Exercises CBA and risk

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Outline

1. Discussion on the EU Directive on electric cars and CBA
2. Computation of NPV
3. Exercises on risk

1. EV cars
Background (1)

- This directive is part of the European Union's "Fit for 55" package, which is aimed at reducing net greenhouse gas emissions by at least 55% by 2030, compared to 1990 levels. One of the key measures in this package includes a de facto ban on the sale of new combustion engine cars and vans from 2035, as part of the effort to reach climate neutrality by 2050.
- The legislation mandates a 100% reduction in CO_2 emissions from new cars and vans by 2035, essentially meaning that all new vehicles sold should be zero-emission at that point.

Background (2)

- The reasons behind this move are primarily environmental. The transport sector is one of the largest contributors to greenhouse gas emissions in the EU. Shifting to EVs can significantly reduce the carbon footprint of transportation since EVs typically have lower greenhouse gas emissions compared to combustion engine vehicles. The reduction is even more significant when the electricity used for charging EVs comes from renewable energy sources.
- The EU has also witnessed a significant drop in CO2 emissions from new cars, and the uptake of EVs has been growing, with more than 1 million new electric cars registered in 2020.

Background (3)

- To facilitate this transition, various milestones and targets have been set, and economic support is to be provided for the automotive industry and its employees, acknowledging the substantial economic and social impact of such a change, especially in countries with a strong automotive industry presence. By 2025, the EU plans to create a support fund for the automotive industry to assist in this transition. Additionally, there will be a reassessment in 2026 to evaluate the progress of the transition and potentially adjust targets and deadlines.

Background (4)

- The agreement reached between EU countries and the European Parliament also allows for the potential use of carbon-neutral fuels, which could enable new combustion engine vehicles to be on the market after 2035, provided that their fuels do not contribute to pollution (Energy Industry Review).

Questions

A European Directive (made public June 2022) bans the sale of combustion-powered cars after 2035 and the use of combustion-powered cars after 2050.

1. What are the reasons for this directive?
2. Which stakeholders will be affected and what will be the impacts ?
3. Which CBA computations are needed?
4. Which accompanying measures need to be taken?

2. NPV

Question 1:

- Consider an infinite annual return, given on a yearly basis. The interest rate is *r*. What is the present value of this payment? There is a bond that gives constant coupon $1 at the beginning of each year.

(a) If the first payment is provided at time t=0

(b) If the first payment is provided at time t=1

Assume the discount (interest) rate decreases suddenly at once.
An investors has some bonds which yields a constant coupon, as described before.

(c) Does this investor get richer or poorer if the interest rate suddenly decreases?

Question 2:

- A project, has benefit at time *t* are denoted by B_t, and costs at time t, are denoted by C_t, t = 0,1,..,n. The initial investment is I_0. Denote the discount rate (interest rate) by *r*.

(a) Let $B_t = 1$, $C_t = 0$; $I_t = 0$. Compute the Net Present Value (NPV) of the project? (write a compact formula).

Corrections

Group discussion

Question 1: [Once in your life time]

(a) The first payment is given at $t = 0$.
Consider the following infinite series:

$$\sum_{i=0}^{\infty} x^i = ?$$

x=1/(1+r)

How to make this summation?
Just do the division, as in the high school. So we get:

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$$

Recognize for 0 < x < 1, this is a geometric series that can be rewritten as follows.
Substitute x = 1/1+r leads:

$$\frac{1}{1-x} = \frac{1+r}{r}$$

(b) The first payment is given at $t = 1$.
Then:

$$\sum_{i=1}^{\infty} x^i = \frac{1}{1-x} - 1 = \frac{x}{1-x}$$

If we invest on euros, he discount value over an infinite period of time is:

$$\sum_{i=1}^{\infty} \frac{1}{(1+r)^i} = \frac{\frac{1}{(1+r)}}{1 - \frac{1}{(1+r)}} = \frac{1}{r}$$

Therefore (please try to recall this key formula):

$$\sum_{i=1}^{\infty} \frac{1}{(1+r)^i} = \frac{1}{r}.$$

(c) That is, if the interest rate increases, investors
get poor,

Question 2:

We can apply the formula in question 1 to get:

$$\sum_{i=0}^{n} \frac{1}{(1+r)^i} = \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} - \sum_{i=n+1}^{\infty} \frac{1}{(1+r)^i} = (1 + \tfrac{1}{r}) - \frac{1}{(1+r)^{n+1}} \sum_{i=0}^{\infty} \frac{1}{(1+r)^i}$$

$$= (1 + \tfrac{1}{r}) - \frac{1}{(1+r)^{n+1}}(1 + \tfrac{1}{r}) = (1 + \tfrac{1}{r})\left(1 - \frac{1}{(1+r)^{n+1}}\right) = \frac{\left(1 - \frac{1}{(r+1)^{n+1}}\right)}{1 - \frac{1}{1+r}}$$

So

$$\sum_{i=0}^{n} \frac{1}{(1+r)^i} = \frac{\left(1 - \frac{1}{(r+1)^{n+1}}\right)}{1 - \frac{1}{1+r}}.$$

NPV & IRR

Question 3 (continuation of question 2)

- Assume B_t=4 and C_t=3, t=0,1,…n, I_0=8. Let n=20 years. What are the NPV of the project, n=20, for

(a) An interest rate of 4%?

