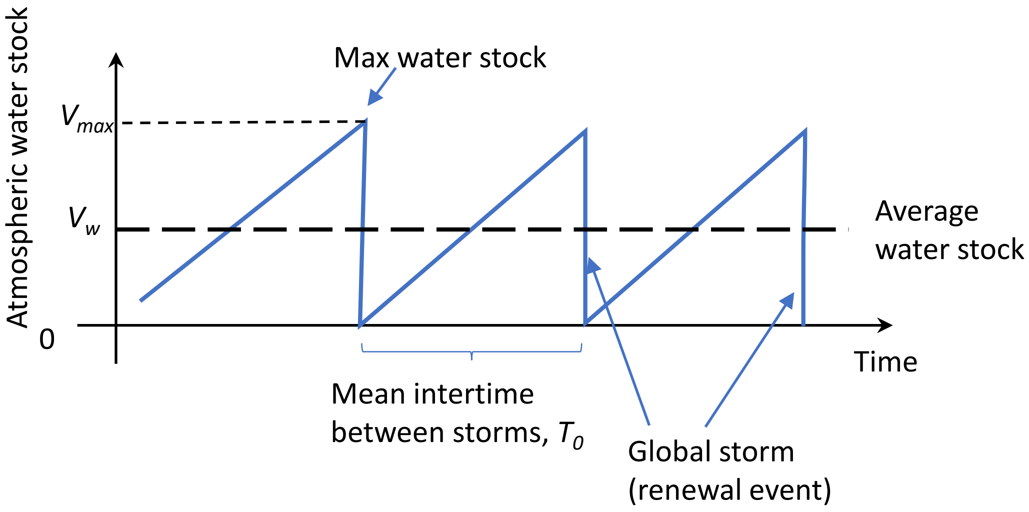
# Water Resources Engineering and Management

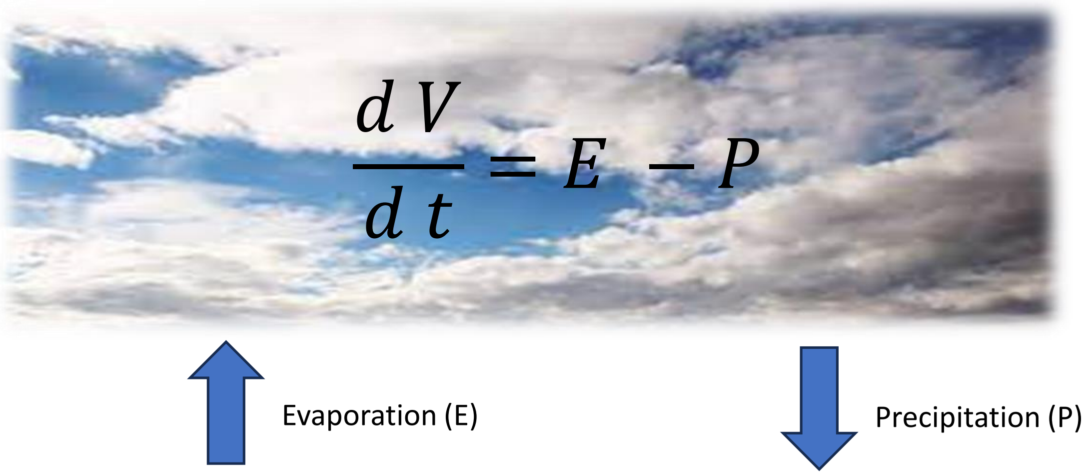
## Exercises Lecture 2: global circulation; groundwater dynamics

