FORMULAE – LECTURE 5

Hardening Elasto-Plasticity

Basic ingredients:

1. Strain decomposition $$\delta\varepsilon_{ij} = \delta\varepsilon_{ij}^{e} + \delta\varepsilon_{ij}^{p}$$
2. Elastic relationship $$\sigma'_{ij} = D_{ijhk}^{e}\varepsilon_{hk}^{e}$$
3. Yield function $$F = F(\sigma_{ij}, p_k) \quad p_k = p_k(\varepsilon_{ij}^{p}) \longrightarrow F = F(\sigma_{ij}, \varepsilon_{ij}^{p})$$
4. Plastic potential function $$g(\sigma_{ij}, r_k) \qquad \delta\varepsilon_{ij}^{p} = \mu\frac{\partial g}{\partial\sigma_{ij}}$$

Consistency condition
$$dF = \left.\frac{\partial F}{\partial\sigma_i}\right|_{p_k}\delta\sigma_i + \left.\frac{\partial F}{\partial p_k}\right|_{\sigma_i}\frac{\partial p_k}{\partial\varepsilon_i^{p}}\delta\varepsilon_i^{p} = 0$$

Plastic multiplier
$$\mu = \frac{1}{H}\frac{\partial F}{\partial\sigma_i}\delta\sigma_i = \left(\frac{\frac{\partial F}{\partial\sigma_i}D_{ij}^{e}}{H + \frac{\partial F}{\partial\sigma_i}D_{ij}^{e}\frac{\partial g}{\partial\sigma_j}}\right)\delta\varepsilon_j$$

Hardening modulus
$$H = -\frac{\delta F}{\delta p_k}\frac{\delta p_k}{\delta\varepsilon_j^{p}}\frac{\delta g}{\delta\sigma_j}$$

General incremental stress-strain relationship for plastic strain isotropic hardening elasto-plasticity
$$\delta\sigma_i = \underbrace{\left(D_{ij}^{e} - \frac{D_{ik}^{e}\frac{\partial g}{\partial\sigma_l}\frac{\partial F}{\partial\sigma_k}D_{kl}^{e}}{H + \frac{\partial F}{\partial\sigma_i}D_{ij}^{e}\frac{\partial g}{\partial\sigma_j}}\right)}_{D_{ij}^{ep}}\delta\varepsilon_j$$