EPFL
Static Models in Transport

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March 4, 2025

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  - First-best optimal capacity
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Fundamentals of traffic congestion

Small and Verhoef (2007, Section 3.3)

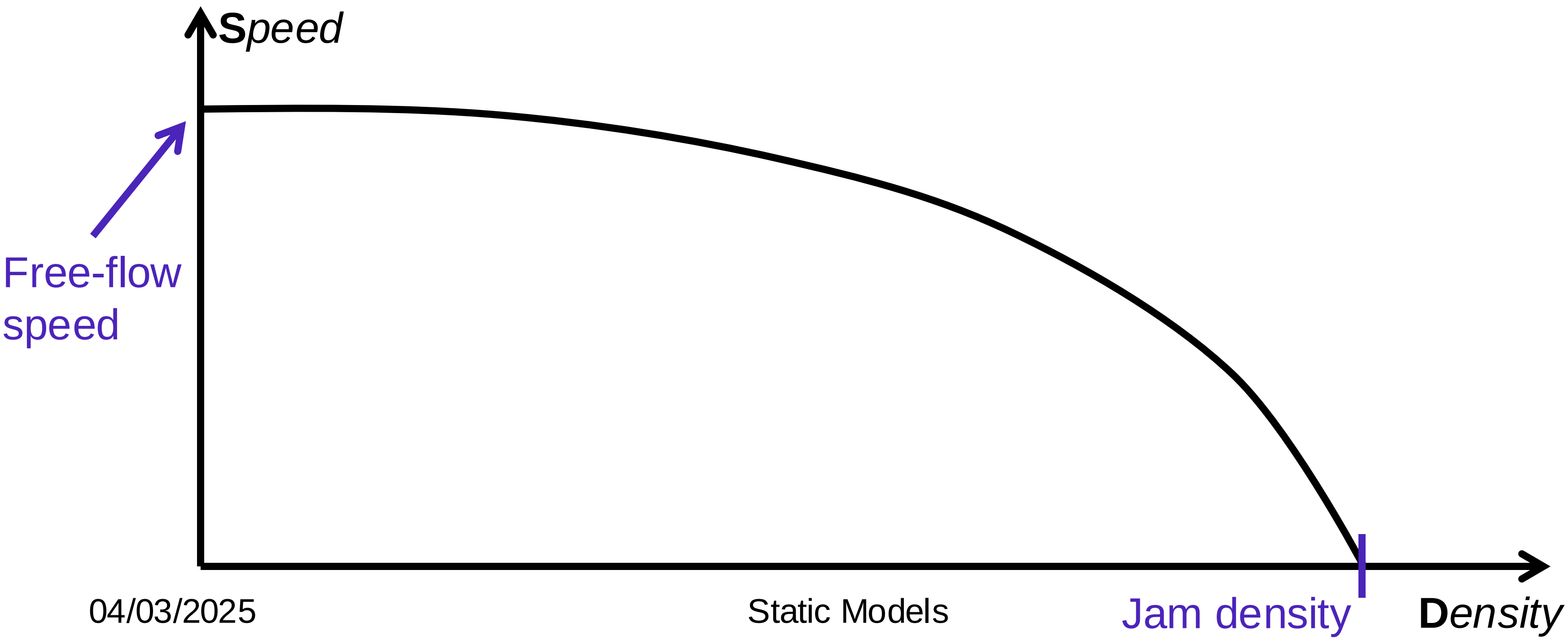
D = Density [vehicles/km.]

S = Speed [km/hr.]

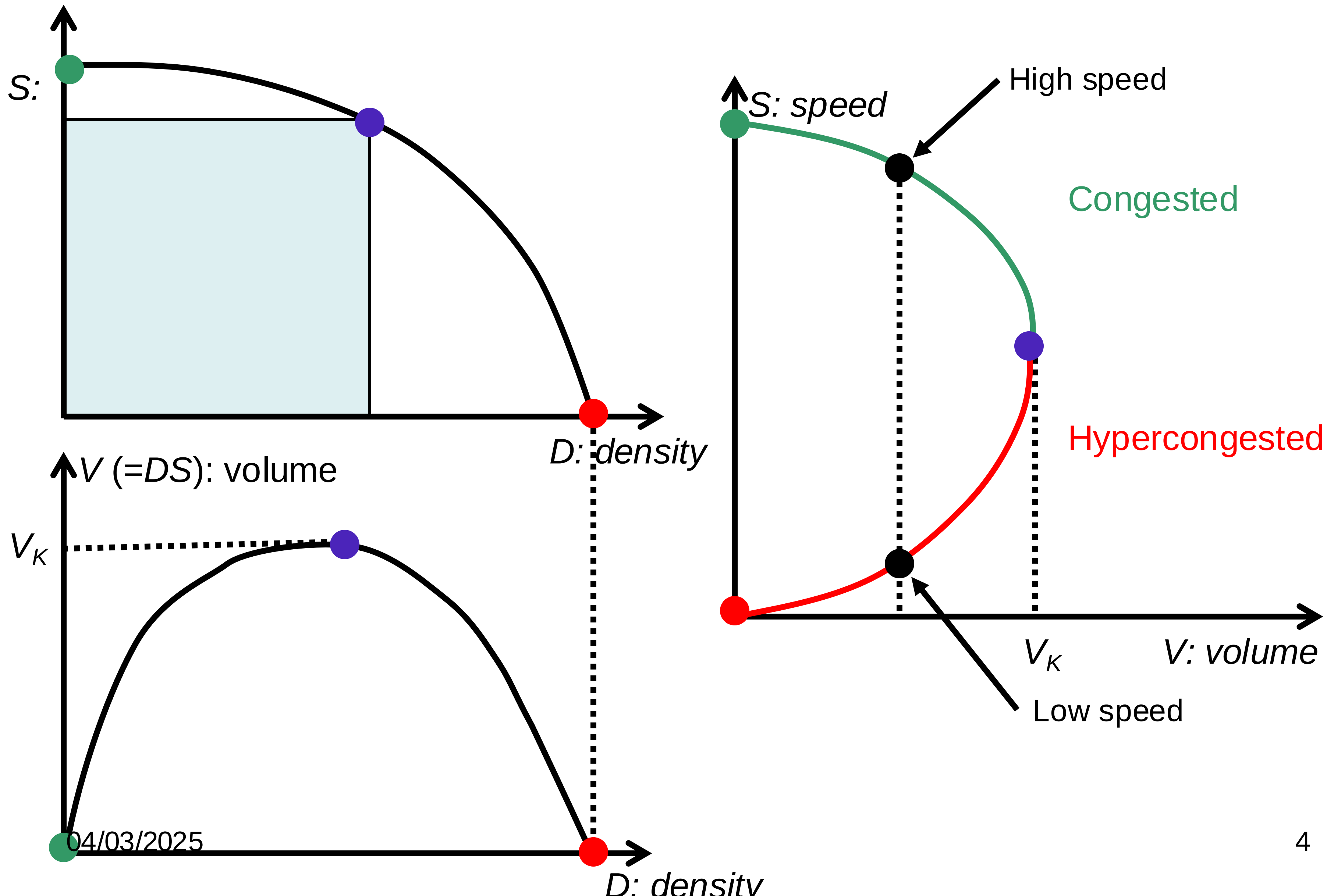
V = volume (flow) [vehicles/hr.]

V = DS e.g. 40 veh/km*50 km/hr = 2,000 veh/hr (1 lane)

Here: safety distance = 25 meters – Length of the car.



