Water Resources Engineering and Management

(CIVIL-466, A.Y. 2024-2025)

5 ETCS, Master course

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Platform of hydraulic constructions

Lecture 3-1 Water engineering: review of pipeline and pipe network hydraulics

General hydraulics

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

Rationales

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

We study the flow of a Newtonian fluid (i.e. water) within circular pipes of relevant length (pipeline), i.e. several thousand of time the value of the pipe diameter.

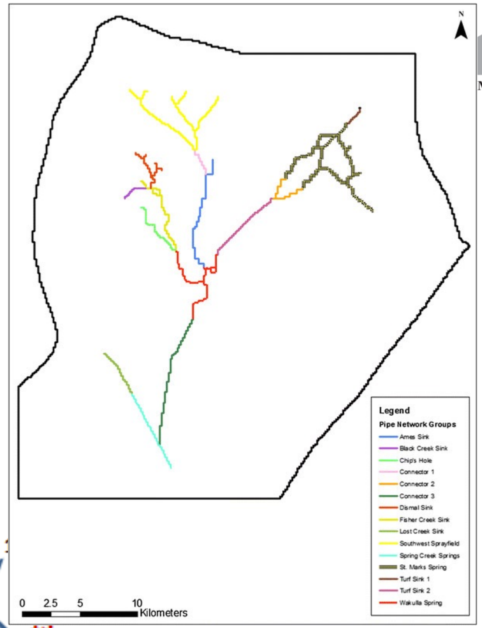
Why?

Water service or in particular the hydraulics of water distribution systems is generally assumed to be reliable and utility customers expect high-quality service. Therefore design and operation of water systems require a deep understanding of the flow in complex systems and the associated energy losses.

Pipelines are widely and typically used in several fields of engineering:

- Generally to convoy a fluid from supply to storage reservoirs;
- Aqueducts;
- Hydroélectrique plants, i.e. penstokes, etc. ;
- Oil-ducts;
- Etc.

Practically, all cases that are relevant to water engineering and management



ME DE CONSTRUCTIONS HYDRAULIQUES

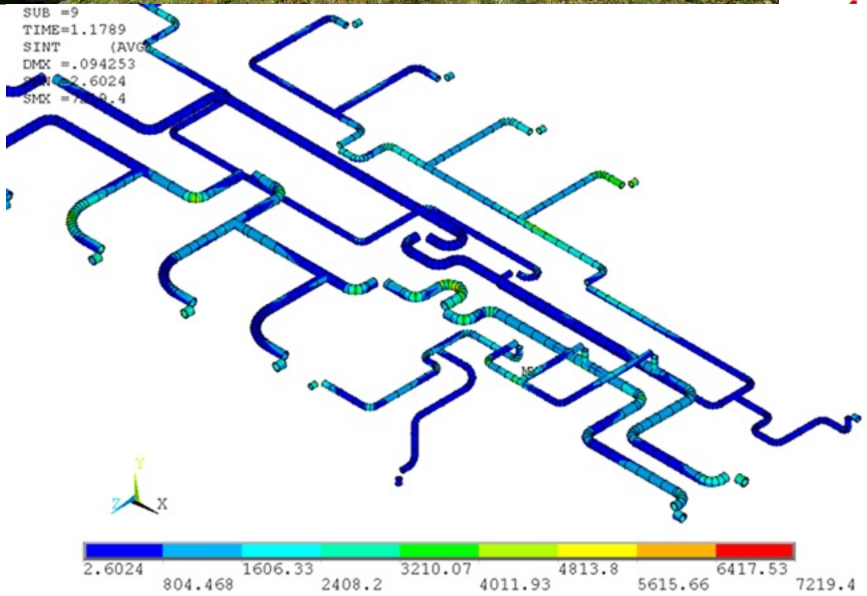
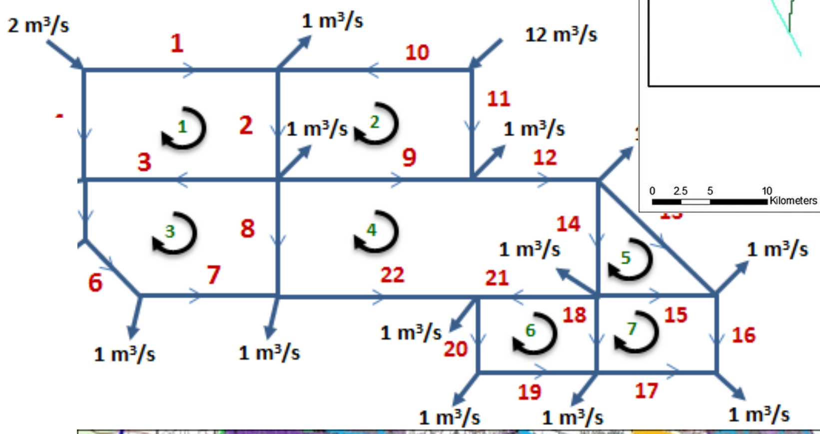
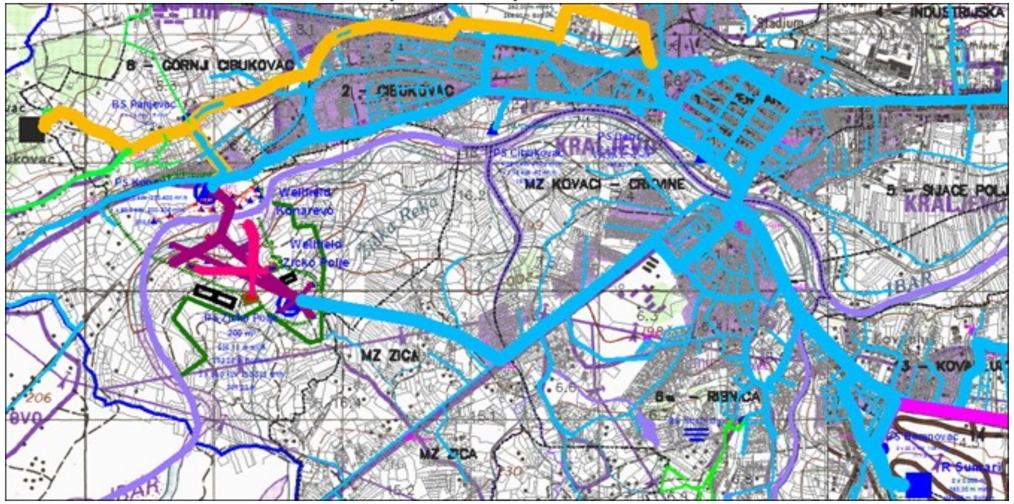


