CIVIL 449: Nonlinear Analysis of Structures

School of Architecture, Civil & Environmental Engineering
Civil Engineering Institute

Material nonlinearities – Concentrated plasticity models

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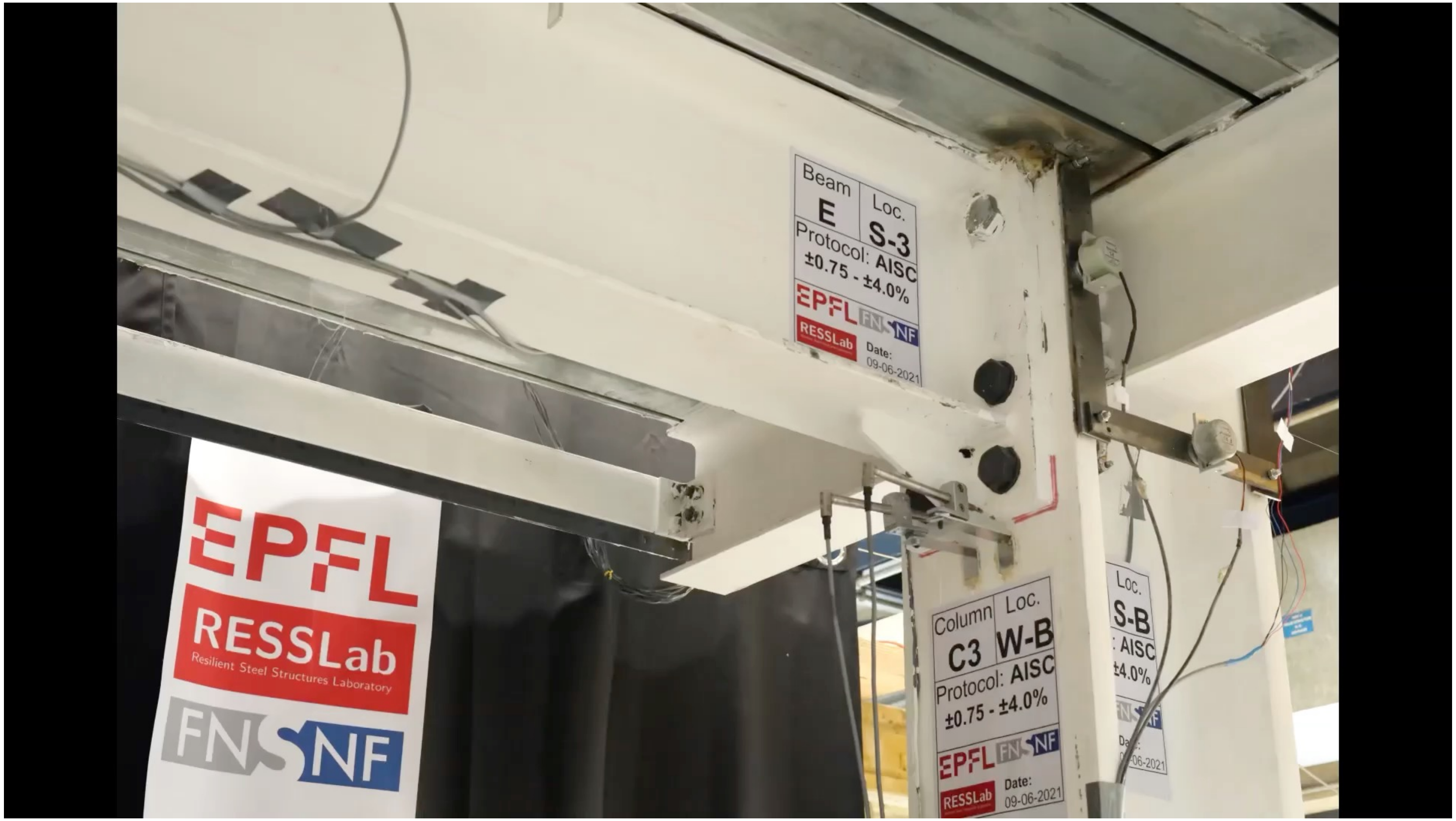
Objectives of Today’s Lecture

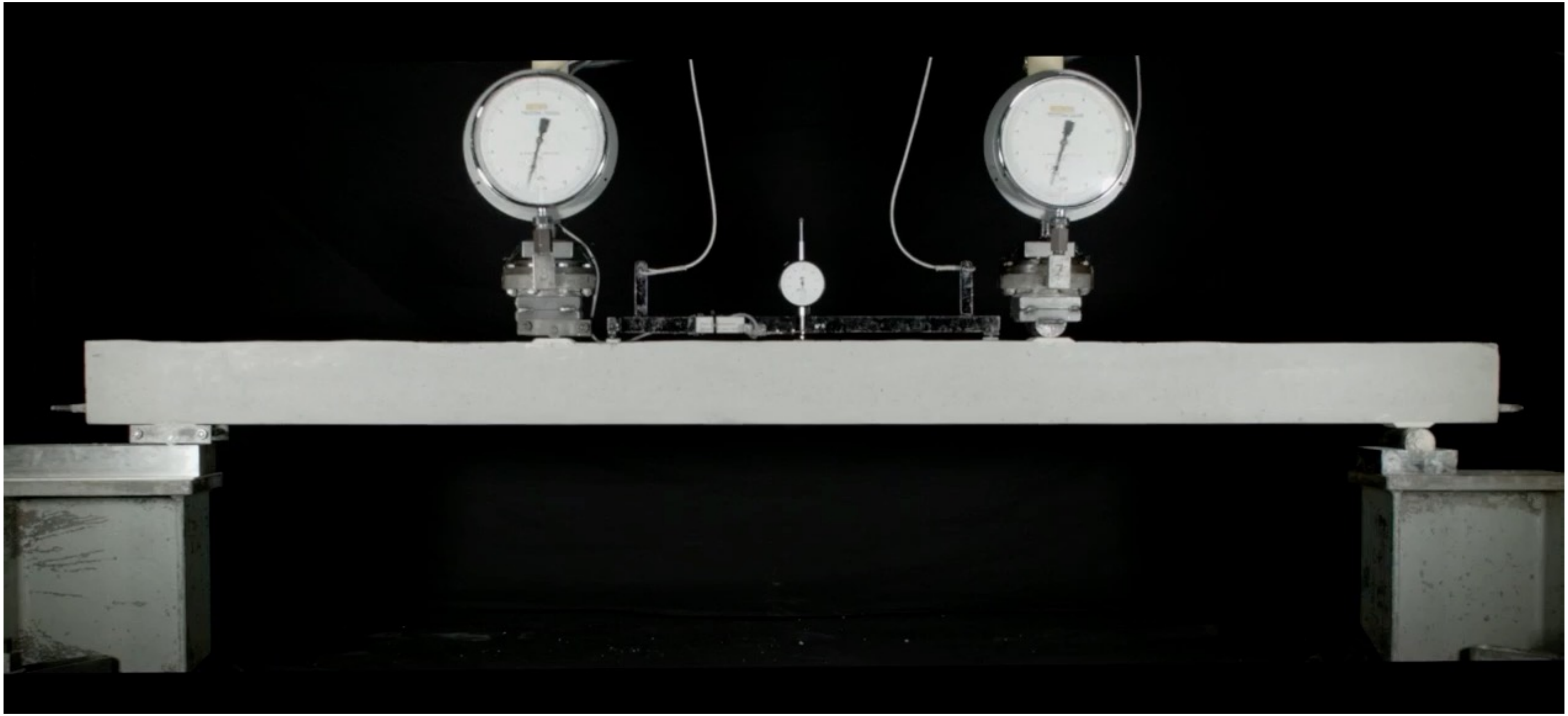
- To introduce:
  - Material nonlinearity
  - Concept of ”plastic hinge”
  - Element formulations for tracing material nonlinearity
  - Constitutive formulations for concentrated plasticity models

Damage sequence in structural members



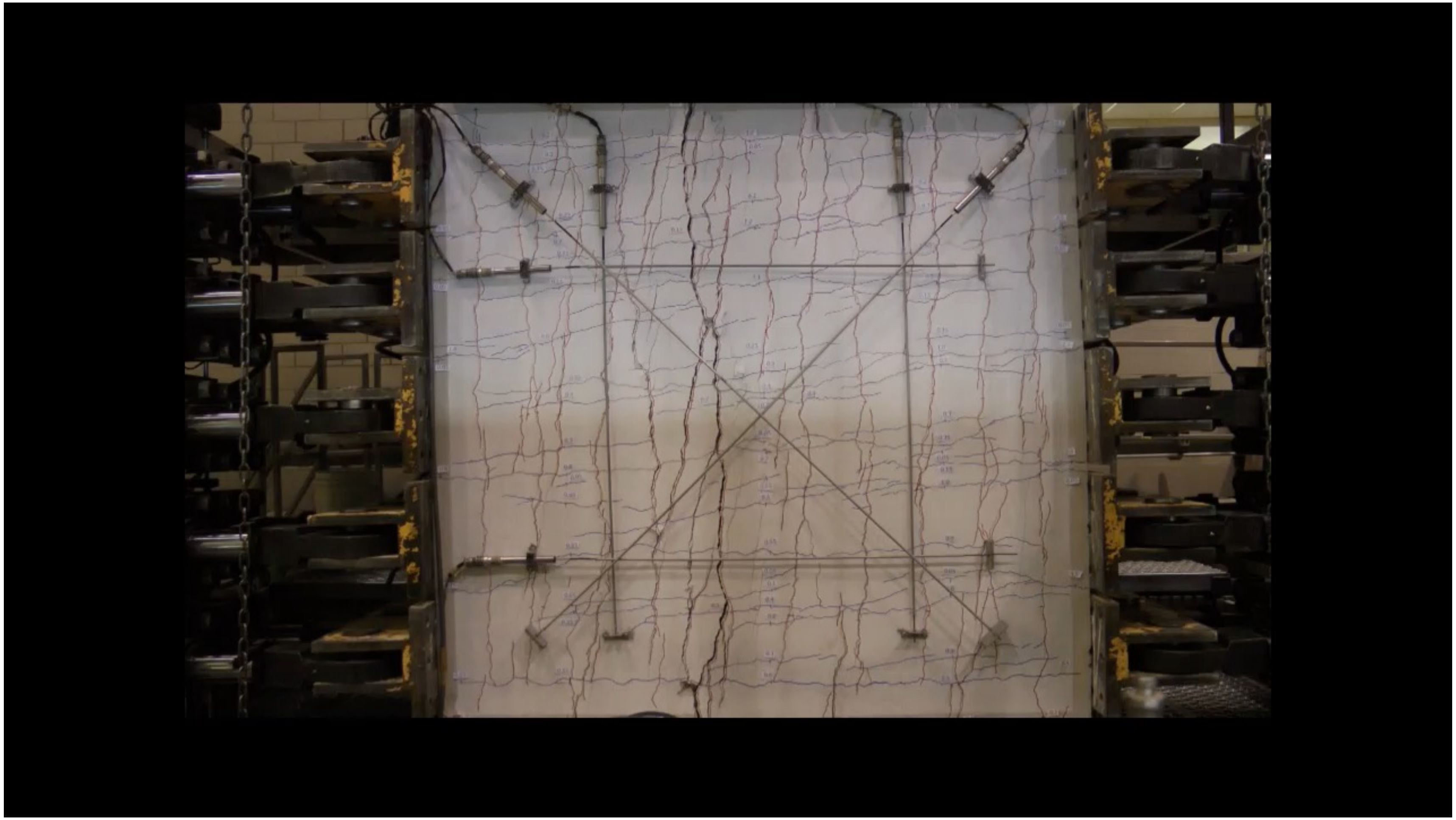
Source: El Jisr and Lignos (2021)

Damage sequence in structural members (2)



Source: Prof. Tim Ibell

Damage sequence in structural members (3)



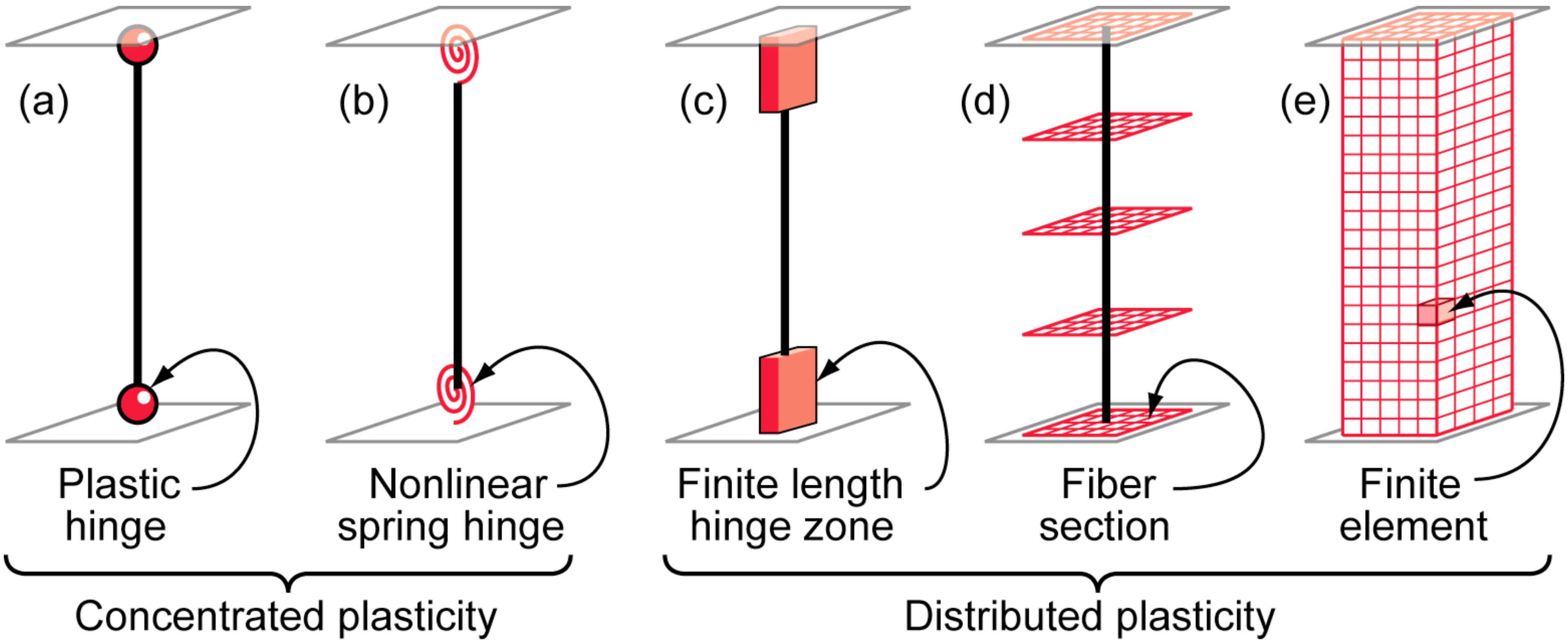
Source: Prof. David Ruggiero

Incorporating material nonlinearities into frame analysis

- From the previous lectures, the global stiffness matrix of a structure, $\mathbf{K}_g$, is given as follows,

$$\mathbf{K}_g = \underbrace{\mathbf{T}^T \underbrace{\left(\underbrace{\mathbf{L}^T\bar{\mathbf{k}}\mathbf{L}}_{\textbf{Material stiffness matrix}} + \underbrace{\bar{q}_1 \frac{\mathbf{z}\mathbf{z}^T}{L_n} + \frac{1}{L_n^2}\left(\mathbf{r}\mathbf{z}^T + \mathbf{z}\mathbf{r}^T\right)\left(\bar{q}_2 + \bar{q}_3\right)}_{\text{Geometric stiffness matrix}} \right)}_{\text{Tangent stiffness matrix in the local reference system}} \mathbf{T}}_{\text{Tangent stiffness matrix in the global reference system}}$$