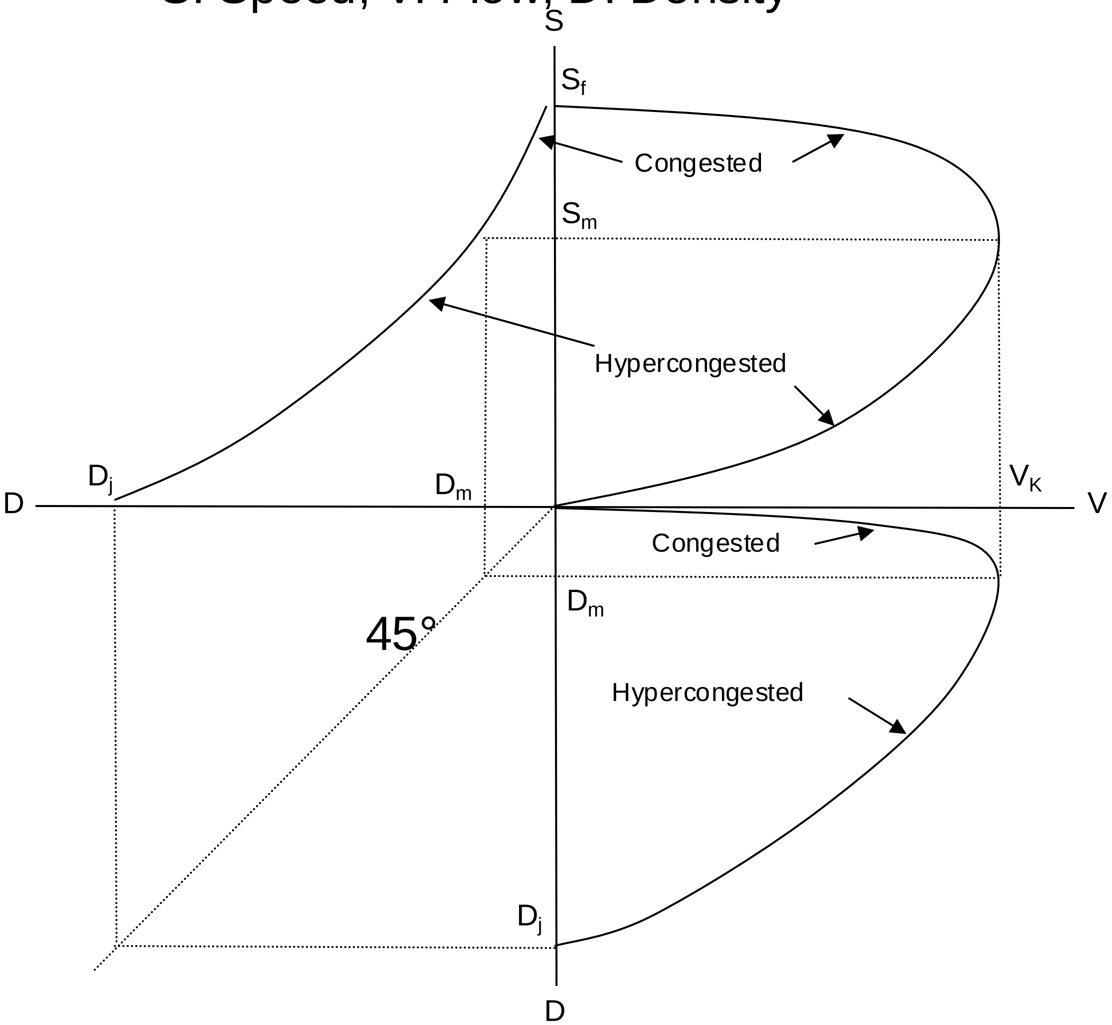
Fundamentals of traffic congestion



Fundamentals of traffic congestion

Small and Verhoef (2007, Figure 3.1)

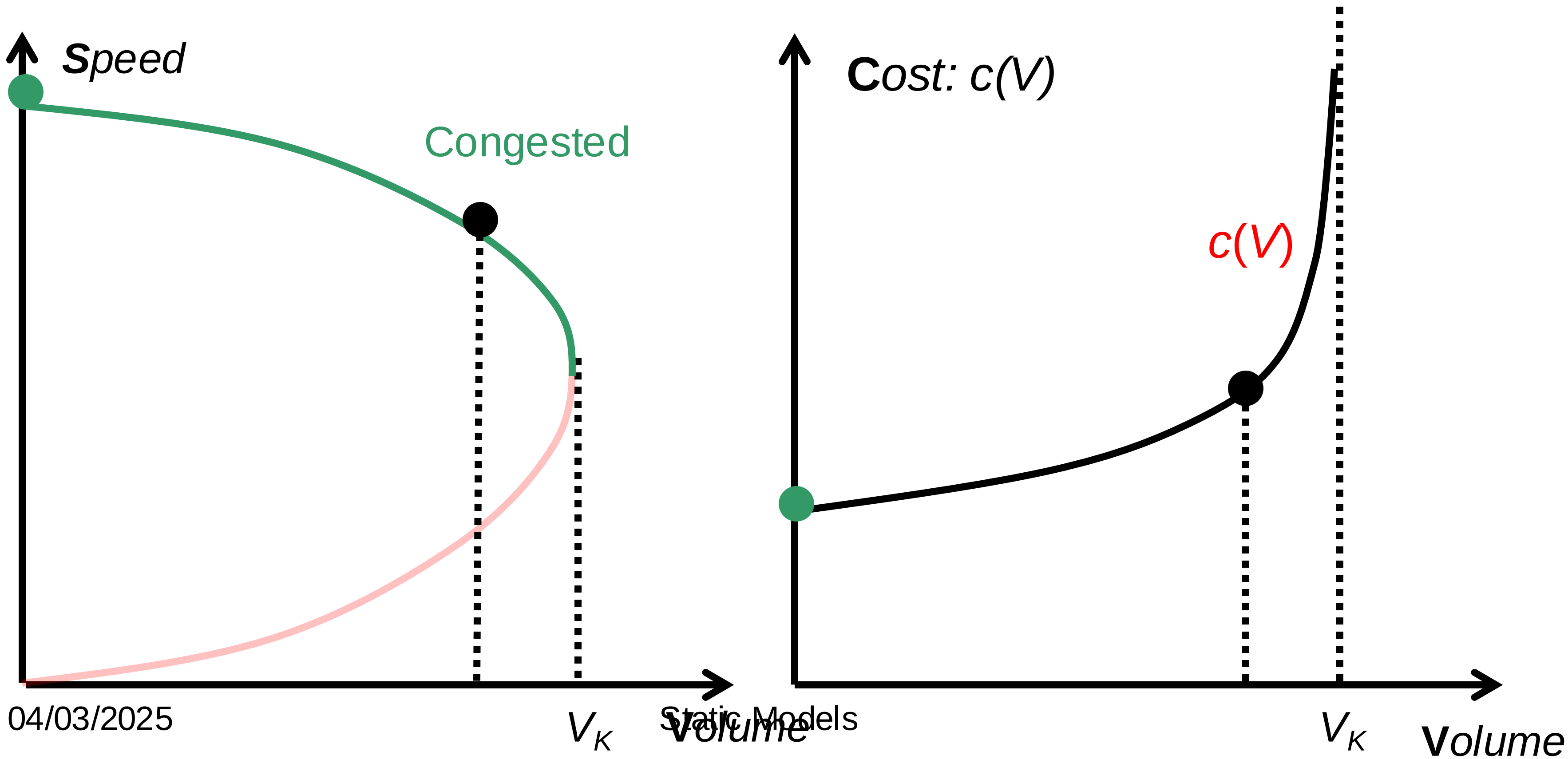
S: Speed; V: Flow; D: Density



Fundamentals of traffic congestion

Construct travel cost curve from speed-flow curve

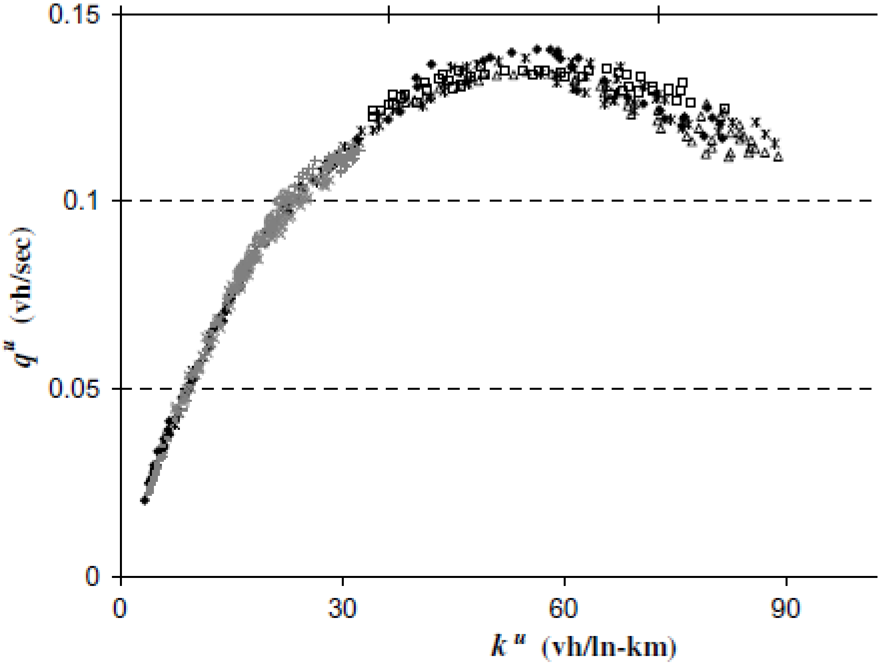
1. Travel time inversely proportional to speed (inversely proportional to travel time).
2. Assume cost (monetary & opportunity) of travel time is proportional to travel time.
3. Limit attention to the **congested** portion of speed-flow curve.



