PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

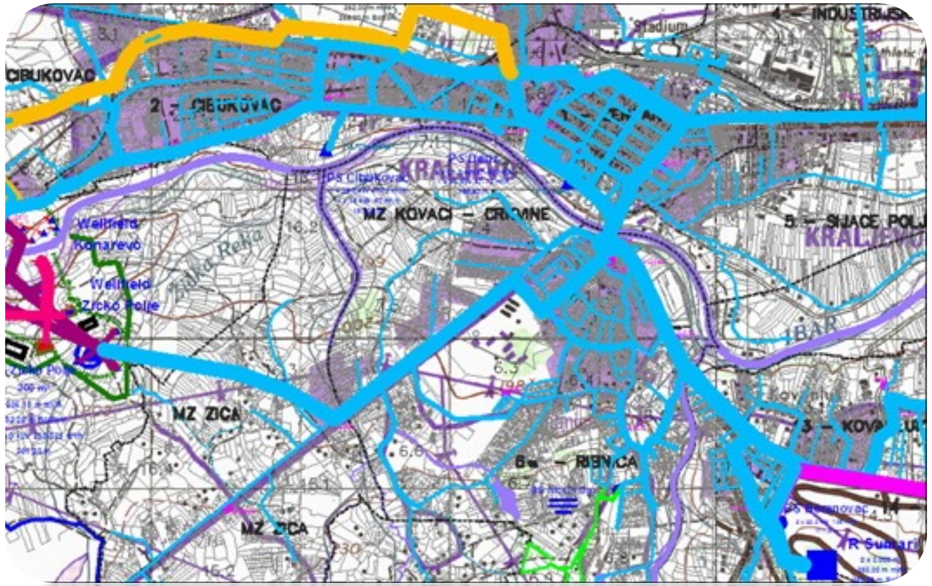
Water Resources Engineering and Management

(CIVIL-466, A.Y. 2024-2025)

5 ETCS, Master course

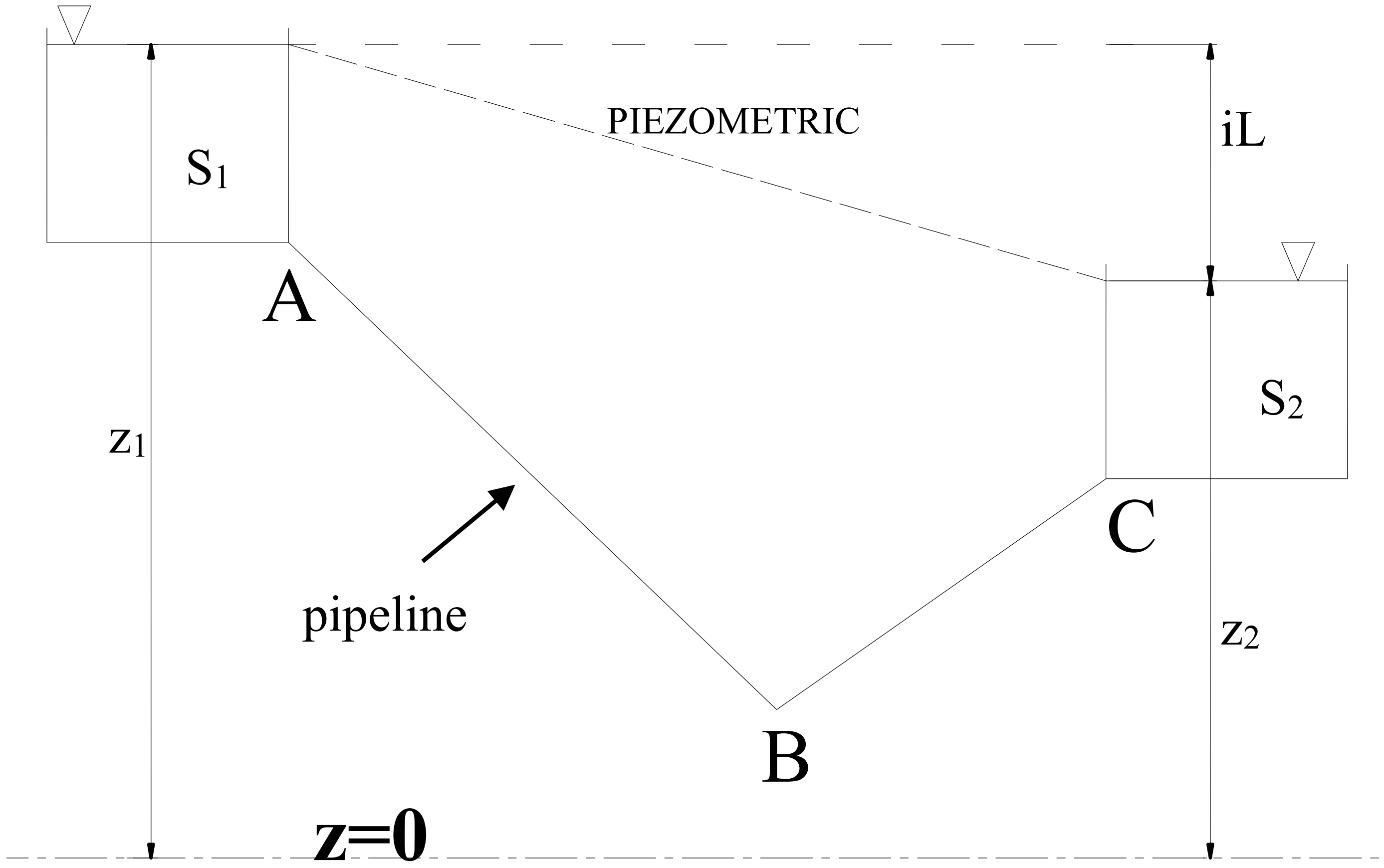
Prof. P. Perona

Platform of hydraulic constructions



Lecture 3-2 Water engineering and management of pipelines and networks

The ‘used pipes’ criterion for pipelines design



Energy balance equation $z_1 - z_2 = iL$ $z_1 - z_2 = \beta' \frac{Q^2}{D^{5.33}} L$

The diameter D is therefore

$$i = \beta_{used} \cdot \frac{Q^2}{D^{5,33}} \longrightarrow D$$

$D_1<D$

$D_2>D$

Choose the two closest pipe diameters D_1, D_2 that are commercially available

Back calculate the new lengths L_1, L_2

$$i_1 = \beta_{used} \cdot \frac{Q^2}{D_1^{5,33}}$$

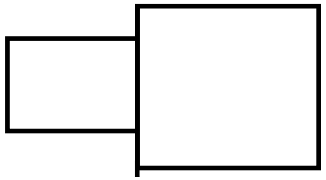
$$i_2 = \beta_{used} \cdot \frac{Q^2}{D_2^{5,33}}$$

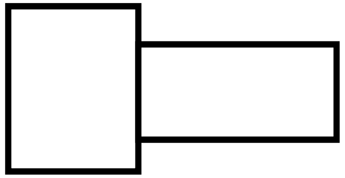
$$\begin{cases} L = L_1 + L_2 \\ z_1 - z_2 = i_1 L_1 + i_2 L_2 \end{cases},$$

$$\longrightarrow L_1, L_2$$

NOTE: since we are dealing with pipe hydraulically long the local head loss due to the discontinuity in presence of the different diameters, is not considered.