Element formulations for nonlinear material response



Source: NIST GSR 10-917-5

Concentrated plasticity models

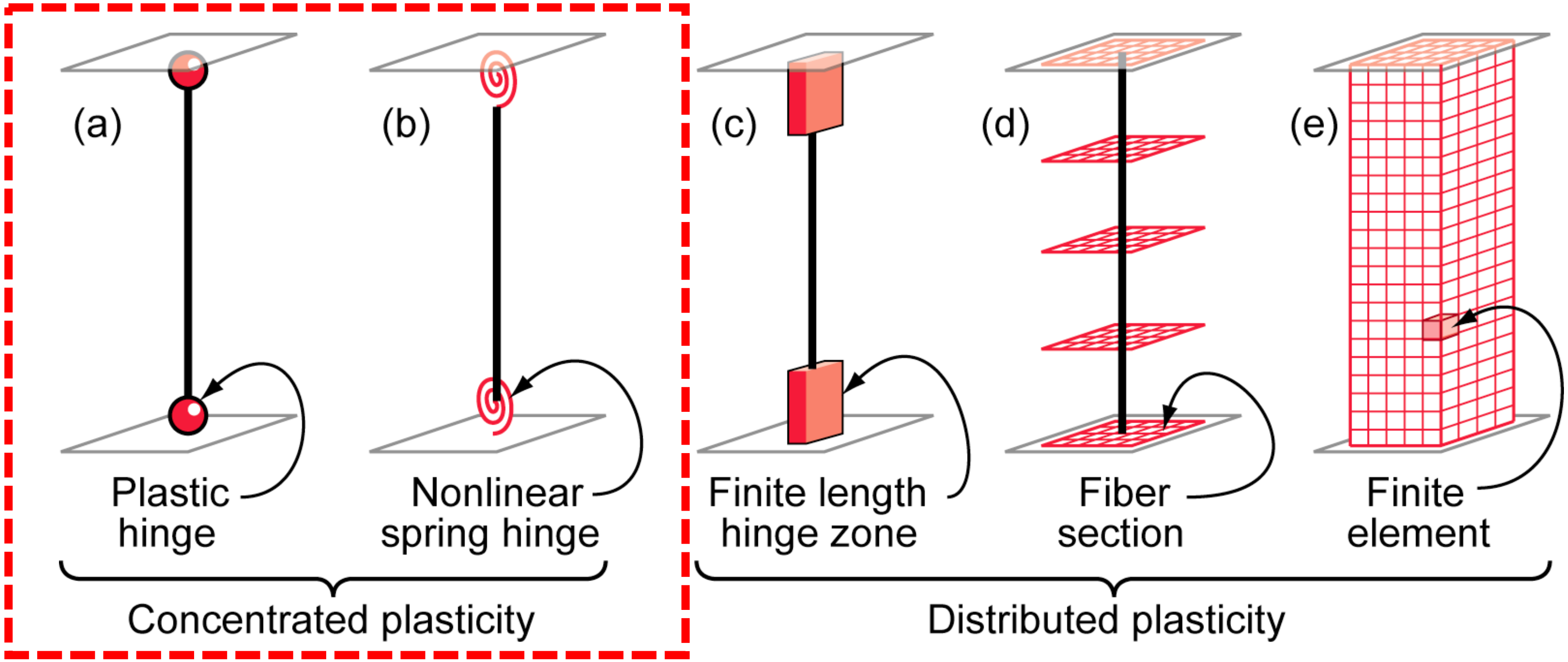


Image Source: NIST GSR 10-917-5

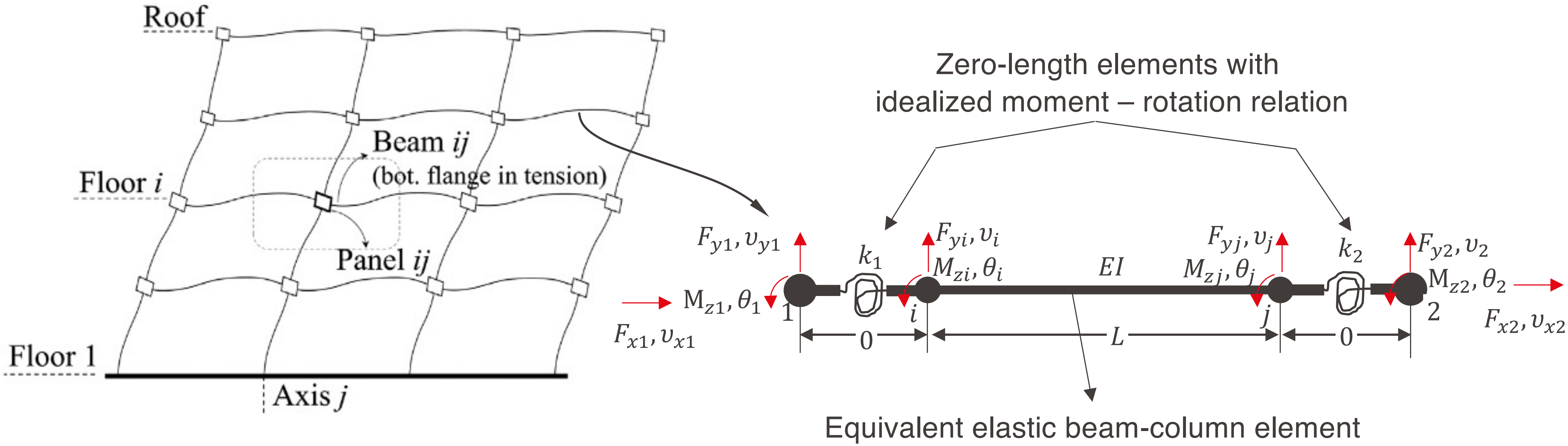
Advantages

- Fairly simple
- Effective for interface effects
- Computationally efficient

Disadvantages

- Require the load-displacement relationship (as opposed to engineering stress-strain)
- They generally don't capture interactive effects
- Member instabilities (see later on)

Within a numerical model of a frame structure



Elastic beam element with two rotational springs

Let's assume the following: $k_1 = n\dfrac{6EI_e}{L}$ $\qquad k_2 = n\dfrac{6EI_e}{L}$

The equivalent element stiffness matrix becomes:

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ M_{z1} \\ F_{x2} \\ F_{y2} \\ M_{z2} \end{Bmatrix} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{(S_{ii}+S_{jj}+2S_{ij})EI_e}{L^3} & -\frac{(S_{ii}+S_{ji})EI_e}{L^2} & 0 & -\frac{(S_{ii}+S_{jj}+2S_{ij})EI_e}{L^3} & -\frac{(S_{jj}+S_{ij})EI_e}{L^2} \\ 0 & -\frac{(S_{ii}+S_{ji})EI_e}{L^2} & \frac{S_{ii}EI_e}{L} & 0 & \frac{(S_{ii}+S_{ij})EI_e}{L^2} & \frac{S_{ji}EI_e}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{(S_{ii}+S_{jj}+2S_{ij})EI_e}{L^3} & \frac{(S_{ii}+S_{ij})EI_e}{L^2} & 0 & \frac{(S_{ii}+S_{jj}+2S_{ij})EI_e}{L^3} & \frac{(S_{jj}+S_{ij})EI_e}{L^2} \\ 0 & -\frac{(S_{jj}+S_{ij})EI_e}{L^2} & \frac{S_{ji}EI_e}{L} & 0 & \frac{(S_{jj}+S_{ij})EI_e}{L^2} & \frac{S_{jj}EI_e}{L} \end{bmatrix} \begin{Bmatrix} \upsilon_{x1} \\ \upsilon_{y1} \\ \theta_1 \\ \upsilon_{x2} \\ \upsilon_{y2} \\ \theta_2 \end{Bmatrix}$$

Elastic beam element with two rotational springs (2)

'Equivalent' moment of inertia:

$$I_e = \frac{n+1}{n} I$$

Stiffness coefficients:

$$S_{ij} = S_{ji} = \frac{6(1+n)}{2+3n}$$

$$S_{ii} = S_{jj} = \frac{(1+2n)}{(1+n)} S_{ij}$$

Elastic beam element with two rotational springs (Option 2)

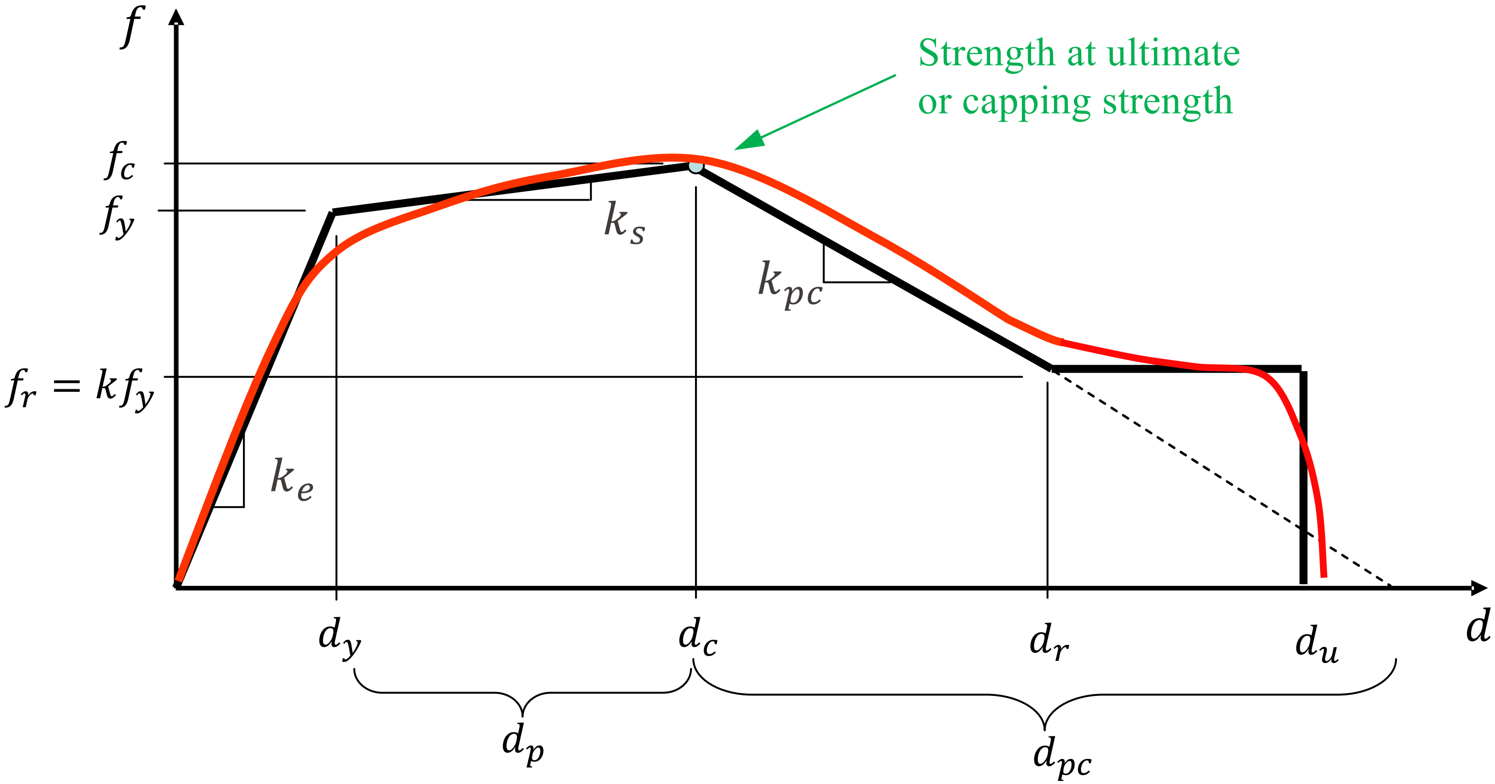
Let's assume the following:

$$k_1 = a_1 EI_e/L \qquad k_2 = a_2 EI_e/L \qquad a = \frac{a_1 a_2}{a_1 a_2 + 4a_1 + 4a_2 + 12}$$

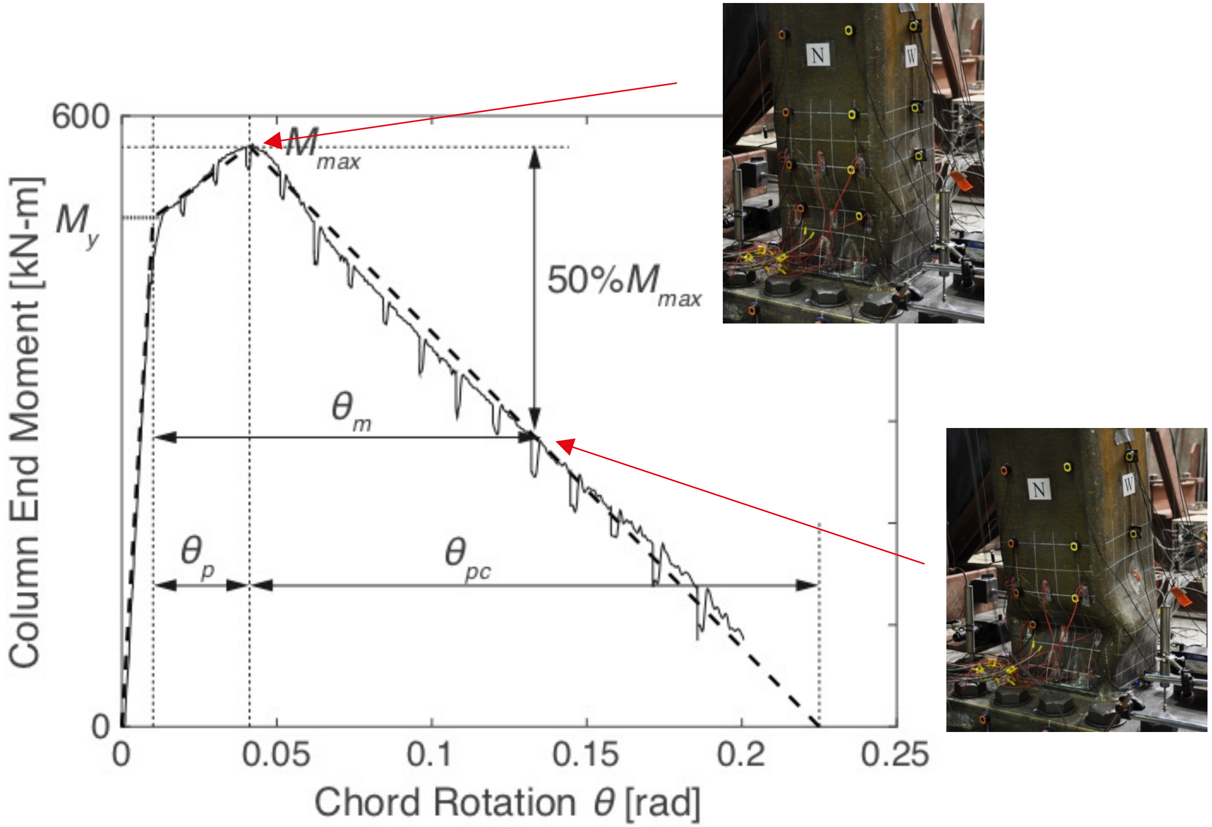
The "spring and beam" element stiffness matrix becomes:

