Problem Set 4

CIVIL-425: Continuum Mechanics and Applications

27 March 2024

Exercise 1, Principal directions and stretches

Consider a local deformation gradient $\mathbf{F}$. Let the spectral decomposition of the right Cauchy-Green deformation tensor be:

$$\mathbf{C} = \sum_{\alpha=1}^{3} c_\alpha \mathbf{N}_\alpha \otimes \mathbf{N}_\alpha$$

here c_α and $\mathbf{N}_\alpha, \alpha = 1, 2, 3,$

$$(\mathbf{C} - c_\alpha \mathbf{I}) \cdot \mathbf{N}_\alpha = 0, \quad \|\mathbf{N}_\alpha\| = 1, \quad \alpha = 1, 2, 3$$

Let $\lambda_\alpha = \sqrt{c_\alpha}$ be the principal stretches; $\mathbf{F} = \mathbf{V}\mathbf{R} = \mathbf{R}\mathbf{U}$ the polar decompositions of $\mathbf{F}$, the $\mathbf{R} \in SO(3), \mathbf{U} = \mathbf{U}^T = \sqrt{\mathbf{C}}, \mathbf{V} = \mathbf{V}^T = \sqrt{\mathbf{B}}$; and set $\mathbf{n}_\alpha = \mathbf{R}\mathbf{N}_\alpha$. Prove the identities:

i.1) $\mathbf{n}_\alpha = \lambda_\alpha^{-1} \mathbf{F}\mathbf{N}_\alpha$.

i.2) $\mathbf{F} = \sum_\alpha \lambda_\alpha \mathbf{n}_\alpha \otimes \mathbf{N}_\alpha$.

i.3) $\mathbf{R} = \sum_\alpha \mathbf{n}_\alpha \otimes \mathbf{N}_\alpha$.

Exercise 2, Linearization

The volumetric-deviatoric decomposition of the deformation gradients:

$$\mathbf{F} = \mathbf{F}^{\text{vol}} \mathbf{F}^{\text{dev}}, \quad \mathbf{F}^{\text{vol}} \equiv J^{1/3} \mathbf{I}, \quad \mathbf{F}^{\text{dev}} \equiv J^{-1/3} \mathbf{F}$$

where J is the Jacobian of the deformation and $\mathbf{I}$ is the identity tensor. Linearize with respect to a small displacement $\mathbf{u}$ field superposed on the spatial configuration.