Problem Set 8

CIVIL-425: Continuum Mechanics and Applications

01 May 2025

Exercise 1: Thermodynamic potentials

Consider a thermoelastic material in the Lagrangian description, with internal energy per unit mass:

$$U(\boldsymbol{C}, N, \boldsymbol{Q}),$$

where:

- $\boldsymbol{C}$ is the right Cauchy-Green deformation tensor,
- N is the entropy per unit mass,
- $\boldsymbol{Q}$ is a vector of internal variables.

(a) Define the thermodynamic potentials: Helmholtz free energy $A(\boldsymbol{C}, T, \boldsymbol{Q})$, enthalpy $H(\boldsymbol{S}^e, N, \boldsymbol{Q})$, and Gibbs free energy $G(\boldsymbol{S}^e, T, \boldsymbol{Q})$ via Legendre transforms of the internal energy $U(\boldsymbol{C}, N, \boldsymbol{Q})$.

(b) Derive expressions for A, H, and G, and then compute H from G, and A from H.

(c) Let the internal variable be a scalar damage variable $d \in [0, 1]$. Let the Helmholtz free energy per unit mass be:

$$A(\boldsymbol{C}, T, d) = \frac{1}{2R}(1-d)^2 \boldsymbol{C} : \mathbb{C}_0 : \boldsymbol{C} - \left[c\,T \ln\left(\frac{T}{T_0}\right) - c(T - T_0) \right].$$

- Write the corresponding thermodynamic forces
- State the dissipation inequality for this material.