Figure 3. ANSYS Piping Stress Plot





EPFL

Glossary

L, l = pipe length

d = pipe diameter

k = pipe roughness

p = relative fluid pressure

z = geodetic height

g = gravitational acceleration

$\bar{u}$= mean stream velocity

Q = flow rate

ρ = water density

γ = specific weight

R= hydraulics radius

α_c = Coriolis averaging coefficient

μ = dynamic viscosity

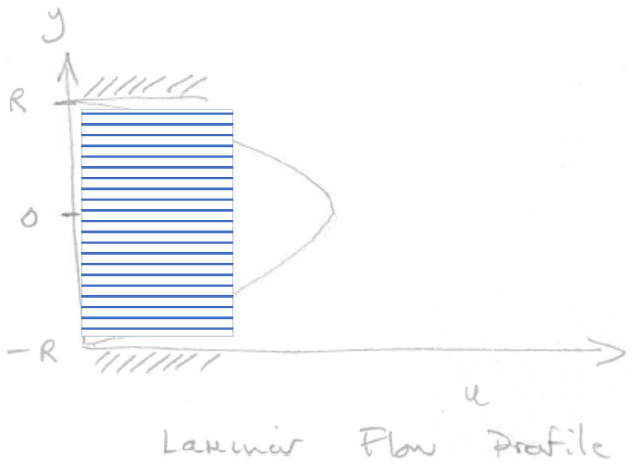
Re = Reynolds number

f = friction factor

C = Chezy coefficient

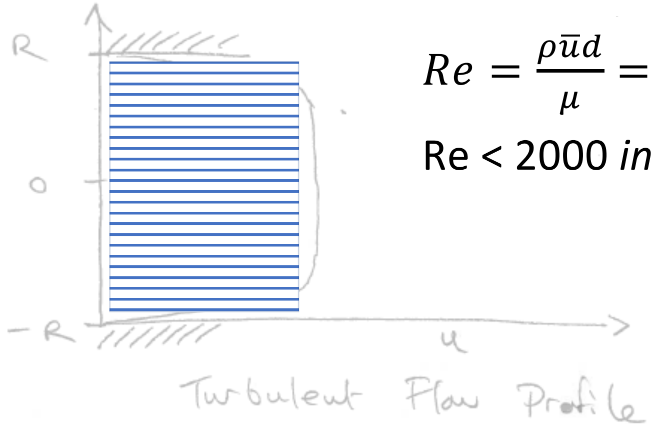
β = Darcy roughness

Laminar vs turbulent flow



$Q = \bar{u}A$

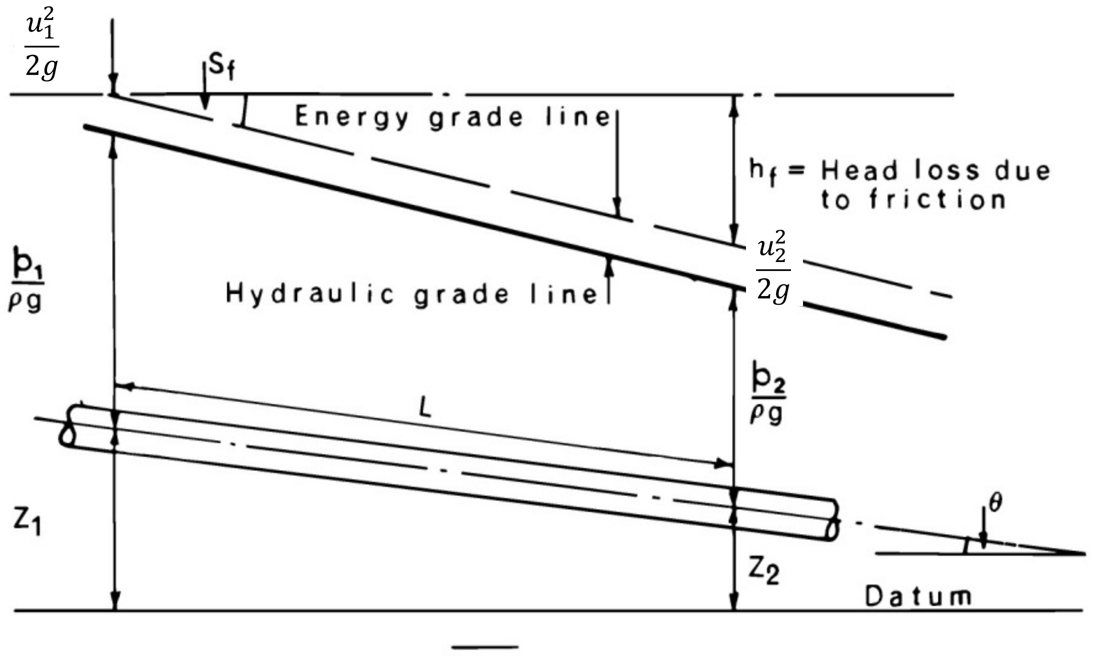
Flow rate is the product of mean flow velocity (occasionally u, U_m or v and many other things) times the cross sectional area



$$Re = \frac{\rho \bar{u} d}{\mu} = \frac{\bar{u} d}{\nu}$$

Re < 2000 *in a pipe* > laminar flow; Re > 4000 *in a pipe* > turbulent flow.

Pipe flow



$$H_u = \frac{\bar{u}^2}{2g}$$

$$H_p = \frac{p}{\gamma} = \frac{p}{\rho g}$$

$$\frac{\bar{u}_1{}^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{\overline{u_2}^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_f$$

Hydraulic gradient is the plot of potential energy, i.e. $\frac{p}{\rho g} + z = h_p$ against distance

Energy gradient is the hydraulic gradient plus the velocity head, i.e. $H = \frac{u^2}{2g} + \frac{p}{\rho g} + z$

The motion of a fluid implies that a distributed dissipation *j* per unit length *x*,

$j = -\frac{\partial H}{\partial x}$ → $h_f = j\,L$

Friction losses

$$\tau_w = f(\rho, \mu, \bar{u}, d, k_s),$$

Now use dimensional analysis…

$$\frac{\tau_w}{\rho \bar{u}^2} = \phi(Re, \frac{k_s}{d}).$$

$$f = \psi(Re, \frac{k_s}{d}).$$

Introduce the friction factor *f* (or *λ*)

$$f = \frac{Dj}{\frac{U_m^2}{2g}}$$

This refers the amount of distributed dissipation over one pipe diameter reach length to the kinetic term. It is therefore a dimensionless quantity

Hydraulic Radius

$$R = \frac{A}{P} = \frac{\frac{\pi d^2}{4}}{\pi d} = \frac{d}{4}.$$

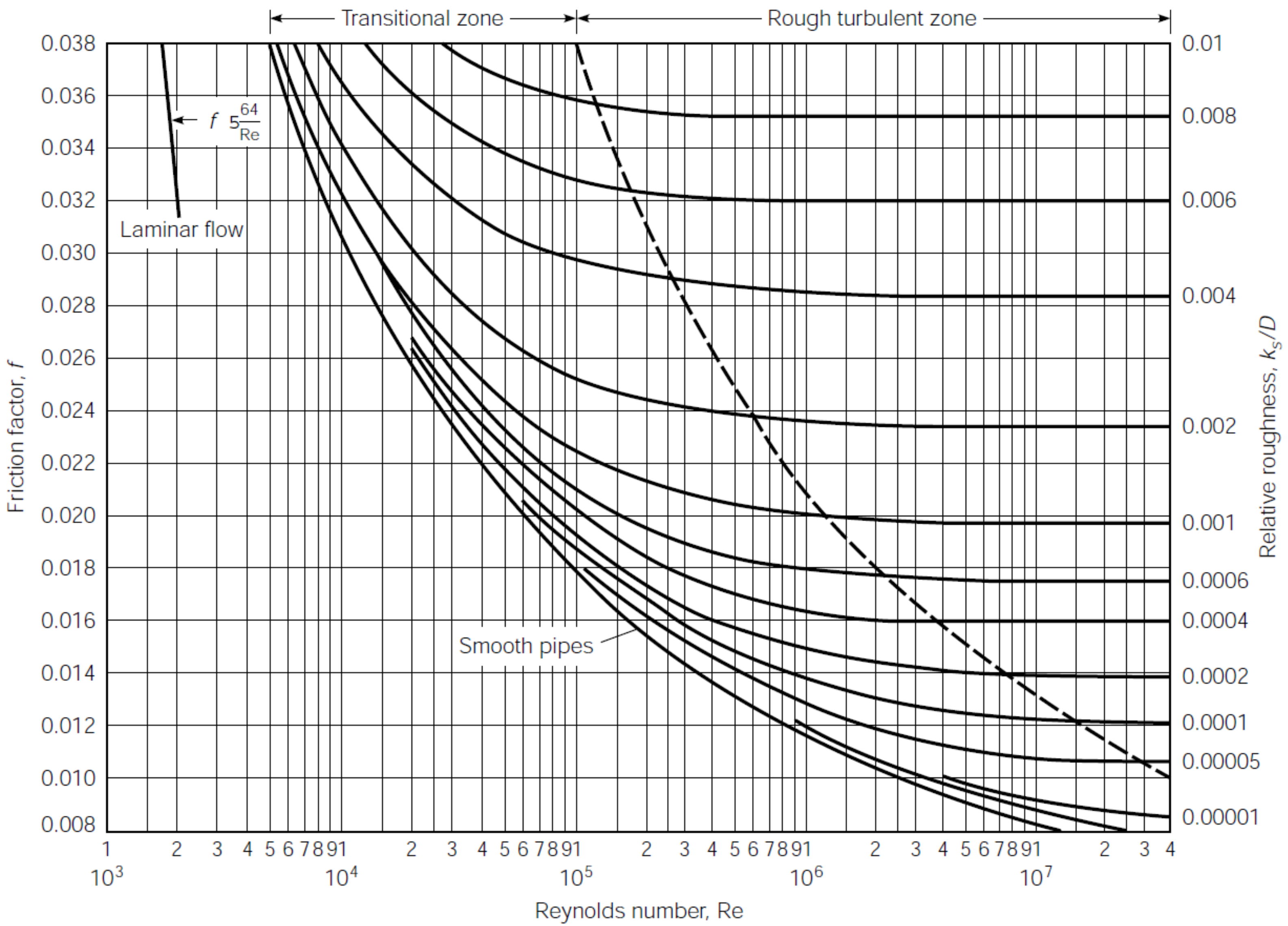
The wall shear stress can always be expressed as $\tau_w = \gamma R j$