24/02/2025

# Exercise 1. Assess the mean period for the renewal of water in the atmosphere (residence time)



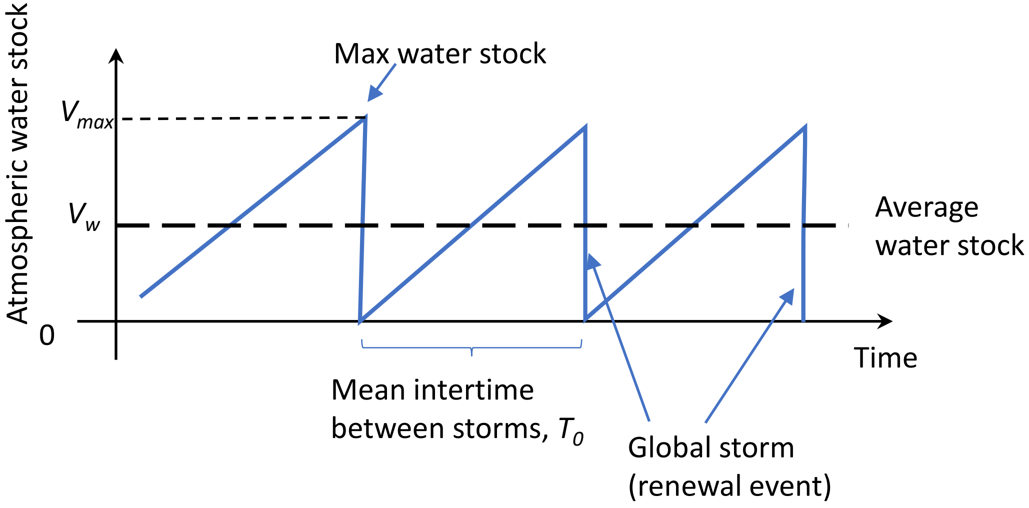


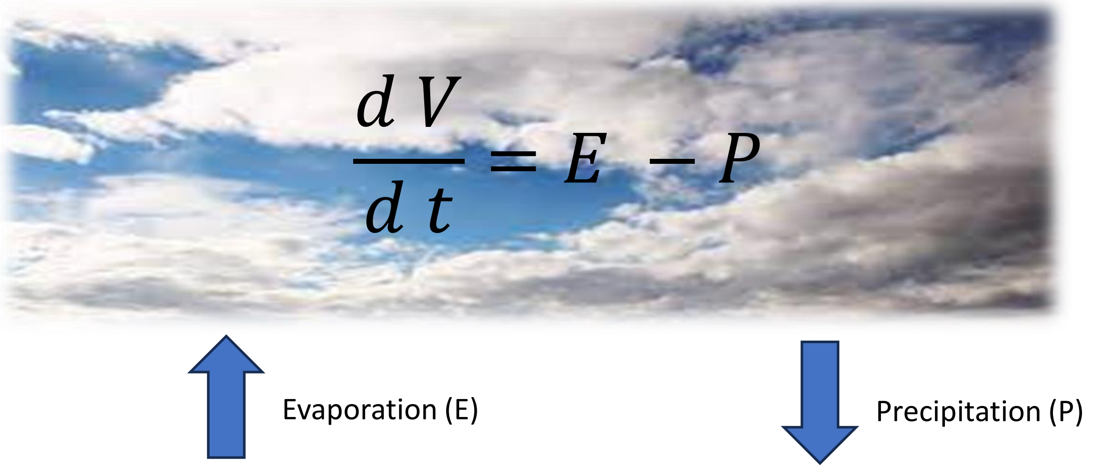




QUESTION

- The residence time, $T_W$
- The relationship between $T_W$ and $T_0$

# Exercise 1. Assess the mean period for the renewal of water in the atmosphere (residence time) - SOLUTION




DATA - L2.1 (slide 14)

- World average $H_2O$ stock (mainly as water vapour) → $V_W = 1.3 \cdot 10^{13}$ m$^3$
- Total world precipitation → $P = 5.77 \cdot 10^{14}$ m$^3$/yr

HYPOTHESIS

- Permanent regime $dV/dt=0$ → $E=P$ , $V= V_w$

$$E = \frac{V_W}{T_W} = P$$

$$T_w = \frac{V_W}{P} = \mathbf{8.22}\ \text{d}$$

$$T_w = \frac{V_W}{\left(\frac{V_{\max}}{T_0}\right)} = T_0 \frac{\frac{1}{2} V_{\max}}{V_{\max}} = \frac{T_0}{2}$$