We must decide about the arrangement of the two reaches. Each solution has both advantages and disadvantages.





CASE 1. First L_1 D_1 and then L_2 D_2 (Figure 4.2)

CASE 2. First L_2 D_2 and then L_1 D_1 (Figure 4.3)

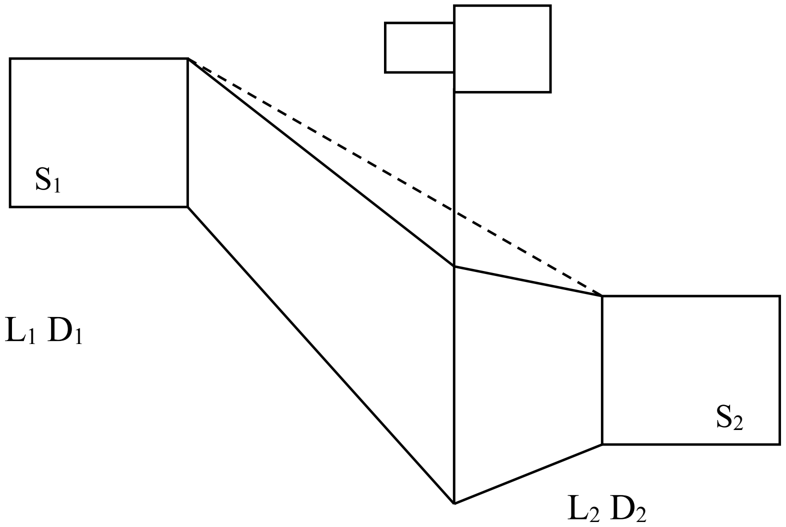


Figure 4.2

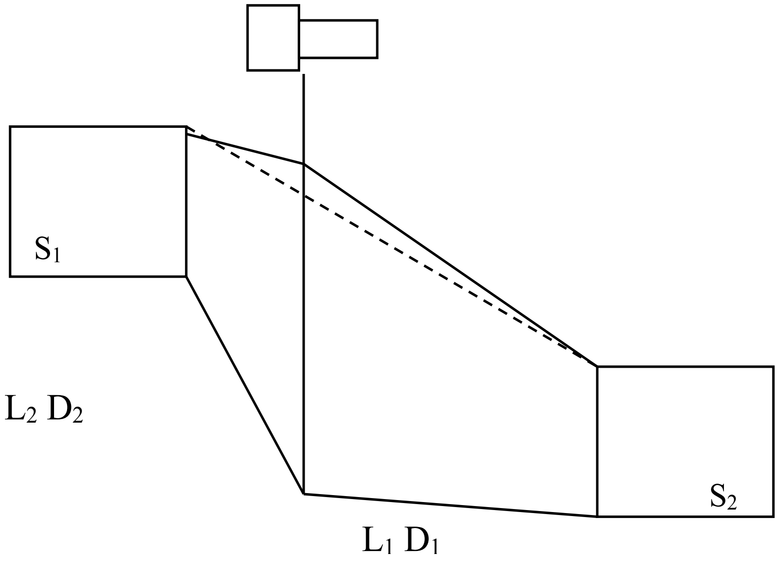
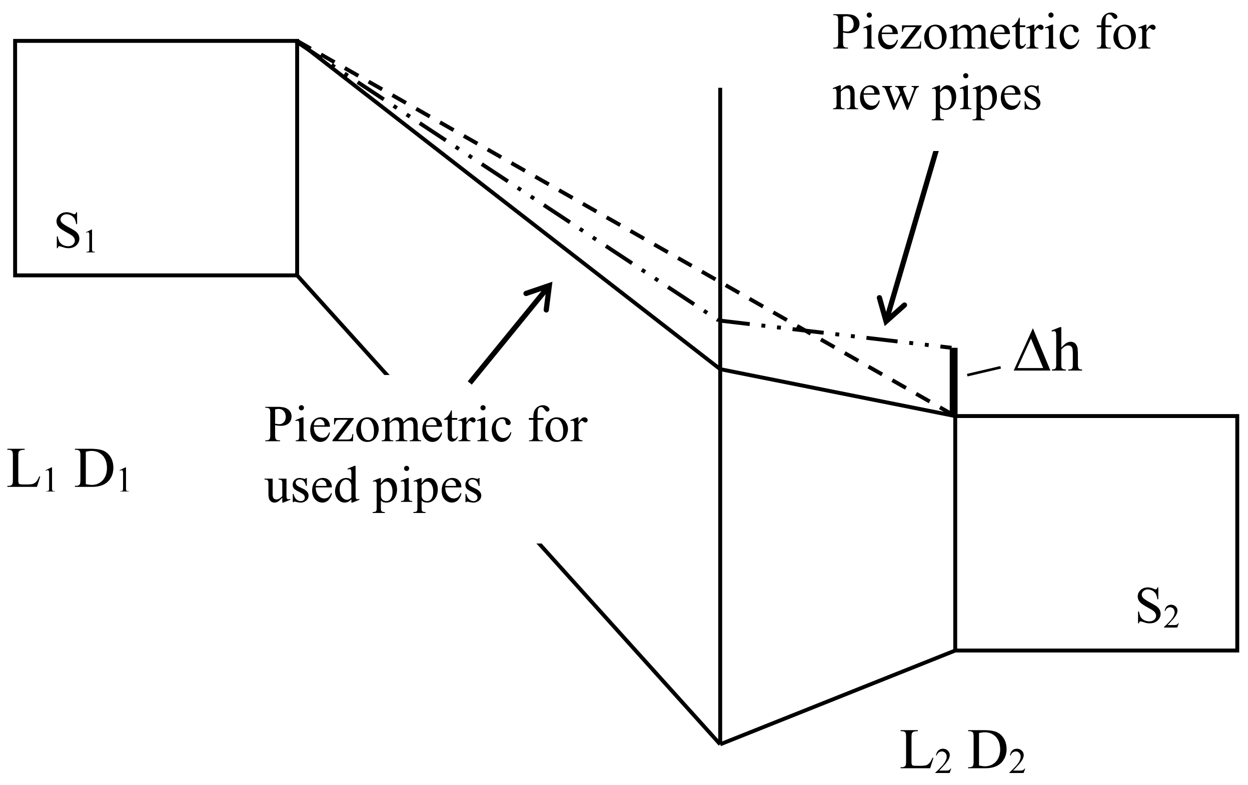