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ M_{z1} \\ F_{x2} \\ F_{y2} \\ M_{z2} \end{Bmatrix} = \frac{aEI_e}{L} \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12}{L^2}\left(1+\frac{a_1+a_2}{a_1 a_2}\right) & \frac{6}{L}\left(1+\frac{2}{a_2}\right) & 0 & -\frac{12}{L^2}\left(1+\frac{a_1+a_2}{a_1 a_2}\right) & \frac{6}{L}\left(1+\frac{2}{a_1}\right) \\ 0 & \frac{6}{L}\left(1+\frac{2}{a_2}\right) & 4\left(1+\frac{3}{a_2}\right) & 0 & -\frac{6}{L}\left(1+\frac{2}{a_2}\right) & 2 \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12}{L^2}\left(1+\frac{a_1+a_2}{a_1 a_2}\right) & -\frac{6}{L}\left(1+\frac{2}{a_2}\right) & 0 & \frac{12}{L^2}\left(1+\frac{a_1+a_2}{a_1 a_2}\right) & -\frac{6}{L}\left(1+\frac{2}{a_1}\right) \\ 0 & \frac{6}{L}\left(1+\frac{2}{a_1}\right) & 2 & 0 & -\frac{6}{L}\left(1+\frac{2}{a_1}\right) & 4\left(1+\frac{3}{a_1}\right) \end{bmatrix} \begin{Bmatrix} \upsilon_{x1} \\ \upsilon_{y1} \\ \theta_1 \\ \upsilon_{x2} \\ \upsilon_{y2} \\ \theta_2 \end{Bmatrix}$$

Typical component behavior under monotonic loading



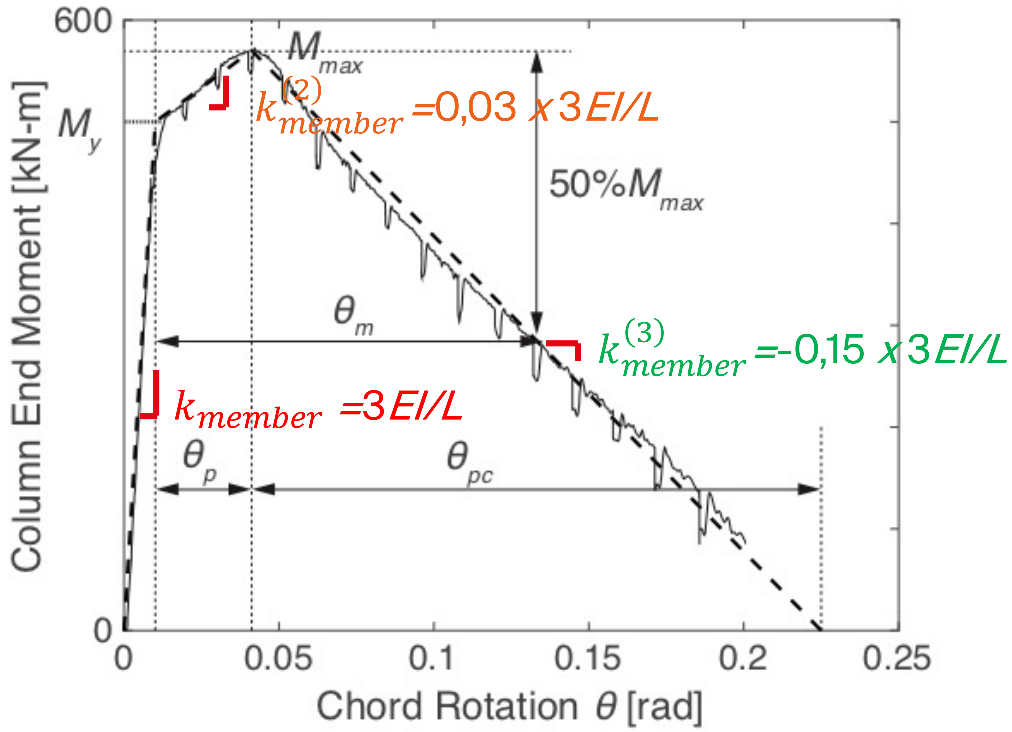
Typical component behavior
-Example: Cantilever steel column



Source: Suzuki and Lignos (2021)

Typical component behavior (2)

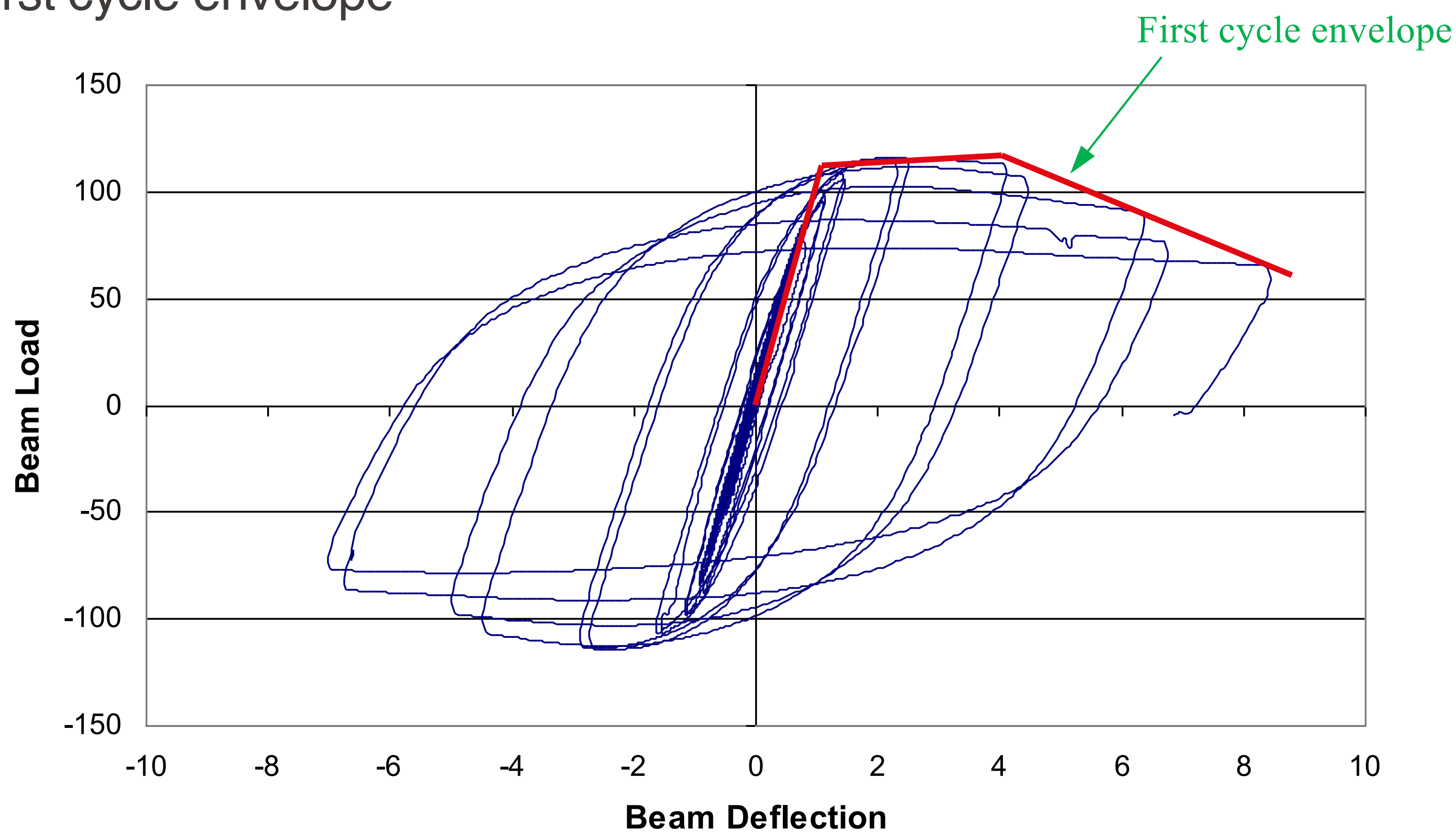
-Example: Cantilever steel column



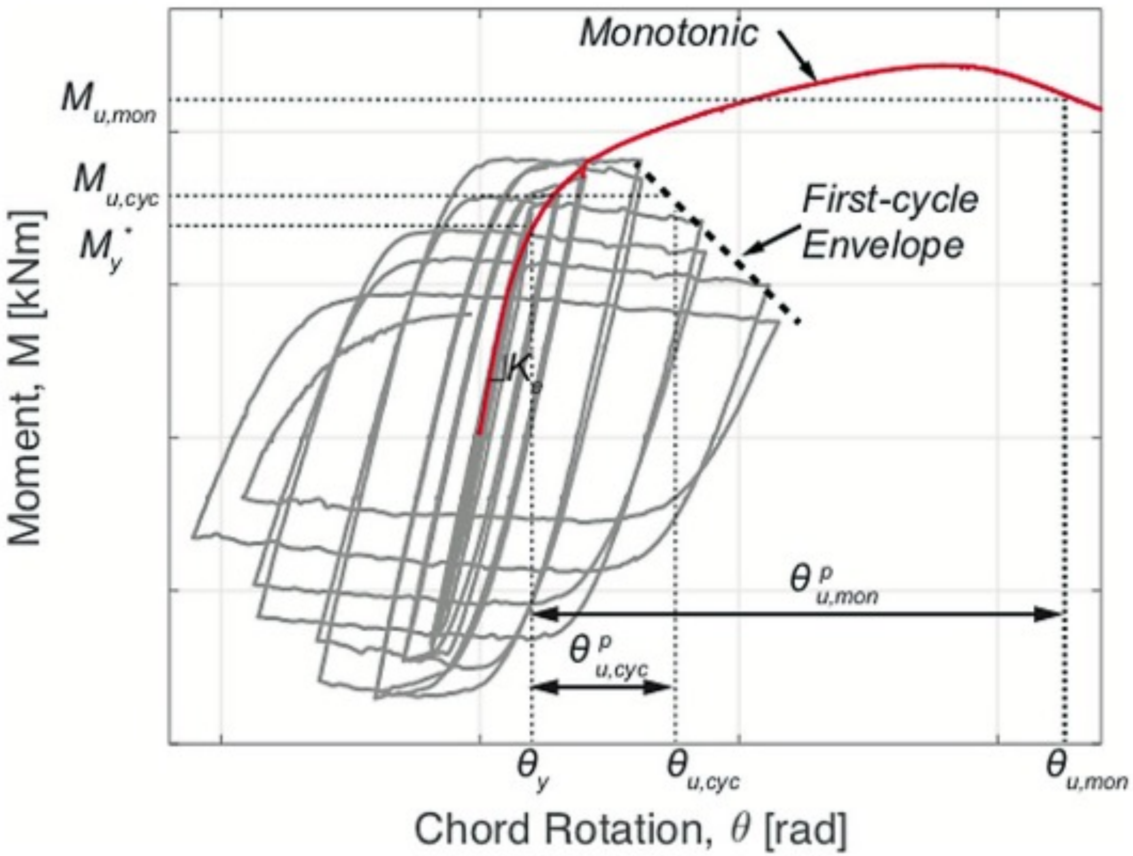
Source: Suzuki and Lignos (2018)

Typical component behavior under cyclic loading

-First cycle envelope

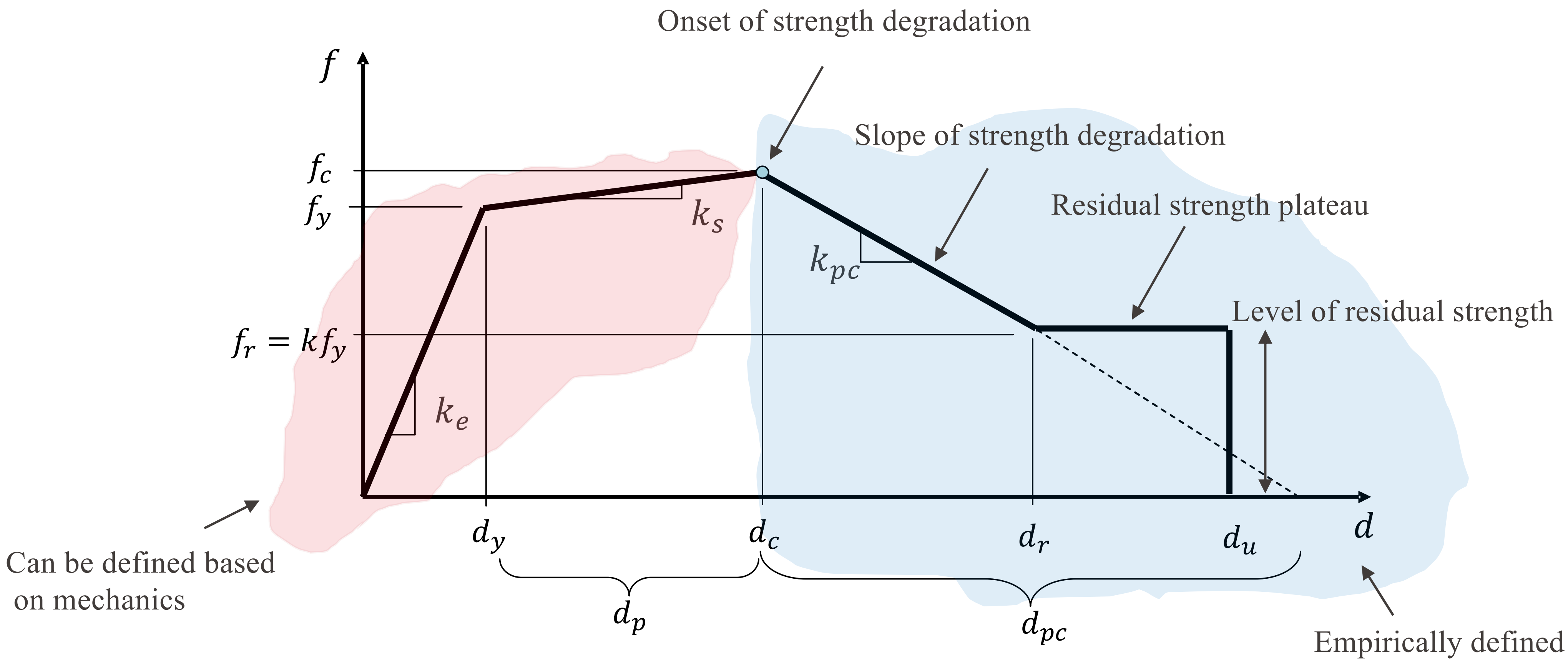


Comparison of monotonic and first cycle envelopes



Source: Lignos and Hartloper (2020)

Component behavior
-Some general remarks



Basic workflow

Increment n

$i = 1$: set $\lambda^{n,i=1} = \lambda^{n-1}$, $\mathbf{F}_{unb}^{n,i=1} = \mathbf{F}_{int}^{n-1} - \mathbf{F}_{ext}^{n-1}$, $\mathbf{v}^{n,i=1} = \mathbf{v}^{n-1}$ and $\mathbf{K}_{struct}^{n,i=1} = \mathbf{K}_{struct}^{n-1}$

Determine $\delta\mathbf{v}_r^{n,i}$ and $\delta\mathbf{v}_p^{n,i}$ → compute $\delta\lambda^{n,i}$ and $\delta\mathbf{v}^{n,i}$ → update $\lambda^{n,i}$, $\mathbf{v}^{n,i}$ and $\mathbf{F}_{ext}^{n,i}$