Where *j* is the dissipation per unit length

Moody diagram

k$_s$ is equivalent to ε; λ is equivalent to f

Moody diagram: graphical representation of the Colebrook equation



Formulae for friction factor λ (or *f*)

- Colebrook-White equation: $$\frac{1}{\sqrt{f}} = -2\ Log\left(\frac{\varepsilon}{3.71D} + \frac{2.51}{Re\sqrt{f}}\right)$$

- Moody equation: $\lambda = 0.0055\left[1 + \left(\frac{20000k_s}{D} + \frac{10^6}{R_e}\right)^{1/3}\right]$

±5% accuracy for Re 4000 to 10^7 & $k_s/D < 0.01$

- Barr formula: $\frac{1}{\sqrt{\lambda}} = -2log\left(\frac{k_s}{3.7D} + \frac{5.1206}{{R_e}^{0.89}}\right)$

1% accuracy for Re > 10^5

Explicit (simplified) formulae for λ

- Darcy-Weisbach formula for h_f: $h_f = \frac{\lambda L u^2}{2gD}$ ←

The Darcy-Weissbach comes directly from the definition of the friction factor, which is often the unknown

Typical value for the equivalent roughness (new pipes)

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Pipe Material	k_s (mm)
Brass, copper, glass, perspex	0.003
Asbestos cement	0.03
Wrought iron	0.06
Galvanised iron	0.15
Plastic	0.03
Bitumen-lined ductile iron	0.03
Spun concrete lined ductile iron	0.03
Concrete sewer in poor condition	6.0

Hazen-Williams

- Has the advantage of simplicity but less accurate than using λ.
- Note that the friction factor C is completely different from λ – a different concept.

$$u = 0.355 D^{0.63} C \left(\frac{h_f}{L} \right)^{0.54}$$

$$h_f = \frac{6.78 L}{D^{1.165}} \left(\frac{u}{C} \right)^{1.85}$$

$$70 \leq C \leq 150$$

Formulae for the dissipation per unit length, j

- These are practical formulas only valid in the fully turbulent regime (rough turbulent flow region)

DARCY-WEISBACH

$j = \beta \frac{Q^2}{d^m}$ *where if*

- $\beta = \beta'$ => $m = 5.33$
- $\beta = \beta_i(d)$ => $m = 5$

CHEZY

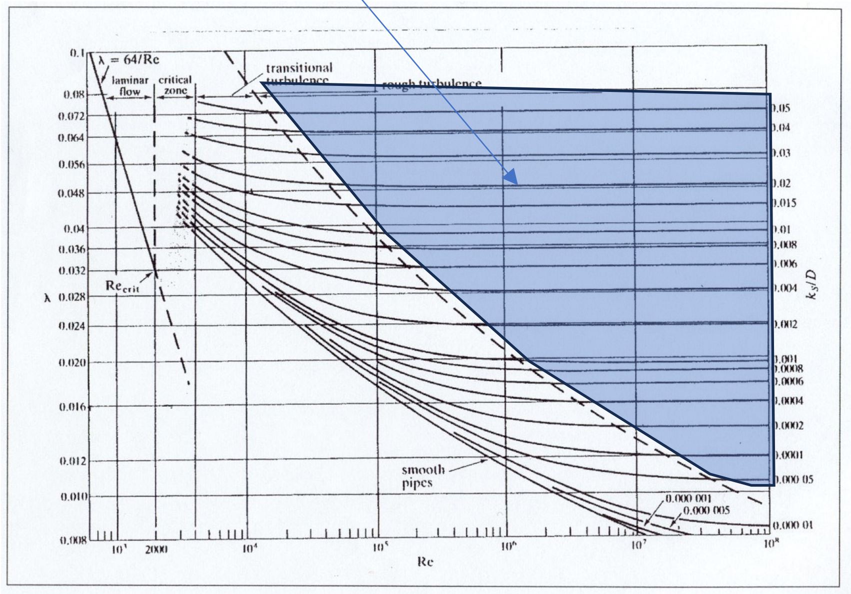
$\bar{u} = C\sqrt{R\,j}$ *where*

$C = \frac{87}{1+\frac{\gamma_B}{\sqrt{R}}}$ *Bazin*

$C = \frac{100}{1+\frac{m_K}{\sqrt{R}}}$ *Kutter*

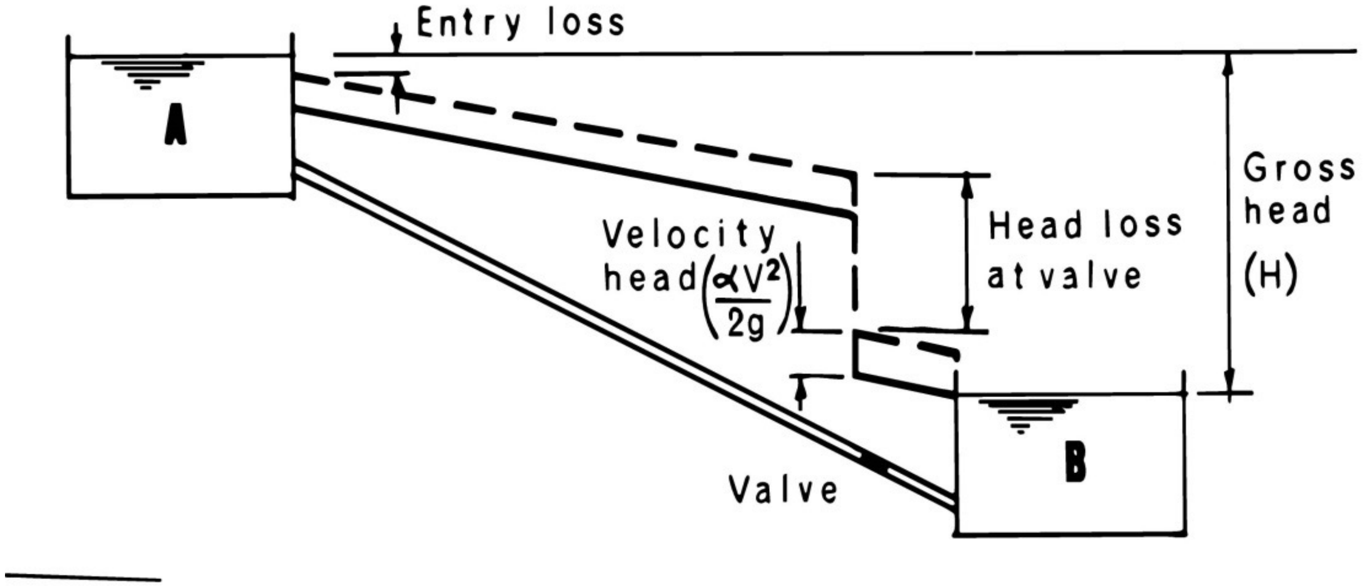
$C = c_{GS} R^{1/6}$ *Gaucler-Strikler*

$n = 1/c_{GS}$ *Manning*



In the rough-turbulence regime the Re does not play a role anymore, whence it does not appear in the formulae