In a Macroscopic Fundamental Diagram (MFD)

A flow-density relationship at the level of an area rather than a link.

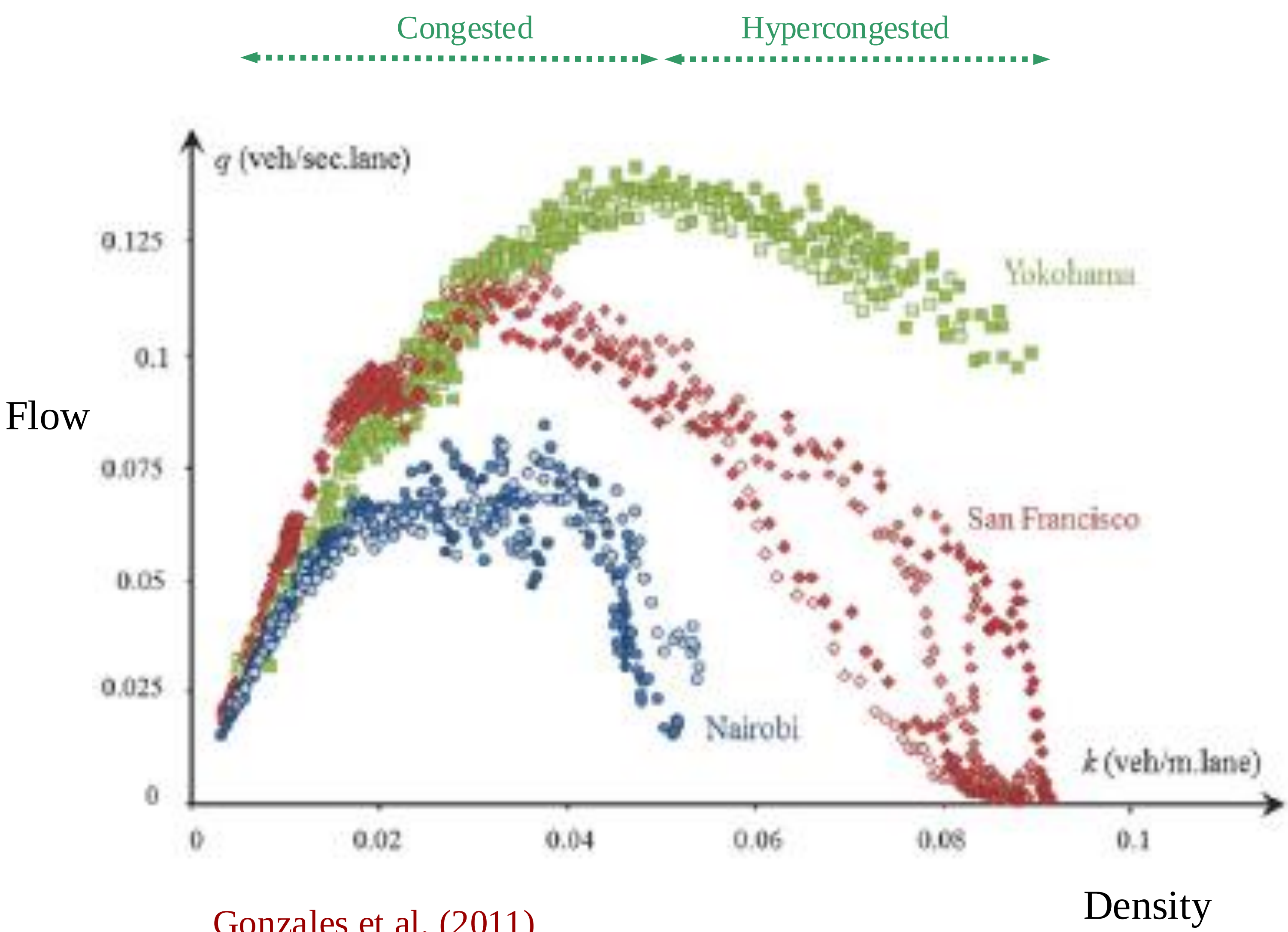
Flow



Density

Geroliminis and Daganzo (2008)

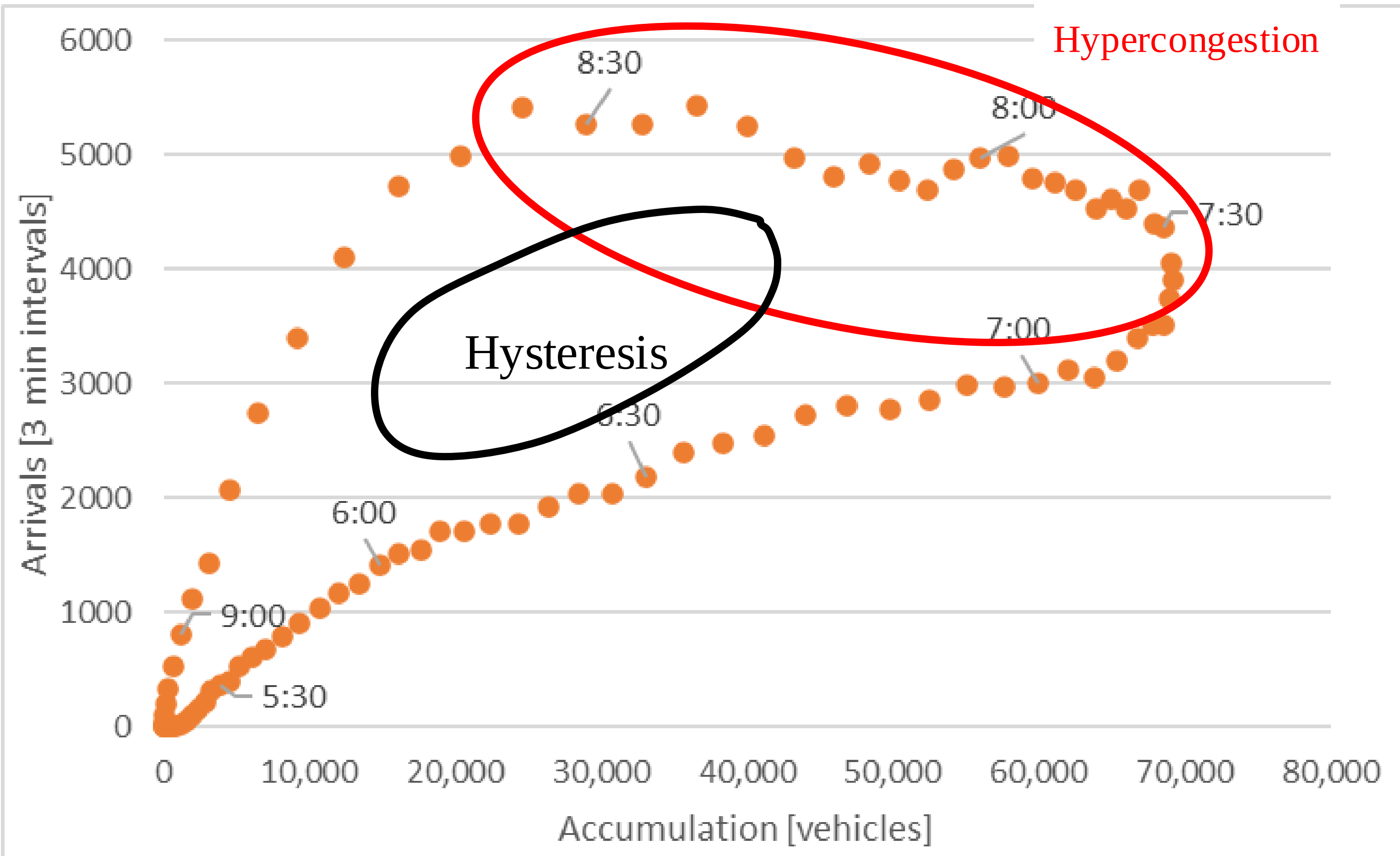
In a Macroscopic Fundamental Diagram (MFD)