For all elements: element state determination

Assemble the new structure tangent stiffness matrix $\mathbf{K}_{structure}^{n,i}$ and structure resisting force vector $\mathbf{F}_{int}^{n,i}$

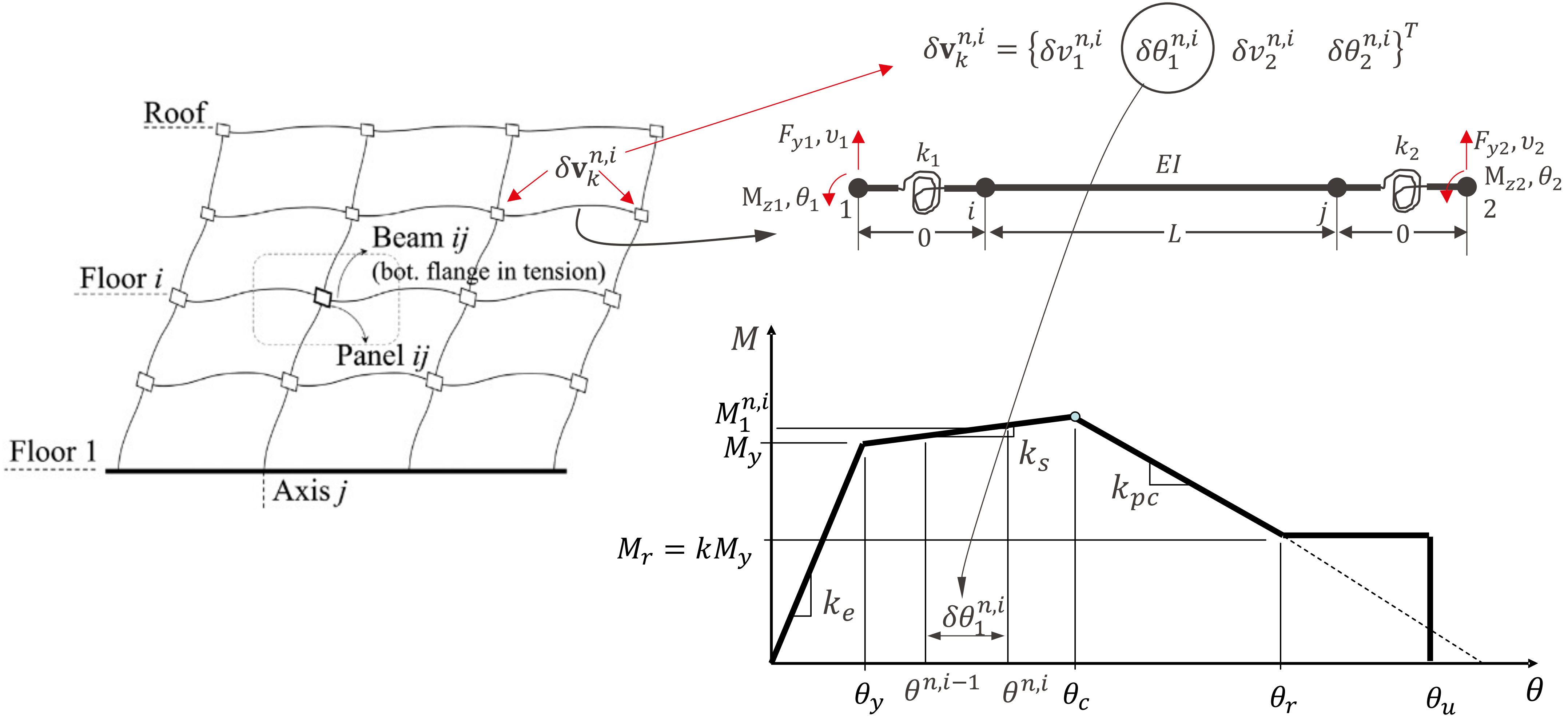
Compute the structure unbalanced force vector $\mathbf{F}_{unb}^{n,i} = \mathbf{F}_{int}^{n,i} - \mathbf{F}_{ext}^{n,i}$

Has the procedure converged?

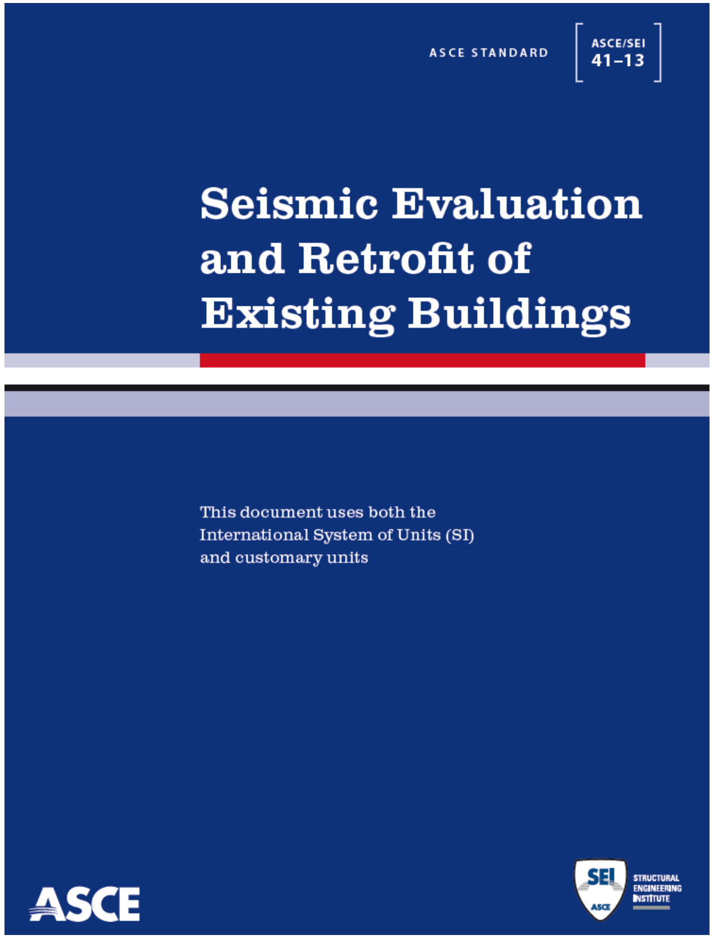
No → $i+1$ → Set $\mathbf{F}_{unb}^{n,1} = \mathbf{F}_{unb}^{n,i-1}$

Yes → $n+1$ → Next increment

Basic workflow (2)



Assessment of existing structures
-Readily available load-deformation curves



EUROPEAN STANDARD
NORME EUROPÉENNE
EUROPÄISCHE NORM

EN 1998-3

June 2005

ICS 91.120.25

Supersedes ENV 1998-1-4:1996
Incorporating corrigendum March 2010

English version

Eurocode 8: Design of structures for earthquake resistance -
Part 3: Assessment and retrofitting of buildings

Eurocode 8: Calcul des structures pour leur résistance aux séismes - Partie 3: Evaluation et renforcement des bâtiments

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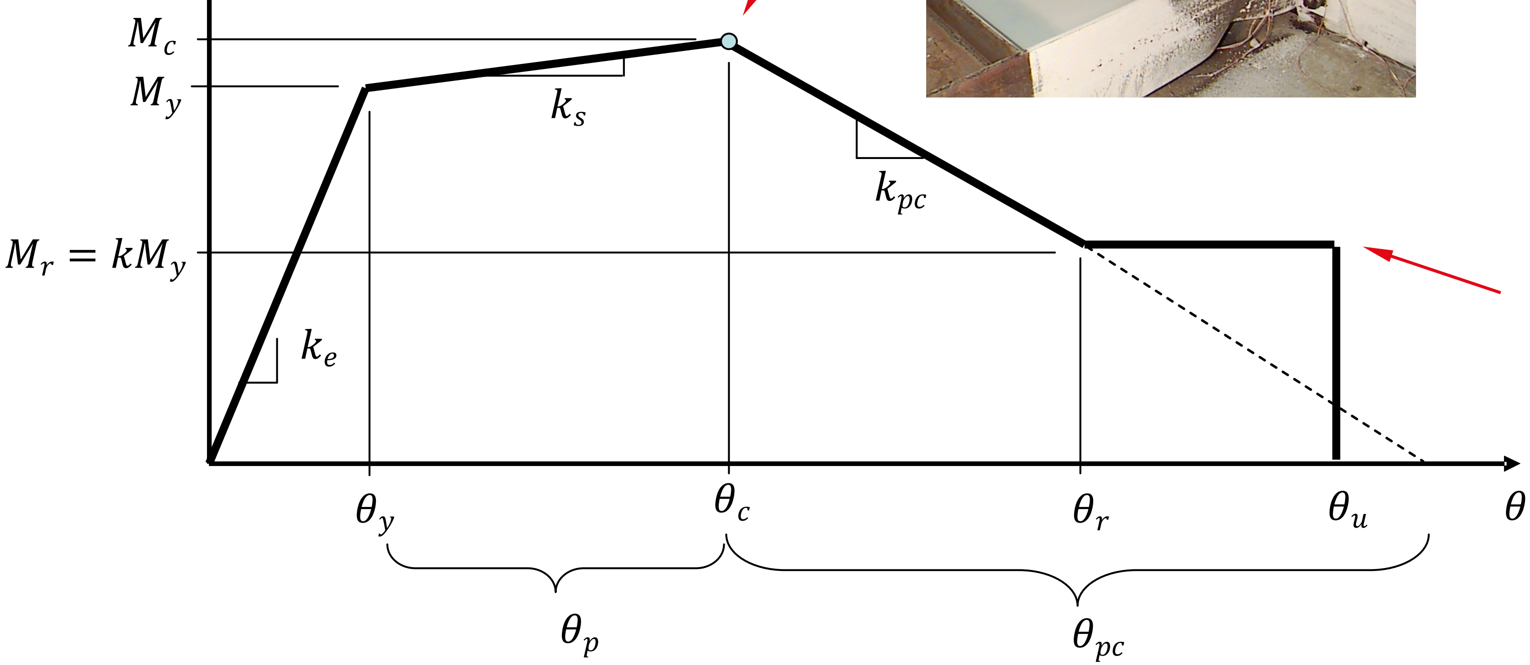
Some challenges with concentrated plasticity models

Source: Elkady and Lignos (2018)

