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In-class Exercise Week #3: Truss, frame and zero length elements

Exercise #1:

What is the magnitude and direction of the force P at point a required to displace that point a vertically downward 5mm without any horizontal displacement? Assume that $E = 200\,GPa$; the rotational stiffness $k_R = 1000\frac{kNmm}{rad}$; the translational stiffness $k_T = 50kN/mm$.

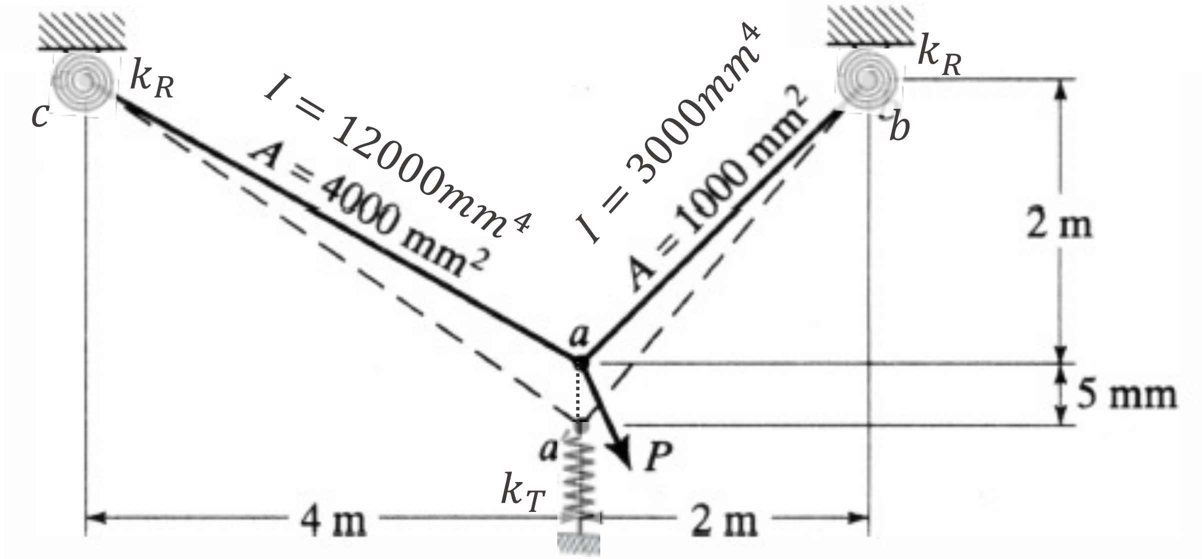

Figure 1. Schematic representation of the planar frame when subjected to the force P

Exercise #2:

Using the quartic polynomial, $v = a_1 + a_2x + a_3x^2 + a_4x^3 + a_5x^4$, construct the shape function for the illustrated flexural element.

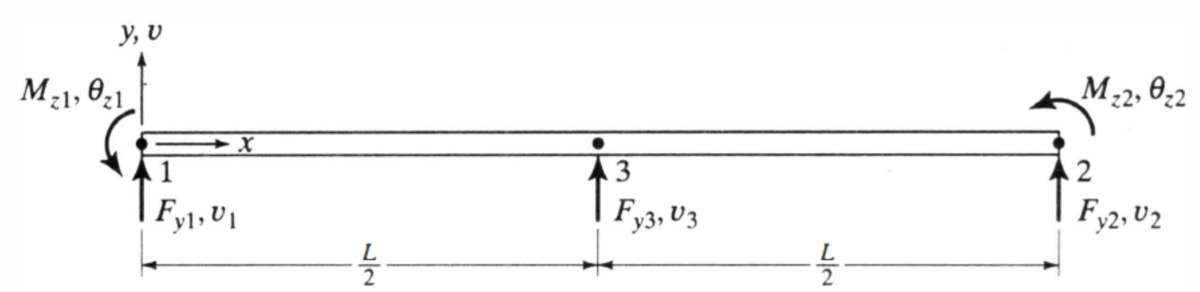

Figure 2. Flexural element

Exercise #3:

Formulate the stiffness matrix for a four-node torsional beam-column element (Young's modulus, E and shear modulus, G), using for the rotational displacement θ_x the following shape function,

$$\theta_x = N_1\theta_{x1} + N_2\theta_{x2} + N_3\theta_{x3} + N_4\theta_{x4}$$

Where,

$$N_1 = \frac{(L-x)(2L-x)(3L-x)}{6L^3} \qquad N_3 = \frac{x(L-x)(x-3L)}{2L^3}$$

$$N_2 = \frac{x(2L-x)(3L-x)}{2L^3} \qquad N_4 = \frac{x(L-x)(2L-x)}{6L^3}$$

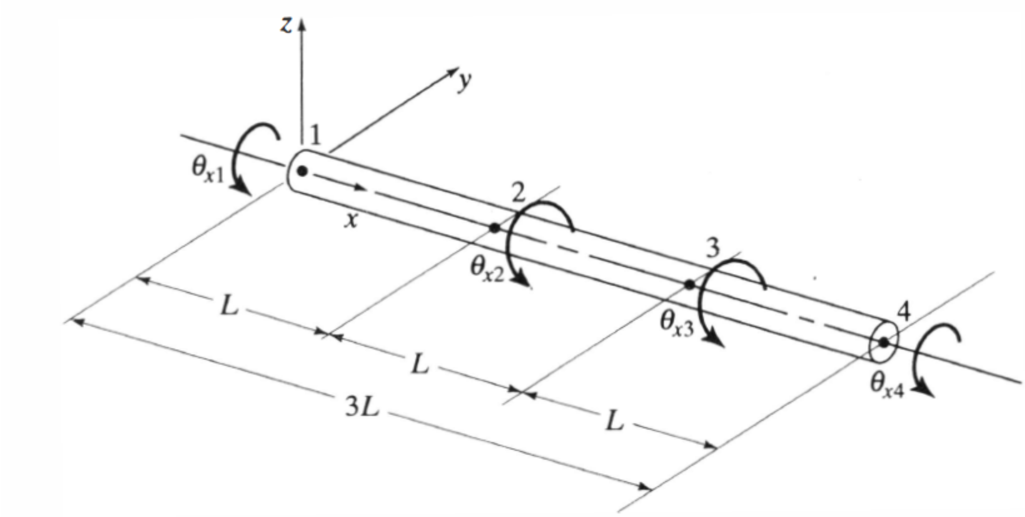

Figure 2. Beam-column under torsion

It can be proven that the stiffness matrix of a uniform beam-column element of length L under uniform torsion is as follows:

$$[k] = \int_0^L \{N'\} G \lfloor N' \rfloor J dx$$

Where J is the torsional constant of the cross section