# Exercise 2. Where would all the water go?

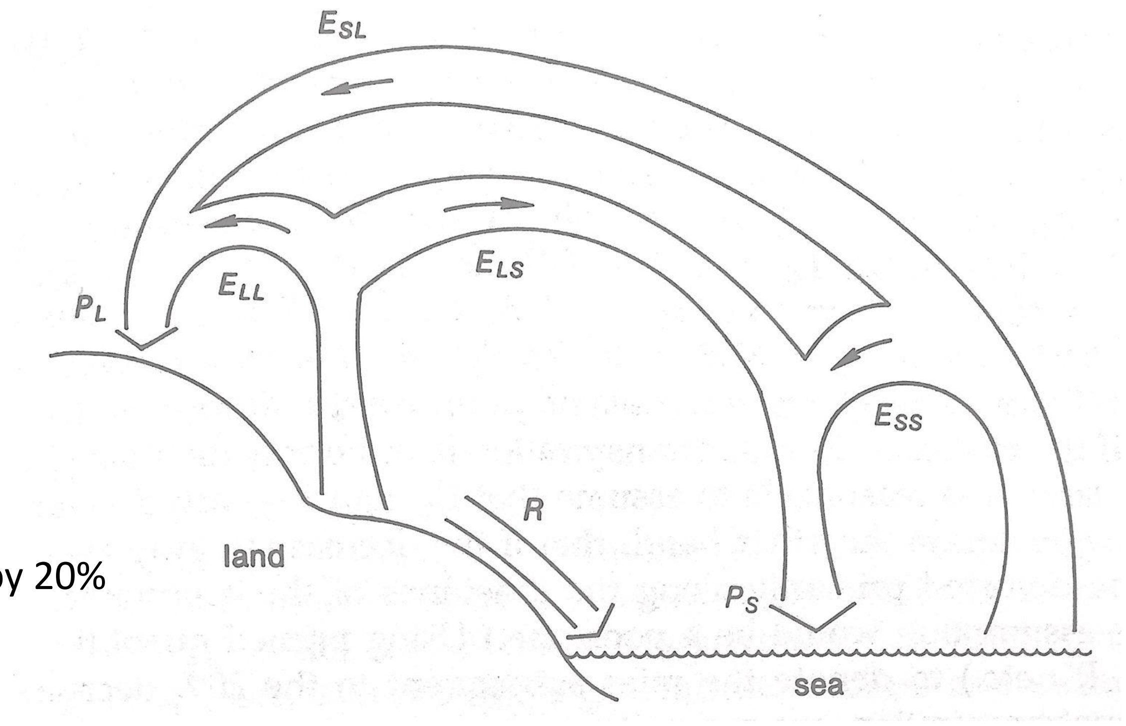
Assuming that evapotranspiration from the land surface decreases uniformly by 20% as a result of widespread deforestation, what changes would occur in global precipitation at the land surface and in average runoff from the land to the oceans?

This is not a residence time problem, but rather a box model problem

If you think that precipitation changes by 20%, you're wrong

QUESTION

- The fluxes at the current state
- The altered fluxes if evaporation from land diminishes by 20%




Source: John Harte. Consider a spherical cow. University Science Book, 1988

# Exercise 2. Where would all the water go? - SOLUTION

DATA - L2.1 (slide 14)

- Total precipitation on land → $P_L$= 110 · 10³ Km³/yr
- Total precipitation on sea → $P_S$= 458 · 10³ Km³/yr
- Total flow from land to sea → $R$= 44.8 · 10³ Km³/yr

<u>CURRENT STATE – BALANCING EQUATION</u>

Sea: $P_S + R = E_{SS} + E_{SL}$ (1)

Earth: $P_L = E_{LL} + E_{LS} + R$ (2)

Atmosphere: $E_{SL} + E_{LL} = P_L$ (3a)

$E_{SS} + E_{LS} = P_S$ (3b)

Constitutive Eq.: $E_{LL} = 3E_{LS}$ (4)

L2.1 (slide 14)

