Problem Set 10

CIVIL-425: Continuum Mechanics and Applications

15 May 2025

Exercise 1: Constitutive laws

For the following strain-energy functions, write: a) the stress-strain relations in material form (i. e., in terms of the second PK stress tensor $\mathbf{S}$ and the right Cauchy-Green deformation tensor $\mathbf{C}$) and in spatial form (i. e., in terms of the Cauchy stress tensor $\boldsymbol{\sigma}$ and the left Cauchy-Green deformation tensor $\mathbf{C}$); b) the material and spatial elastic moduli D_{IJKL} and D_{ijkl}, respectively; c) the relation between the Cauchy stress σ and the stretch λ in uniaxial tension. If possible, plot the uniaxial stress against the logarithm of the stretch (i. e., the logarithmic strain). Is the predicted stress-strain behavior sensible?

1. Ideal compressible fluid:
$$W = W(J)$$
2. Saint Venant-Kirchhoff solid:
$$W = \frac{1}{2}\left[\lambda_0 \left(E_{KK}\right)^2 + 2\mu E_{IJ}E_{IJ}\right]$$
3. Neo-Hookean solid:
$$W = \frac{\lambda}{2}\log^2(J) - \mu\log(J) + \frac{\mu}{2}I_1$$
4. *Bonus:* Compressible Mooney-Rivlin solid:
$$W = \frac{\lambda}{2}\log^2(J) - \mu\log(J) + \frac{1}{2}\left[\alpha_1\left(I_1 - 3\right) + \alpha_2\left(I_2 - 3\right)\right]$$

In the above expressions, λ, μ, α_1 and α_2 are constants. Consider the case $\alpha_1 = \alpha_2 = \mu$.