Fundamentals of Traffic Operations and Control
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Exercise solutions
Parking management

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a) We can visualize the dynamics as three reservoirs in series, with levels representing accumulations, as depicted in fig. 1.

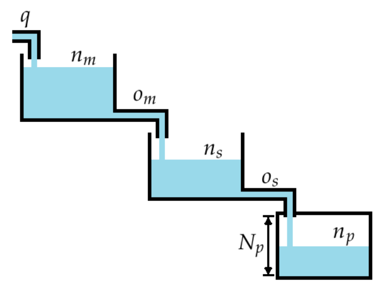

Figure 1: Reservoir system representing cruising-for-parking dynamics.

We write the continuous mass conservation equations as:

$$\begin{aligned} \dot{n}_m(t) &= q - o_m(n_m(t), n(t)) \\ \dot{n}_s(t) &= o_m(n_m(t), n(t)) - o_s(n_s(t), n(t)) \\ \dot{n}_p(t) &= o_s(n_s(t), n(t)), \end{aligned} \tag{1}$$

where the transfer flows (i.e., trip completion rates) are:

$$\begin{aligned} o_m(n_m(t), n(t)) &= \frac{n_m(t)}{n(t)} \frac{P(n(t))}{l_m} \\ o_s(n_s(t), n_p(t), n(t)) &= \frac{n_s(t)}{n(t)} \frac{P(n(t))}{l_s(n_p(t))} = \frac{N_p - n_p(t)}{N_p d_1} \frac{n_s(t)}{n(t)} P(n(t)). \end{aligned}$$

Discretizing eq. (1) using the forward Euler method, we obtain:

$$\begin{aligned} n_m(k+1) &= n_m(k) + T \cdot (q - o_m(n_m(k), n(k))) \\ n_s(k+1) &= n_s(k) + T \cdot \big(o_m(n_m(k), n(k)) - o_s(n_s(k), n_p(k), n(k))\big) \\ n_p(k+1) &= n_p(k) + T \cdot o_s(n_s(k), n_p(k), n(k)), \end{aligned} \tag{2}$$

where T is the step size.

b) Simulating using eq. (2) we obtain the trajectory of $n_p(k)$ given in fig. 2, which shows that all spots will be filled in 27 minutes.

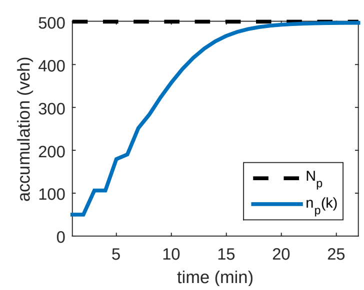

Figure 2: Trajectory of $n_p(k)$.

c) When N_p is infinite we can assume that the dynamics of $n_s(k)$ disappears (as drivers would not spend any time cruising for parking spots), resulting in the following model (where $n(k) = n_m(k)$ since $n_s(k)$ is always 0):

$$\begin{aligned} n(k+1) &= n(k) + T \cdot (q - o_m(n(k))) \\ n_p(k+1) &= n_p(k) + T \cdot o_m(n(k)). \end{aligned} \tag{3}$$

Simulating the system using eq. (3), we obtain the trajectory of $n(k)$ and compare it with that of part b) as shown in fig. 3. The gray shaded area between the two curves (5645 veh.min) is the extra delay due to parking space limitation. Average trip lengths are compared in fig. 4, showing that limitation results in increased trip lengths, whereas for infinite availability the value is fixed at l_m by definition.

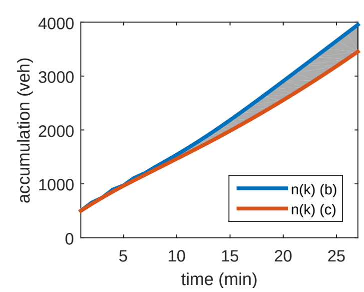

Figure 3: Trajectories of $n(k)$.

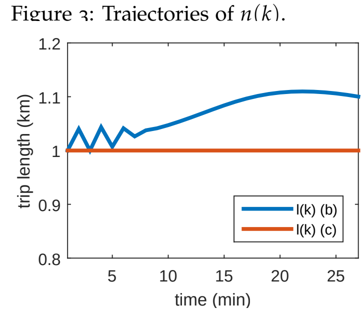

Figure 4: Trajectories of $l(k)$.

d) Perimeter control can be used (changing the term q in eq. (1) to $u(t) \cdot q$, where $u(t)$ is the perimeter control input, with $0 \leq u(t) \leq 1$) to operate the city center at maximum production $P_{\max}$.