Source:Prof. Jack Moehle

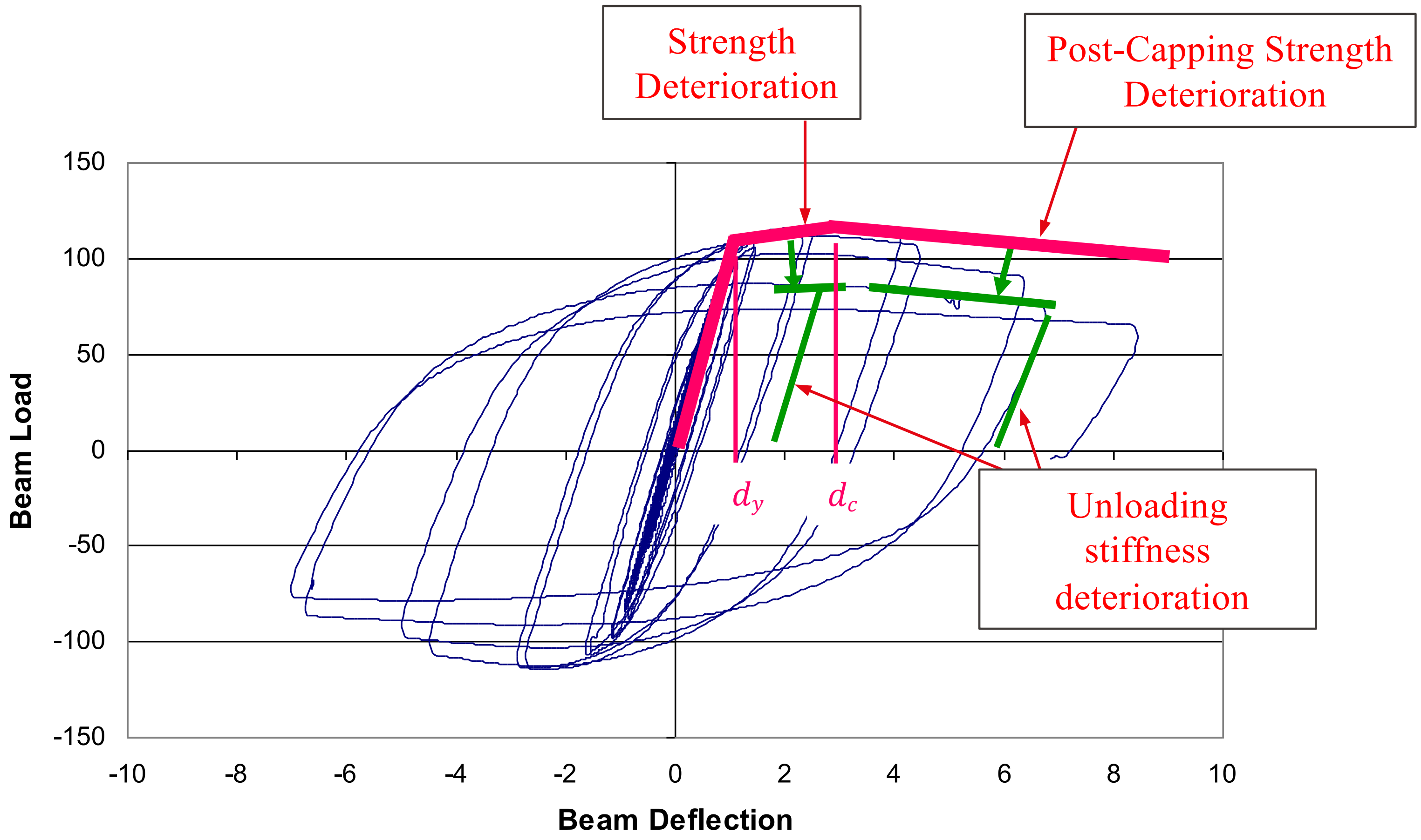
Illustration for steel beams

Local buckling



Tearing

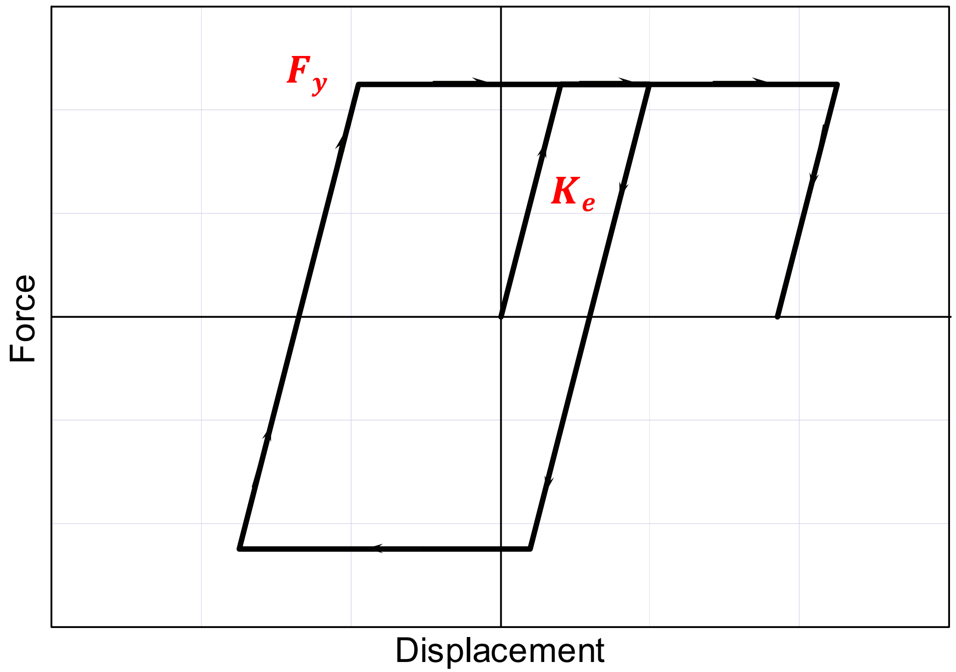
Deterioration modes in steel members



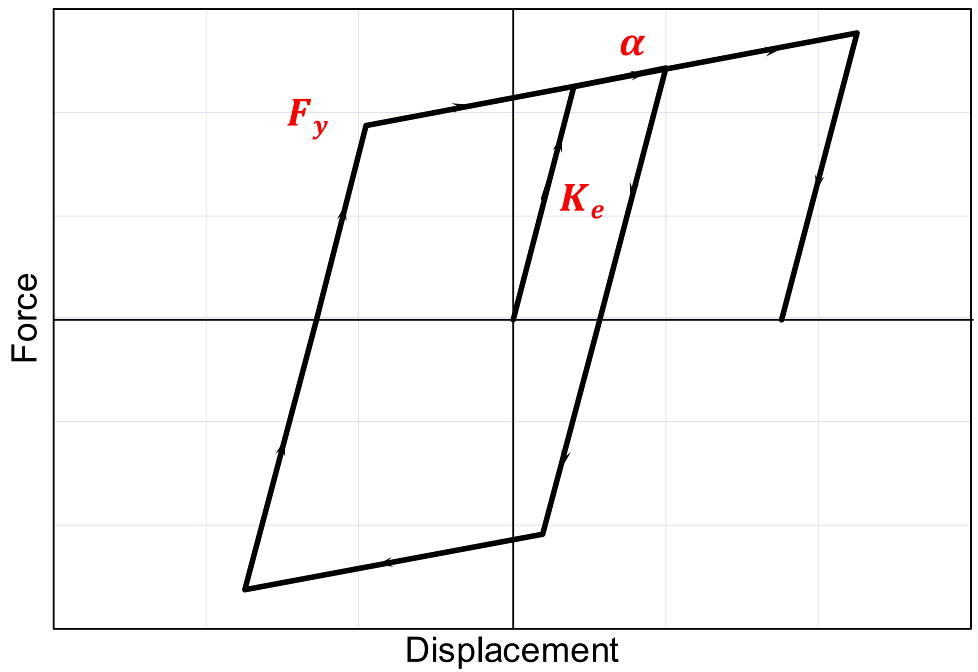
Source: Prof. Helmut Krawinkler

Typical models to trace the hysteretic response

Elastic Perfectly Plastic Model

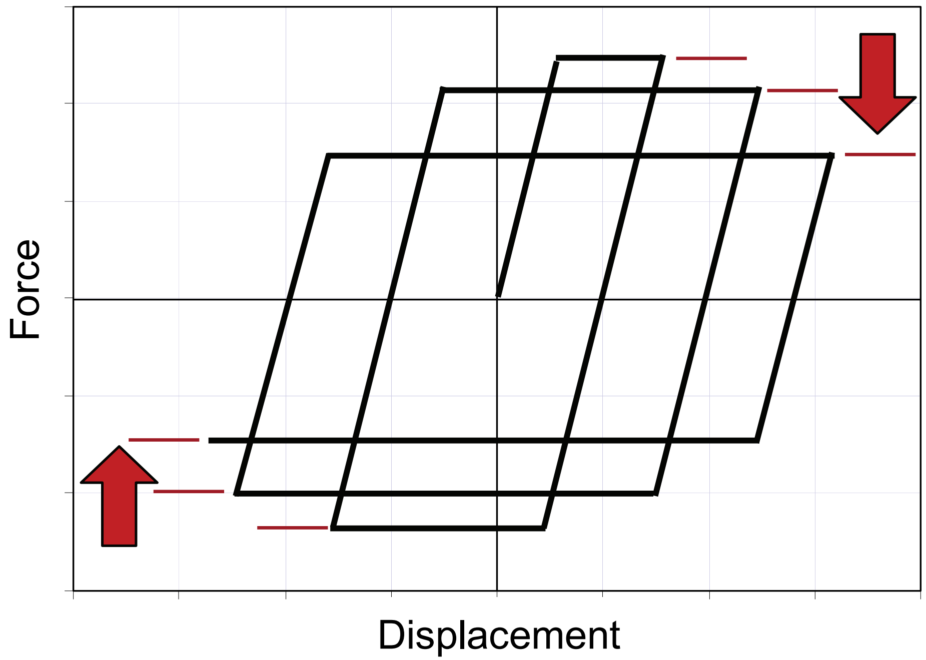


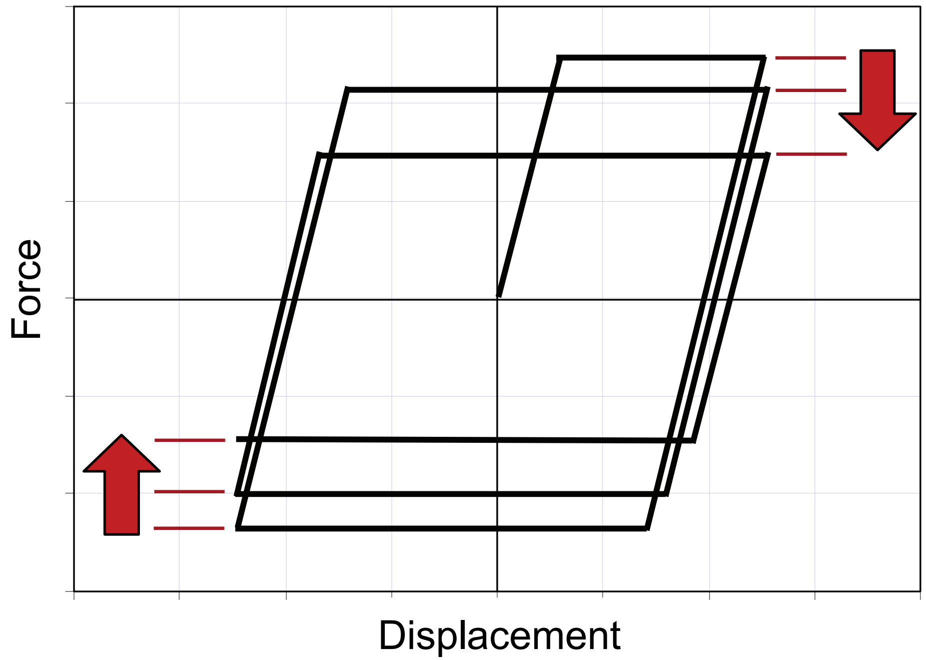
Simple Bilinear Model



ADVANTAGE: You only need K_e, F_y (or M_y) and α

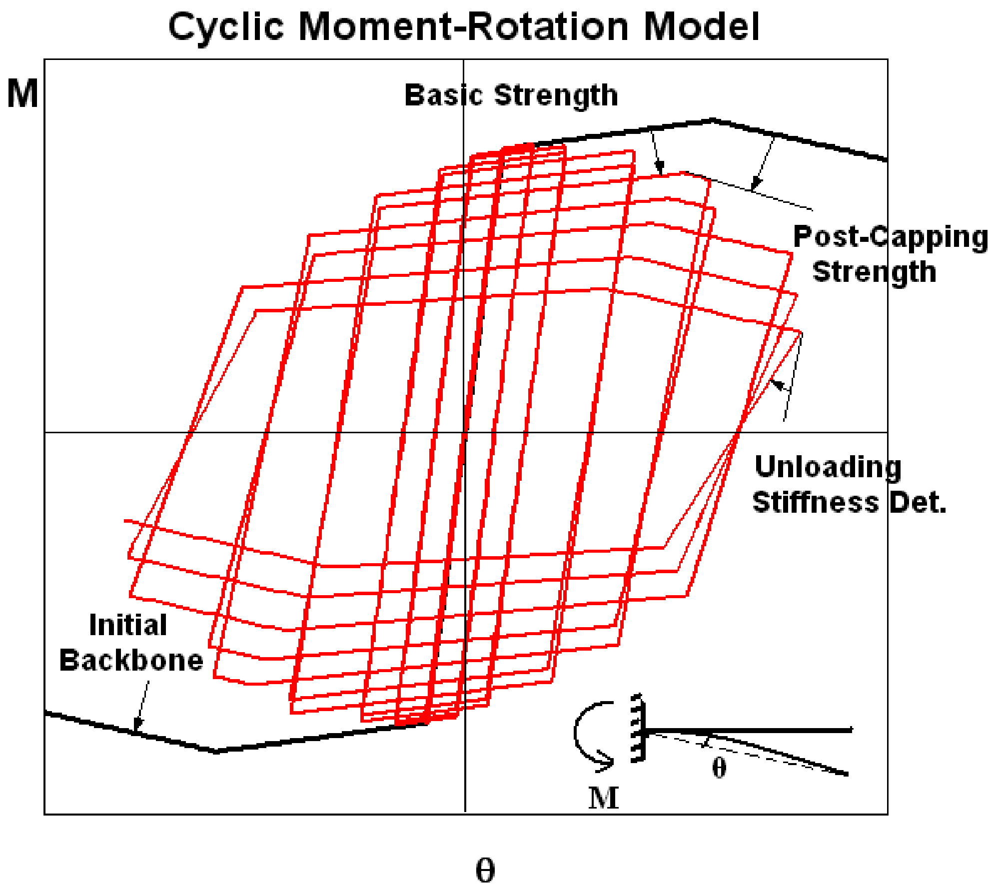
Models that capture cyclic deterioration





Notice that they do not capture post-capping strength deterioration

Models that capture cyclic deterioration



-Reference energy dissipation capacity

$$E_t = \lambda \cdot \theta_p \cdot M_y$$

-Rule for modeling deterioration

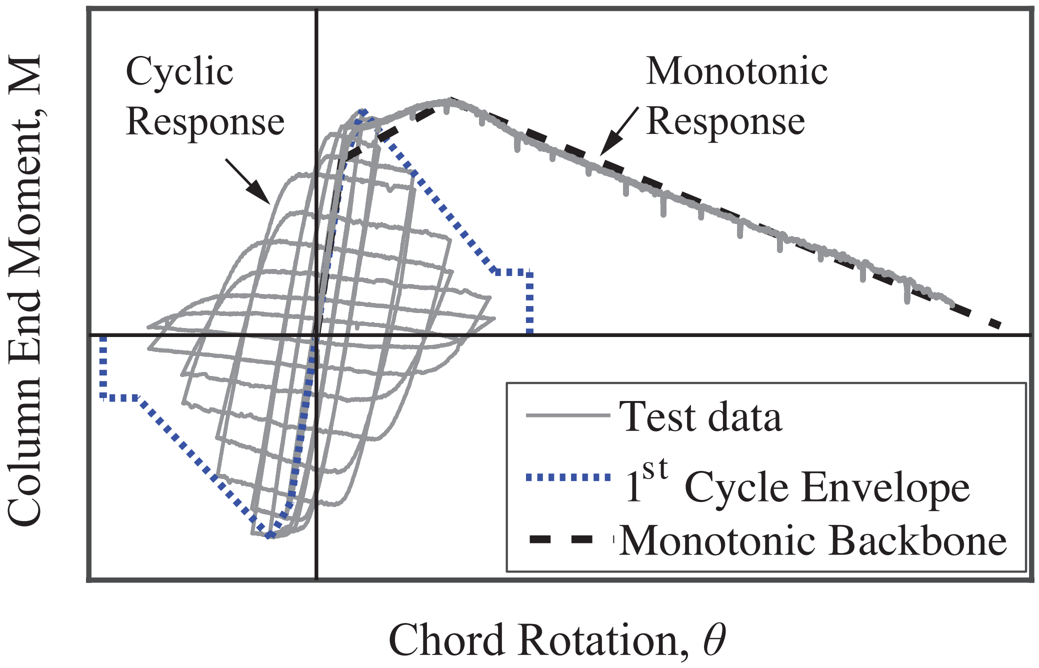
$$M_i = (1 - \beta_i) \cdot M_{i-1}$$

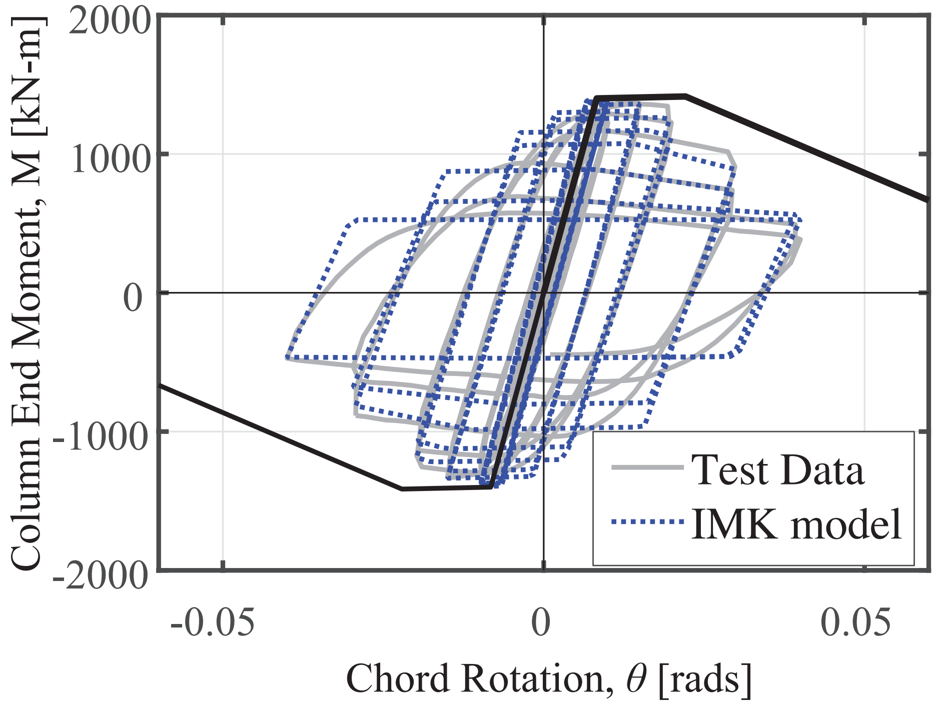
$$K_i = (1 - \beta_i) \cdot K_{i-1}$$

$$\beta_i = \left(\frac{E_i}{E_t - \sum_{j=1}^{i-1} E_j} \right)^c$$

(Source: Lignos and Krawinkler 2011)

Calibration with available experimental data

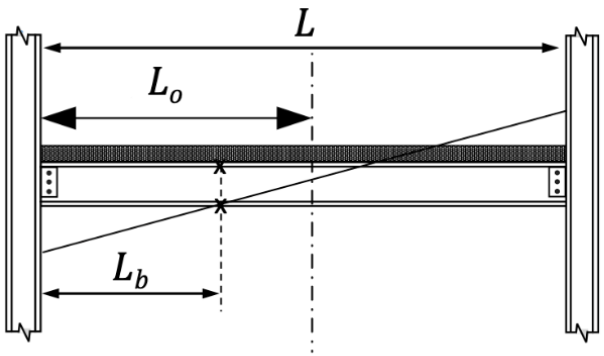




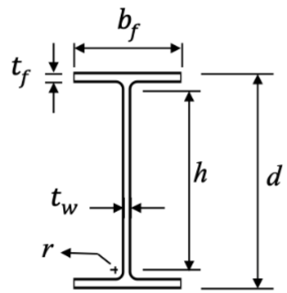
(Source: Lignos et al. 2019)