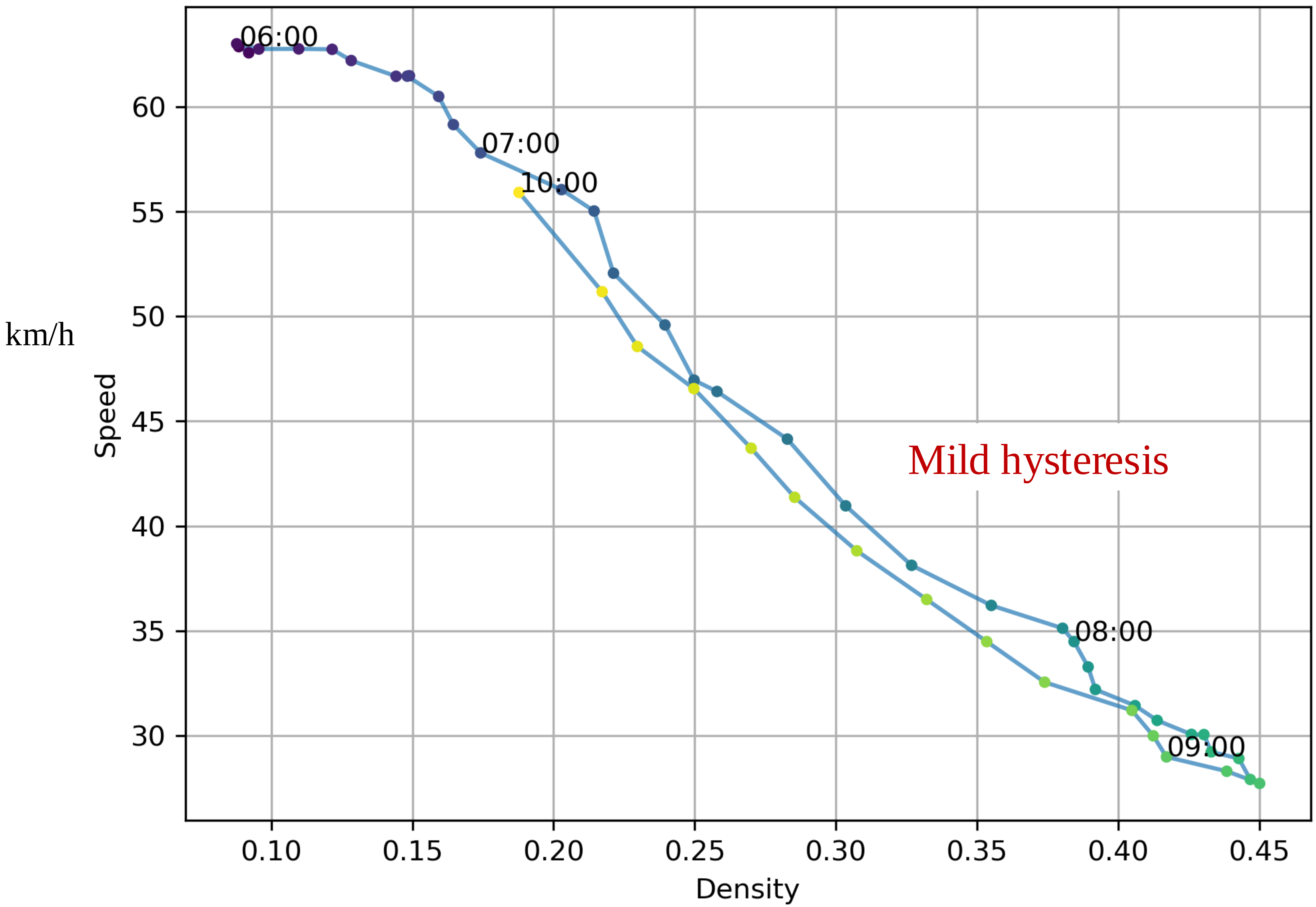
Gonzales et al. (2011)