Figure 4.3

Hydraulic management of the pipeline

New versus Old (worn out) pipe hydraulic grade line



$$i_{1N} = \beta_{new} \cdot \frac{Q^2}{D_1^{5,33}}$$

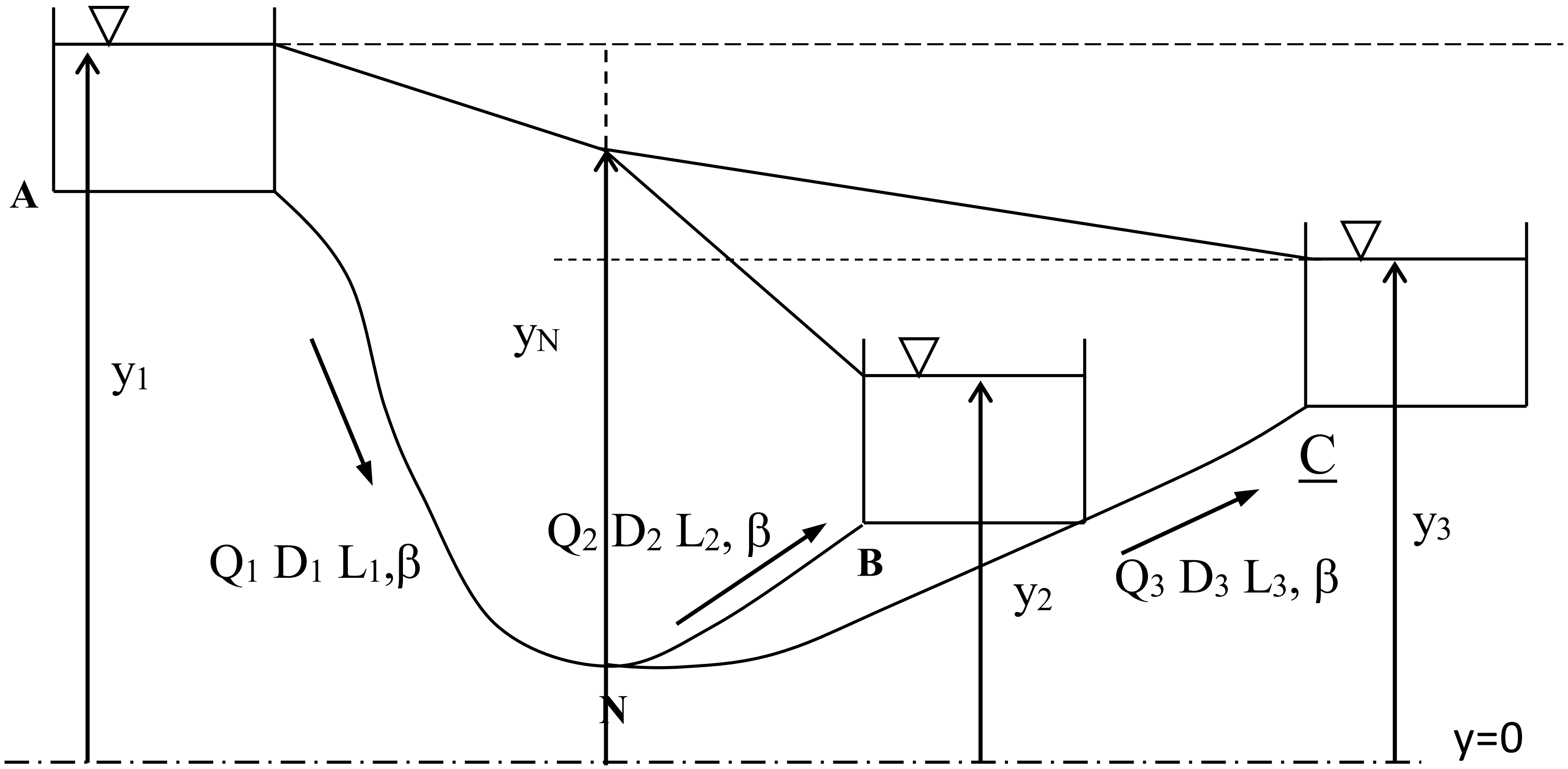
$$i_{2N} = \beta_{new} \cdot \frac{Q^2}{D_2^{5,33}}$$

$$\Delta z = \Delta h + iL = \Delta h + i_{1N} L_1 + i_{2N} L_2 \,,$$

$\rightarrow$ Δh

To be induce by pressure valve, micro-turbine, etc.

Multiple pipelines: hydraulic design



Calculate the set of diameters D_1, D_2, D_3 to carry the assigned flow rates Q_1, Q_2, Q_3 given y_1, y_2, y_3.

Write the equations of motion

$$\begin{cases} y_1 - y_N = \beta \dfrac{Q_1^2}{D_1^n} L_1 \\ y_N - y_2 = \beta \dfrac{Q_2^2}{D_2^n} L_2 \\ y_N - y_3 = \beta \dfrac{Q_3^2}{D_3^n} L_3 \end{cases}$$

Notice the number of unknowns: D_1, D_2, D_3, y_N!

We need one additional equation: any idea?