(b) An interest rate of 10%?

(c) An interest rate of 15%?

Question 4 (continuation of questions 2 and 3)

The Internal rate of return, or IRR is the discount rate r, which sets NPV equal to zero

(a) What is the IRR for a 20 years investment (same parameters as before)

Corrections

Question 3:

recall $B_t = 4$ and $C_t = 3$ so we have

$$PNV = \underbrace{1}_{=B_t - C_t} \frac{\left(1 - \frac{1}{(1+r)^{21}}\right)}{1 - \frac{1}{1+r}} - 8$$

(a) For an interest rate of $r = 4\%$, we get:

$$PNV = \frac{\left(1 - \frac{1}{(1.04)^{21}}\right)}{1 - \frac{1}{1.04}} - 8 = 6.5903 > 0$$

(b) For an interest rate of $r = 10\%$, we get:

$$PNV = \frac{\left(1 - \frac{1}{(1.1)^{21}}\right)}{1 - \frac{1}{1.1}} - 8 = 1.5136 > 0$$

(c) For an interest rate of $r = 15\%$, we get:

$$PNV = \frac{\left(1 - \frac{1}{(1.15)^{21}}\right)}{1 - \frac{1}{1.15}} - 8 = -0.74067 < 0.$$

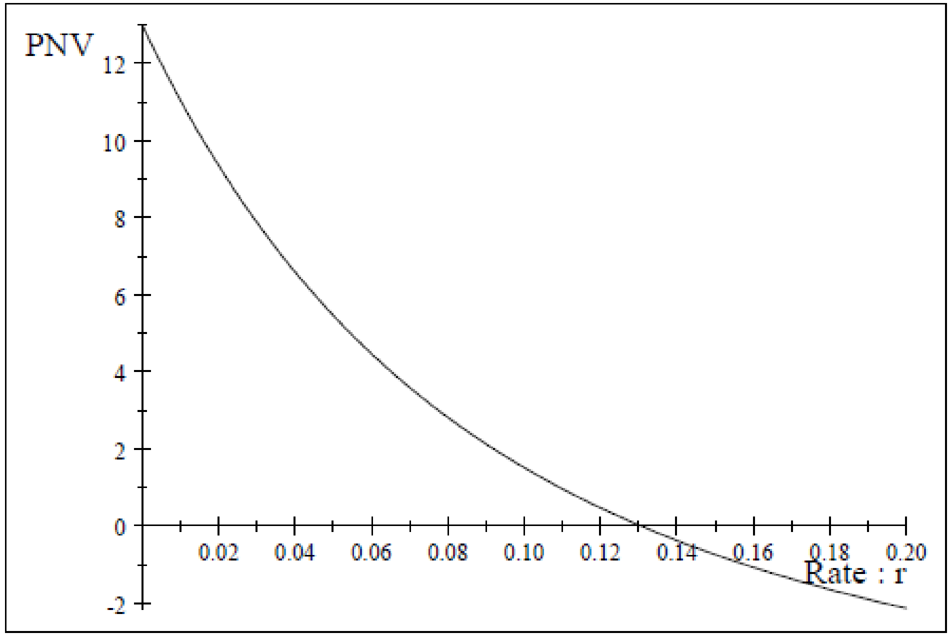
→ Key role of the interest rate!

Question 4:

What is the internal rate of return? It is the interest which solves:

$$PNV = \frac{\left(1 - \frac{1}{(1+r)^{21}}\right)}{1 - \frac{1}{1+r}} - 8 = 0$$

What is the rate, r, so that the PNV is zero?
Let draw the PNV for different rates of returns.
We get:



Therefore, the internal rate of return is about 13%, since:

$$\frac{\left(1 - \frac{1}{(1.13)^{21}}\right)}{1 - \frac{1}{1.13}} - 8 = 2.4752 \times 10^{-2}$$

3. Risk

- Consider a CARA utility function:

$$U(x) = \frac{-\exp(-\theta x)}{\theta}, \theta \geq 0.$$

- Absolute risk aversion is denoted by ARR=-U''/U'. Note that u(x) is increasing and concave.

(a) (i) Compute the absolute risk aversion and interpret it.
(ii) What is the intuition for $\theta=0$? And what does it imply for the utility function U(x)?

Question

- Recall:

$$U(x) = \frac{-\exp(-\theta x)}{\theta}, \theta \geq 0.$$

- If a consumer gets a lottery: “W1” with probability 0.5 and “W2” with probability 0.5, his/her expected utility is :

EU(W1, W2; 0.5) = 0.5 U(W1) + 0.5 U(W2).

(b) Draw the curve U(x) versus x, to explain what is the safe amount, w, with Utility U(w), equivalent to this lottery, i.e. EU(W1, W2; 0.5) =U(w).