METROPOLIS
Morning, no overtaking, spillback

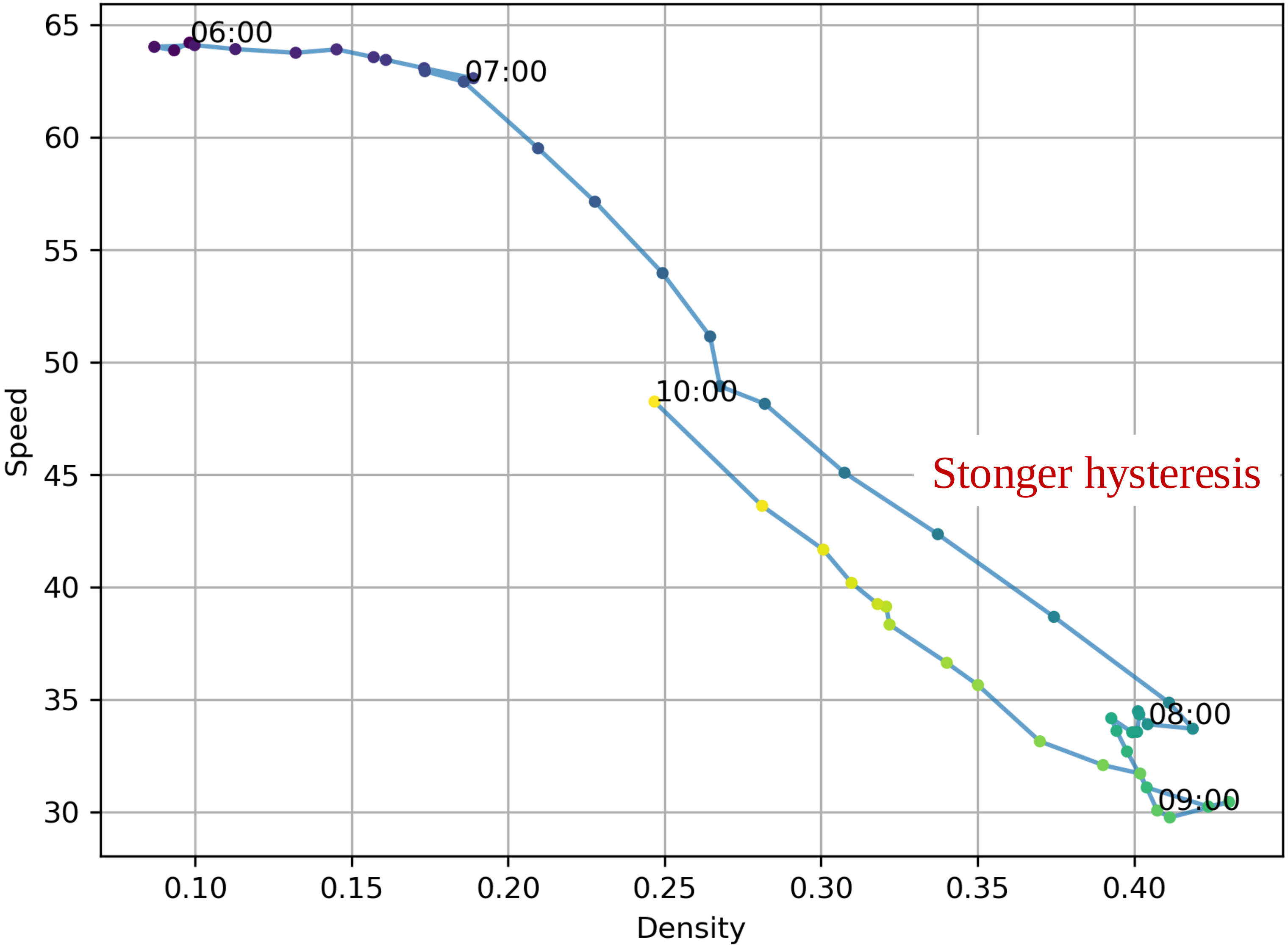
Arrival rate vs. accumulation



METROPOLIS BP Intérieur morning

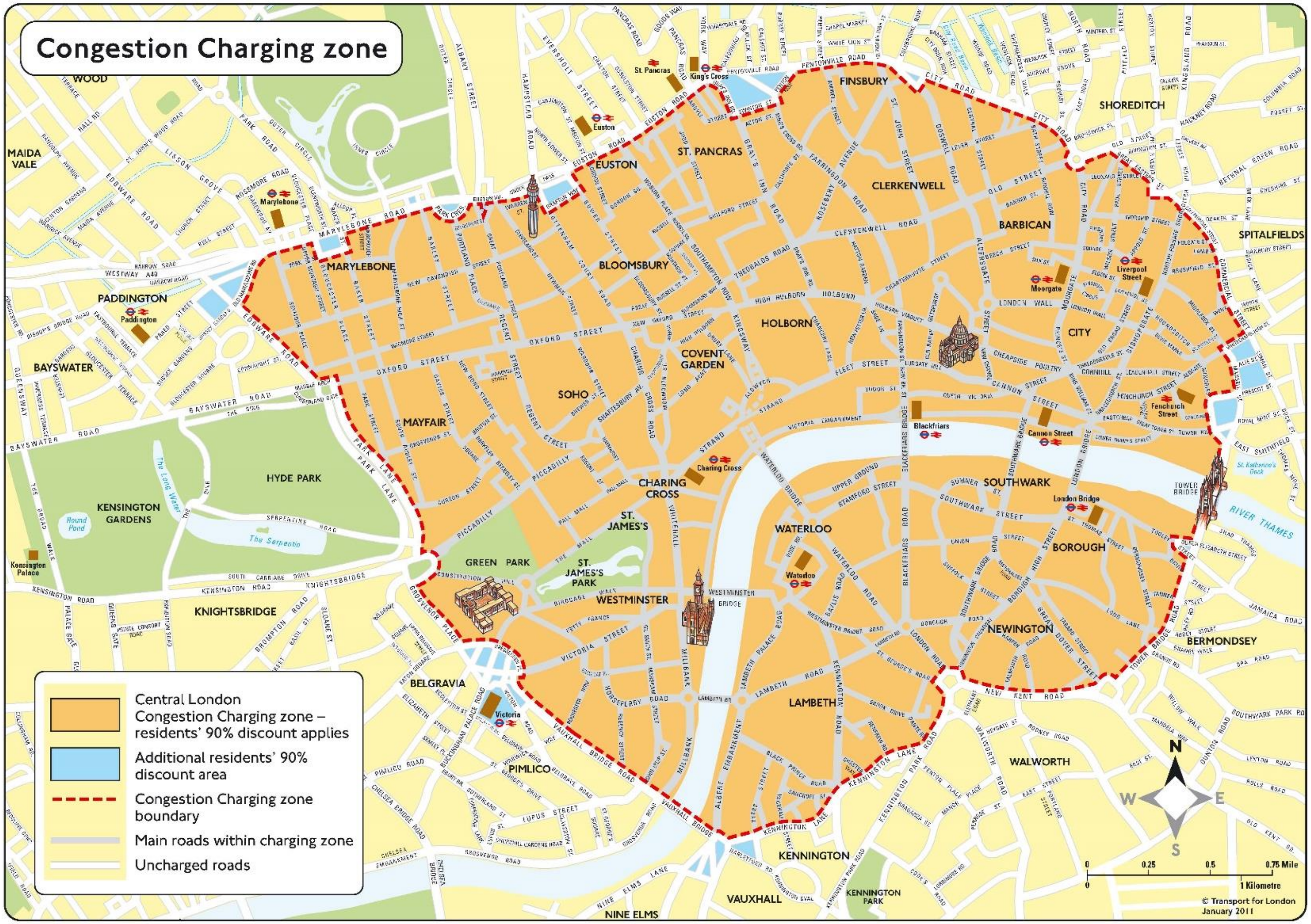


BP Extérieur morning



Congestion Pricing in Practice

London Congestion Charge

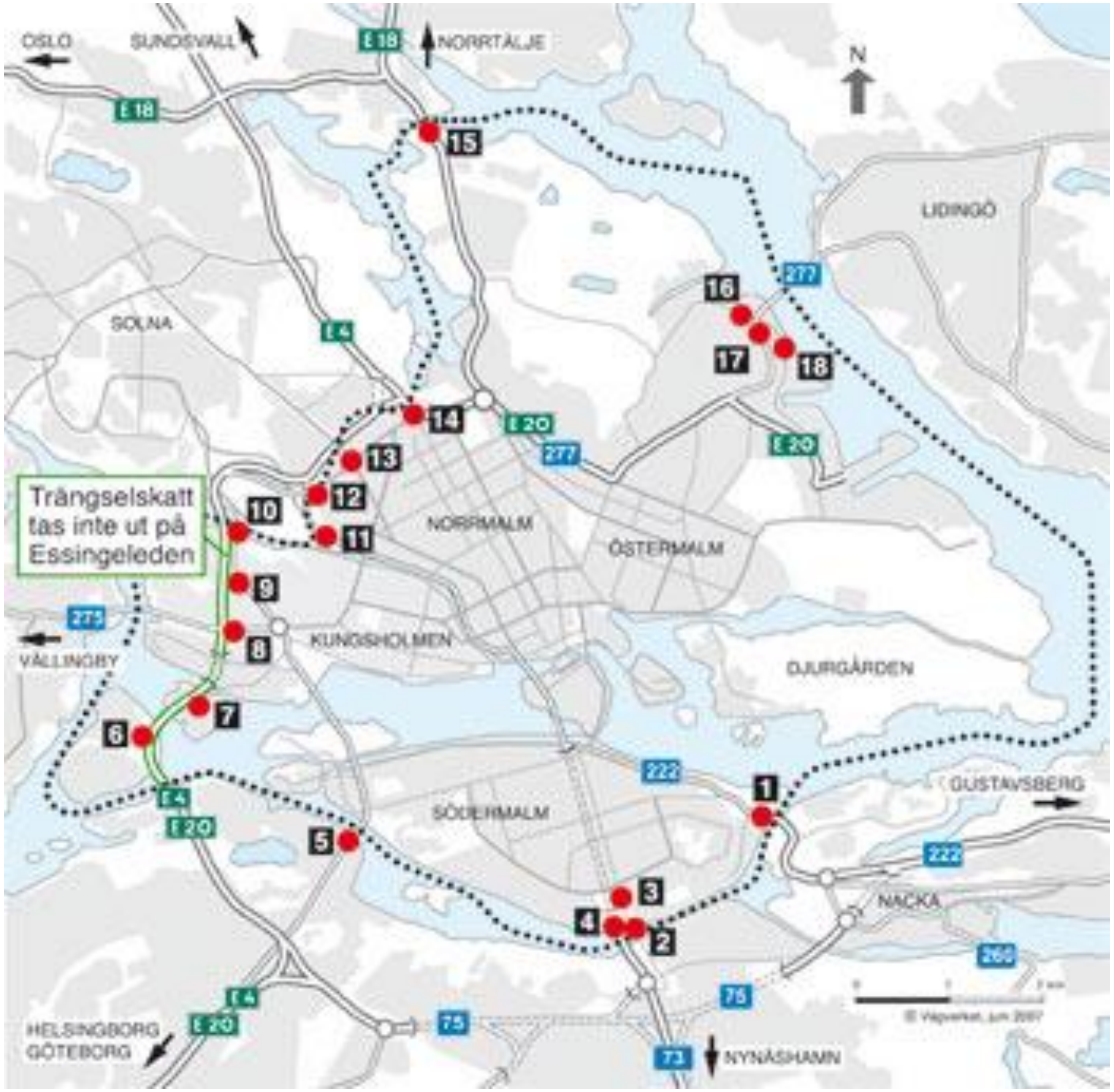


Enforcement

London

An earlier time

Stockholm Congestion Charge



Singapore Electronic Road Pricing (ERP1; ERP 2)

Land Transport Authority (LTA) initiative:
toll charges are levied on vehicles according to time and congestion levels.