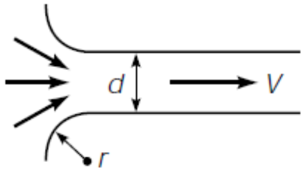
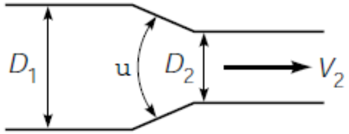
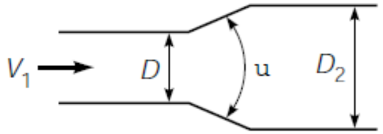
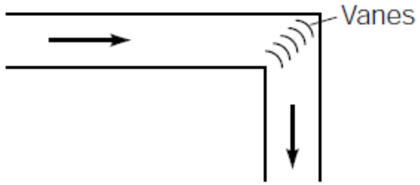
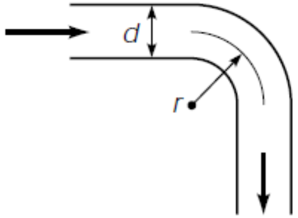
Localized losses



For "hydraulically long" pipes, then localized losses may be negligible with respect to distributed losses. This happens when a length unit is L >> 1000 d (see ahead)

Local losses: $h_{fl} = K\frac{u^2}{2g}$ (bends, entrances, exits, changing sections, etc)

K is often indicated as ξ in the European literature

Description	Sketch	Additional Data	K	
Pipe entrance		r/d	K	
		0.0	0.50	
		0.1	0.12	
		>0.2	0.03	
Contraction		D_2/D_1	K	K
			u = 60°	u = 180°
		0.0	0.08	0.50
		0.20	0.08	0.49
		0.40	0.07	0.42
		0.60	0.06	0.27
		0.80	0.06	0.20
		0.90	0.06	0.10
Expansion		D_1/D_2	K	K
			u = 20°	u = 180°
		0.0		1.00
		0.20	0.30	0.87
		0.40	0.25	0.70
		0.60	0.15	0.41
		0.80	0.10	0.15
90° miter bend		Without vanes	K = 1.1	
		With vanes	K = 0.2	
90° smooth bend		r/d	K	
			u = 90°	
		1	0.35	
		2	0.19	
		4	0.16	
		6	0.21	

Pipe flow hydraulics

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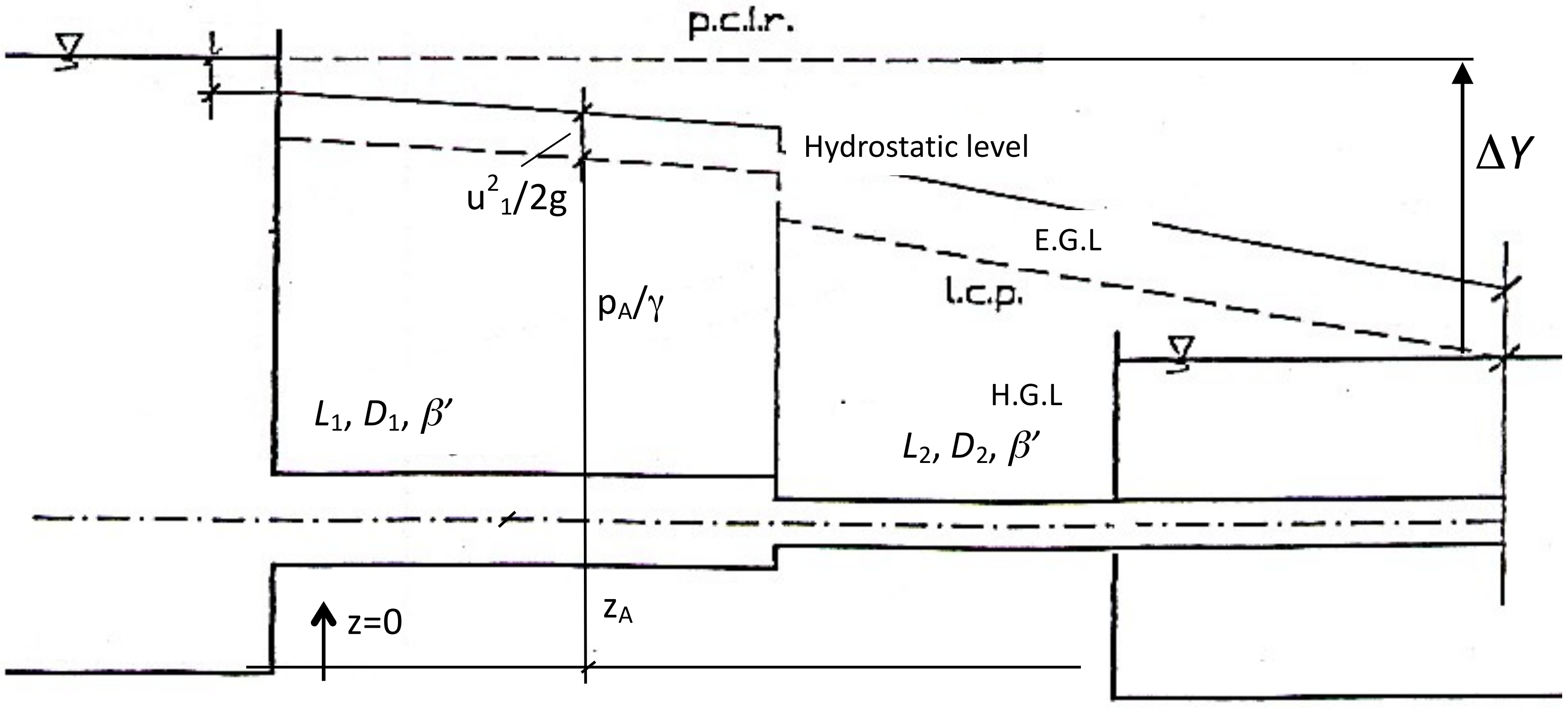
Energy vs hydrualic grade lines



