Problem Set 9

CIVIL-425: Continuum Mechanics and Applications

08 May 2025

Exercise 1: Traction test of Elastomer Specimen

We are interested in studying the response of an incompressible elastomeric specimen subjected to homogeneous deformations. We consider a specimen whose reference underformed configuration is a cube, whose edges of length L are directed along the axes. The specimen is an elastomer whose constitutive law has the form:

$$\underline{\underline{\sigma}} = -p\underline{\underline{\mathbb{I}}} + f(I_1)\underline{\underline{F}} \cdot \underline{\underline{F}}^T$$

with

$$\begin{cases} f(I_1) = \mu = \rho c k_B T & \text{Neo-Hookean model} \\ f(I_1) = \mu \left(3\sqrt{\frac{I_1}{3N}}\right)^{-1} \mathcal{L}^{-1}\left(\sqrt{\frac{I_1}{3N}}\right) & \text{8-chain model} \end{cases}$$

where ρ is the mass density, $I_1 = (\boldsymbol{C} : \boldsymbol{I})$ is the first strain invariant and $\mathcal{L}$ is the Langevin function defined as $\mathcal{L}(x) = \coth x - 1/x$. It is assumed that the deformations are quasi-static, isothermal, and that the volumetric strains are negligible.

We consider that the specimen is subjected to an extension by a factor of α in the direction $\underline{e}_1$ while its lateral faces (with external normal $\underline{e}_2$ and $\underline{e}_3$) are stress-free. By denoting β as the contraction factor in the directions $\underline{e}_2$ and $\underline{e}_3$, we are interested in transformations of the form:

$$\underline{\phi}(\underline{X}) = \alpha X_1 \underline{e}_1 + \beta\left(X_2 \underline{e}_2 + X_3 \underline{e}_3\right)$$

1. Explicit the deformation gradient tensor $\underline{\underline{F}}$ and show that $\beta = \frac{1}{\sqrt{\alpha}}$. Calculate $I_1 = \underline{\underline{C}} : \underline{\underline{\mathbb{I}}}$.
2. Determine the Cauchy stress tensor using the local equilibrium and the boundary conditions. Is p free to vary?
3. Deduce the resultant $\underline{R}$ of the force exerted on the deformed face from the face $(X_1 = L)$ in the initial configuration as a function of α, then plot $\frac{R \cdot e_1}{L^2}$. Consider the change of area from reference to deformed configuration.