Calibration with available experimental data (2)

- **Trends between geometric & material parameters on input model parameters**
  - Dimensionless slenderness parameters:
    - flange local buckling, $\lambda_f = b_f / 2t_f$
    - web local buckling, $\lambda_w = h/t_w$
    - lateral torsional buckling, $\lambda_t = L_b / i_z \quad \left(i_z = \sqrt{\frac{I_z}{A}} \right)$
    - Shear span ratio, L_o/h
    - Yield stress, f_y



Section C-C



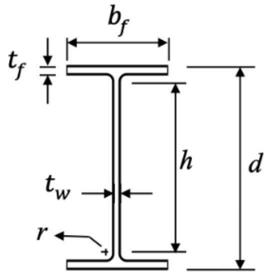
Empirical relations for steel beams

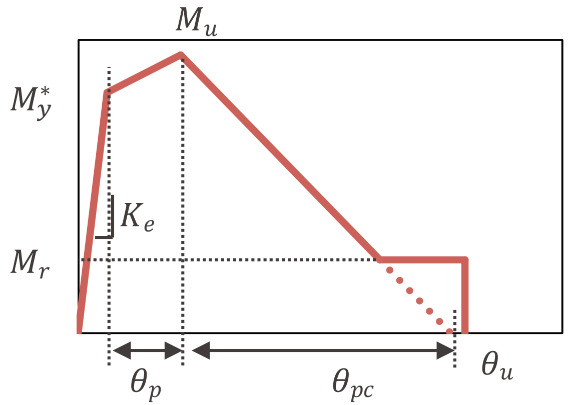
Non-composite steel beams (other than RBS)

$$M_y^* = 1.17M_{pl} = 1.17 \cdot \gamma_{rm} \cdot W_{pl,y} \cdot f_y \quad (\gamma_{rm} = 1.25\ for\ S355)$$

$$M_u = 1.11M_y^*\ (COV = 0.1)$$

$$M_r = 0.4M_y^*$$





Plastic deformation parameters (for monotonic loading)

$$\theta_p = 0.0865\left(\frac{h}{t_w}\right)^{-0.365}\left(\frac{b}{2t_f}\right)^{-0.140}\left(\frac{L_o}{d}\right)^{0.340}\left(\frac{d}{533}\right)^{-0.721}\left(\frac{f_y}{355}\right)^{-0.230} \quad (\sigma_{ln} = 0.32)$$

$$\theta_{pc} = 5.63\left(\frac{h}{t_w}\right)^{-0.565}\left(\frac{b}{2t_f}\right)^{-0.800}\left(\frac{d}{533}\right)^{-0.280}\left(\frac{f_y}{355}\right)^{-0.430} \quad (\sigma_{ln} = 0.25)$$

$$\theta_u = 0.20rad$$

Range of applicability

$$20 \leq h/t_w \leq 55$$

$$4 \leq b_f/2t_f \leq 8$$

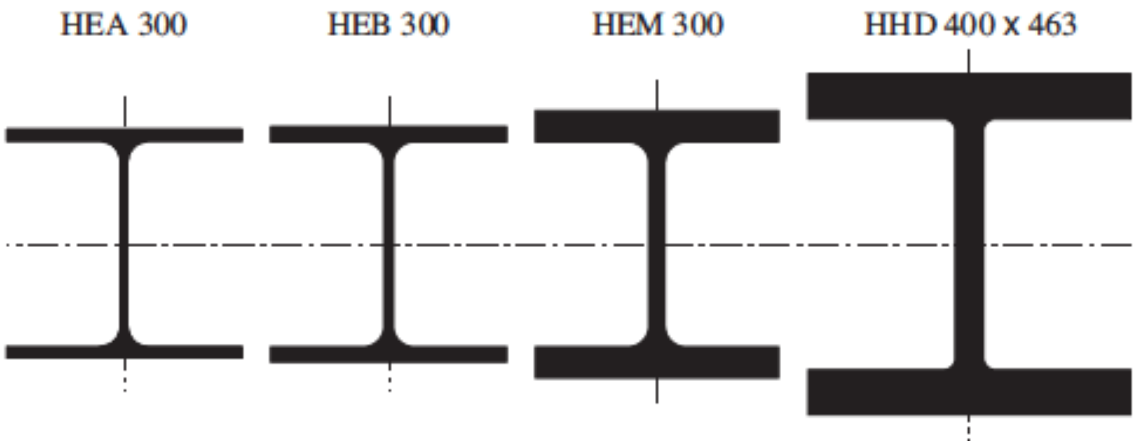
$$2.5 \leq L_o/d \leq 7$$

$$102mm \leq d \leq 914mm$$

$$240MPa \leq f_y \leq 450MPa$$

Source: Lignos and Krawinkler (2011)

Some remarks on multivariate-regression models

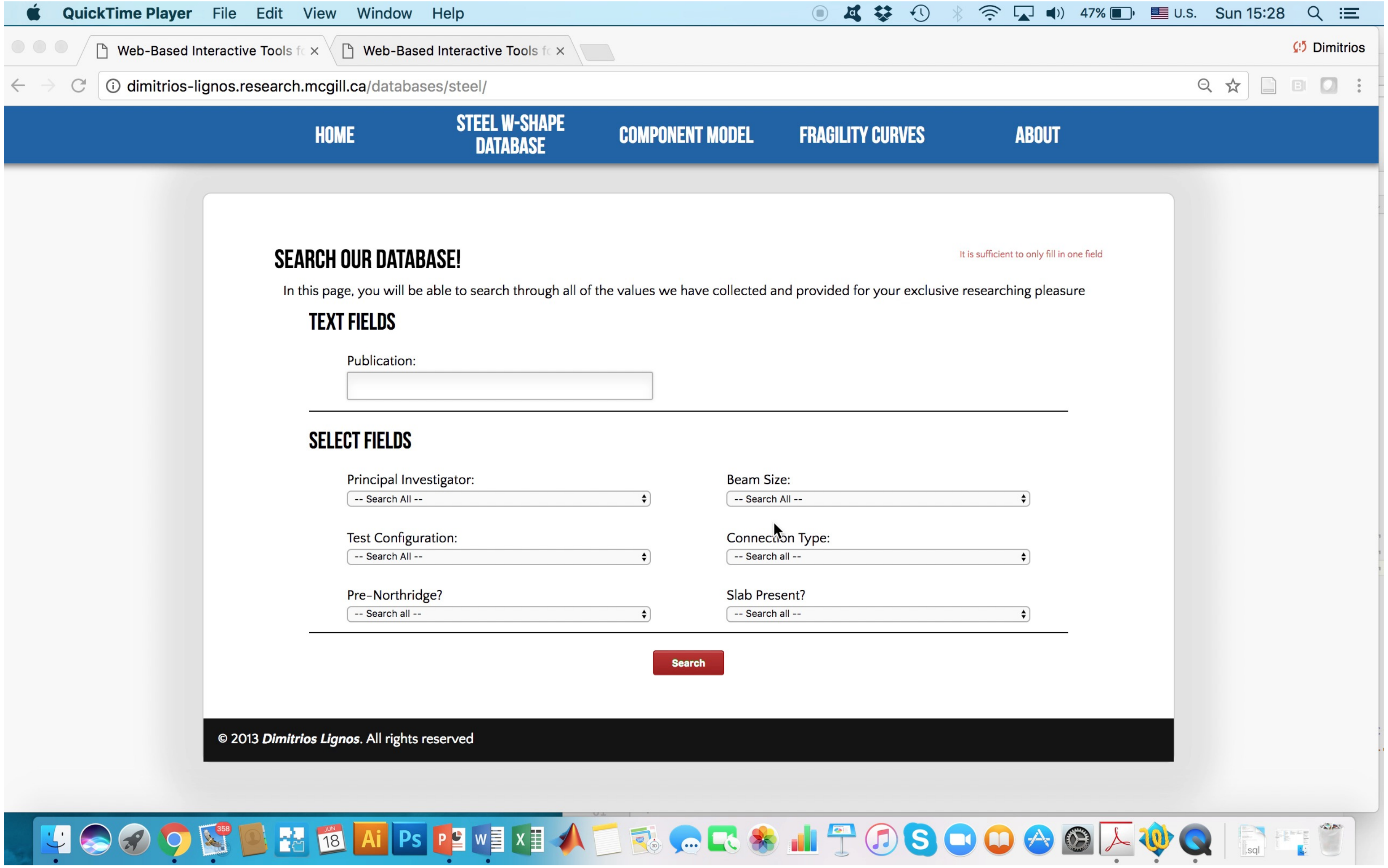


$$\frac{h}{t_w} = 24.5 \qquad \frac{h}{t_w} = 18.9 \qquad \frac{h}{t_w} = 9.9 \qquad \frac{h}{t_w} = 8.1$$

$$\theta_p = 0.0885\left(\frac{h}{t_w}\right)^{-0.365}\left(\frac{b}{2t_f}\right)^{-0.14}\left(\frac{L_o}{d}\right)^{0.34}\left(\frac{d}{533}\right)^{-0.721}\left(\frac{f_y}{355}\right)^{-0.23}$$

Databases and model availability

-Publicly available from: resslabtools.epfl.ch



Empirical equations for steel columns

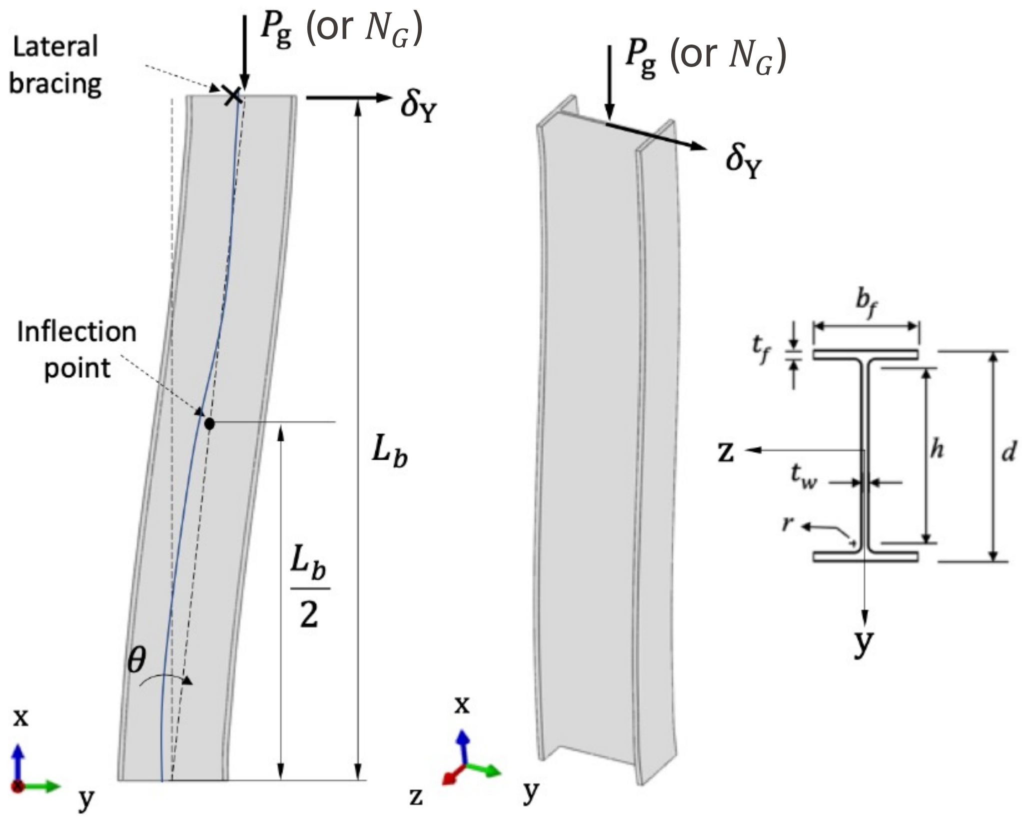
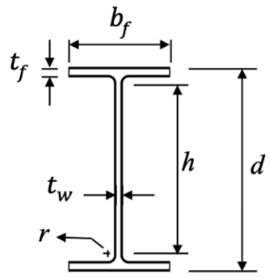
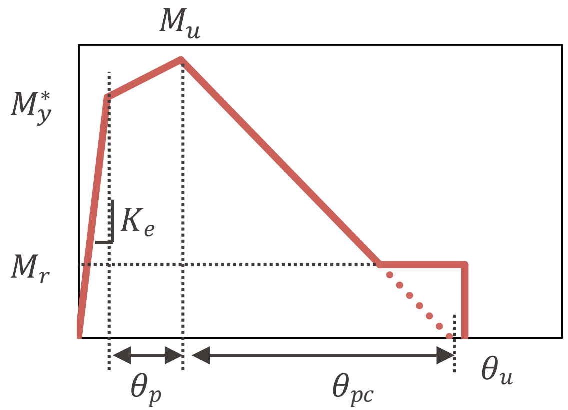


Image adopted from RESSLab-hub: https://resslab-hub.epfl.ch/ (Source: Lignos et al. 2019)