NOTE: The continuity equation $Q_1=Q_2+Q_3$ is in this case useless as it declares an identity

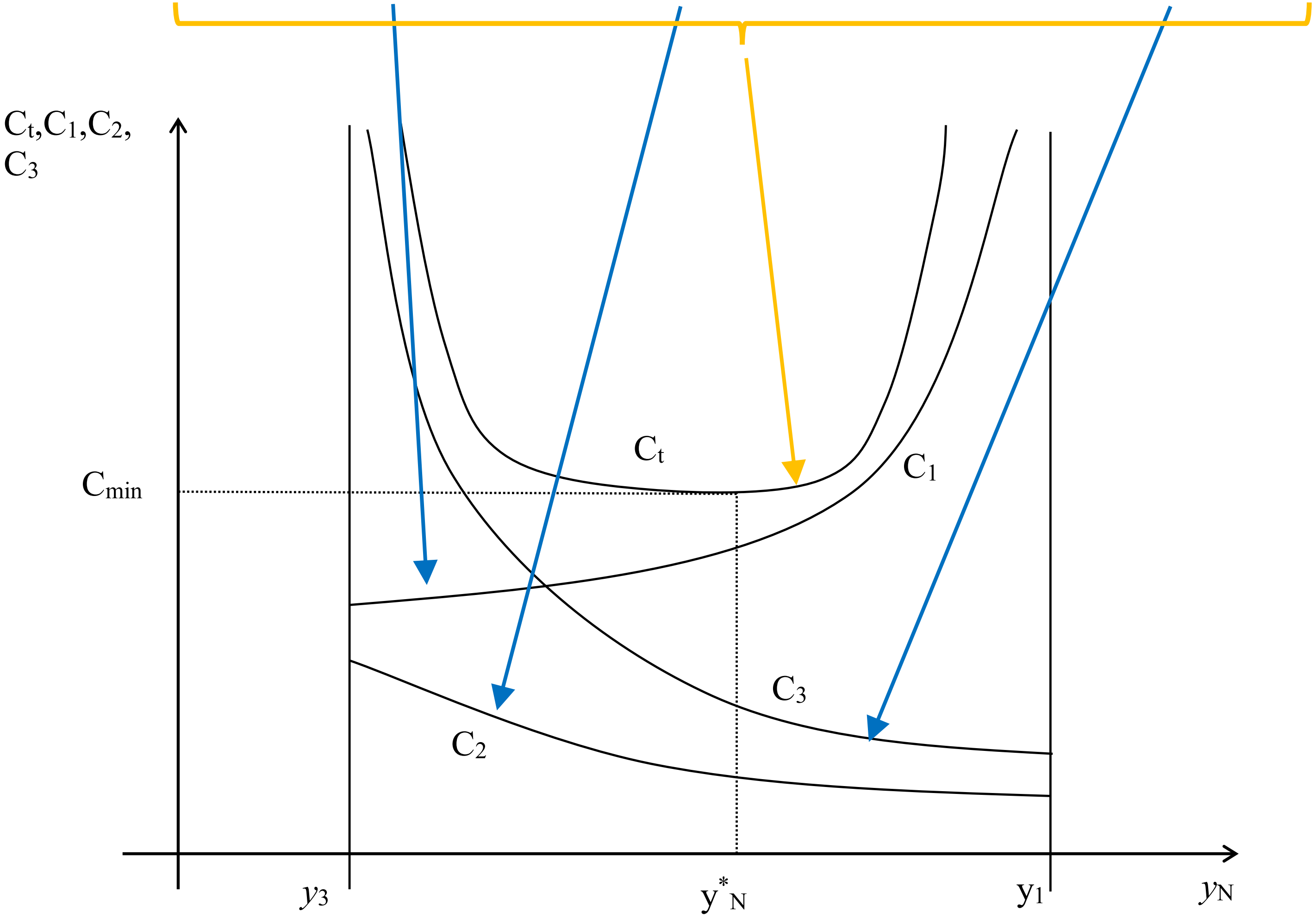
Economic criterion to solve the problem

$$C_i = c \cdot D_i^{\alpha} \cdot L_i$$ Cost of a single pipe

$$C_t = C_1 + C_2 + C_3$$ Total cost

$$D_i = \sqrt[n]{\frac{\beta Q_i^2 L_i}{|y_i - y_N|}}$$ Express D from motion equations and substitute in C_t

$$C_t = cL_1\left(\frac{\beta Q_1^2 L_1}{y_1 - y_N}\right)^{\alpha/n} + cL_2\left(\frac{\beta Q_2^2 L_2}{y_N - y_2}\right)^{\alpha/n} + cL_3\left(\frac{\beta Q_3^2 L_3}{y_N - y_3}\right)^{\alpha/n}$$

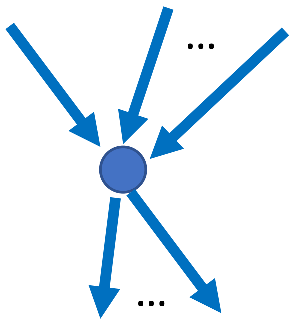


PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

The minimum of the total cost function found by imposing its first derivatives with respect to y_N is zero returns the additional equation that can be used to solve the problem

$$\sum_i {}_{in} \frac{D_i^{\alpha+n}}{Q_i^2} = \sum_j {}_{out} \frac{D_j^{\alpha+n}}{Q_j^2}$$

Branches entering node



Branches exiting node

$$D_i = \sqrt[n]{\frac{\beta Q_i^2 L_i}{y_1 - y_N}} \quad \Rightarrow \quad D_i$$

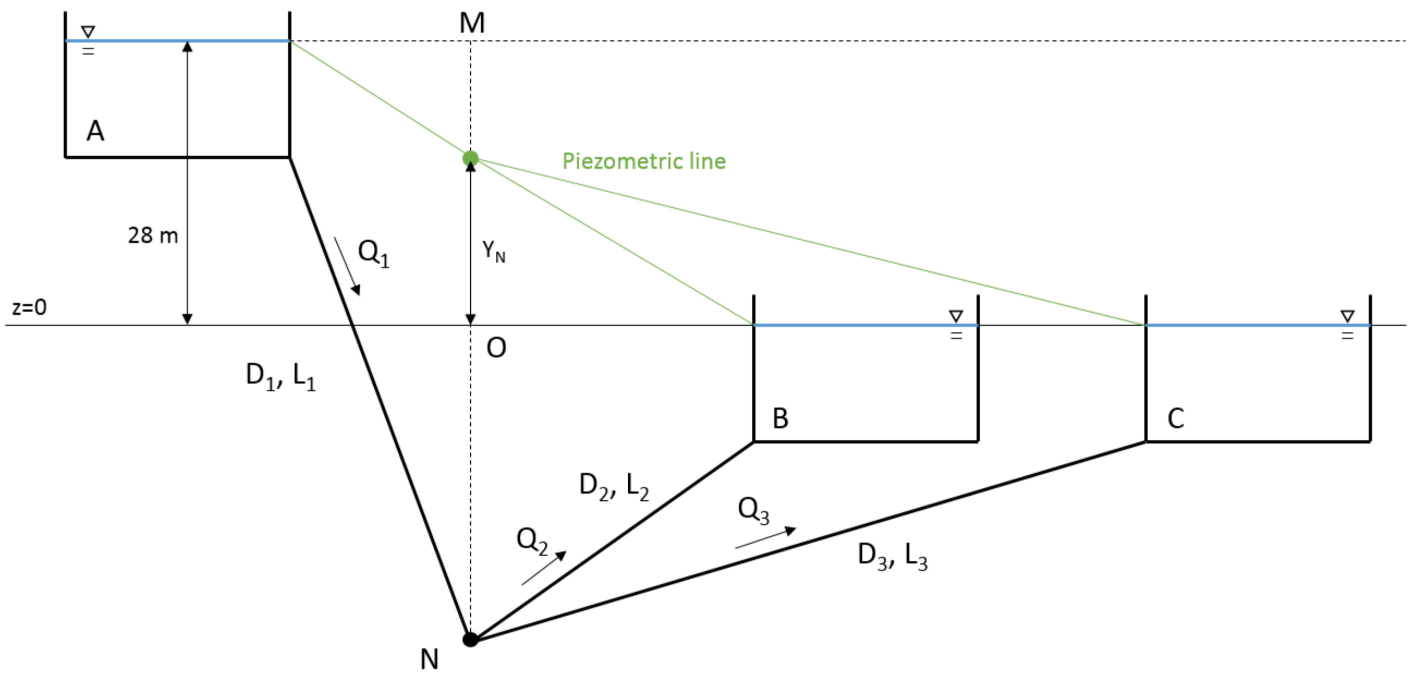
One single equation in the unknown y_N.