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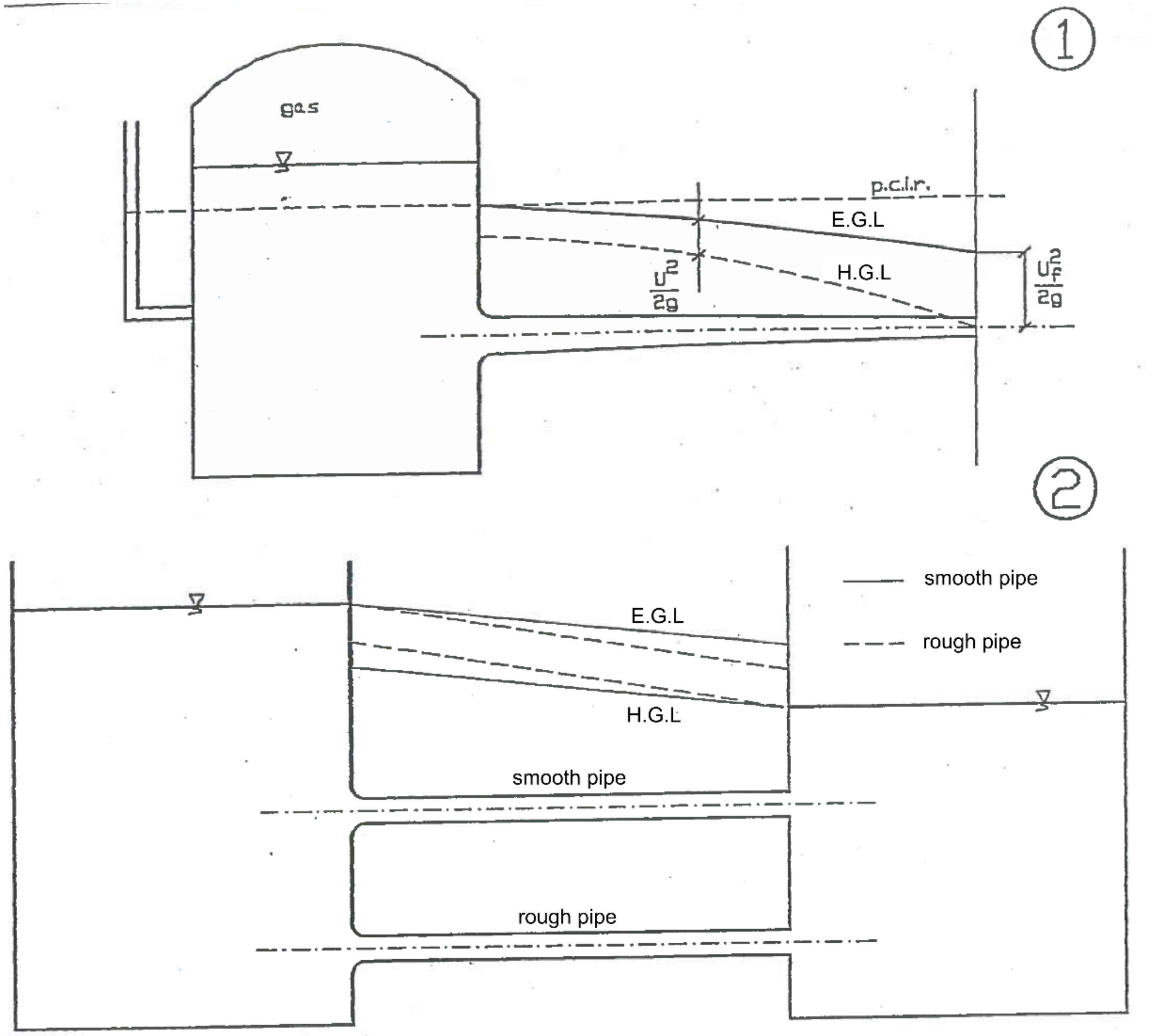
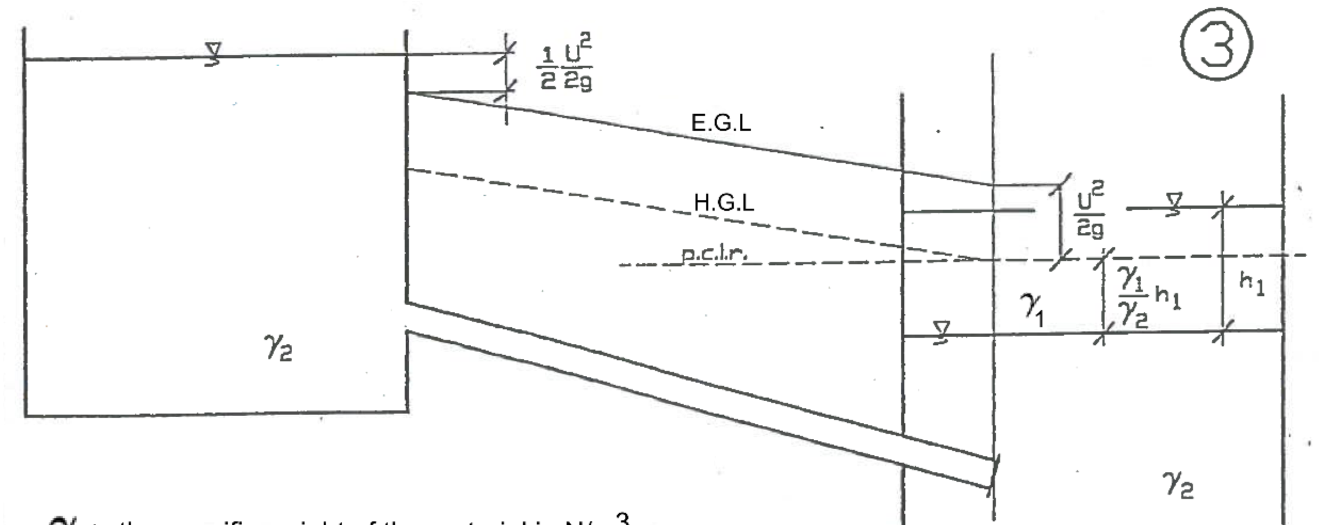
Energy balance eqaution

$$\Delta Y = 0.5\,\frac{u_1^2}{2g} + \beta'\,\frac{Q^2}{d_1^{5.33}}L_1 + \xi\,\frac{u_1^2}{2g} + \beta'\,\frac{Q^2}{d_2^{5.33}}L_2 + \frac{u_2^2}{2g}$$