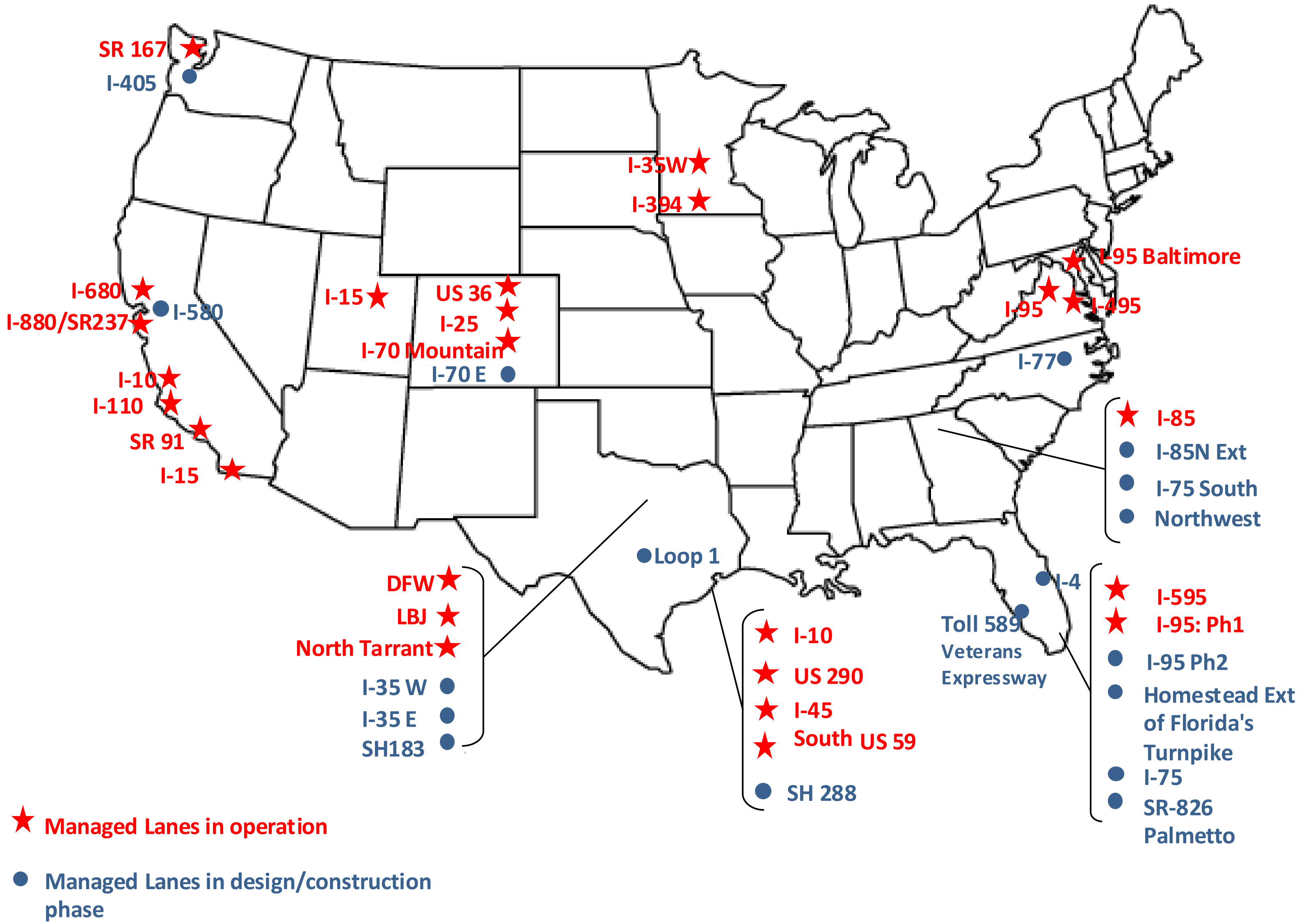
$0.50 to $6 per trip

High Occupancy Toll (HOT) lanes on State Route 91, Orange County, CA

Managed lane (adapted from Wiki)

- Type of highway lane operated by a transport agency with a management scheme. On such lanes, the operator uses restrictions or variable tolling, to optimize traffic flow, vehicle throughput, or both. Goals: improve traffic speed and throughput traffic, reduce air pollution and improve safety.
- Managed lanes include Express Toll Lanes, High Occupancy Vehicle (HOV) lanes (e.g. during peak travel time), High-occupancy toll lanes (free for high-occupancy vehicle, but tolled for other vehicles), reversible lanes, or bus lanes
- LEZ (Low Emission Zones)

Managed Lanes Across US



Source: Sánchez (2016)

Static model, single link

Walters (1961)

- One origin and one destination
- One road link connecting origin to destination
- One traveler per vehicle
- V = volume: the number of trips taken per unit time
- Inverse demand curve: $d(V)$, downward-sloping
- Travelers identical except for willingness to pay for a trip

Static model, single link

Average (= private) cost of trip: $c(V)$

Total trip Costs: $TC = c(V)*V$

Marginal social trip Cost: MC

$$(1) \quad MC =: \frac{dTC}{dV} = \frac{d\left[c(V)*V\right]}{dV} = \underbrace{c(V)}_{Private\ cost} + \underbrace{c'(V)*V}_{External\ cost}$$

Static model, single link

No-toll equilibrium number of trips, V_E solves:

(2) $$d\left(V_E\right) = c\left(V_E\right).$$

Optimal number of trips, V_O solves:

(3) $$d\left(V_O\right) = MC\left(V_O\right) = c\left(V_O\right) + c'\left(V_O\right)V_O.$$

with $c'\left(V_O\right) > 0 \;\Rightarrow\; V_O < V_E$ → Too many vehicles at equilibrium

