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In-class Exercise – Week #10: Sectional analysis

The steel girder shown in Figure 1a is made of S355 (nominal yield strength, $f_y = 355MPa$, Young's modulus, $E = 200GPa$) steel plates welded together. The geometry of the cross section is as follows: $b_f = 200mm, t_f = 12mm, d = 300mm, t_w = 10mm.$

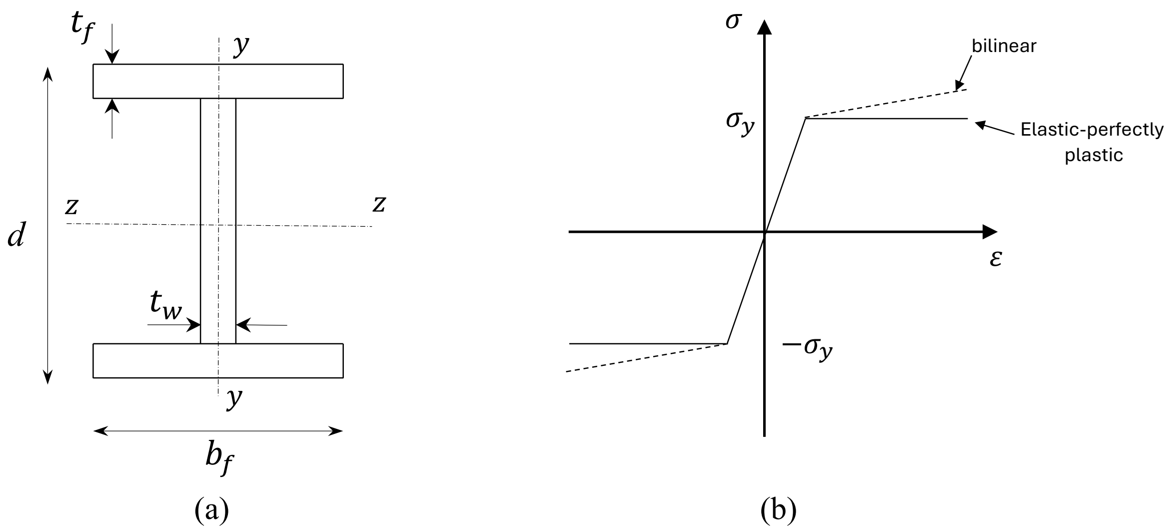

Figure 1. (a) Steel plate girder cross section; (b) stress-strain constitutive formulation

Answer to the following questions:

1. Assume an elastic-perfectly plastic constitutive material law as shown in Figure 1b:

   a. Calculate the yield and plastic bending resistance of the cross section based on structural mechanics and draw the moment-curvature relationship for these two points.

   b. Write a script in Python or MATLAB to discretize the cross section into fiber blocks and to conduct moment curvature analysis for multiple curvature values.

2. Assume a bilinear stress-strain constitutive formulation with a 3% strain hardening ratio as shown in Figure 1b:

a. Assume the same fiber block discretization of the cross section with that in Question 1-b and conduct a moment curvature analysis for multiple curvature values.
b. Compare your answer with Questions 1-a and 1-b in the same graph.

Solution:
1)a) First, the elastic section modulus W_{el} and the plastic section modulus W_{pl} are computed:

$$W_{el} = \frac{b_f d^2}{6} - \frac{(b_f - t_w)(d - 2t_f)^3}{6d} = \frac{200 \cdot 300^2}{6} - \frac{(200 - 10)(300 - 2 \cdot 12)^3}{6 \cdot 300} = 7.81 \cdot 10^5 mm^3$$

$$W_{pl} = \frac{b_f d^2}{4} - \frac{(b_f - t_w)(d - 2t_f)^2}{4} = \frac{200 \cdot 300^2}{4} - \frac{(200 - 10)(300 - 2 \cdot 12)^2}{4} = 8.82 \cdot 10^5 mm^3$$

The yield and plastic bending resistance can then be computed:

$$M_y = W_{el} \cdot f_y = 7.81 \cdot 10^5 \cdot 355 = 277.16\ kNm$$

$$M_{pl} = W_{pl} \cdot f_y = 8.82 \cdot 10^5 \cdot 355 = 312.98\ kNm$$

The curvature at yield is given by

$$\varphi_y = \frac{\varepsilon_y}{\frac{d}{2}} = \frac{\frac{355}{200000}}{\frac{300}{2}} = 1.18 \cdot 10^{-5}$$

The curvature when the full cross-section is plastified becomes infinite

2) The section is discretized into fibers, as shown in the figure below

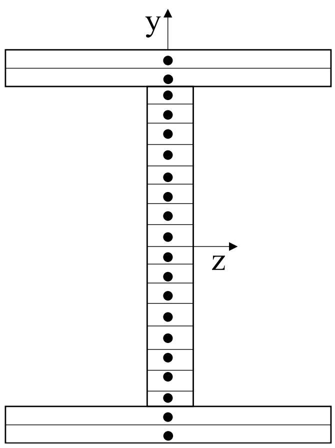

Figure 2. Fiber section

To conduct the moment-curvature analysis using the fiber discretization, the following steps can be taken:
1) The analysis is divided into different steps (denoted by the index n), at each of these steps the curvature value is denoted by φ^n
2) For every increment in curvature φ^n, the code iterates over each fiber $iFib$ of the section:

2.1) For a fiber $iFib$, the strain is computed using:

$$\varepsilon_{iFib}^{n} = y_{iFib} \cdot \varphi^{n}$$

Where y_{iFib} is the coordinate of the centroid of fiber $iFib$

2.2) Using the constitutive law, the stress σ_{iFib}^{n} is computed at every fiber

2.3) The fiber stresses are integrated to compute the bending moment M^{n}:

$$M^{n} = \int_{A} y_{iFib} \cdot \sigma_{iFib}^{n} dA = \sum_{iFib=1}^{nFibers} y_{iFib} \cdot \sigma_{iFib}^{n} \cdot A_{iFib}$$

Where A_{iFib} is the area of fiber $iFib$

Figure 3 below compares the results obtained when considering an elastic-perfectly plastic constitutive material law and a bilinear stress-strain constitutive formulation with a 3% strain hardening ratio

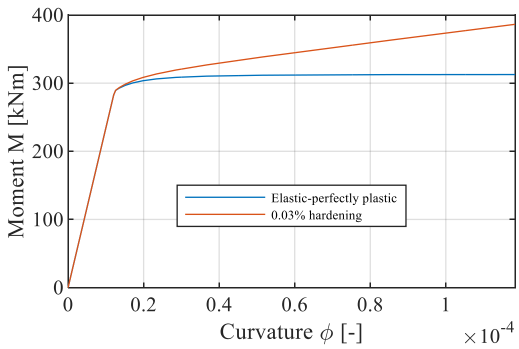

Figure 2. Results moment-curvature analysis

From this figure, it is evident that the results obtained using an elastic-perfectly plastic material law approach the theoretical value of the cross-section's plastic bending resistance. In contrast, when a bilinear stress-strain constitutive model with a 3% strain hardening ratio is employed, the bending moment increases linearly.