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Exercise #7: Frame stability

Problem 1

The 7-storey moment resisting frame shown in Figure 1 is subjected to a concentrated load at the top.

Q1: Prove that for the concentrated load, the resulting drift in a single storey, i, is given by,

$$\delta_i = \frac{V_i \cdot h_i^2}{12 \cdot E} \cdot \left(\frac{1}{G} + \frac{1}{C}\right)_i$$

In which, $C_i = \sum_{j=1}^{N} \left(\frac{I_c}{h}\right)_{i,j}$ and $G_i = \sum_{k=1}^{N-1} \left(\frac{I_g}{L}\right)_{i,k}$.

Q2: Assume that the inflection point of the columns in the first storey is located at $3/4h_1$ (not at mid-height). Prove that for the concentrated load, the drift of the first storey may be estimated as follows,

$$\delta_1 = \frac{V_1 \cdot h_1^2}{12 \cdot E} \cdot \left(\frac{3}{8 \cdot G} + \frac{7}{4 \cdot C}\right)_1$$

Q3: What is the minimum moment of inertia of the girders I_g of the moment resisting frame to satisfy a lateral drift ratio limit $\delta_i/h_i = 0.02$ at all stories? Assume that in all cases, the girders control the relative deformations, therefore, $\psi \cong 1$ (>>0.5). Assume that all members are made of steel ($E = 200GPa$).

I_g: moment of inertia of girders

I_c: moment of inertia of columns

$I_g = 1.5 \cdot I_c$

$F_7 = 1000kN$

$h_1 = h_i = 3.5m$

$L = 8m$

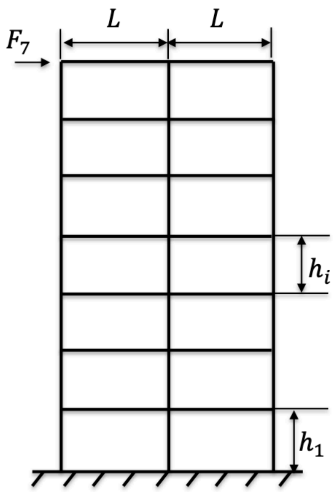


Figure 1. Moment resisting frame with rigid connections and concentrated lateral load

Problem 2

The 7-storey moment resisting frame shown in Figure 2 is subjected to a uniform wind load of F_i per storey, i.

Q1: Prove that for the uniform wind load, the resulting drift in a single storey, i, is given by,

$$\delta_i = \frac{h_i^2}{24E} \cdot \left[\frac{(V_i + V_{i+1})}{G_i} + \frac{2 \cdot V_i}{C_i}\right]$$

In which, $C_i = \sum_{j=1}^{N} \left(\frac{I_c}{h}\right)_{i,j}$, $G_i = \sum_{k=1}^{N-1} \left(\frac{I_g}{L}\right)_{i,k}$, V_i and V_{i+1} are the storey shears in storeys i and $i+1$, respectively.

Q2: Assume that the inflection point of the columns in the first storey is located at $3/4h_1$ (not at mid-height). Prove that for the uniform wind load, the drift of the first storey may be estimated as follows,

$$\delta_1 = \frac{h_1^2}{48E} \cdot \left[\frac{(V_1 + 2 \cdot V_2)}{2G_1} + \frac{7 \cdot V_1}{C_1}\right]$$

Q3: What is the minimum moment of inertia of the girders I_g of the moment resisting frame to satisfy a lateral drift ratio limit $\delta_i/h_i = 0.02$ at all stories? Assume that in all cases, the girders control the relative deformations, therefore, $\psi \cong 1$ (>>0.5). Assume that all members are made of steel ($E = 200GPa$).

