Thermo-Pile

A software for the geotechnical design of energy piles

THEORETICAL ASPECTS

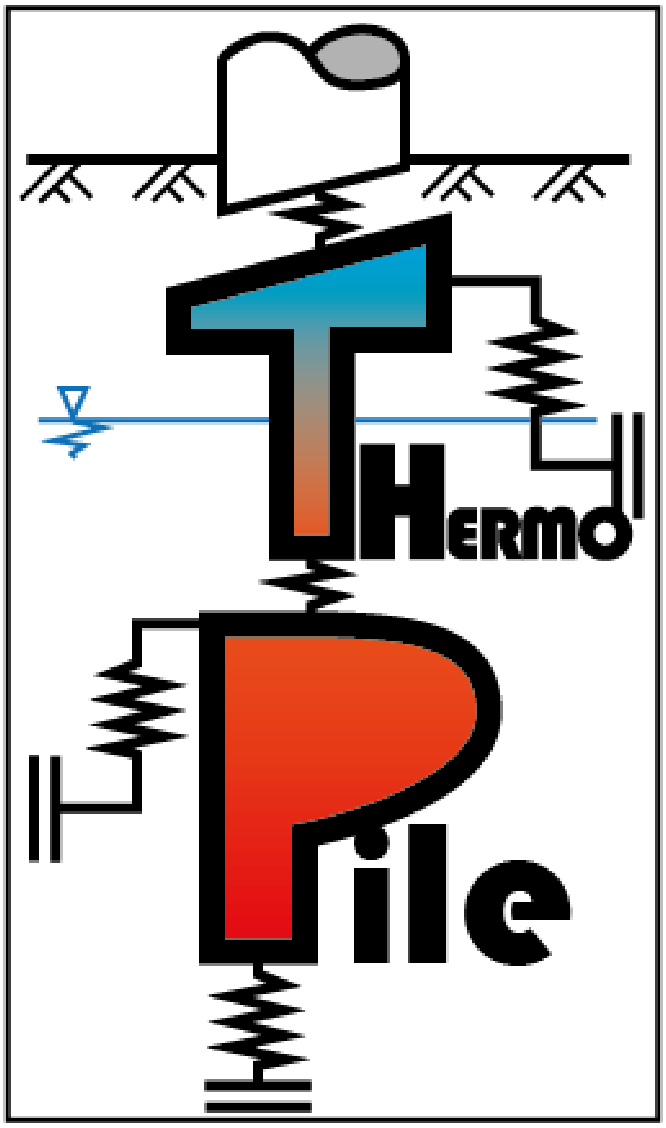

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TABLE OF CONTENT

1 INTRODUCTION

This document is a complement to the user's manual of the software *Thermo-Pile*, and contains the theoretical aspects. The chapters are related to the different theories used by the software to assess the ultimate bearing capacity and the pile soil interactions. A description of the code is given in the last chapter, detailing how the pile mechanical response to building weight and thermal loading is computed.
The theoretical concepts are presented in the paper: Knellwolf C., Peron H. and Laloui L. "Geotechnical analysis of heat exchanger piles". Journal of Geotechnical and Geoenvironmental Engineering, doi: 10.1061 / (ASCE) GT.1943-5606.0000513, 2011.
In the following, the main used parameters are described.

2 BEARING CAPACITY AFTER LANG & HUDER THEORY

2.1 ULTIMATE BEARING CAPACITY AT THE BASE OF THE PILE

Granular soil:

$$q_b = \left(c' N_c + \sigma'_v \cdot N_q\right)\chi \tag{1}$$

$$N_q = \exp\left(\pi \cdot \tan\varphi'\right) \cdot \tan^2\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right) \tag{2}$$

$$N_c = (N_q - 1)tan^{-1}(\varphi') \tag{3}$$

q_b	Ultimate bearing capacity
N_q	Bearing factor
N_c	Bearing factor
χ	Correction factor depending on the shape and length of the pile
c	Cohesion
φ'	Internal friction angle
σ'_v	Vertical stress in the soil at pile base level

2.2 ULTIMATE SHEAR STRESS

Granular soil:

$$q_s = c' + \sigma'_v \cdot k \cdot \tan\delta \tag{4}$$

In the code, k is set to k_0 which reads:

$$k_0 = 1 - \sin\varphi' \tag{5}$$

δ	Interface friction angle
k	Ratio between the horizontal and vertical soil stresses