Empirical equations for steel columns (2)

$$M_y^* = \begin{cases} 1.15 W_{pl,y}\gamma_{rm} f_y (1 - N_G/2N_{pl,e}), & if\ \ N_G/N_{pl,e} \le 0.20 \\ 1.15 W_{pl,y}\gamma_{rm} f_y \left[\frac{9}{8}(1 - N_G/N_{pl,e})\right], & if\ \ N_G/N_{pl,e} > 0.20 \end{cases}$$





$$M_u = \alpha M_y^* \qquad \alpha = 12.5\left(\frac{h}{t_w}\right)^{-0.2}\left(\frac{L_b}{i_z}\right)^{-0.4}\left(1 - \frac{N_G}{N_{pl,e}}\right)^{0.4}, 1.0 \le \alpha \le 1.3\ (COV = 0.10)$$

$$M_r = \left(0.5 - 0.4\frac{N_G}{N_{pl,e}}\right) M_y^* \ge 0\ (COV = 0.27)$$

Plastic deformation parameters (for monotonic loading)

$$\theta_p = 294\left(\frac{h}{t_w}\right)^{-1.7}\left(\frac{L_b}{i_z}\right)^{-0.7}\left(1 - \frac{N_G}{N_{pl,e}}\right)^{1.6} \le 0.20\ (COV = 0.39)$$

$$\theta_{pc} = 90\left(\frac{h}{t_w}\right)^{-0.8}\left(\frac{L_b}{i_z}\right)^{-0.8}\left(1 - \frac{N_G}{N_{pl,e}}\right)^{2.5} \le 0.30\ (COV = 0.26)$$

$$\theta_u = 0.15\ (COV = 0.46)$$

Range of applicability

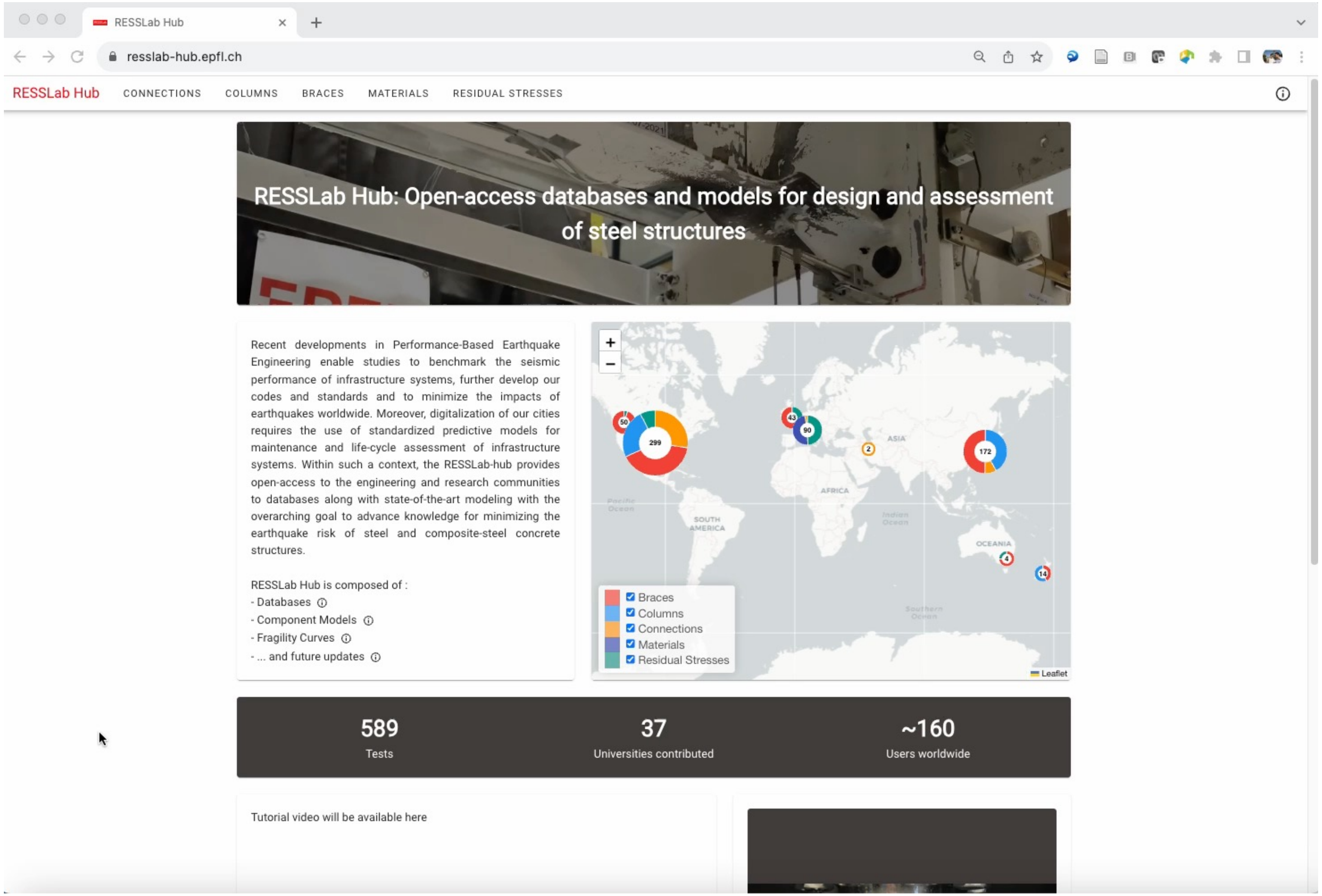
$$3.71 \le h/t_w \le 57.5$$

$$38.4 \le L_b/i_z \le 120$$

$$0 \le N_G/N_{pl,e} \le 0.75$$

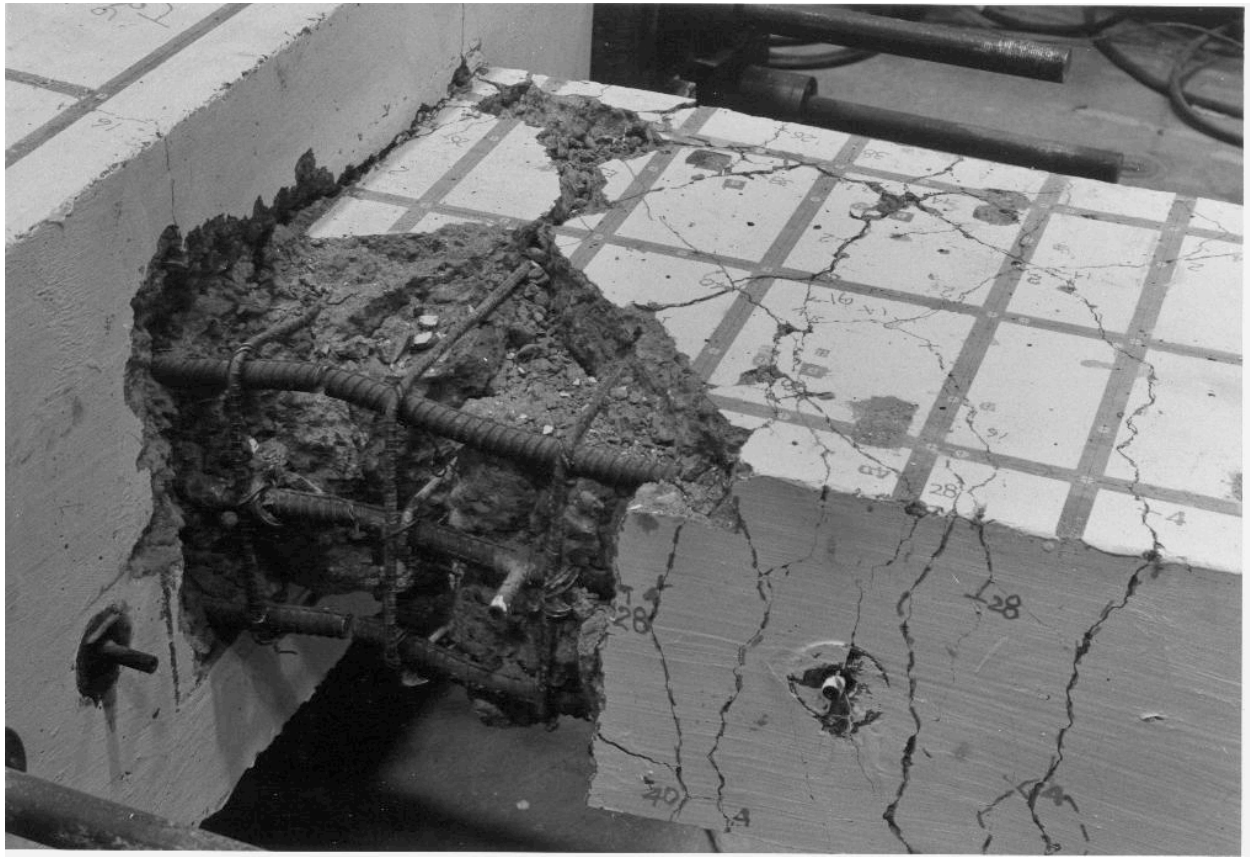
Source: Lignos et al. (2019)

The RESSLab-Hub (https://resslab-hub.epfl.ch/)



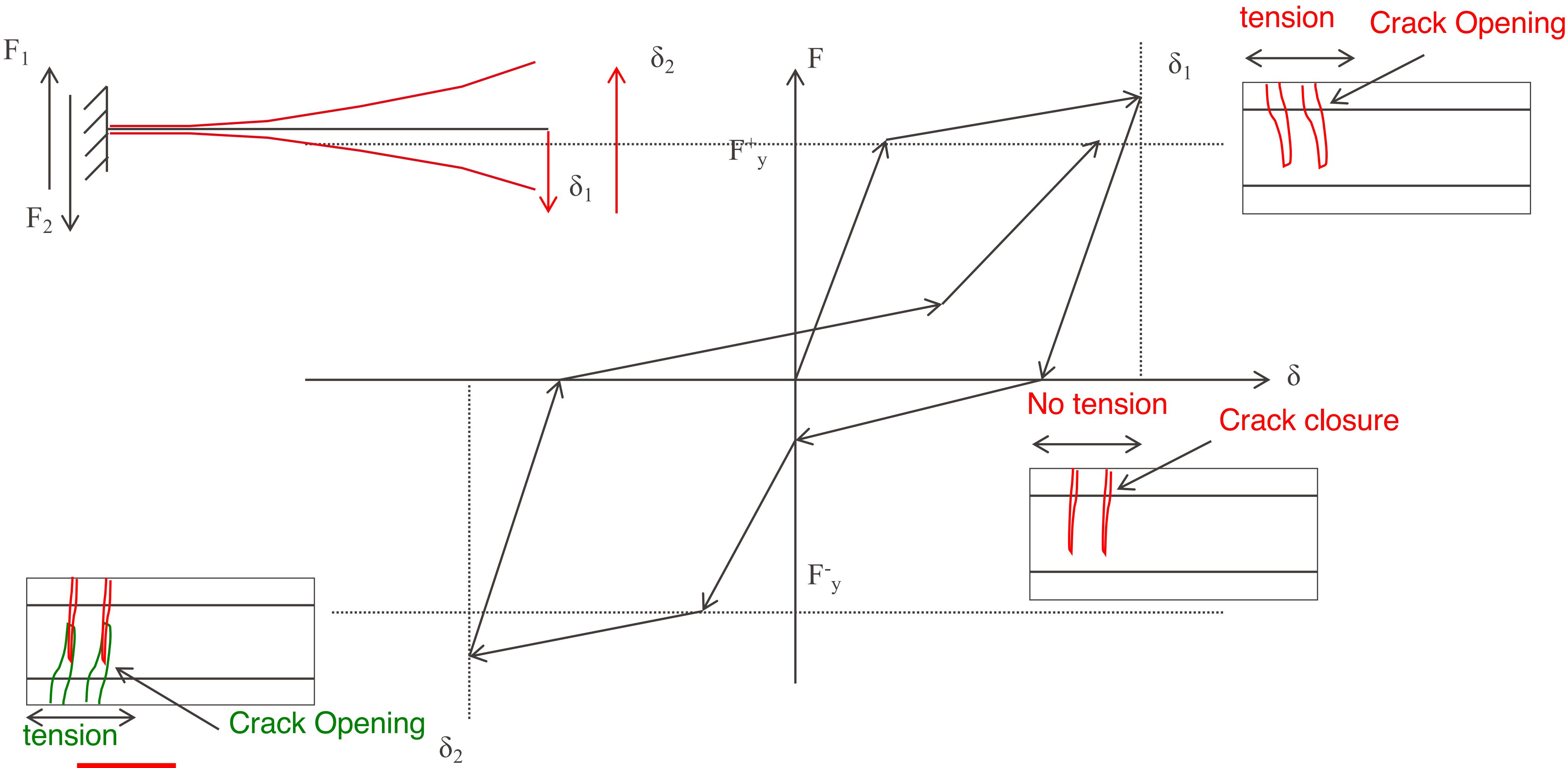
(Source: Lignos et al. 2019)

Nonlinear behavior of reinforced concrete members



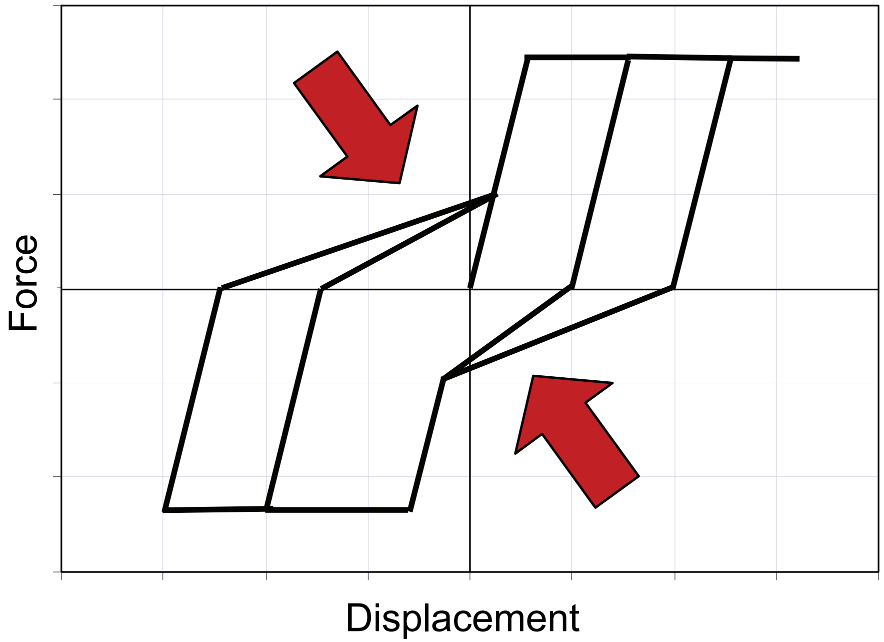
Source: Krawinkler et al. (1978)

Nonlinear behavior of reinforced concrete members (2)

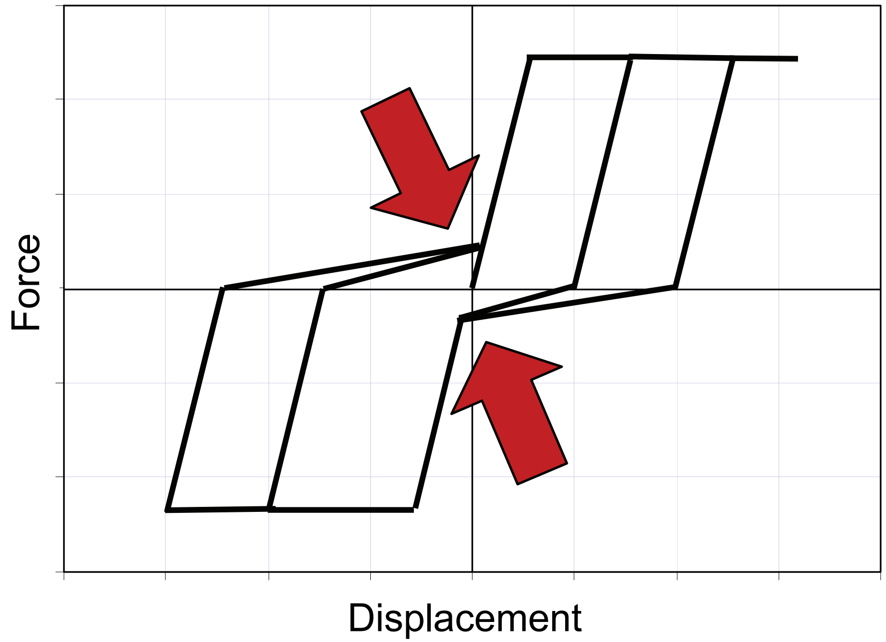


Nonlinear behavior of reinforced concrete members (3)

