FACULTE ENVIRONNEMENT NATUREL, ARCHITECTURAL ET CONSTRUIT ENAC
INSTITUT D'INGÉNIERIE CIVILE IIC
Laboratoire des Structures Métalliques Résilientes RESSLab

Téléphone : +41 21 693 24 27
Fax : +41 21 693 28 68
E-mail : dimitrios.lignos@epfl.ch
Site web : http://resslab.epfl.ch
Address: EPFL ENAC IIC RESSLAB
GC B3 485, Station 18,
CH-1015, Lausanne

Exercise #8: Buckling analysis of wall-frame structures

Problem 1

Figure 1 shows the plan view of a 8-storey tube-in-tube structure. The inner and outer tube represent the wall system and the frame system, respectively. The storey height is $3.20m$. The inner tube wall thickness is $0.20m$ with a modulus of elasticity $E_w = 2.5x10^7 kN/m^2$. The outer tube consists of $0.30x0.50m^2$ column cross-sections and $0.30x0.70m^2$ girder cross-sections and a modulus of elasticity $E = 3.0x10^7 kN/m^2$.

Determine the buckling loads of the first storey of the structure and verify if the structure is safe against P-Delta gravity effects. The structure's dead and live load together is $12kN/m^2$.

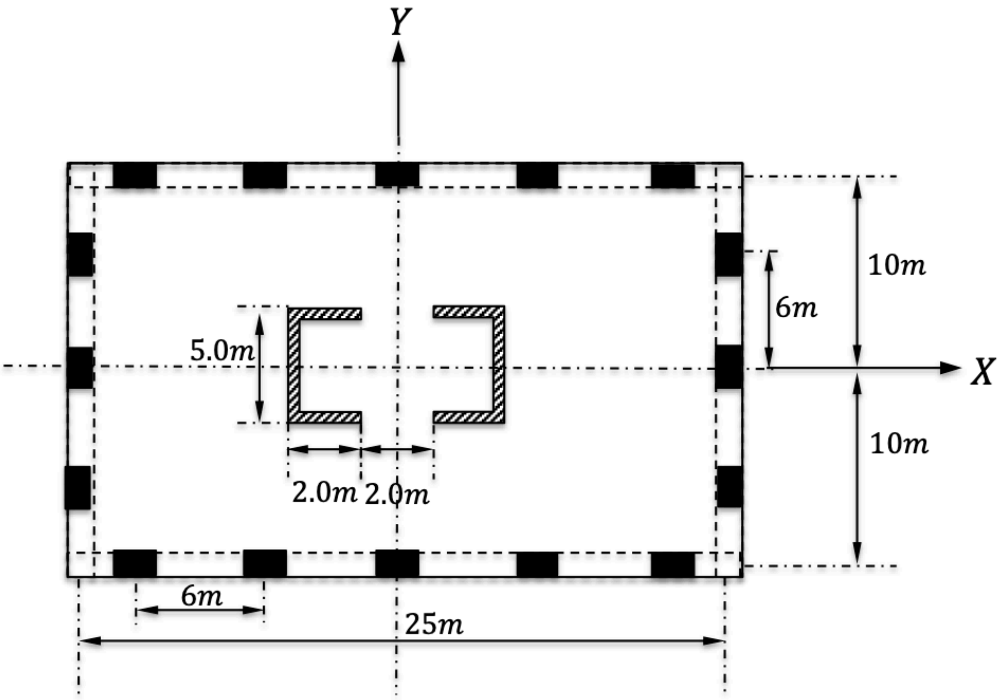

Figure 1. Tube-in-tube structure

Solution

Step 1: Member Properties

-Inertia of a single wall in the X direction: $I_{X,wall} = \frac{200 \cdot 2000^3}{12} = 1.33 \cdot 10^{11} mm^4$

-Inertia of a single wall in the Y direction: $I_{Y,wall} = \frac{200 \cdot 5000^3}{12} = 2.08 \cdot 10^{12} mm^4$

The inertia of the walls with respect to their weak axis is neglected; hence we assume that the walls do not resist lateral loads with their weak axis (common assumption).

-Inertia of a single column about its strong axis: $I_{y,c} = \frac{300 \cdot 500^3}{12} = 3.125 \cdot 10^9 mm^4$

-Inertia of a single column about its weak axis: $I_{z,c} = \frac{500 \cdot 300^3}{12} = 1.125 \cdot 10^9 mm^4$

- Inertia of a girder: $I_g = \frac{300 \cdot 700^3}{12} = 8.575 \cdot 10^9 mm^4$

Step 2: Translational Parameters

-For the walls in the X direction:

$$(EI)_{tX}(4\ walls) = 4 \cdot 25 \cdot 1.33 \cdot 10^{11} = 1.33 \cdot 10^{13} kNmm^2$$

-For the frames in the X direction:

$$(GA)_{tX}(2\ frames) = 2 \cdot \frac{12 \cdot E}{h \cdot [(1/C) + (1/G)]}$$

Where:

$$C = \sum \frac{I_{y,C}}{h} = 5 \cdot \frac{3.125 \cdot 10^9}{3200} = 4.88 \cdot 10^6 mm^3$$

And

$$G = \sum \frac{I_g}{L} = 4 \cdot \frac{8.575 \cdot 10^{19}}{6000} = 5.72 \cdot 10^6 mm^3$$