3 BEARING CAPACITY AFTER DOCUMENT TECHNIQUE UNIFIÉ (DTU)

3.1 ULTIMATE BEARING CAPACITY AT THE BASE OF THE PILE

<u>Granular soil:</u>

$$q_b = 50N_q + \lambda c' N_c \tag{6}$$

$$N_q = 10^{3.04 \tan\varphi'} \tag{7}$$

$$N_c = (N_q - 1)tan^{-1}(\varphi') \tag{8}$$

$\lambda = 1 + 0.3$ for a pile with a cross section of a circle

q_b Ultimate bearing capacity
N_q Bearing factor
N_c Bearing factor
λ Factor of shape of the pile

3.2 ULTIMATE SHEAR STRESS;

<u>Analytic, Granular soil:</u>

$$q_s = k\sigma'_v \tan\delta \tag{9}$$

q_s Ultimate shear friction
k Ratio between the horizontal and vertical stresses in soil
δ Interface friction angle

k varies for cast-in-place piles from k_a to k_0 and for precast piles from k_0 to k_a. In the code, the value $k = k_0$ is used.

$k_0 = 1 - \sin\varphi'$

φ' Internal friction angle

<u>Empiric</u>

q_s is directly defined by the user

4 SOIL PILE INTERACTION, MODIFIED AFTER FRANK AND ZHAO 1982

4.1 MENARD PRESSUREMETER ELASTIC MODULUS

The Menard pressuremeter modulus E_M is used in the code to compute the stiffness at the pile / soil interaction. It is related to the modulus measured in an oedometer test E_{oed}, by the rheological coefficient α:

$$E_{oed} = E_M / \alpha \tag{10}$$

Table 1 – Values of the rheological coefficient.

Type	Turf α	Clay α	Silt α	Sand α	Cobble α
Normally consolidated / normally dense	1	2/3	1/2	1/3	1/4

(Source: Amar and Jézéquel 1998, Amar et al. 1991)

4.2 T_S – Z AND T_B – Z CURVES

The shapes of the curves of the mobilized shaft friction and base reaction, respectively, versus pile displacements as proposed by Frank and Zhao (1982) are illustrated in Fig. 1. The shapes of the curves, with two linear parts and a plateau equal to the ultimate value, conform to the behaviour often observed in pile in-situ loading tests (Frank and Zhao 1982). The slopes of first linear branches of the load transfer curves are related to the Menard pressure meter modulus E_M. The following empirical expressions are elaborated for fine-grained soils and weak rocks (clays, marls and limestone):

$$K_s = \frac{2E_M}{D} \quad \text{and} \quad K_b = \frac{11E_M}{D} \tag{11}$$

The same parameters for granular soils read:

$$K_s = \frac{0.8E_M}{D} \quad \text{and} \quad K_b = \frac{4.8E_M}{D}. \tag{12}$$

D stands for the pile diameter. K_s and K_b are the slopes of the shaft and base load transfer functions for the first linear branches, respectively (i.e., the stiffness of the shaft and base springs in Fig. 1).

In the present software, an unloading branch has been added with regards to the original theory, accounting for the irreversible behaviour of the soil. The slopes of the unloading branch are K_s and K_b, for shaft friction and the reaction at the base, respectively. Therefore, only the first branch of the load transfer curves defines a linear elastic behaviour.

The ultimate shaft friction q_s and the ultimate bearing capacity q_b define the plateau of the load transfer curves. They can be empirically or analytically related to the soil parameters via conventional methods (Lang and Huder 1978, Legrand et al. 1993 see chapter 1.1 and 1.2).