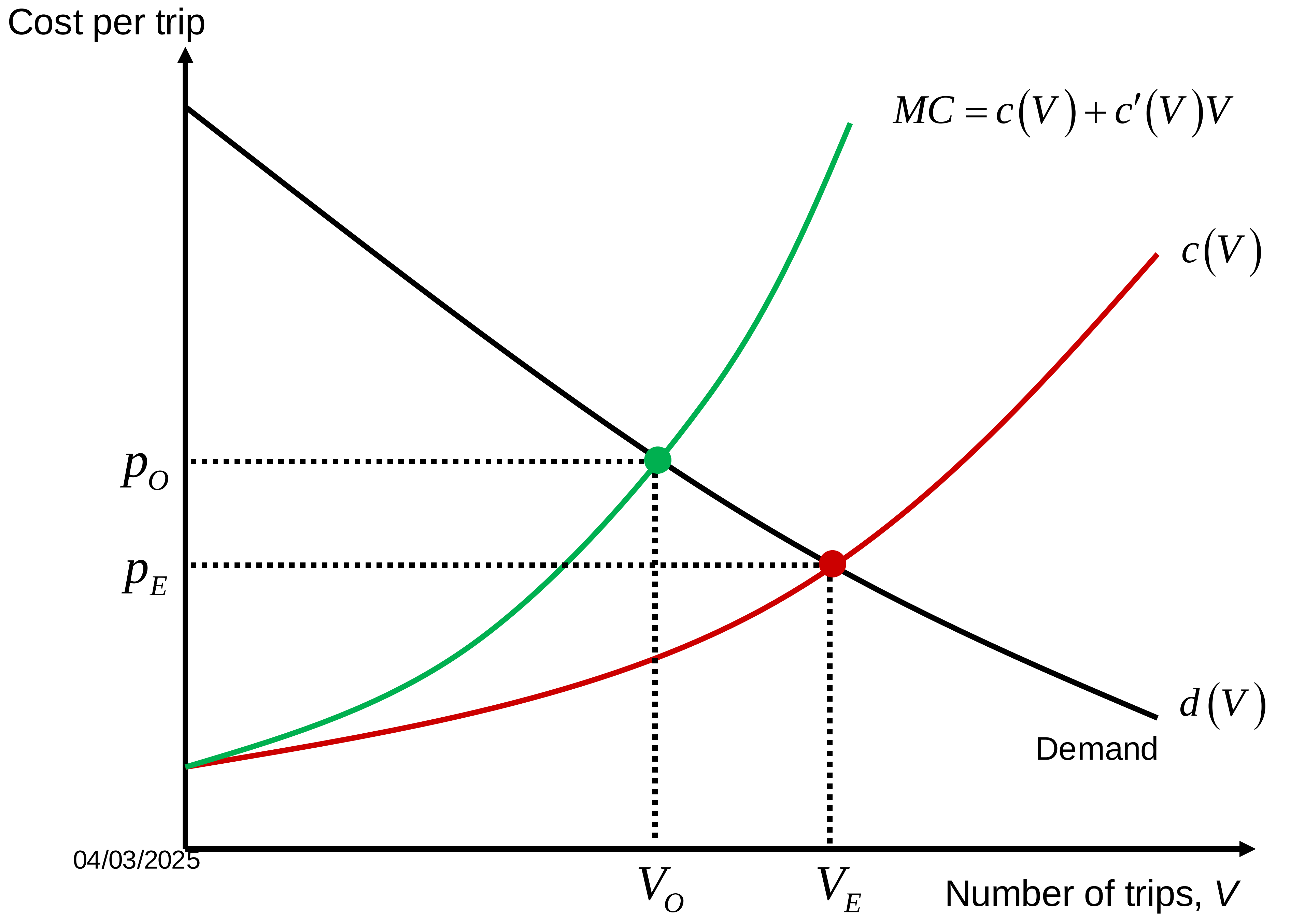
Optimal congestion toll: τ

$$d\left(V_O\right) = c\left(V_O\right) + \tau,\; so...$$

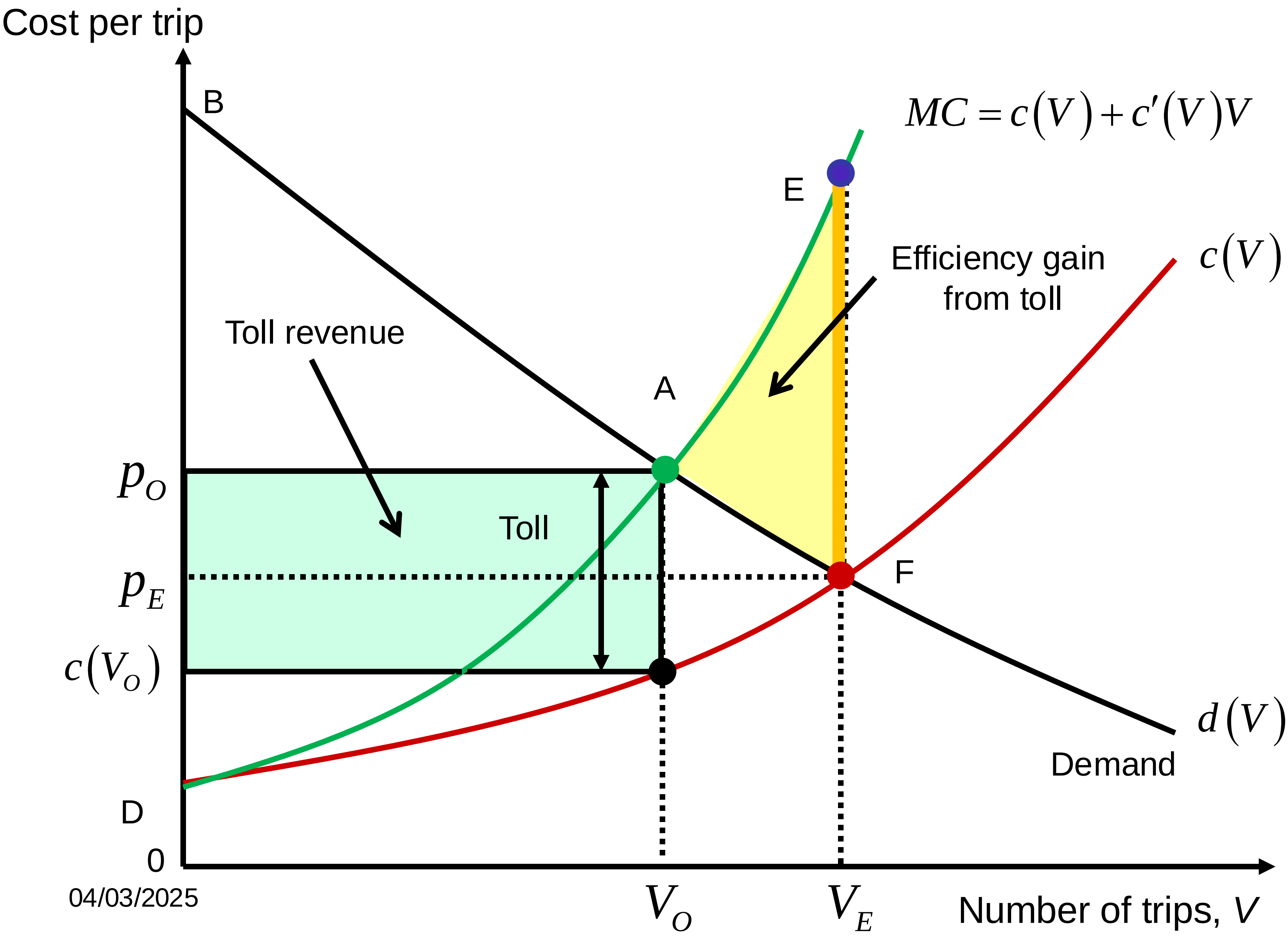
(4) $$\tau = MC(V_O) - c(V_O) = c'(V_O)V_O.$$ → External cost

Pigouvian toll (Pigou, 1920).

Equilibrium and optimal numbers of trips



Congestion toll, efficiency gain and toll revenue



Some challenges in implementing the toll

Optimal congestion toll: $\tau = c'(V_O)V_O.$

Challenge 1: Computation

- The toll is evaluated at traffic volume V_O, not V_E. To deduce the optimal toll it is necessary to know, or estimate, both the demand curve and the cost curve. The toll can be determined by trial and error, but *c*(V) must still be used to compute the marginal external cost.

Challenge 2: Acceptability

- Tolling does not eliminate congestion
- The toll increases drivers' private costs. Drivers are thus unlikely to accept tolling unless (at least some of) the toll revenues are used in ways that benefit them.

Challenge 3: Implementation cost

- Toll collection infrastructure and operations are costly. If the costs exceed the benefits, tolling is not worthwhile.

Extensions of the simple static model

Road networks

Heterogeneity of travelers & vehicles

Constraints on toll differentiation (complexity, equity issues)

Revenue generation incentives.

Because government revenue is costly to raise (the marginal cost of public funds), a "premium" value may be attached to toll revenues. In France it is about 30%.

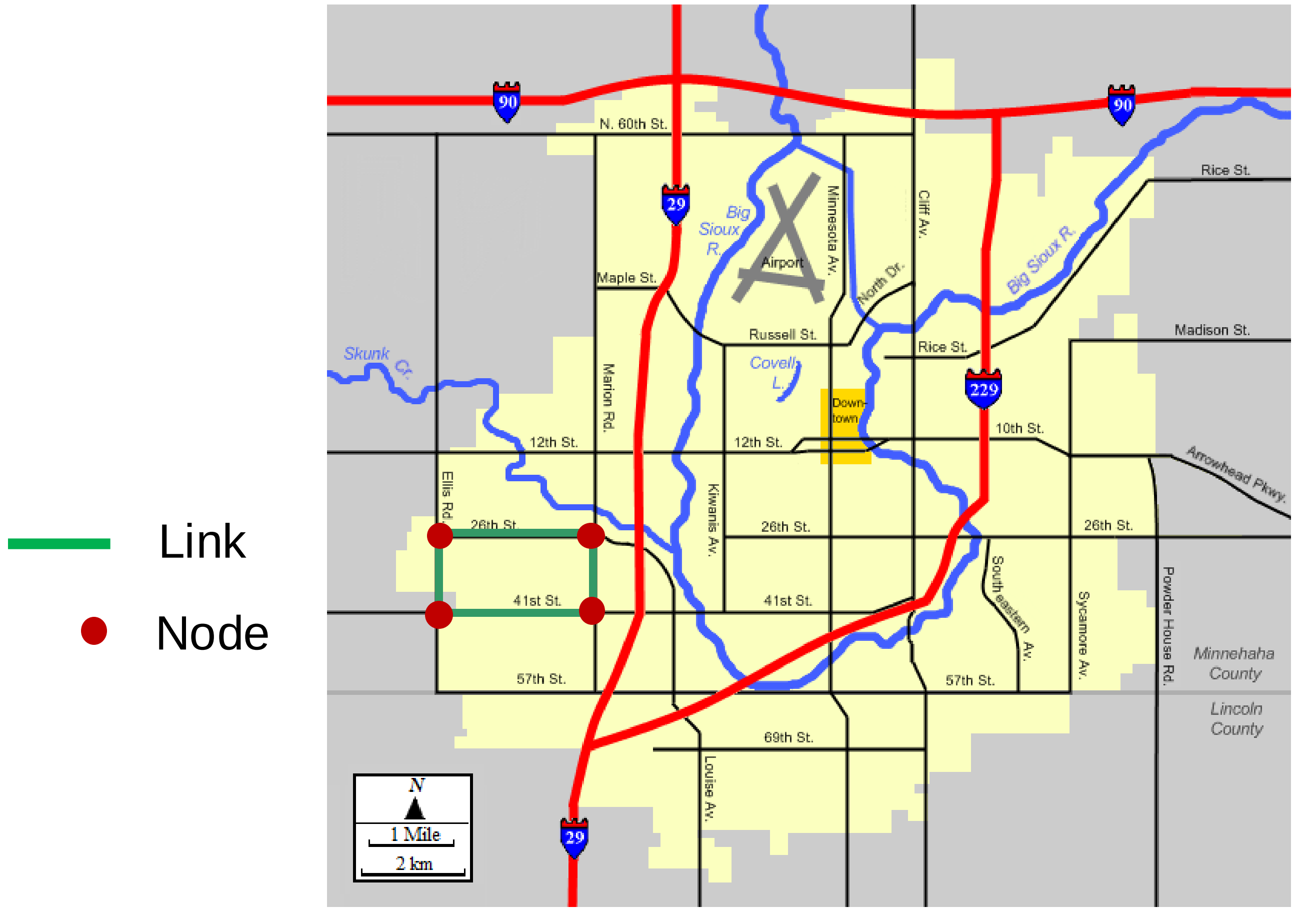
Mobility permits (Seshadri, et al. 2022). How do permits work? Permits are an alternative to road pricing. (Based on emission permits : M. Weitzman, 1974, in the environmental context)

Many debates issues (especially in CBA). Often tolls are "regressive" : poor are worse-off, rich are better-off.