Therefore,

$$(GA)_{tX}(2\ frames) = 2 \cdot \frac{12 \cdot 30}{3200 \cdot \left[\left(\frac{1}{4.88 \cdot 10^6}\right) + \left(\frac{1}{5.72 \cdot 10^6}\right)\right]} = 5.93 \cdot 10^5 kN$$

Then

$$(aH)_X = H \cdot \sqrt{\frac{(GA)_{tX}}{(EI)_{tX}}} = 8 \cdot 3200 \cdot \sqrt{\frac{5.93 \cdot 10^5}{1.33 \cdot 10^{13}}} = 5.40$$

-For the walls in the Y direction:

$$(EI)_{tY}(2\ walls) = 2 \cdot 25 \cdot 2.08 \cdot 10^{12} = 1.04 \cdot 10^{14} kNmm^2$$

-For the frames in the Y direction:

$$(GA)_{tY}(2\ frames) = 2 \cdot \frac{12 \cdot E}{h \cdot [(1/C) + (1/G)]}$$

Where:

$$C = \sum \frac{I_{y,C}}{h} = 3 \cdot \frac{3.125 \cdot 10^9}{3200} = 2.93 \cdot 10^6 mm^3$$

And

$$G = \sum \frac{I_g}{L} = 2 \cdot \frac{8.575 \cdot 10^9}{6000} = 2.86 \cdot 10^6 mm^3$$

Therefore,

$$(GA)_{tY}(2\ frames) = 2 \cdot \frac{12 \cdot 30}{3200 \cdot \left[\left(\frac{1}{2.93 \cdot 10^6}\right) + \left(\frac{1}{2.86 \cdot 10^6}\right)\right]} = 3.26 \cdot 10^5 kN$$

Then

$$(aH)_Y = H \cdot \sqrt{\frac{(GA)_{tY}}{(EI)_{tY}}} = 8 \cdot 3200 \cdot \sqrt{\frac{3.25 \cdot 10^5}{1.04 \cdot 10^{14}}} = 1.43$$

Step 3: Torsional Parameters

-For the walls:

$$(EI_w)_t = 4 \cdot (EI)_X \cdot y^2 + 2 \cdot (EI)_Y \cdot x^2$$

$$= 1.33 \cdot 10^{13} \cdot (2500 - 100)^2 + 1.04 \cdot 10^{14} \cdot (1000 + 2000 - 100)^2$$

$$= 9.51 \cdot 10^{20} kNmm^4$$

-For the moment resisting frames:

$$(GK)_t = 2 \cdot (GA)_X \cdot y^2 + 2 \cdot (GA)_Y \cdot x^2$$

$$= 5.93 \cdot 10^5 \cdot 10000^2 + 3.26 \cdot 10^5 \cdot 12500^2$$

$$= 1.10 \cdot 10^{14} kNmm^2$$

Then,

$$(aH)_\theta = H \cdot \sqrt{\frac{(GK)_t}{(EI_w)_t}} = 8 \cdot 3200 \cdot \sqrt{\frac{1.10 \cdot 10^{14}}{9.51 \cdot 10^{20}}} = 8.71$$

Step 4: Weight Distribution Parameter

Dividing the floor plan at a typical level into 25 regions of 5x4m, each carrying $p = 240kN$ gravity load, and taking the distance from the center of each region to the center of the structure as r, the following parameters are obtained:

$$\sum p \cdot r^2 = 492000kNm^2 = 492000000000kNmm^2$$

And

$$\sum p = 25 \cdot 240 = 6000kN$$

Hence,

$$R = \frac{\sum p \cdot r^2}{\sum p} = \frac{492000000000}{6000} = 82000000mm^2$$

Step 5: Computation of the critical loads for buckling at storey 1

-Transverse buckling:

Using the table from Slide 47, for $(aH)_X = 5.40 \rightarrow s_X = 69.0$

Then,

$$N_{1,cr,X} = \frac{s_X (EI)_{t,X}}{H^2} = \frac{69.0 \cdot 1.33 \cdot 10^{13}}{(8 \cdot 3200)^2} = 1.40 \cdot 10^6 kN$$

For $(aH)_Y = 1.43 \rightarrow s_Y = 13.74$ (based on linear interpolation from Slide 47 of the lecture notes).

Then,

$$N_{1,cr,Y} = \frac{s_Y (EI)_{t,Y}}{H^2} = \frac{13.74 \cdot 1.04 \cdot 10^{14}}{(8 \cdot 3200)^2} = 2.18 \cdot 10^6 kN$$

-Torsional buckling:

For $(aH)_\theta = 8.71 \rightarrow s_X = 142.46$(based on linear interpolation from Slide 47 of the lecture notes).

Then,

$$N_{1,cr,\theta} = \frac{s_\theta (EI_w)_t}{R \cdot H^2} = \frac{142.46 \cdot 9.51 \cdot 10^{20}}{8.2 \cdot 10^7 \cdot (8 \cdot 3200)^2} = 2.52 \cdot 10^6 kN$$

The actual maximum value of the total loading over 8 storeys is:

$$N_1 = 8 \cdot 6000 = 4.8 \cdot 10^4 kN < min\{N_{1,cr,X}, N_{1,cr,Y} N_{1,cr,\theta}\} = 1.40 \cdot 10^6 kN$$

Which leaves adequate margin of safety against overall buckling in both the translational and torsional modes.