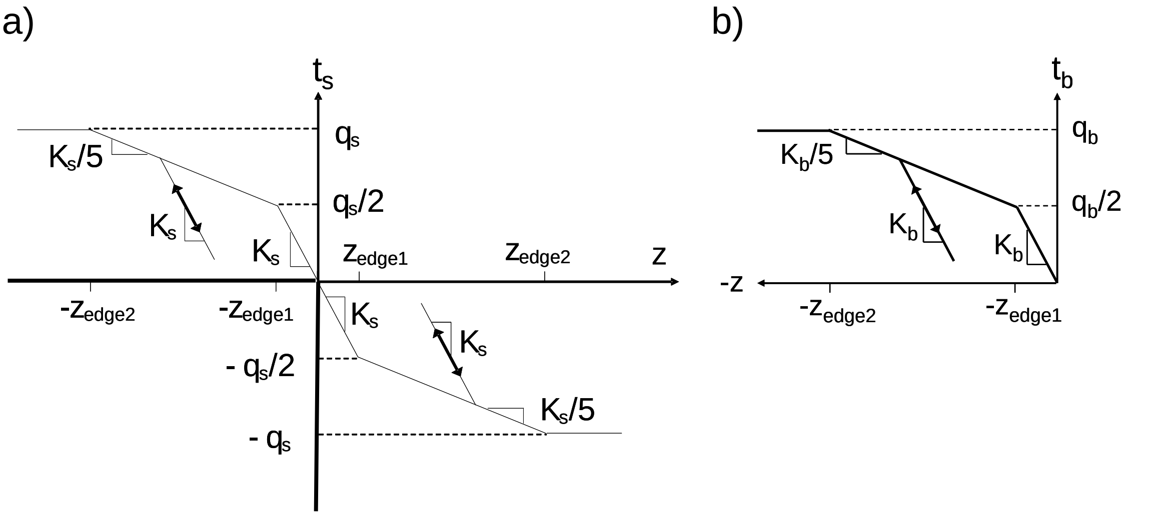

Figure 1- Load transfer curves modified after Frank and Zhao (1982). a) Evolution of the mobilized shaft friction t_s with respect to pile displacements. b) Evolution of the mobilized reaction on the base of the pile t_b with respect to pile displacements

5 CALCULATION METHOD

5.1 BASIC ASSUMPTIONS

The method relies on the following basic assumptions:

1. The pile displacement calculation is done using a one dimensional finite difference scheme (only the axial displacements are considered).
2. The properties of the pile, namely its diameter D, Young modulus E_{pile} and coefficient of thermal expansion α, remain constant along the pile and do not change with temperature.
3. The relationships between the shaft friction/shaft displacement, head stress/head displacement and base stress/base displacement are known.
4. The soil and soil-pile interaction properties do not change with temperature.

Upwards movements are taken as positive; downwards movements are negative. Tensile stresses are taken as positive.

For the sake of clarity, the following notation conventions are adopted: capital letters are for forces while lower case letters are for stresses. The letter T is used for the forces applied by the soil along the pile shaft and at the pile base and the force induced by the upper structure at the pile head. Q stands for the ultimate values of lateral friction and bearing capacity at the pile base. P stands for the weight of the building. The indexes “*mec*” and “*th*” indicate the mechanical and thermal origins of the reactions forces, respectively. Another set of indexes is used to specify where a reaction acts: index “*b”* is for the pile base, index “*h”* is for the pile head and index “*s”* is for the pile shaft.

5.2 PILE DISPLACEMENT CALCULATION

The pile displacement for a mechanical load P is done by the load transfer method (Seed and Reese 1957, Coyle and Reese 1966). In this method, the pile is subdivided into several rigid elements, which are connected by springs representing the pile stiffness. Each rigid element experiences along its side an elasto-plastic interaction with the surrounding soil. The pile base element is supported by the reaction of the substrate, the pile/soil interaction being elasto-plastic as well (Fig 2a). The relation between the shaft friction and pile displacements, as well as the relation between the normal stresses and the pile displacements at the base, are described by load transfer functions. Such a discretization of the system enables to consider various soil layers with distinct properties and/or the variation of the soil properties with depth.