After reorganization:

$$E_{LL} = \frac{3}{4}(P_L - R) = 48.9 \cdot 10^3 \text{ km}^3/\text{yr}$$

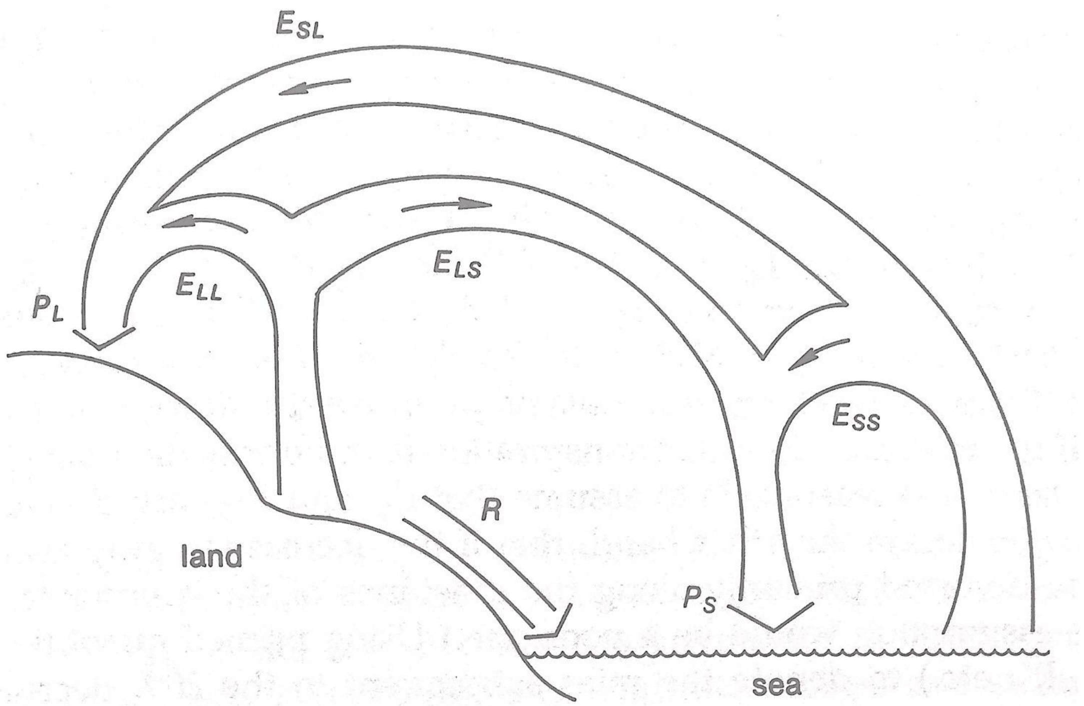
$$E_{LS} = \frac{1}{4}(P_L - R) = 16.3 \cdot 10^3 \text{ km}^3/\text{yr}$$

$$E_{SL} = \frac{1}{4}P_L + \frac{3}{4}R = 61.1 \cdot 10^3 \text{ km}^3/\text{yr}$$

$$E_{SS} = P_S - \frac{1}{4}(P_L - R) = 441.7 \cdot 10^3 \text{ km}^3/\text{yr}$$

