

FORMULAE – LECTURE 4

Perfect plasticity model – Basic ingredients

1. Strain decomposition $\delta \varepsilon_{ij} = \delta \varepsilon_{ij}^e + \delta \varepsilon_{ij}^p$
2. Elastic relationship $\sigma'_{ij} = D_{ijhk}^e \varepsilon_{hk}^e$
3. Yield function $F = F(\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}, p_k)$
4. Plastic potential function $g(\sigma_{ij}, r_k) \quad \delta \varepsilon_{ij}^p = \mu \frac{\partial g}{\partial \sigma_{ij}}$

Von Mises

$$F = J_{2D} - k^2$$

Drucker-Prager

$$F = \sqrt{J_{2D}} - \alpha \cdot J_1 - k \quad \alpha = \frac{2 \cdot \sin \phi}{\sqrt{3} \cdot (3 - \sin \phi)} \quad k = \frac{6 \cdot c \cdot \cos \phi}{\sqrt{3} \cdot (3 - \sin \phi)}$$

Mohr-Coulomb

$$\tau_f = c' + \sigma_n' \tan \varphi'$$

$$F = -\frac{\sigma_1 - \sigma_3}{2} + \frac{\sigma_1 + \sigma_3}{2} \cdot \sin \phi + c \cdot \cos \phi = 0$$

$$F = J_1 \cdot \sin \phi + \sqrt{J_{2D}} \cdot \cos \theta - \frac{\sqrt{J_{2D}}}{3} \cdot \sin \phi \cdot \sin \theta - c \cdot \cos \phi = 0$$

$$\theta = -\frac{1}{3} \cdot \sin^{-1} \left(-\frac{3 \cdot \sqrt{3}}{2} \cdot \frac{J_{3D}}{J_{2D}^{3/2}} \right)$$

$$q_f = \frac{6 \sin \varphi'}{3 - \sin \varphi'} p_f' + \frac{6 \cos \varphi'}{3 - \sin \varphi'} c' = M p_f' + a'$$

$$q_f = -\frac{6 \sin \varphi'}{3 + \sin \varphi'} p_f' - \frac{6 \cos \varphi'}{3 + \sin \varphi'} c' = M_{ext} p_f' + a'_{ext}$$

Mohr-Coulomb elasto-perfectly plastic model

1. Strain decomposition $\delta \varepsilon_{ij} = \delta \varepsilon_{ij}^e + \delta \varepsilon_{ij}^p$
2. Elastic relationship $\begin{pmatrix} \delta p' \\ \delta q \end{pmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{pmatrix} \delta \varepsilon_{vol}^e \\ \delta \varepsilon_q^e \end{pmatrix}$
3. Yield function $F(p', q, p_k) = q - Mp'$
4. Plastic potential function $g(p', q, r_k) = q - M^*p' + k$

$$\begin{pmatrix} \delta p' \\ \delta q \end{pmatrix} = \frac{3GK}{MKM^* + 3G} \begin{bmatrix} 1 & M^* \\ M & MM^* \end{bmatrix} \begin{pmatrix} \delta \varepsilon_v \\ \delta \varepsilon_d \end{pmatrix}$$