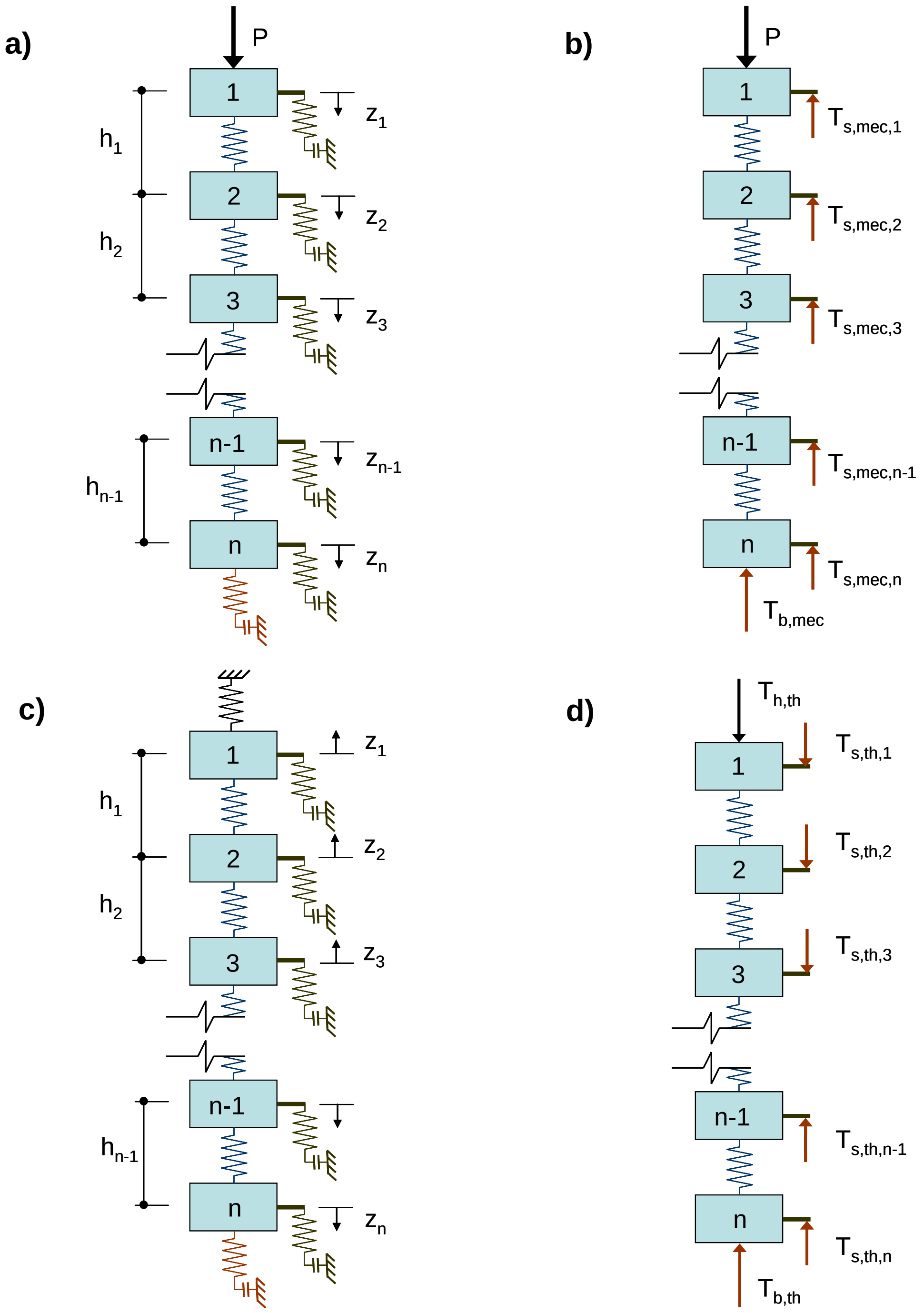

Figure 2 Finite difference model for heat exchanger pile load and displacement computation. a) Model for mechanical load (z_i: displacement of pile segment i). b) External forces $T_{s,mec,i}$ and $T_{b,mec}$ mobilized by mechanical loading. c) Model for thermal loading (z_i: displacement of pile segment i). d) External forces $T_{h,th}$, $T_{s,th,i}$ and $T_{b,th}$ mobilized by thermal loading.

In the case of thermal loading, the heat exchanger pile moves, while the weight *P* of the upper structure remains unchanged. The originality of the present approach is to introduce an additional spring linked to the pile head element (Fig. 2c), which represents the restraining effect of the upper structure. This additional spring is considered only when a thermal loading is applied.

For the load transfer functions, one can use the curves proposed by Frank and Zhao (1982) or manually defined curves with three branches and a plateau.