EPFL

Example

Reservoir A supplies water to the reservoirs B and C with the following proportions: $Q_2 = \frac{4}{10} \cdot Q_1$ and $Q_3 = \frac{6}{10} \cdot Q_1$.

Calculate the diameters D_1, D_2 and D_3 of the relative pipes using the minimum cost condition. A similar example is explained in the Lecture 4.



$Q_1 = 40$ l/s ; $L_1 = 1300$ m ; $L_2 = 600$ m ; $L_3 = 1000$ m

Steps:

- For each pipe, express the diameter as a function of head loss.

  The head loss can be expressed for each pipe using the Darcy-Weisbach formula:

$$Y_A - Y_H = i_1 \cdot L_1 = \beta' \cdot \frac{{Q_1}^2}{{D_1}^{5,33}} \cdot L_1$$

$$Y_H = i_2 \cdot L_2 = \beta' \cdot \frac{{Q_2}^2}{{D_2}^{5,33}} \cdot L_2$$

$$Y_H = i_3 \cdot L_3 = \beta' \cdot \frac{{Q_3}^2}{{D_3}^{5,33}} \cdot L_3$$

By rearranging the equations, we find:

$$\Rightarrow D_1 = \sqrt[5.33]{\frac{\beta' \cdot {Q_1}^2 \cdot L_1}{28 - Y_N}} = \frac{0.361}{\sqrt[5.33]{28 - Y_N}}$$

$$\Rightarrow D_2 = \sqrt[5.33]{\frac{\beta' \cdot {Q_2}^2 \cdot L_2}{Y_N}} = \frac{0.2214}{\sqrt[5.33]{Y_N}}$$

$$\Rightarrow D_3 = \sqrt[5.33]{\frac{\beta' \cdot {Q_3}^2 \cdot L_3}{Y_N}} = \frac{0.284}{\sqrt[5.33]{Y_N}}$$

- Use the minimum cost condition to compute the piezometric head at the node Y_N.

  The minimum cost condition is expressed as follows:

$$\sum_{i,in} \frac{{D_i}^{1.09+5.33}}{{Q_i}^2} = \sum_{j,out} \frac{{D_j}^{1.09+5.33}}{{Q_j}^2}$$

Thus,

$$\frac{{D_1}^{1.09+5.33}}{{Q_1}^2} = \frac{{D_2}^{1.09+5.33}}{{Q_2}^2} + \frac{{D_3}^{1.09+5.33}}{{Q_3}^2}$$

$$\frac{1.44 \cdot 10^{-3}}{(28 - Y_N)^{1.2} \cdot 1.6 \cdot 10^{-3}} = \frac{61.8 \cdot 10^{-6}}{{Y_N}^{1.2} \cdot 256 \cdot 10^{-6}} + \frac{309 \cdot 10^{-6}}{{Y_N}^{1.2} \cdot 576 \cdot 10^{-6}}$$

$$\frac{0.9}{(28 - Y_N)^{1.2}} = \frac{0.241}{{Y_N}^{1.2}} + \frac{0.536}{{Y_N}^{1.2}}$$