Exemplary (didactic) cases

Use the following cases to train yourself in drawing the EGL and HGL

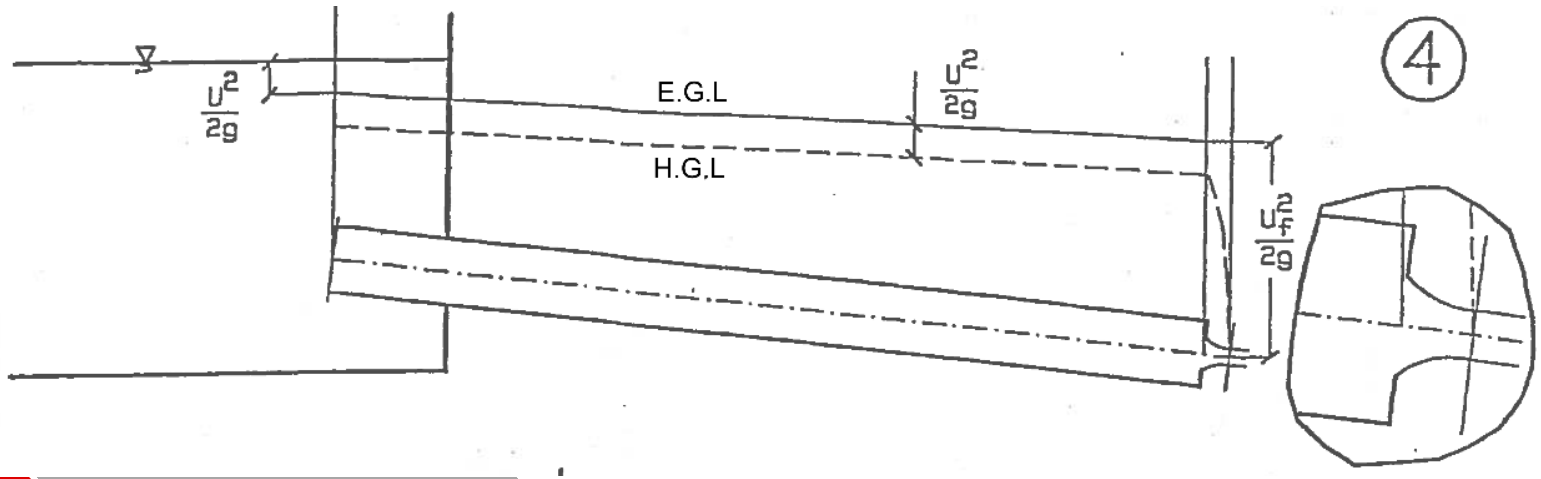


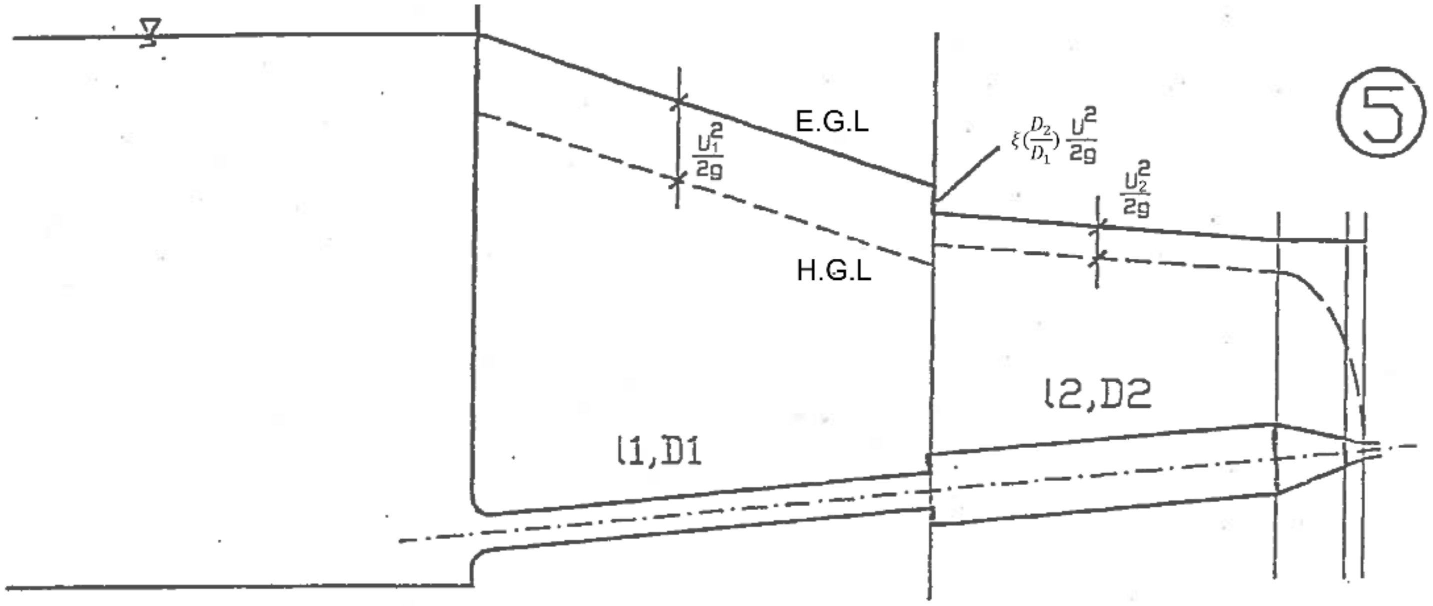


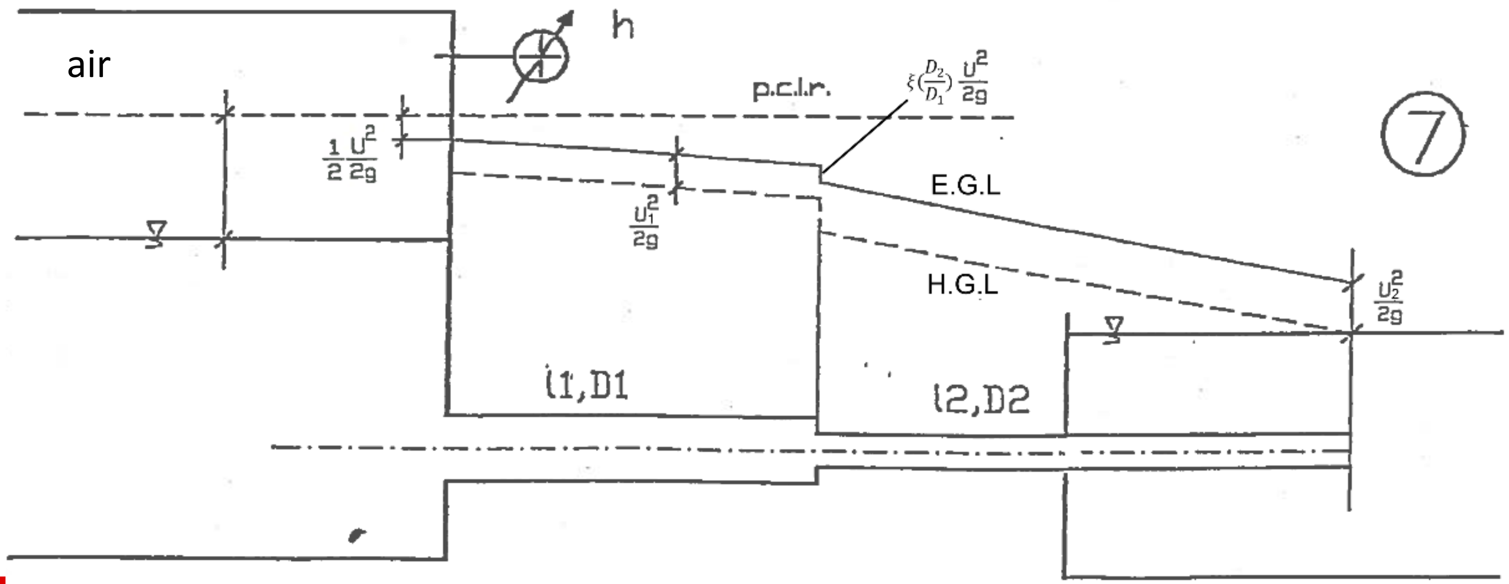
γ is the specific weight of the material in N/m^3

ρ is the density of the material in kg/m^3

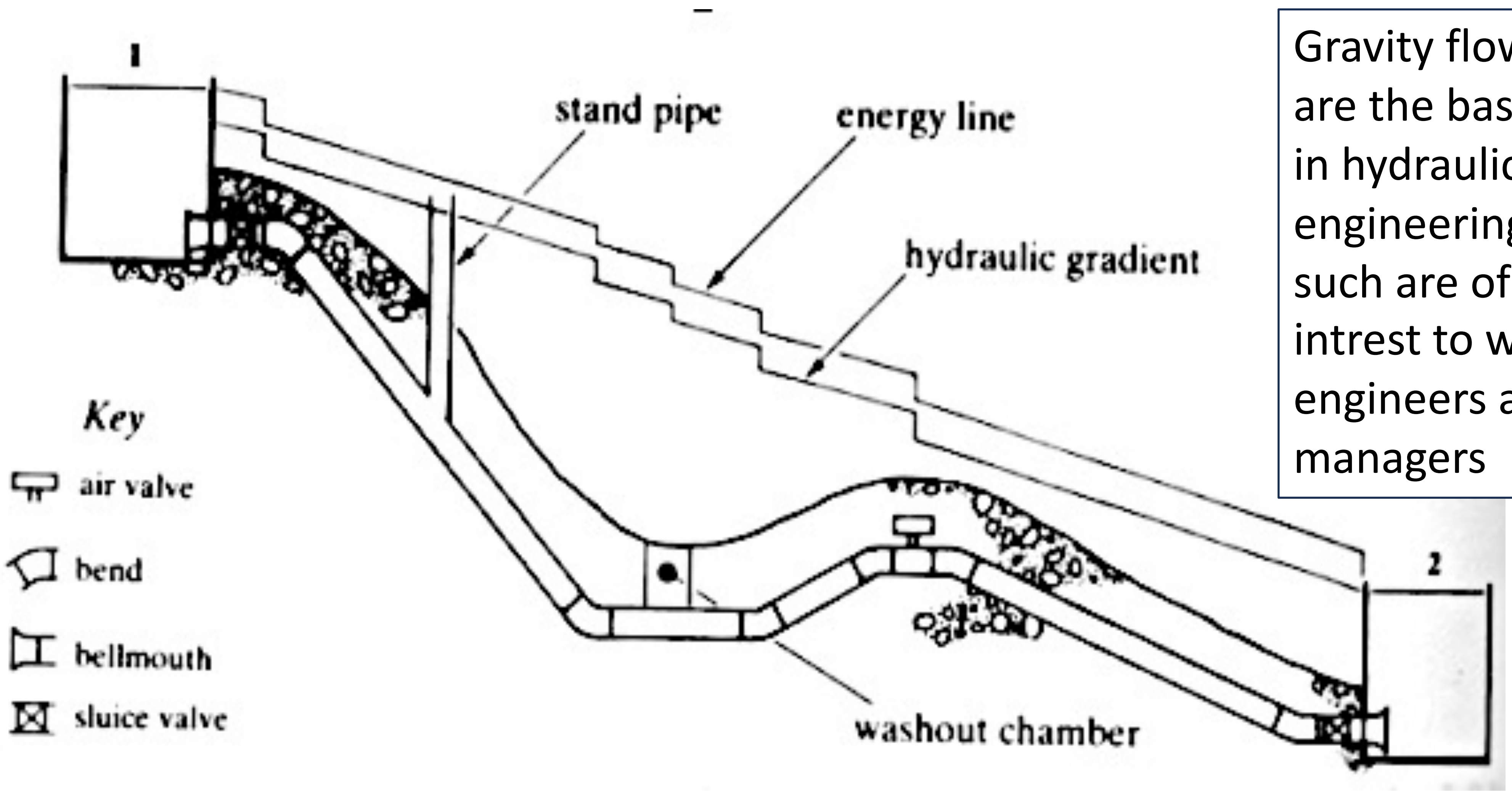
g is the gravitational constant, 9.81 m/s^2