The load transfer curves describe the mobilized stress or a given displacement. Fig. 1 shows the load transfer curves used for the soil-pile interaction, $t_{s,i}$ - z and $t_{b,i}$ - z. The pile-supported structure interaction is considered linear elastic as well and is represented by the spring constant K_h,: $t_h = K_h \cdot z_1$. K_h ultimately depends on many factors, such as the supported structure rigidity, the type of contact between the pile and the foundation raft, the position and the number of heat exchanger piles.

The mobilized external forces $T_{s,i}$, $T_{h,i}$ and $T_{b,i}$, as illustrated in Fig. 2b and 2d, are obtained by multiplying the considered stress by the surface on which it is acting.

The springs between two adjacent pile elements represent the rigidity of the pile. The pile behaviour is considered linear and elastic. The rigidity $K_{pile,n}$ of a given spring connecting two elements of the pile of length h_n is therefore (one dimension hypothesis) $K_{pile,n} = E_{pile} / h_n$.

On the basis of the soil-pile interaction models defined above, the calculation of the thermo-mechanical response of the heat exchanger pile is made as follows. First, the stress state and the pile displacements induced by the imposed mechanical loading are calculated; this state is further referred to as the initialization state and corresponds to effects due to the weight of the building. Then, from the initialization state, the pile response due to the thermal loading (heating or cooling occurring during heat exchange) is calculated.

5.2.1 Initialization by mechanical loading

The displacement of the pile under a given mechanical load P is computed with the load transfer method, as described by Coyle and Reese (1966). The element i, of length h_i, diameter D and section A, is sketched in Fig. 3. $F_{B,i}$ is the axial force acting at the element base, $F_{M,i}$ is the axial force acting in the middle and $F_{H,i}$ is the axial force acting at the element head; the respective axial displacements are $z_{B,i}$, $z_{M,i}$ and $z_{H,i}$. $t_{s,i}$ is the average side friction of the element.

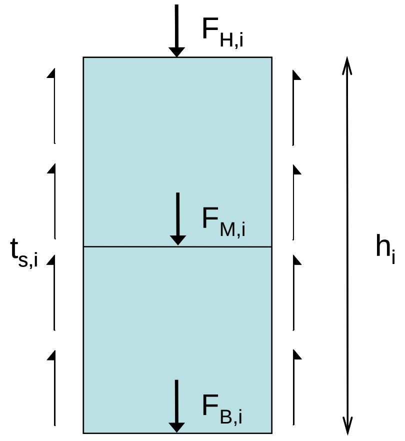

Figure 3- Sketch of a pile element. $F_{B,i}$ is the axial force acting at the bottom, $F_{M,i}$ the is the axial force acting in the middle, $F_{H,i}$ is the axial force acting at the pile head and $t_{s,i}$ is the average shaft friction.

Knowing the value of $F_{B,i}$ and assuming a constant shear stress along the lower half side of the element, direct application of Hooke's law yields a relative displacement Δz in the middle of element i :

$$\Delta z_i = \frac{F_{B,i} + F_{M,i}}{2} \cdot \frac{1}{AE} \cdot \frac{h_i}{2} = \left(F_{B,i} + \frac{1}{2} \cdot \frac{h_i D\pi}{2} \cdot t_{s,i}(z_{M,i}) \right) \cdot \frac{1}{AE} \cdot \frac{h_i}{2} \tag{13}$$

With $\Delta z_i = z_{M,i} - z_{B,i}$, one obtains:

$$z_{M,i} = z_{B,i} + \left(F_{B,i} + \frac{1}{2} \cdot \frac{h_i D\pi}{2} \cdot t_{s,i}(z_{M,i}) \right) \cdot \frac{1}{AE} \cdot \frac{h_i}{2} \tag{14}$$

To obtain the value of $z_{M,i}$, Equation 14 is solved iteratively until the required precision is reached. Once $z_{M,i}$ is known, one can deduce the axial force in the middle of the element, $F_{M,i}$:

$$F_{M,i} = F_{B,i} + \frac{h_i D\pi}{2} \cdot t_{s,i}(z_{M,i}) \tag{15}$$

The force $F_{H,i}$, at the head of the element, is finally:

$$F_{H,i} = F_{B,i} + 2(F_{M,i} - F_{B,i}) \tag{16}$$

While $z_{H,i}$, the corresponding displacement, is:

$$z_{H,i} = z_{B,i} + \frac{F_{M,i}}{AE} h_i \tag{17}$$

The displacement at the head of an element is then used as the bottom displacement of the upper element. The procedure is repeated for each successive element up to the head of the pile.

