Transportation Economics

CIVIL-455

Exercise of Week 6

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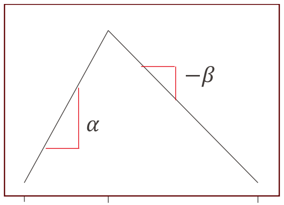
Exercise

Consider a Macroscopic Fundamental Diagram (MFD) represented by the **production vs accumulation** relationship defined in the right graph. The values of $\alpha, \beta, n_{critical}$, and n_{jam} are known.



1. Derive the maximum production $(\mathrm{km/hr})$ from a network with this MFD.

2. Write the speed v as a function of the accumulation n.

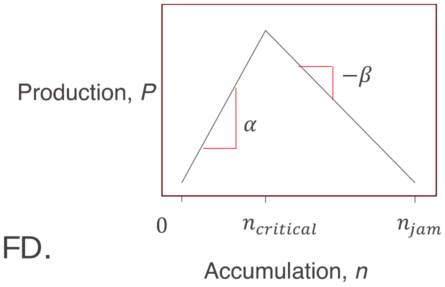
3. Suppose a fraction γ of road space is allocated to buses exclusively, and the fraction $1-\gamma$ is left for cars. What are the maximum productions of the car network? *(hint: rescale the original production MFD according to the value of γ.)*

4. Consider that demand for buses is η $(\mathrm{pkm/hr})$. If the targeted occupancy of each bus is O_b and the designed operating speed is v_b^*, what is the fleet size to fulfill the demand? And what are the speed and the passenger production of the bus network, respectively?

5. If η is fixed, what is the optimal γ to maximize the total production?

6. Let $\alpha = 35, \beta = 19,\ n_{jam} = 58831\ \mathrm{veh}, O_b = 20, O_c = 1.2, v_b^* = 20\ km/h$. Write the optimal γ as a function of η.

Exercise



Consider a Macroscopic Fundament Diagram (MFD) represented by the **production vs accumulation** relationship defined in the right graph. The values of $\alpha, \beta, n_{critical}$, and n_{jam} are known.

1. Derive the maximum production ($\mathrm{km/hr}$) from a network with such an MFD.

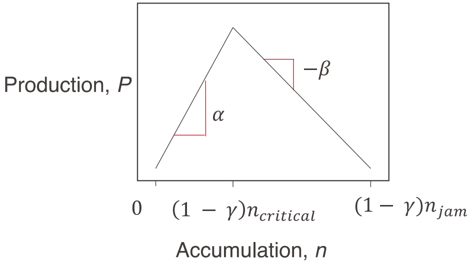
$\alpha\, n_{critial}$

2. Write the speed v as a function of accumulation n.

When $n \le n_{critial}$, we have $v = \alpha$.
When $n > n_{critial}$, we have $v = \frac{\beta\,(n_{jam} - n)}{n}$.

3. Suppose a fraction γ of road space is allocated to buses exclusively, and the fraction $1-\gamma$ is left for cars. The average occupancy of cars is O_c. What are the maximum passenger productions of the car network? *(hint: rescale the original production MFD according to the value of γ.)*

$(1-\gamma)\,\alpha\, n_{critial}\, O_c$



Exercise

Consider a Macroscopic Fundament Diagram (MFD) represented by the **production vs accumulation** relationship defined in the right graph. The values of $\alpha, \beta, n_{critical}$, and n_{jam} are known.

4. Consider that demand for buses is η (pkm/hr). If the targeted occupancy of each bus is O_b and the designed operating speed is v_b^*, what is the fleet size to fulfill the demand? And what are the speed and the passenger production of the bus network, respectively?

The fleet size n_b is given by $n_b = \frac{\eta}{v_b^* O_b}$.

Bus speed: if $n_b \le \gamma\, n_{critical}$, then the speed is $\mathrm{v_b} = \alpha$; Otherwise, $\mathrm{v_b} = \frac{\beta\,(\gamma\, n_{jam} - n_b)}{n_b}$.

Production: $\mathrm{v_b} n_b\, O_b$

5. If the demand for buses η is fixed, what is the optimal γ to maximize the total production?

$\mathrm{TP} = \mathrm{v_b} n_b\, O_b + (1 - \gamma)\, \alpha\, n_{critial}\, O_c$.

When $n_b \le \gamma\, n_{critical}$, i.e., $\gamma \ge \bar{\gamma} = \frac{\eta}{n_{critical} v_b^* O_b}$, the term $\mathrm{v_b} n_b\, O_b$ does not depend on γ; Thus, TP is decreasing in γ when $\gamma \ge \bar{\gamma}$.

When $\gamma < \bar{\gamma}$, $\mathrm{TP} = \frac{\beta\,(\gamma\, n_{jam} - n_b)}{n_b} n_b O_b + (1 - \gamma)\, \alpha\, n_{critial}\, O_c$.

Then we have $\frac{\partial\, TP}{\partial\, \gamma} = \beta\, n_{jam} O_b$- $\alpha\, n_{critial}\, O_c$. If $\beta\, n_{jam} O_b$- $\alpha\, n_{critial}\, O_c$ > 0, the optimal γ is $\bar{\gamma}$. Otherwise, the optimal γ is zero.

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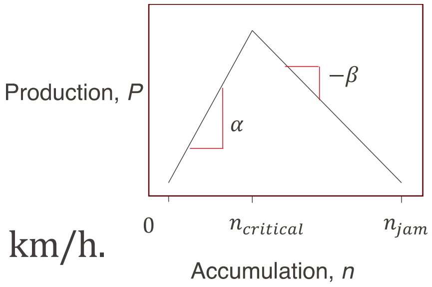

6. Let $\alpha = 35, \beta = 19,\ n_{jam} = 58831 \text{ veh}, O_b = 20, O_c = 1.2, v_b^* = 20 \text{ km/h}$. Write the optimal γ as a function of η.

Given that $\alpha = 35, \beta = 19,\ n_{jam} = 58831 \text{ veh}$, we have $n_{critial} = 20700$;

Then we have
$\beta\, n_{jam} O_b$- $\alpha\, n_{critial}\ O_c = 21486380$ > 0.

Then the optimal γ is $\bar{\gamma} = \frac{\eta}{n_{critical} v_b^* O_b} = \frac{\eta}{20700 * 20 * 20}$