4 unknowns fluxes: $E_{LL}, E_{LS}, E_{SL}, E_{SS}$

→ 4 equations are required

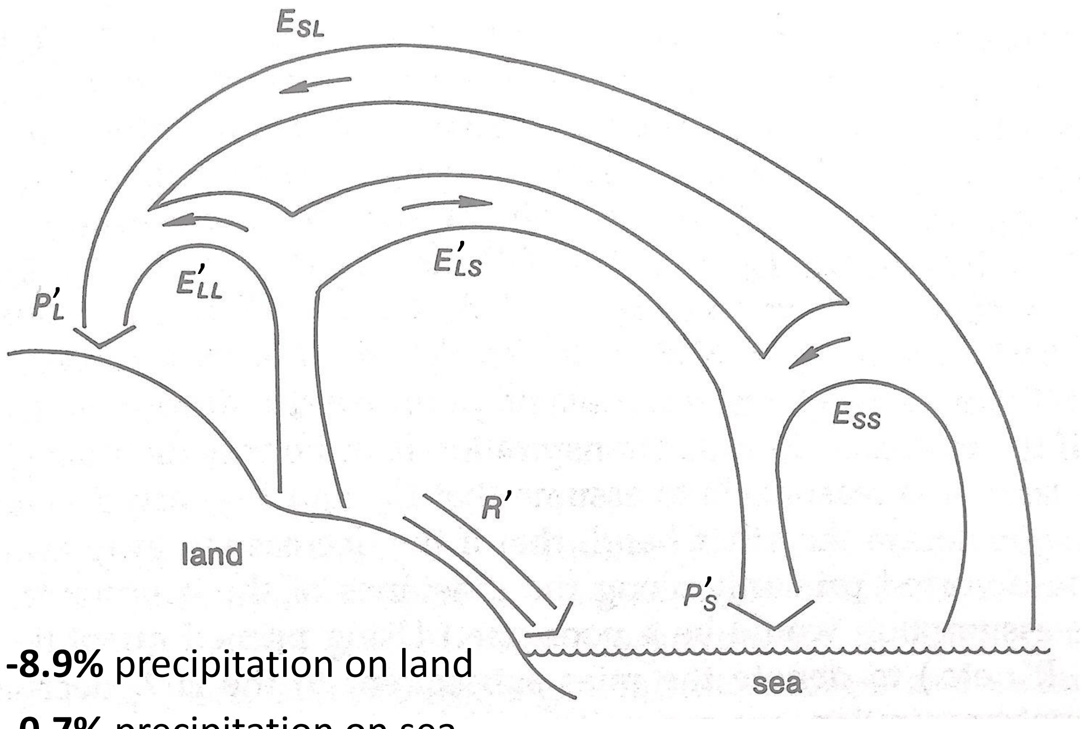


# Exercise 2. Where would all the water go? - SOLUTION

DATA - L2.1 (slide 14)

- Reducing factor for evaporation from land → $\alpha = 0.8$
- Modified total evaporation from land to land → $E'_{LL} = \alpha E_{LL}$
- Total evaporation from land to sea → $E'_{LS} = \alpha E_{LS}$
- Total evaporation from sea to land → $E'_{SL} = E_{SL}$
- Total evaporation from sea to sea → $E'_{SS} = E_{SS}$

3 unknowns fluxes: $P'_L$, $P'_S$, $R'$ → 3 equations are required




MODIFIED STATE – BALANCING EQUATION

Sea: $P'_S + R' = E_{SS} + E_{SL}$ (1)

Earth: $P'_L = E'_{LL} + E'_{LS} + R'$ (2)

Atmosphere: $E_{SL} + E'_{LL} = P'_L$ (3a)

After reorganization:

$P'_L = E_{SL} + \alpha E_{LL} = P_L - (1-\alpha) E_{LL} = 100.2 \cdot 10^3$ km³/yr → **-8.9%** precipitation on land

$P'_S = E_{SS} + \alpha E_{LS} = P_S - (1-\alpha) E_{LS} = 454.7 \cdot 10^3$ km³/yr → **-0.7%** precipitation on sea

$R' = E_{SL} - \alpha E_{LS} = R + (1-\alpha) E_{LS} = 48.1 \cdot 10^3$ km³/yr → **+7.4%** flow from land to sea (runoff)

# Exercise 3. Pumping test to define the soil conductivity, *K*

The average saturated hydraulic conductivity, *K*, of a given soil region is derived from field measurements. The following scheme is usually adopted for the phreatic groundwater flow.