$$1.11 \cdot (28 - Y_N)^{1.2} = 1.287 \cdot {Y_N}^{1.2}$$

$$\frac{(28 - Y_N)^{1.2}}{{Y_N}^{1.2}} = 1.1583$$

$$\frac{28 - Y_N}{Y_N} = 1.193$$

➔ $Y_N = 12.77$ m

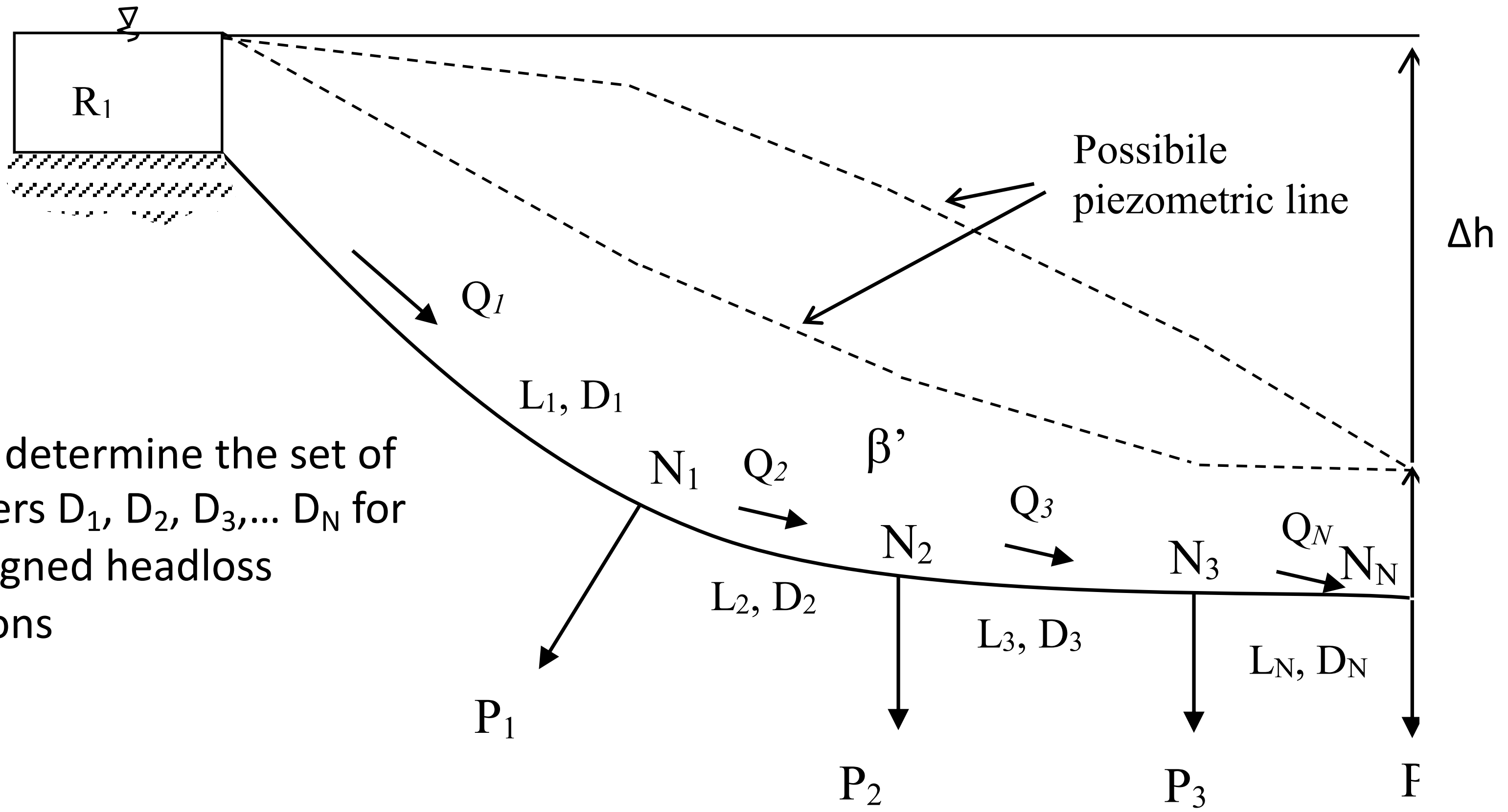
- Use Y_N to compute the diameters D_1, D_2 and D_3.

➔ $D_1 = \frac{0.361}{\sqrt[5.33]{28-12.77}} = 0.216\,m$

➔ $D_2 = \frac{0.2214}{\sqrt[5.33]{12.77}} = 0.137\,m$

➔ $D_3 = \frac{0.284}{\sqrt[5.33]{12.77}} = 0.176\,m$

Design of open branched networks



Aim: to determine the set of diameters D_1, D_2, D_3,… D_N for the assigned headloss conditions

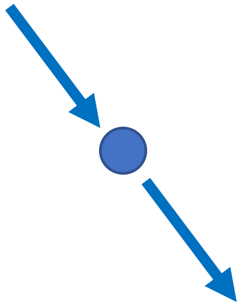
Again this is an undetermined problem, so make use of the economic condition at each node

Hypothesis: neglect the cost of the secondary branches at each node.

$Q_{i+1}+P_i=Q_i$

Continuity at each node

This transitive property leads therefore to the following relation



$$\frac{D_1^{\alpha+n}}{Q_1^2} = \frac{D_2^{\alpha+n}}{Q_2^2} = \frac{D_3^{\alpha+n}}{Q_3^2} = \dots = \frac{D_N^{\alpha+n}}{Q_N^2} = A$$

$$\boxed{D_i = a\sqrt[3]{Q_i}}$$

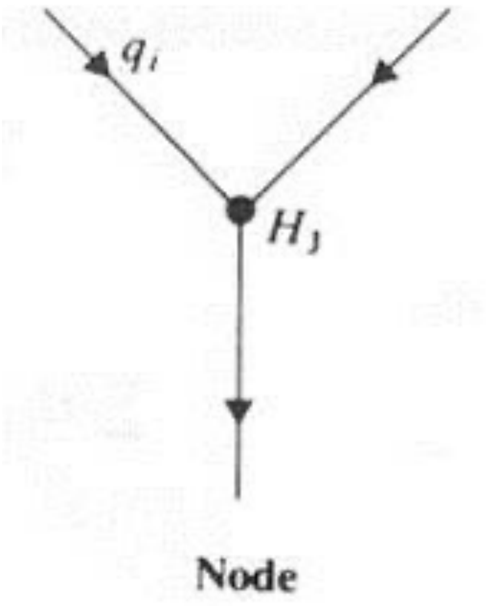
assuming

n=5 and α=1

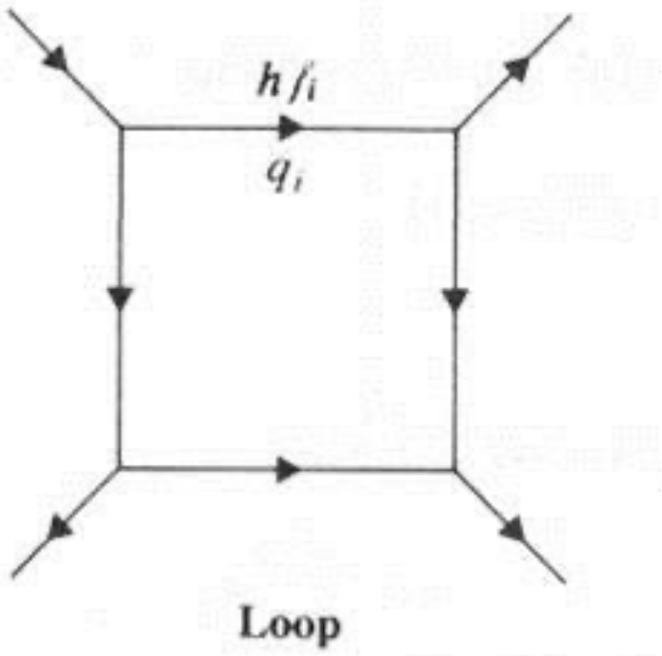
where $a = \sqrt[6]{A}$.