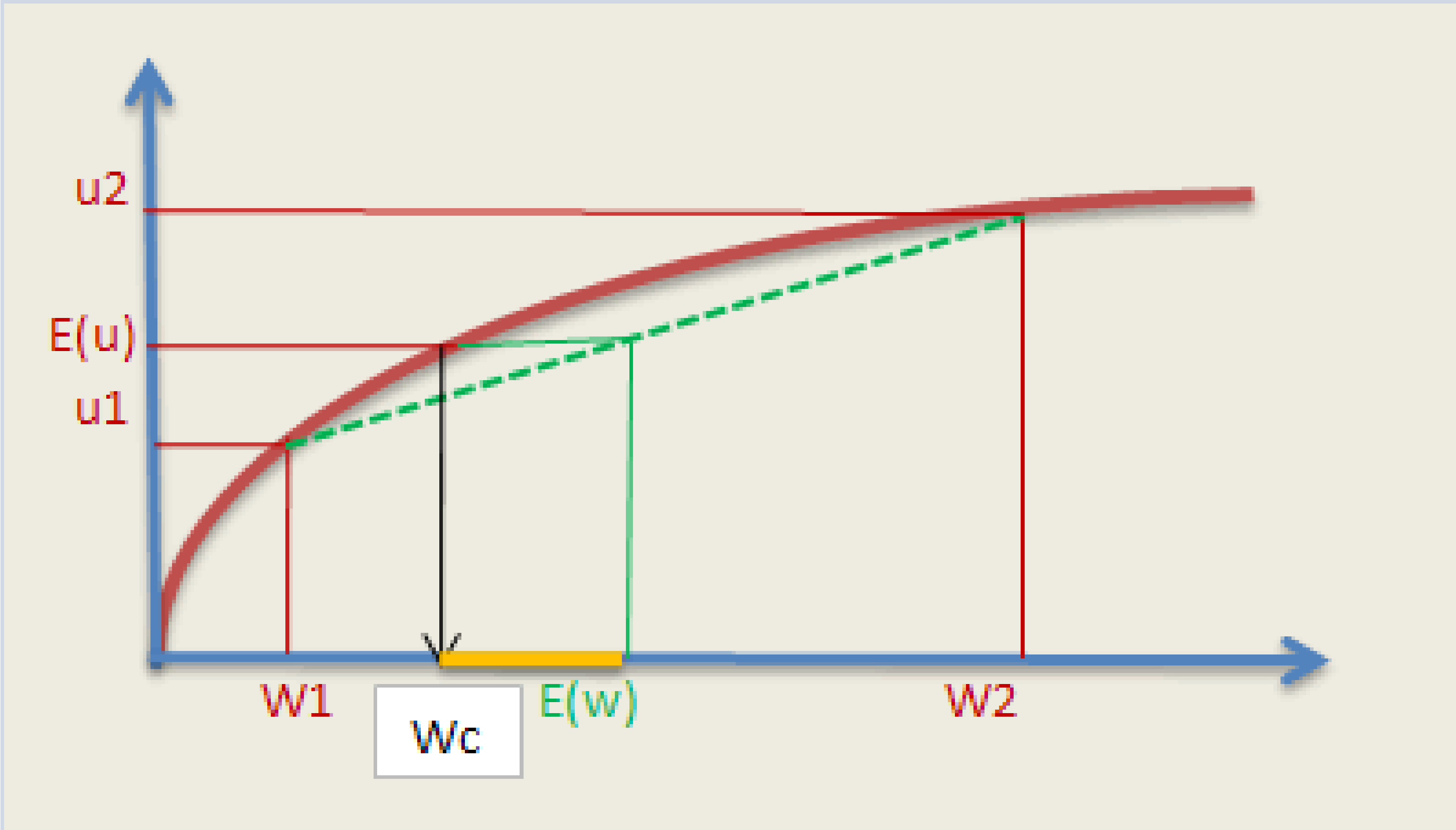
Corrections

RISK (a)

(a) CARA utility function (Constant Absolute risk aversion):

- $U(x) = -\exp(-\theta x)/\theta$; $U'(x) = \exp(-\theta x) > 0$; $U''(x) = -\theta \exp(-\theta x) < 0$ if $\theta > 0$.
- So the Absolute Risk Aversion is: $ARA = -U''(x)/U'(x) = \theta \geq 0$.
- If $\theta = 0$, the consumer is risk neutral (ARA=0), and utility is linear: $U(x) = x$. This is (*hélas*) the standard case, where expectations of costs and benefits can be used.
- But could θ be negative as well (see below).

RISK (b)



W=(W1+W2)/2

Explanations

- U(Wc) is the utility of Wc. Note that Wc is certain equivalent return.

Recall that the expected utility is E(U)=0.5*U(W1) + 0.5*U(W2)

- **"Wc" uniquely solves** : U(Wc) = E(U)=0.5*U(W1) + 0.5*U(W2)
- (W1+W2)/2 – Wc is the risk premium (e.g. insurance premium). It is positive if and only if the function U(x) is concave, i.e. iff U''(x)<0, i.e. ARA>0. Here $\theta>0$ as represented graphically.

Observe that the risk premium is …

- …null if the utility is linear: risk neutrality, $\theta=0$ and ARA=0
- …negative is the utility is convex: risk loving, $\theta<0$, and ARA<0