E.G.L
H.G.L
$\frac{U_1^2}{2g}$
$\xi(\frac{D_2}{D_1})\frac{U^2}{2g}$
$\frac{U_2^2}{2g}$
l1,D1
l2,D2
5


air
h
p.c.l.r.
$\frac{1}{2}\frac{U^2}{2g}$
$\frac{U_1^2}{2g}$
$\xi(\frac{D_2}{D_1})\frac{U^2}{2g}$
E.G.L
H.G.L
$\frac{U_2^2}{2g}$
l1,D1
l2,D2
7

Gravity flow mains



Gravity flow mains are the basic plants in hydraulic engineering and as such are of key intrest to water engineers and managers

Careful in drawing HGL and EGL when pipes are vertical!!

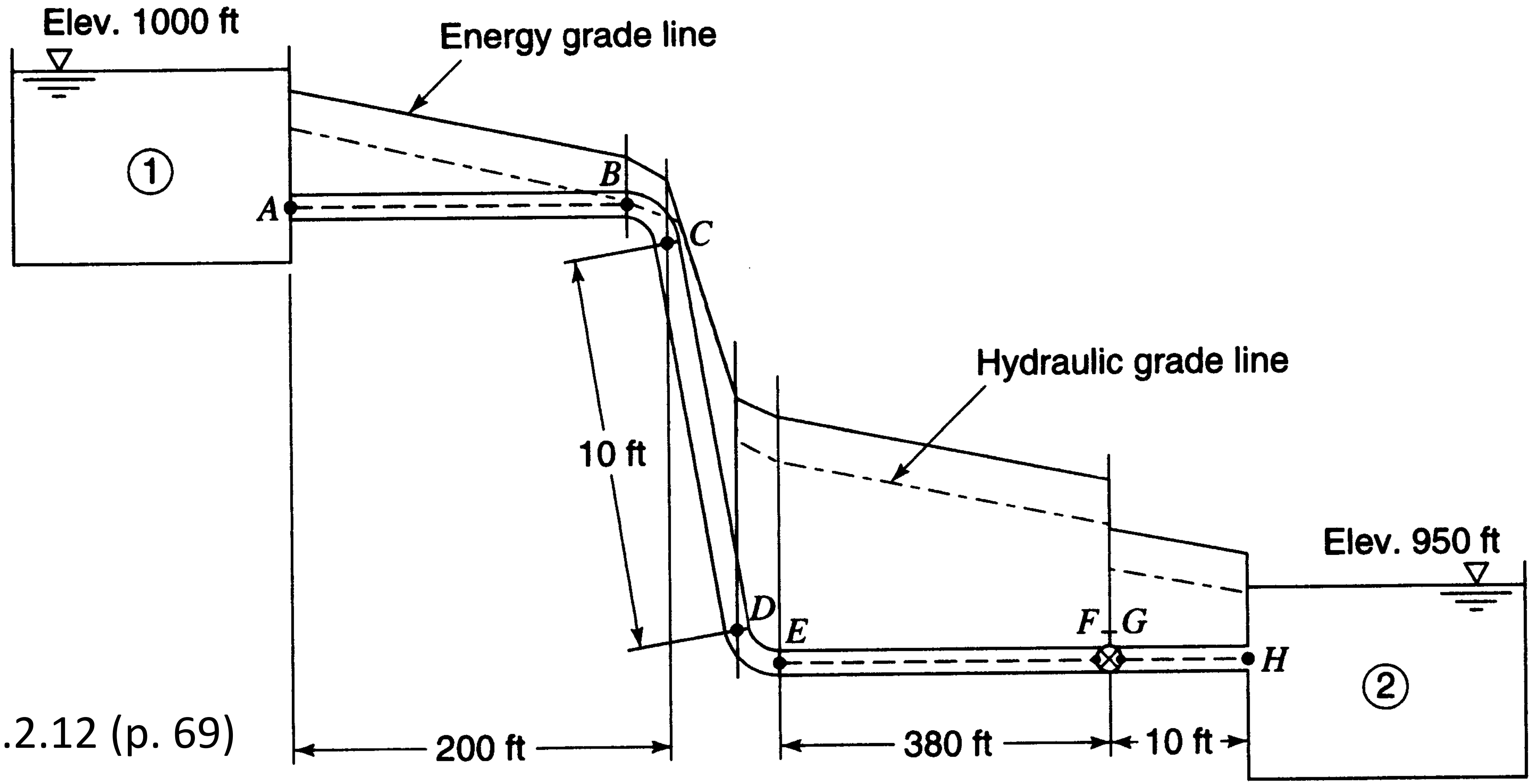


Figure 4.2.12 (p. 69)

Design *vs* diagnostic (verification) problems

- Notice, that by using the continuity equation for the pipe, $Q = u\frac{\pi D^2}{4} = const$, the energy balance equation can be rewritten in terms of flow rate, Q thus allowing for calculating the flow rate once the geometry of the pipe system (lengths, diameters, restrictions, etc), the pipe roughness and the available energy are known (verification problem). Similarly, if the flow rate is known, one can compute the diameter to convey it with the assigned available energy (design problem). We can summarize the two problems as