From energy balance once finds that $a^5 = \beta \frac{\sum_i Q_i^{1/3} L_i}{\Delta h}$ → a → Di

Pipe loops: hydraulic management



$$\sum_{i=1}^{n} q_i = 0$$



$$\sum_{i=1}^{m} h_{fi} = 0$$

Sign Convention is that clockwise flow in loop is positive

- The "loop" or "head balance" method (Hardy Cross, 1936)
  - This is used when the total volume rate of flow through the network is known but the heads or pressures at junctions within the network are unknown.

1. assume values of q_i to satisfy
2. calculate h_{fi} from q_i
3. if then solution is correct
4. if then apply a correction to q_i factor and return to step 2

- Correction factors can be computed from:
- This arises from a binomial expansion.

$$\delta q = -\frac{\sum h_{fi}}{2\sum \frac{h_{fi}}{q_i}}$$

$$h_f = \frac{fLu^2}{2gD}$$

We can say for a given pipe: $h_f = kQ^2$

$$Q = (q_i + \delta q)$$

Introduce a correcting term in Q

the true head loss is then:

$$H_{fi} = k(q_i + \delta q)^2$$

Binomial Theorem

$$H_{fi} = kq_i^2\left[1 + 2\frac{\delta q}{q_i} + \frac{2(2-1)}{2!}\left(\frac{\delta q}{q_i}\right)^2 + \cdots\right]$$

for $\delta q_i << q_i$, we have:

$$H_{fi} = k q_i^2 \left[1 + 2 \frac{\delta q}{q_i} \right]$$

$$\sum H_{fi} = 0 = \sum k Q_i^2 + 2\delta q \sum k \frac{q_i^2}{q_i}$$

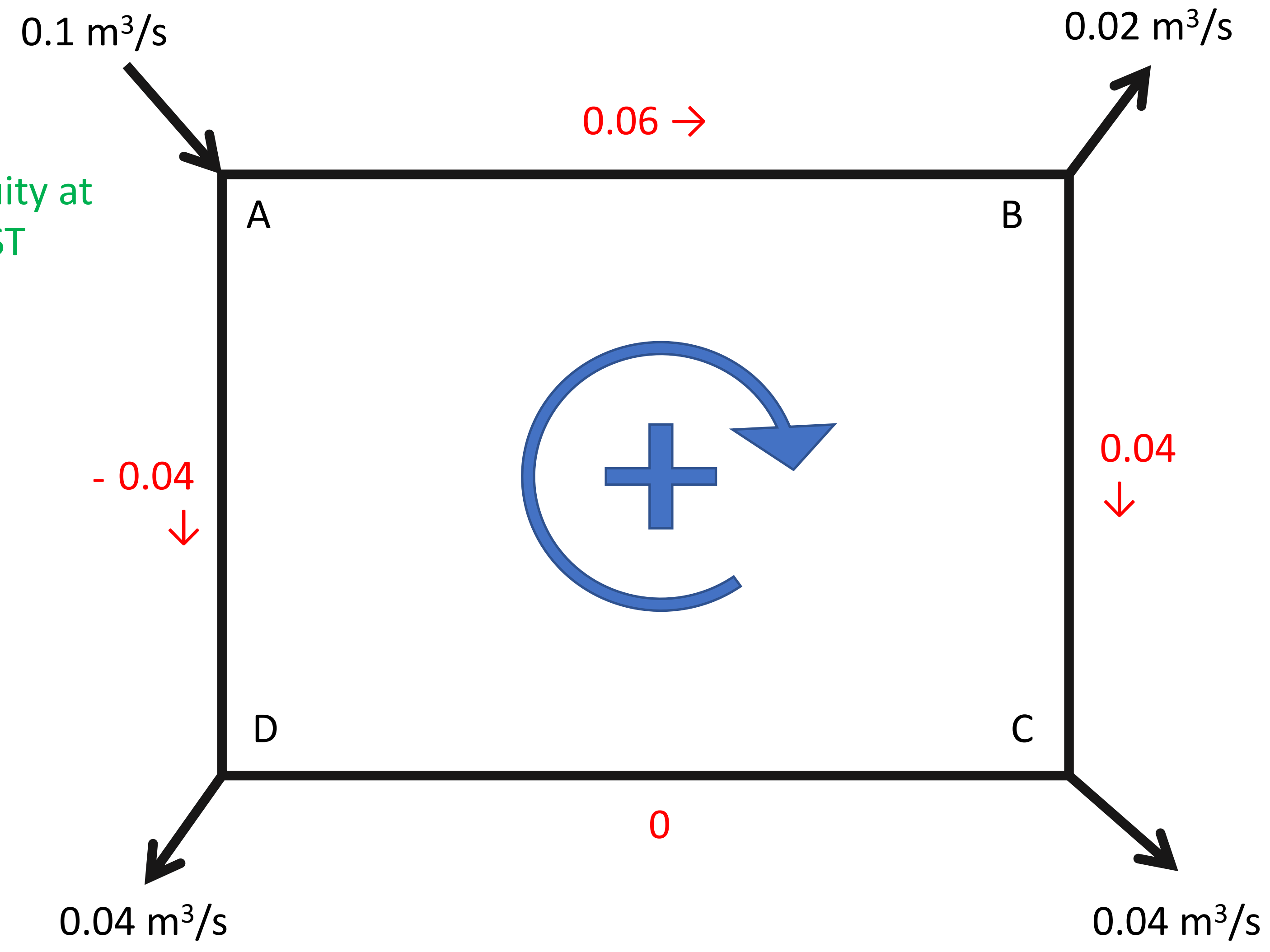
$$\Leftrightarrow \sum h_{fi} + 2\delta q \sum \frac{h_{fi}^2}{q_i}$$

$$\Leftrightarrow \delta q = -\frac{\sum h_{fi}}{2 \sum \frac{h_{fi}}{q_i}}$$

Then introduce the correction (with sign!) and iterate

Example: single loop

Notice: continuity at the nodes MUST match!!



The initial estimate for this example is shown in red on the figure.