Static model, networks

Most roads are links in networks with many other links

Sioux Falls, South Dakota



http://upload.wikimedia.org/wikipedia/commons/3/3c/Sioux_Falls_Map_4.png

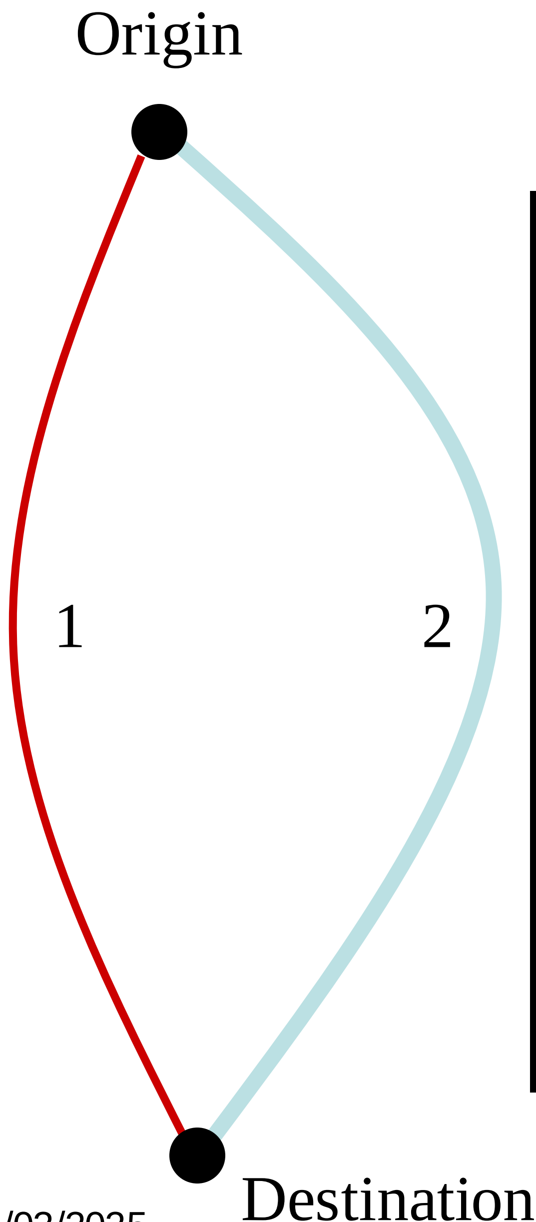
Networks: Second-best tolls

Second-best pricing applicable when first-best conditions do not hold.

Assumptions:

- One origin, one destination
- Elastic demand
- Two routes in parallel. 2nd best because only one route can be tolled.
- What is the second-best optimal toll on the tolled route?

Numerical example (discussion in class linear case)



Tolling regime	Toll on 1	Toll on 2
Both routes tolled	€2	€1
Only 1 can be tolled	[1,2]	**€0**
Only 2 can be tolled	**€0**	[-1,1]

Networks: Second-best tolls

$p(N)$: inverse demand function for trips, $p_N = p'(N)$

$C^i(N_i)$: travel cost on route *i*, *i*=1, 2.

$$C_N^i = \partial C^i / \partial N_i$$

Optimal second-best toll on Route 1 (Verhoef et al., 1996) :

$$\tau_1 = C_N^1 N_1 - \frac{|p_N|}{C_N^2 + |p_N|} C_N^2 N_2 .$$

Networks: Second-best tolls

Recall… $\tau_1 = C_N^1 N_1 - \frac{|p_N|}{C_N^2 + |p_N|} C_N^2 N_2$.

Limiting cases:

Case 1: Perfectly inelastic demand ($p_N = -\infty$):

$$\tau_1 = C_N^1 N_1 - C_N^2 N_2 .$$

Q: What is the intuition for this?

Case 2: Perfectly elastic demand ($p_N = 0$):

$$\tau_1 = C_N^1 N_1 .$$

Q: What is the intuition for this?

Towards general networks

- More complicated toy networks: Lindsey, R., de Palma, A. and P. Rezaeinia (2023).
- General networks (discussion during exercises)

Conditions for self-financing with optimal investment

Motivation

- Cost-benefit analysis is used to determine which projects to undertake and how much to invest
- For infrastructure projects there is a capacity decision
- For congestible facilities there is a trade-off between the cost of capacity and user costs (which decreases with capacity!)
- Usage should be priced efficiently to maximize the value of the investment $\Rightarrow$ needed to account for pricing decisions in choosing capacity

The self-financing question

- Suppose first-best usage charges are imposed. Will user charges suffice to pay for the investment costs, maintenance, and operation?

- Why is this desirable?

Efficiency versus **Equity** oriented policy; tradeoff.

3.1 First-best capacity investment single link

$F(K)$: capacity, K, cost function. Analogous to fixed production cost.

$c(N,K)$: user cost function, $c_N > 0,\ c_K < 0.$

Total cost (construction cost *F*(*K*) added) : $F(K) + c(N,K)N.$

Aggregate benefits to be maximized

$$B = \int_{n=0}^{N} d(n)\,dn - \left(F(K) + c(N,K)N\right) \qquad (5)$$

Cost recovery for a single link

Mohring and Harwitz (1962)

Theorem: Assume:

1. $c(N,K)$ is homogeneous of degree zero.
2. Capacity is perfectly divisible.
3. Capacity cost has unit cost : $F(K)=F_K K$.

Then: **Toll revenue exactly pays for optimal capacity.**

Extensions

If $c(N,K)$ and $F(K)$ have scale *economies* or *diseconomies* then

toll revenues is not equal generally to capacity costs.

Check how the self-financing theorem is changed!

References: Arnott and Kraus (2003); Small and Verhoef (2007, §5.1.1 and §5.1.2; Lindsey (2012, §3.2)); de Palma and lindsey (2007).

Proof of the theorem

If $c(N,K)$ homogeneous of degree zero [$c(N,K) = c(N/K)$]:

Then, homogeneity implies: $Kc_k(N,K) + Nc_N(N,K) = 0$

f.o.c for optimal capacity, $(\text{Max}_K \text{B})$. See (5): $F_K = -c_K(N,K)N$,

so, using homogeneity:

$$F_K = -c_K(N,K)N = \frac{c_N(N,K)N^2}{K} = \frac{\tau N}{K}$$

(Using the opt. toll expression : $\tau = c_N(N,K) * N$ (See (4.))

Or: $F_K * K = F(K)$ (See assumpt. 3.) $= \tau N$

So :Total capacity cost = Total toll revenue, i.e.

Construction cost of capacity K is equal to total toll revenue

References

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3.2 Optimal capacity for a road network

Yang and Meng (2002)

Theorem: Assume the user cost functions, $c_l\left(v_l, K_l\right),\ l = 1,...,L$ are homogeneous of degree zero, and capacity on each link is perfectly divisible and supplied with unit cost elasticity. Then toll revenue on each link just pays for optimal capacity of that link.

3.2 Second-best optimal capacity

Paradoxical results can occur on a network when usage is not efficiently priced:

1. The Braess Paradox (see exercise)

   Building a new link **increases** total user costs.

2. Pigou-Knight-Downs Paradox

   2 routes in parallel, one uncongested. Expanding capacity of the congested route leaves total user costs **unchanged**

3. Downs-Thomson Paradox

   One congested road in parallel with public transit that has scale economies. Expanding the capacity of the road **increases** total user costs. Because decreased demand for public transport decreases PT frequency; Issue also, with MOD (Mobility On Demand) and MaaS (Mobility as a service, or carpooling).

Conclusions: Future considerations (1/2)

Electric and alternative-fueled vehicles

- Reduction in greenhouse gas emissions and pollution: reduce need for road pricing to internalize these externalities (more later this year, when discussing environment)
- But ! Reduction in fuel tax revenue boosts need for other revenue sources (to finance roads or to reduce cost of public funds). EVs are more expensive. See EU *Directive* and:

https://theconversation.com/automobile-est-il-devenu-moins-couteux-dopter-pour-une-voiture-electrique-211958

Conclusions: Future considerations (2/2)

Automated highways and self-driving vehicles

- Will radically increase effective road capacity … eventually
- Weakens case for road pricing for demand management, but may require investments in roadside infrastructure to allow vehicle-to-road and vehicle-to-vehicle communications
  - Potential long run effect → Urban sprawl (not good!)

Conclusions: Future considerations

End of growth in personal travel

Evidence in developed countries

Possible reasons:

1. Long-run economic stagnation
2. Telecommuting (COVID)
3. Increasing urbanization (less sprawl)
4. Environmental concerns
5. Rising fuel prices
6. Population aging
7. Growing health concerns
8. (Young) people travel “virtually”

Implications:

- Reduces the need for new roads & road maintenance

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