DATA

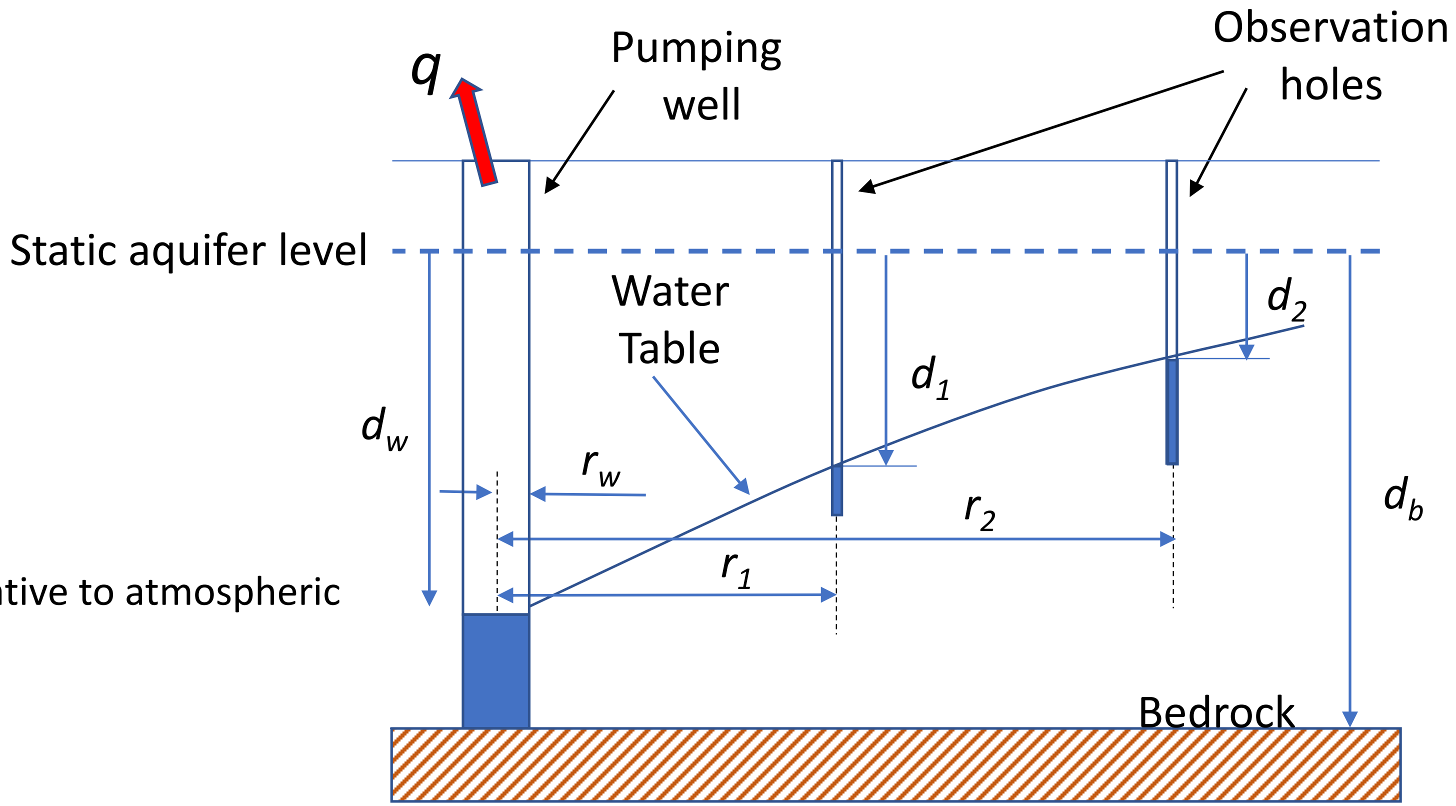
- Pumped flow → $q$ = 5 L/s
- Depth of impermeable bedrock → $d_b$= 32 ± 4.5 m
- Distance of 1st piezometric hole → $r_1$ = 20 m
- Distance of 2nd piezometric hole → $r_2$ = 55 m
- Water depth at 1st hole → $d_1$ = 4.82 m
- Water depth at 2nd hole → $d_2$ = 4.27 m
- Radius of the well → $r_w$ = 0.5 m

HYPOTHESES

- Cylindrical simmetry
- Flow is horizontal (Dupuit-Forchmeier hypothesis)
- Suction above water table is ignored
- Free surface at water table and pressure here is 0 relative to atmospheric
- Hydraulic gradient constant at given radius dh/dr

QUESTIONS

- The hydraulic conductivity, *K*
- The uncertainty in the value of the hydraulic conductivity, $\Delta K$
- The depth within the well, $d_w$