We set up a tabulation of the results and calculate the flow velocities from the estimated flows and pipe diameters:

Pipe	q_i (m^3/s)	L (m)	D (m)	u (m/s)	h_{fi} (m)	h_{fi}/q_i
AB	0.06	1000	0.150	3.40		
BC	0.04	1000	0.125	3.26		
CD	0	1000	0.125	0		
DA	-0.04	1000	0.150	2.26		

Pipe	q_i (m^3/s)	L (m)	D (m)	u (m/s)	h_{fi} (m)	h_{fi}/q_i
AB	0.06	1000	0.150	3.40	59.58	
BC	0.04	1000	0.125	3.26	68.32	
CD	0	1000	0.125	0	0	
DA	-0.04	1000	0.150	2.26	-28.14	

Pipe	q_i (m^3/s)	L (m)	D (m)	u (m/s)	h_{fi} (m)	h_{fi}/q_i
AB	0.06	1000	0.150	3.40	59.58	992.97
BC	0.04	1000	0.125	3.26	68.32	1707.94
CD	0	1000	0.125	0	0	0
DA	-0.04	1000	0.150	2.26	-28.14	703.49

This is given by this divided by this

Note the minus signs cancel in the division

We now work out the correction factor which involves summing each of the last two columns:

Pipe	q_i (m^3/s)	L (m)	D (m)	u (m/s)	h_{fi} (m)	h_{fi}/q_i
AB	0.06	1000	0.150	3.40	59.58	992.97
BC	0.04	1000	0.125	3.26	68.32	1707.94
CD	0	1000	0.125	0	0	0
DA	-0.04	1000	0.150	2.26	-28.14	703.49
				Σ =	99.76	3404.40

The two sums go into the correction factor calculation:

$$\delta q = -\frac{\sum h_{fi}}{2\sum \frac{h_{fi}}{q_i}} = -\frac{99.76}{2\times 3404.40} = -0.015$$

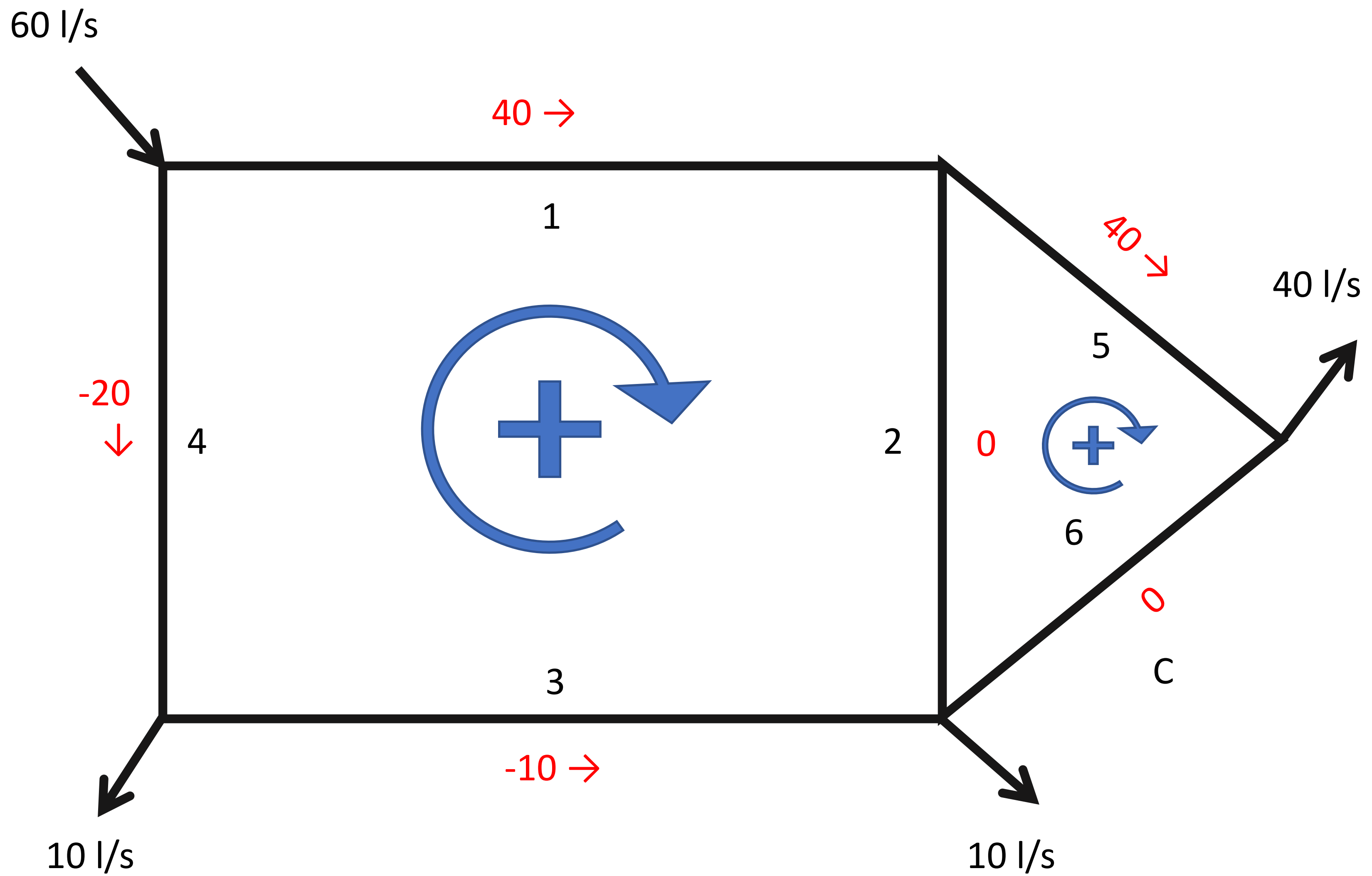
The correction factor is then *added* to the original estimate of flow for *every* pipe in the system. For example the new flow in AB becomes 0.06 + -0.015 = 0.045, and the calculation re-worked:

Pipe	q_i (m^3/s)	L (m)	D (m)	u (m/s)	h_{fi} (m)	h_{fi}/q_i
AB	0.045	1000	0.150	2.57	35.49	782.69
BC	0.025	1000	0.125	2.07	29.38	1159.01
CD	-0.015	1000	0.125	1.19	-10.66	727.30
DA	-0.055	1000	0.150	3.09	-50.13	917.20
				Σ =	4.09	3586.20

This time δq comes out as -0.001.

The process continues until the sum of the penultimate column, Σh_{fi} is close to zero and the correction factor is negligibly small. Usually three or four iterations are sufficient to achieve this in simple loops – in fact the second iteration just above is pretty close already.

Example: complex loops



Left Hand Loop				Right Hand Loop			
Pipe	q_i (litres/s)	h_{fi} (m)	h_{fi}/q_i	Pipe	q_i (litres/s)	h_{fi} (m)	h_{fi}/q_i
1	40	28.5	0.71	5	40	28.5	0.71
2	0	0	0	6	0	0	0
3	-10	-2.2	0.22	2	0	0	0
4	-20	-8	0.40				
	Σ=	18.3	1.33		Σ=	28.5	0.71

These two must always be equal and opposite – same pipe in the two loops, but opposite sign conventions