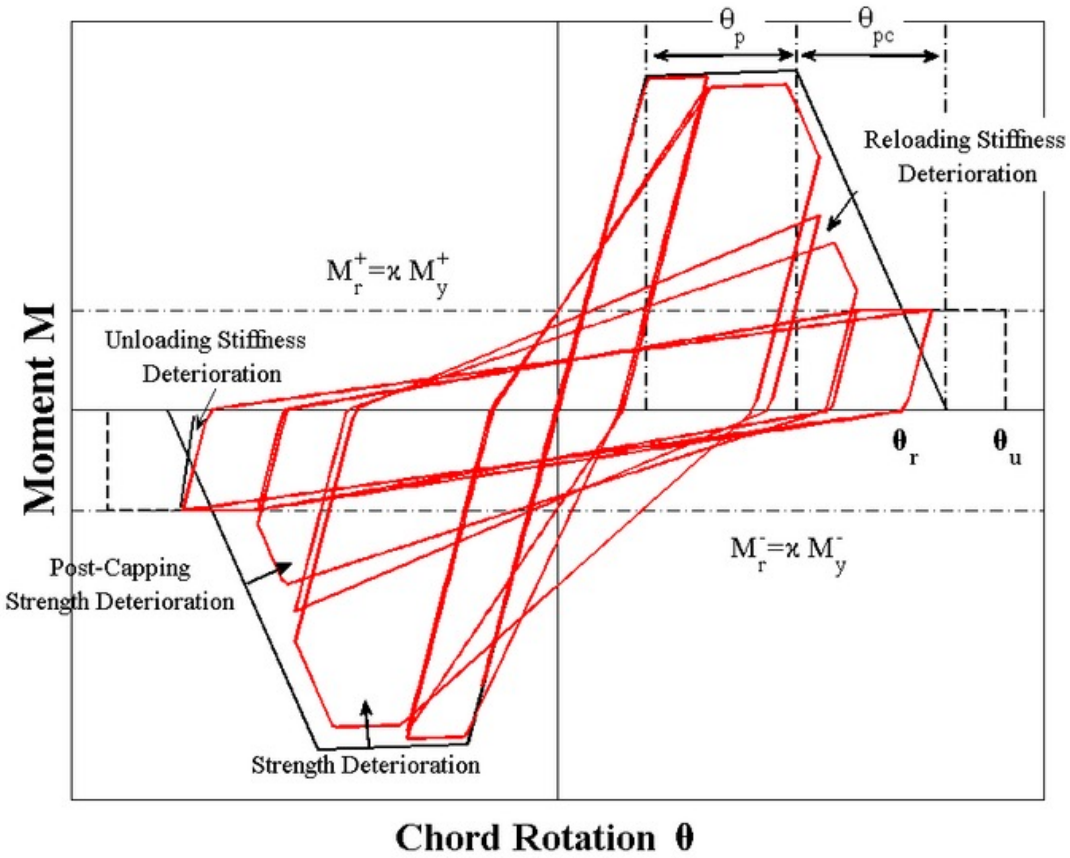
Less pinching (flexure-dominant)



More pinching (shear-dominant)



Nonlinear behavior of reinforced concrete members (4)



Source: Lignos and Krawinkler (2012)

Nonlinear behavior of reinforced concrete members (5)

Key Parameters:

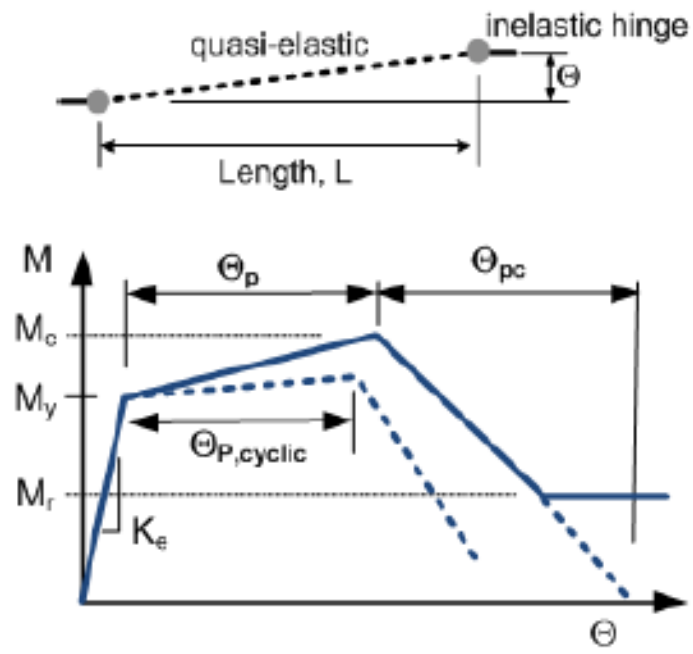
- Strength
- Initial stiffness
- Post-yield stiffness
- Plastic rotation capacity
- Post-capping slope
- Cyclic deterioration

Calibration process:

- 250+ columns
- Flexure & flexure-shear dominant
- Calibrated to expected values

Key assumption:

- Bond slip is incorporated in the beam-column model parameters



Nonlinear behavior of reinforced concrete members (6)

Pre-cracked stiffness (initial stiffness):

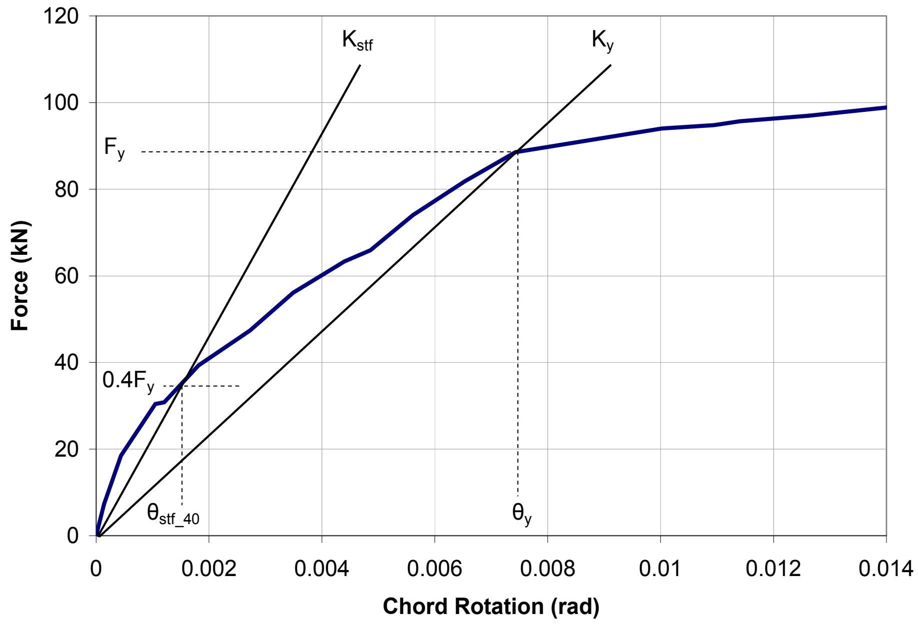
$$K_{stf,40}:\ \frac{E_c I_{stf,40}}{E_c I_g} = -0.02 + 0.98\left(\frac{N}{A_g f_c{}'}\right) + 0.09\left(\frac{L_s}{h}\right), 0.35 \leq \frac{E_c I_y}{E_c I_g} \leq 0.8$$

$$(\sigma_{ln} = 0.42)$$

Post-cracked stiffness (secant stiffness to yield):

$$K_y:\ \frac{E_c I_y}{E_c I_g} = -0.07 + 0.59\left(\frac{N}{A_g f_c{}'}\right) + 0.07\left(\frac{L_s}{h}\right), 0.2 \leq \frac{E_c I_y}{E_c I_g} \leq 0.6$$

$$(\sigma_{ln} = 0.37)$$



$E_c I_g$: Flexural stiffness of the gross cross section

E_c: Elastic concrete modulus

A_g: Gross cross-sectional area

f_c': Concrete compressive strength

L_s: Shear span from point of maximum moment to the inflection point

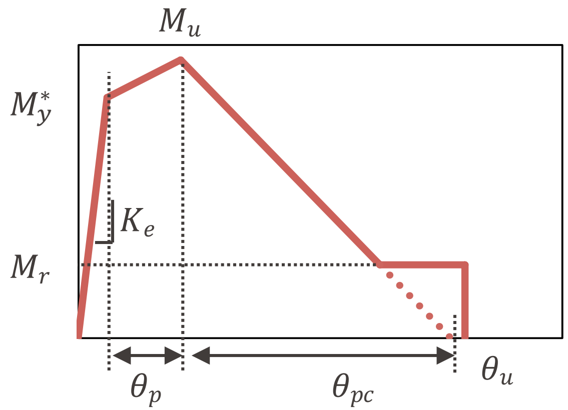
h: Depth of the cross section

Source: Haselton et al. (2008)

Nonlinear behavior of reinforced concrete members (7)

M_y^* Calculated based on standard moment curvature analysis (Panagiotakos and Fardis 2001) or SIA 262 for concrete cross sections (M_{Rd})

$$M_u = 1.25(0.89)^{v}(0.91)^{0.01f_c'}M_y^* \sim 1.13M_y^* \qquad (\sigma_{ln} = 0.12)$$

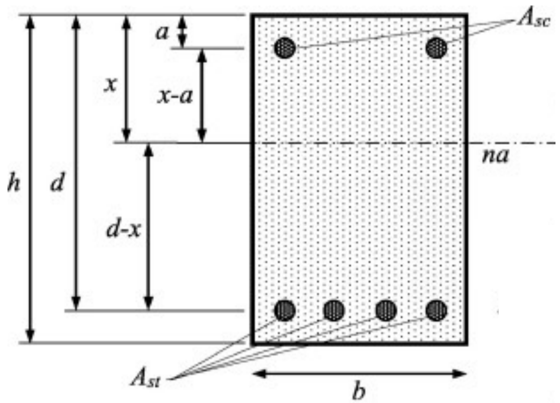


Plastic rotation for symmetric cross-sections (& symmetric reinforcement)

$$\theta_p = 0.13(1 + 0.55a_{sl})(0.13)^{v}(0.02 + 40\rho_{sh})^{0.65}(0.57)^{0.01f_c'} \quad (\sigma_{ln} = 0.69)$$

Plastic rotation for symmetric cross-sections (asymmetric reinforcement)

$$\theta_{p(not-symmetric)} = \frac{\max\left(0.01, \frac{\rho' f_y}{f_c'}\right)}{\max\left(0.01, \frac{\rho f_y}{f_c'}\right)}\theta_p \qquad \left(\rho' = \frac{A_{sc}}{bd}\right) \quad \left(\rho = \frac{A_{st}}{bd}\right)$$



Post-capping plastic rotation

$$\theta_{pc} = 0.76(0.031)^{v}(0.02 + 40\rho_{sl})^{1.02} \leq 0.10 \qquad (\sigma_{ln} = 0.86)$$

Source: Haselton et al. (2008)

Nonlinear behavior of reinforced concrete members (8)

-Definitions

a_{sl} Bond-slip indicator between steel and concrete (accounts for ~ 1/3 of plastic rotation)

$a_{sl} = 1$ If bond-slip is possible

$a_{sl} = 0$ If bond-slip is prevented

$v = \frac{N}{f_c' A_g}$ Axial load ratio

$\rho_{sh} = \frac{A_{sh}}{sb}$ Area ratio of transverse reinforcement (with area A_{sh}) in the plastic hinge region that transverse reinforcement is spaced at distance s

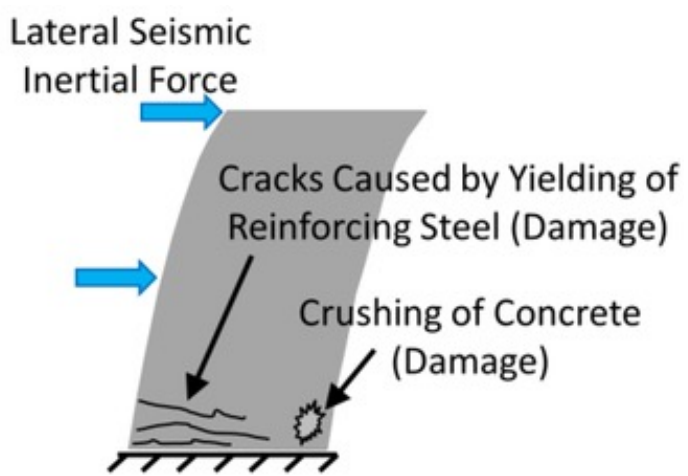
Nonlinear behavior of reinforced concrete members (9)

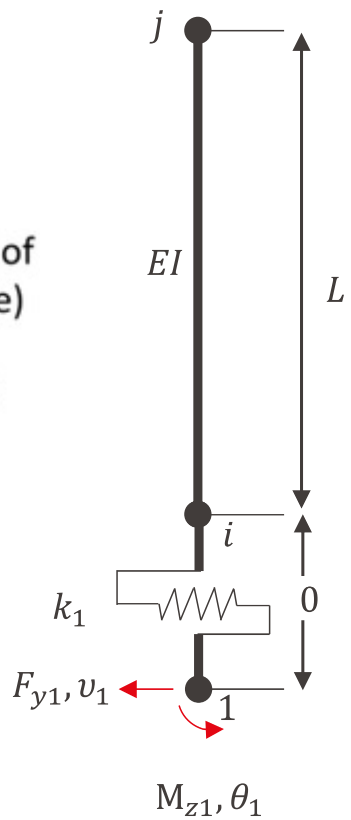
Table 3-3 Empirical Plastic Rotation Values, θ_p and θ_{pc}, for a Representative Column Section (Haselton et al., 2008)

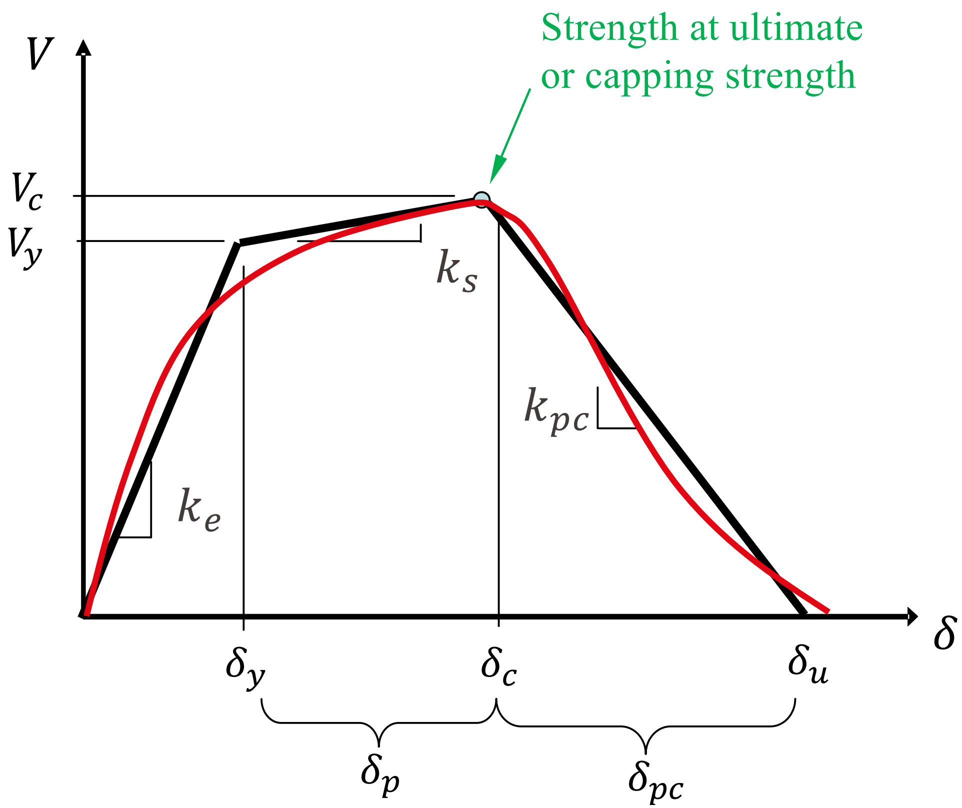
$v = N/f_c' A_g$	ρ_{sh}	θ_p	θ_{pc}
0.1	0.002	0.031	0.052
	0.006	0.047	0.100
	0.020	0.077	0.100
0.6	0.002	0.012	0.009
	0.006	0.019	0.024
	0.020	0.031	0.077

Source: Haselton et al. (2008)

Zero length elements for shear-dominant elements







Interactive effects (e.g., axial load-bending interaction)