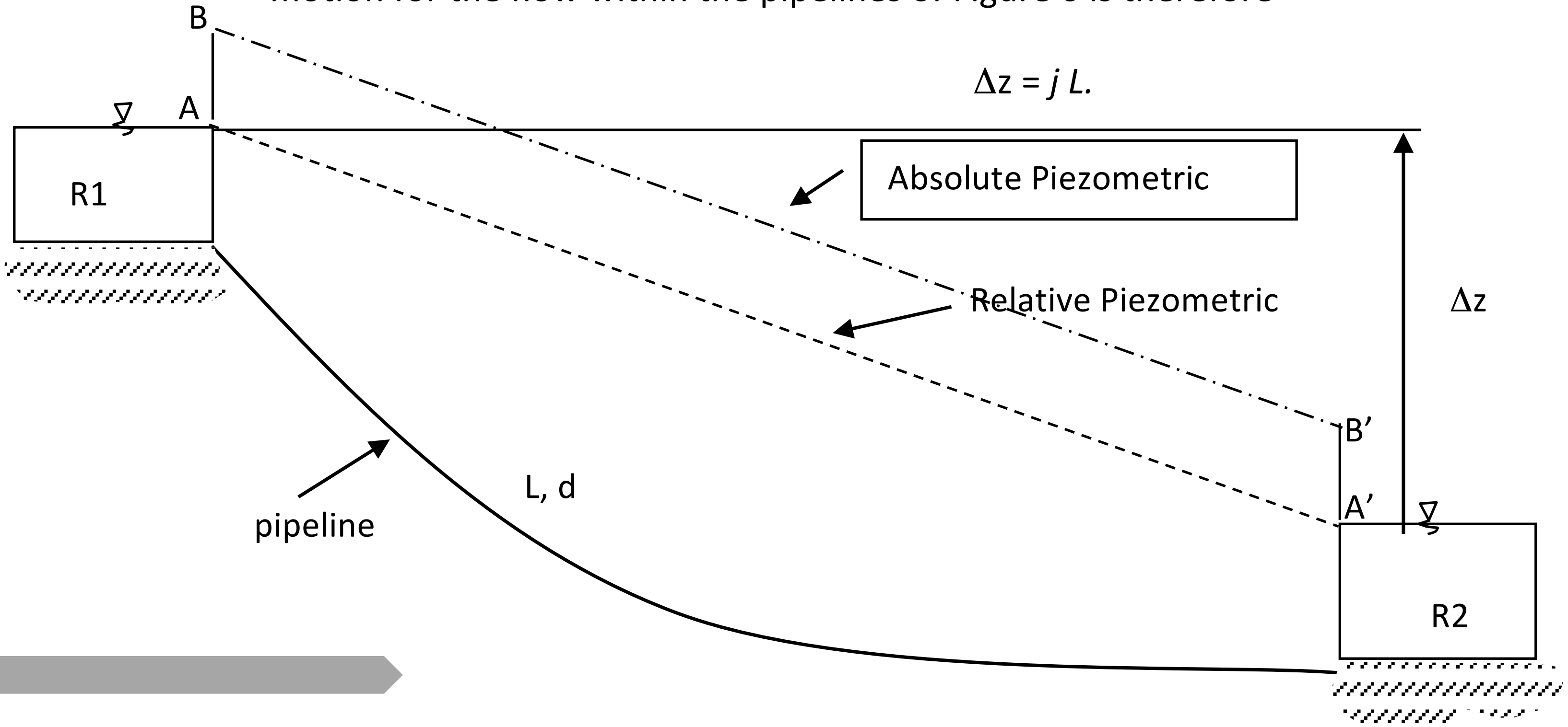
GENERAL VERIFICATION PROBLEMS	GENERAL DESIGN PROBLEM
$\Downarrow$	$\Downarrow$
KNOWN: D, L, i, ε or equivalent roughness	KNOWN: Q, L, i, ε or equivalent roughness
$\Downarrow$	$\Downarrow$
UNKNOWN: effective flow rate Q passing within the pipe of diameter D with the established dissipation per unit lengh i	UNKNOWN: best diameter D (pipe size) to convoy the flow rate Q with the established dissipation per unit length i

Hydraulically long pipes

Definition: A pipe is called ‘hydraulically long’, regardless the presence of a weak discontinuity in the geometry which induces a localized head loss, if the total pipe length is at least $L > 20\,L^{*}$, i.e. $L > 1000d$.

For pipeline systems, the kinetic and energy losses that have order of magnitude $\frac{\overline{u}^2}{2g}$ can be neglected by accepting an approximate solution, i.e. involving a small error. The equation of motion for the flow within the pipelines of Figure 6 is therefore

$$\Delta z = j\,L.$$



Example

Let us now calculate which should be the effective length of a pipe in order to consider it as hydraulically long.

We suppose to accept an approximation of the exact solution of about 4%, i.e 0.04.

The equivalent length *L* thanks to which we would neglect the velocity head, in order to solve the problem with the chosen approximation, can be expressed as a number *n* of time the pipe diameter *d*, i.e. *L=nd* .

Therefore we have that

$$\frac{\bar{u}^2}{2g} \leq 0.04Lf\,\frac{\bar{u}^2}{2gd} = 0.04ndf\,\frac{\bar{u}^2}{2gd},$$

and, as a consequence

$$n = \frac{1}{0.04f}.$$

Assuming now a friction factor f=0.025, the result is therefore *n*=1000. This means that for this system if one neglects the velocity head in the calculation, the result will have an approximation of the 4% only if the pipe length is about 1000 times the pipe diameter. Of course the length of the pipe should be longer as more as higher is the number and kind of the localized head losses.

Hydraulic management: pipeline diagnostic

This is the classic and simplest case of verification problem for two reservoirs (Fig 4.1):

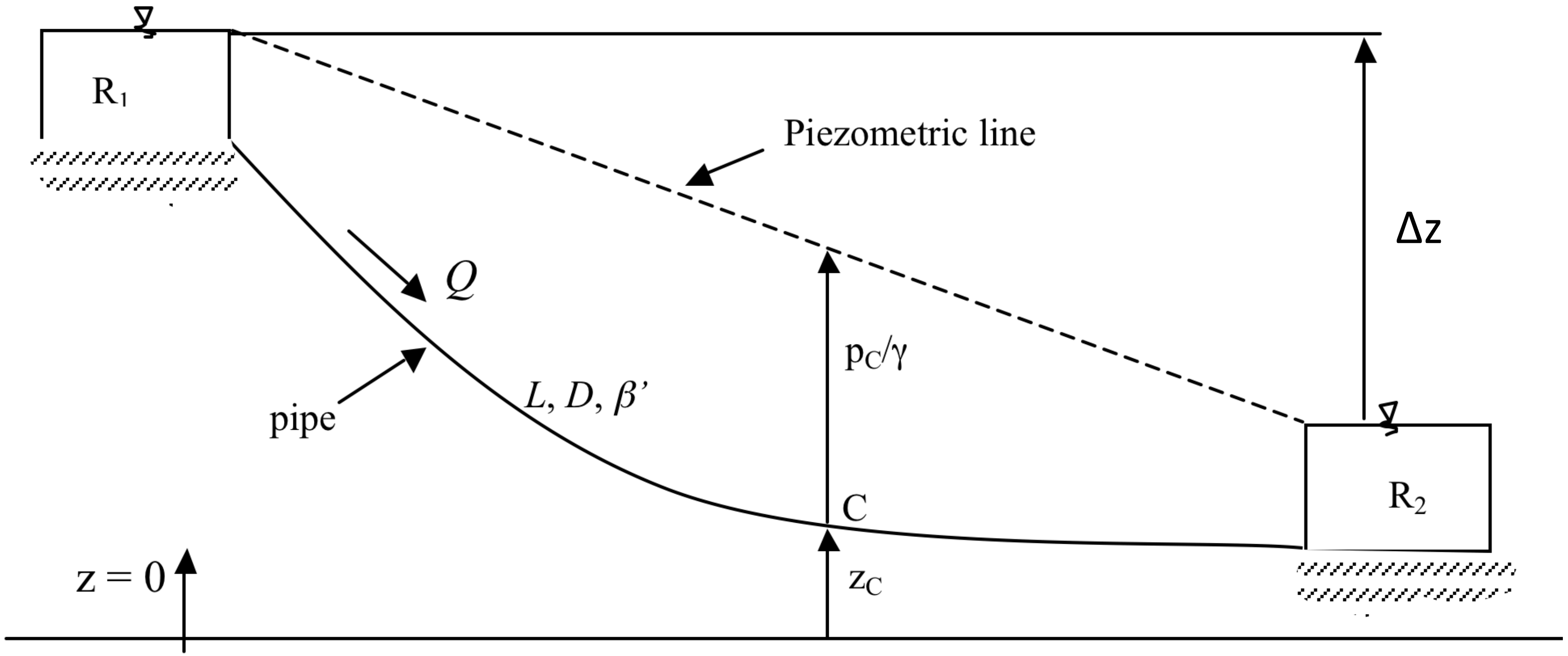


Figure 4.1

Aim: calculate the flow rate *Q* once the geometric characteristics such Δz, *L*, *D*, β' are known.

Using the Darcy's law, the equation of motion is

$$\Delta z = \beta' \frac{Q^2}{D^{5.33}} L$$

and then the flow rate is

$$Q = \sqrt{\frac{\Delta z D^{5.33}}{\beta' L}}$$

NOTE: here the diameter D appears with the exponent 5.33 because β' does not depend on diameter

Notes: the pipe reach is subjected to a positive pressure in that the piezometric height in each section is given by the difference between the piezometric head *h* and the geodetic height *z* of the pipe centreline.