The initial displacement Δz_n at the base of the pile has to be chosen such that the axial force $F_{H,1}$ of the element at the head of the pile is equal to the weight P transferred by the building. The equilibrium of external forces T_b, $T_{s,i}$ and P (see Fig. 2b) is therefore verified:

$$T_b + \sum_{i=1}^{n} T_{s,i} + P = 0 \tag{18}$$

Knowing the value of the axial forces from the above calculation, the strain due to mechanical loading ε_{mec} can be derived.

5.2.2 Thermal loading

When a pile is heated or cooled, it dilates or contracts about a null point (Bourne-Webb et al. 2009). For instance in Fig. 2c and 2d, this specific point is located between elements 3 and n-1. Actually, the null point is situated at that depth NP where the sum of the mobilized friction along the upper part plus the reaction of the structure is equal to the sum of the mobilized friction along the lower part plus the reaction at the base. According to the notations in Fig. 2, this can be expressed by:

$$\sum T_{th,NP} = \sum_{i=1}^{NP} T_{s,th,i} + T_{h,th} + \sum_{i=NP+1}^{n} T_{s,th,i} + T_{b,th} = 0 \tag{19}$$

The position of the null point is found by summation of the external forces listed in Equation 18. The position with the sum closest to zero is taken as null point. The accuracy of calculation depends on the refinement of the finite difference scheme, defined by the user.

Case without mechanical loading

The particular case of heating and cooling without any mechanical load is considered first. In this case, there is no strain prior to temperature change, and the initialization state remains at the origin of the load transfer curves. In order to assess the blocked strain, an iterative procedure is applied following the method described below.

Step 1) Choice of a starting value for the observed deformation

To compute a first set of mobilized resistance (mobilized shaft friction and resistance at the extremities), the pile is initially assumed to be totally free to move. The first displacement calculations are therefore done with $\varepsilon_{th} = \varepsilon_{th,f} = \alpha.\Delta T$.

Step 2) Displacement calculation

By definition, there is no displacement at the null point (thus, $z_{th,NP} = 0$). The displacements $z_{th,i}$ of the upper elements NP-1 to 1 are $z_{th,i} = z_{th,i+1} + h_i \cdot \varepsilon_{th,i}$, while for the lower elements NP+1 to n they are: $z_{th,i} = z_{th,i-1} - h_i \cdot \varepsilon_{th,i}$

Using the t-z curve, a first set of mobilized reaction stresses is obtained. The axial stress in the pile $\sigma_{th,i}$ induced by the thermal free displacement of the pile is the sum of all the external forces divided by the pile section *A*. The summation begins at the base of the pile; j goes from *n* to i.

$$\sigma_{th,i} = \left(T_{th,b} + D\pi \sum_{j=n}^{i} h_j . t_{th,s,j} \right) \frac{1}{A} \tag{20}$$

Step 3) From the mobilized stress, one obtains the blocked thermal strain $\varepsilon_{th,b}$

$$\varepsilon_{th,b} = \frac{\sigma_{th}}{E} \leq \varepsilon_{th,f} \tag{21}$$

Step 4) By subtracting the blocked from the free strain, the observed strain $\varepsilon_{th,o} = \varepsilon_{th,f} - \varepsilon_{th,b}$ is obtained.

Steps 2 to 4 must be repeated with the new set of observed strains $\varepsilon_{th}=\varepsilon_{th,o}$. By repeating steps 2 to 4, the observed strain will converge to the actual values of the blocked and observed strain. Related parameters, such as pile displacement, internal axial stresses, mobilized shaft friction and mobilized reaction at the base and head of the pile, are then deduced.

Case with mechanical loading

In this case, the thermal displacement calculation (step 2 above) is calculated from a non-zero initialized displacement and strain state induced by the mechanical loading. In the case of unloading (uplift), the stress path follows the unloading branch.

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