Problem Set 1

CIVIL-425: Continuum Mechanics and Applications

29 February 2024

Exercise 1, universal static deformations

Look at example 1.1.5 from Ortiz's notes on universal static deformations. Code an interactive notebook with your preferred language (Jupyter notebook, Matlab, Mathematica) to visualize the deformations while changing the parameters.

Exercise 2, cylinder inside out

A cylindrical tube of inner radius a, outer radius b, and length L is turned inside out and subsequently constrained to take the shape of a straight cylindrical tube of the same dimensions. Assuming that the radial fibers do not undergo any stretching, determine the deformation mapping φ. (Hint: use cylindrical coordinates). Show that $\varphi \circ \varphi$ is the identity mapping, i.e. turning the cylinder inside out twice returns it to its initial configuration.

Exercise 3, plate to cylinder

A plate of thickness h, width L and infinite length is bent into a cylindrical shell section (plane strain conditions, see Figure 1). Assume that the plane $X_1 = 0$ remains unstretched, i.e. is the neutral plane. Determine the deformation mapping φ. Find under what conditions the deformation mapping ceases to be invertible.

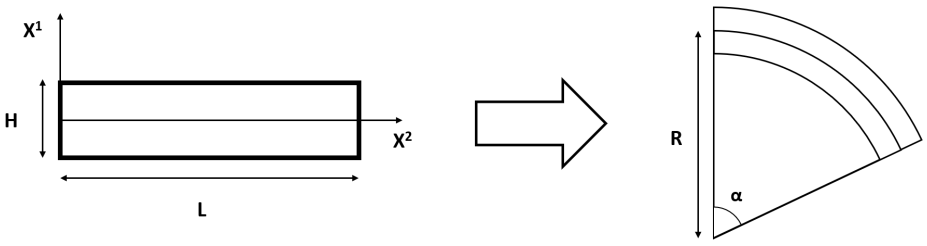

Figure 1: Sketch of deformation of the plate

Exercise 4, composition of mappings

Let $B_0 = [0, a] \times [0, a]$, i.e. a square of side a and let $\varphi_1(X) = \{X_1, \lambda X_2, X_3\}$ (uniaxial stretching) and $\varphi_2(X) = \{X_1 + X_2 \tan\alpha, X_2, X_3\}$ (pure shear). Carry out the compositions of mappings $\varphi_2 \circ \varphi_1$ and $\varphi_1 \circ \varphi_2$. Plot and compare the results. Does composition of mappings commute?

Exercise 5, bug walking on stretching wire

A bug walks on a stretching wire with speed V relative to the material point of the wire on which the bug stands. The bug starts at the pinned end at $t = 0$. The free end of the wire moves with velocity V_0. The initial length of the wire is L. Find the time required for the bug to reach the free end of the wire.

1. write the relationship between the spatial and reference configuration for the stretching wire
2. write the material velocity of a point of the wire
3. write the equation of motion for the bug, with spatial coordinate $x_b(t)$
4. given the solution $x_b(t) = \frac{V}{V_0}(L + V_0 t) \ln\left(\frac{L+V_0 t}{L}\right)$, find the time required for the bug to reach the free end of the wire