# Exercise 3. Pumping test to define *K* - SOLUTION

$$h(r) = d_b - d(r)$$ Height of the aquifer with respect to the bedrock

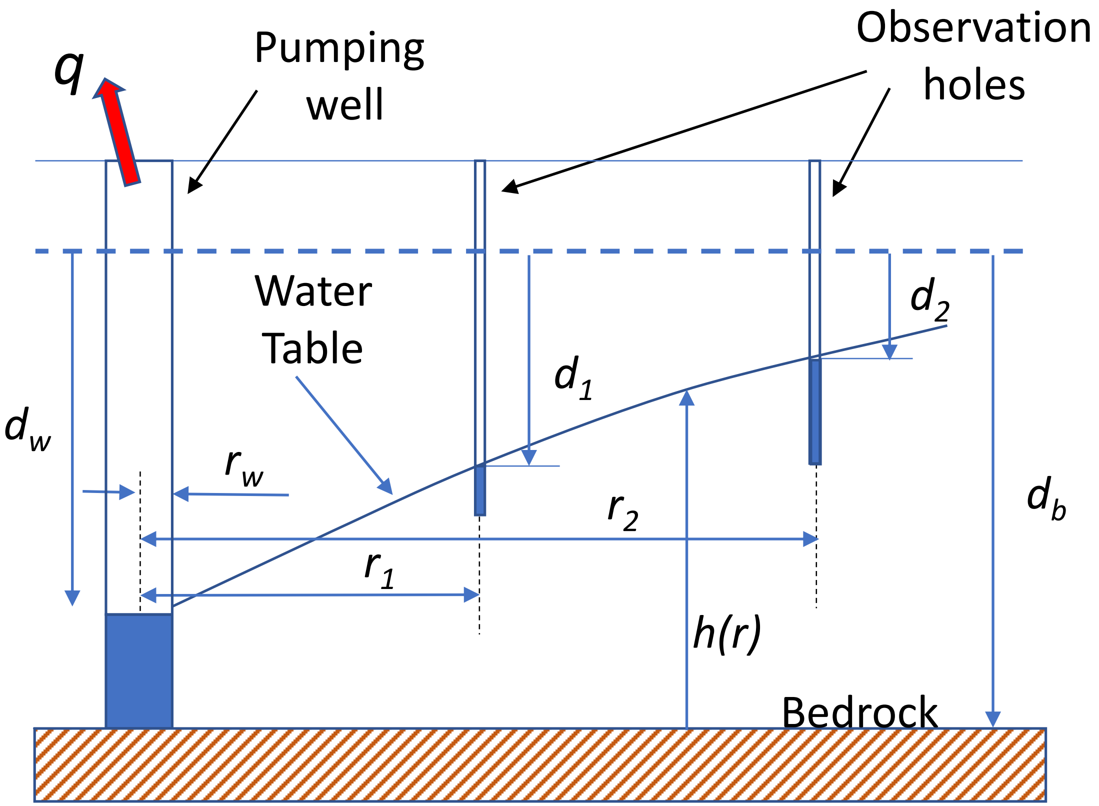
$$q = 2\pi\, r\, h(r)\, U(r)$$ Continuity equation

$$U(r) = K\frac{dh(r)}{dr}$$ Darcy's law

$$q = 2\pi\, r\, h(r) K\frac{dh(r)}{dr} \Rightarrow \frac{q}{2\pi\, K}\frac{dr}{r} = h\, dh$$

$$\frac{q}{2\pi\, K}\int_{r_1}^{r_2}\frac{1}{r}dr = \int_{h_1}^{h_2} h\, dh \Rightarrow \frac{q}{2\pi\, K}\log r\Big|_{r_1}^{r_2} = \frac{h^2}{2}\Big|_{h_1}^{h_2}$$

$$K = \frac{q}{\pi}\frac{\log(r_2/r_1)}{h_2^2 - h_1^2} = 5.33 \cdot 10^{-5}\ \text{m/s}$$



# Exercise 3. Pumping test to define $K$ - SOLUTION

$h(r) = d_b - d(r)$ Height of the aquifer with respect to the bedrock

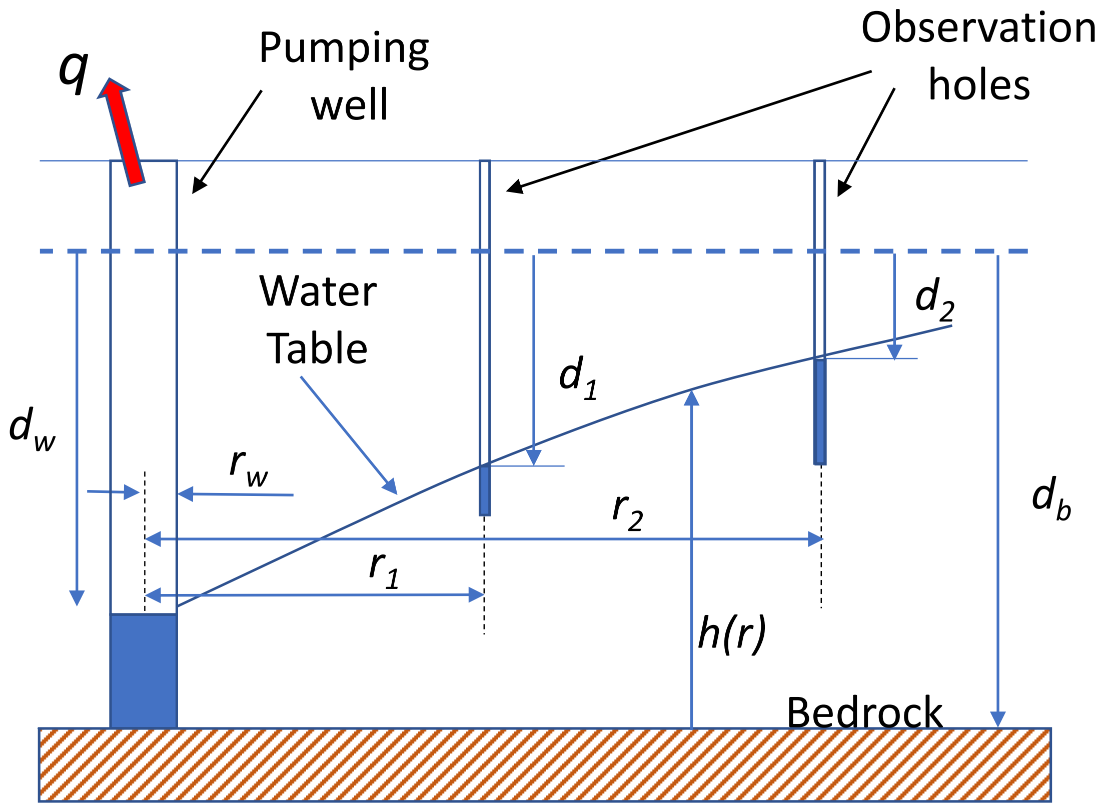
$$K = \frac{q}{\pi} \frac{\log(r_2/r_1)}{h_2^2 - h_1^2}$$

$$K = \frac{q}{\pi} \frac{\log(r_2/r_1)}{(d_b - d_2)^2 - (d_b - d_1)^2}$$

$$K = \frac{q}{\pi} \frac{\log(r_2/r_1)}{d_1 - d_2} \frac{1}{2\, d_b - d_1 - d_2}$$

Error propagation

$$\Delta f = \sqrt{\sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \Delta x_i \right)^2} \Rightarrow \Delta K = \left| \frac{\partial K}{\partial d_b} \Delta d_b \right|$$




$$\Delta K = \frac{q}{\pi} \log\frac{r_2}{r_1} \frac{1}{d_1 - d_2} \left| \frac{-2}{(2\, d_b - d_1 - d_2)^2} \Delta d_b \right| = 8.74 \cdot 10^{-6} \text{ m/s}$$

$$\frac{\Delta K}{K} = 16.4\ \%$$

$$\frac{\Delta\, d_b}{d_b} = 14.1\ \%$$

# Exercise 3. Pumping test to define $K$ - SOLUTION

We can follow the same procedure as for Q1, but between the well and hole 2, then reorganizing for $h_w$

$$K = \frac{q}{\pi} \frac{\log(r_2/r_w)}{h_2^2 - h_w^2}$$

$$h_w = \sqrt{h_2^2 - \frac{q}{\pi K} \log(r_2/r_w)} = 25.07 \text{ m}$$

Similarly, between the well and hole 1

$$K = \frac{q}{\pi} \frac{\log(r_1/r_w)}{h_1^2 - h_w^2}$$

$$h_w = \sqrt{h_1^2 - \frac{q}{\pi K} \log(r_1/r_w)} = 25.07 \text{ m}$$