I_g: moment of inertia of girders

I_c: moment of inertia of columns

$I_g = 1.5 \cdot I_c$

$F_1 = F_i = F_7 = \frac{1000}{7} kN$

$h_1 = h_i = 3.5m$

$L = 8m$

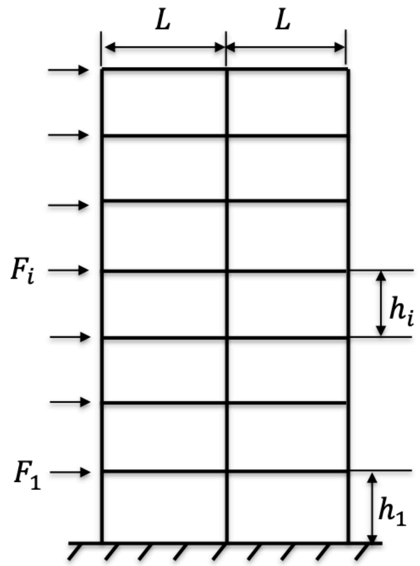
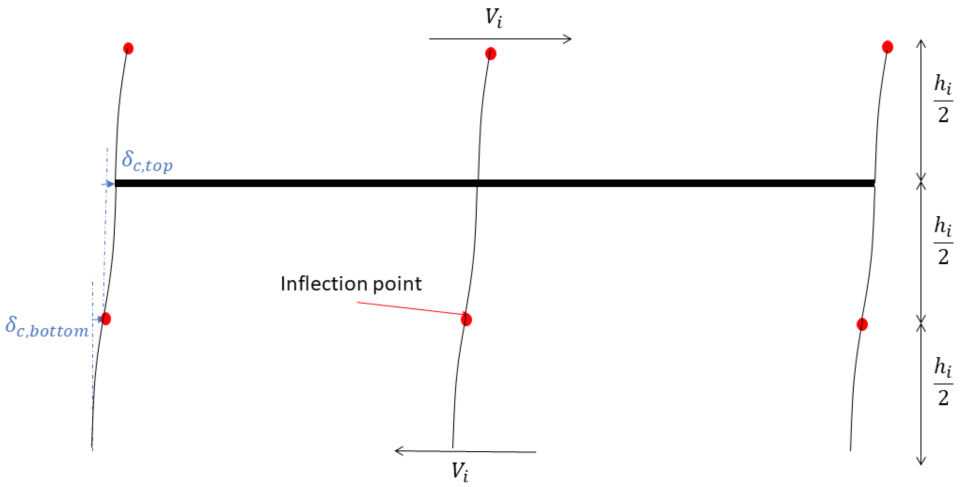


Figure 2. Moment resisting frame with rigid connections

Solution to Problem 1 (Concentrated Lateral Load)

Q1) Using the easy statics approach we can derive the deflections at the top of each storey. Note that the storey shear V_i will be the same in each storey. The deflections will be calculated based on the deflections due to bending in the columns of the storey of interest plus the deflections due to bending in the girders above the storey of interest.

-Deflection due to bending in the columns (assume $I_g = \infty$):



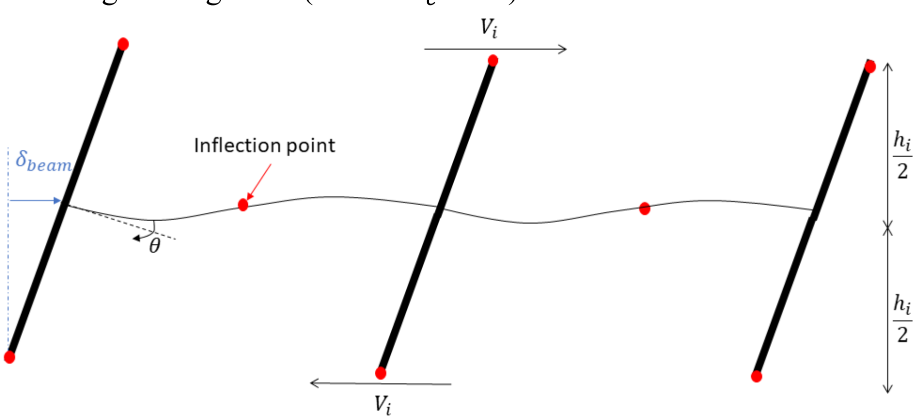
For the bottom part of the column:

$$\delta_{c,bottom} = \frac{V_i}{\frac{3E}{\left(\frac{h_i}{2}\right)^3} \cdot \sum_{j=1}^{N} I_{c,j}} \tag{1}$$

For the top part of the column:

$$\delta_{c,top} = \frac{V_i}{\frac{3E}{\left(\frac{h_i}{2}\right)^3} \cdot \sum_{j=1}^{N} I_{c,j}} \tag{2}$$

-Deflection due to bending in the girders (assume $I_c = \infty$):



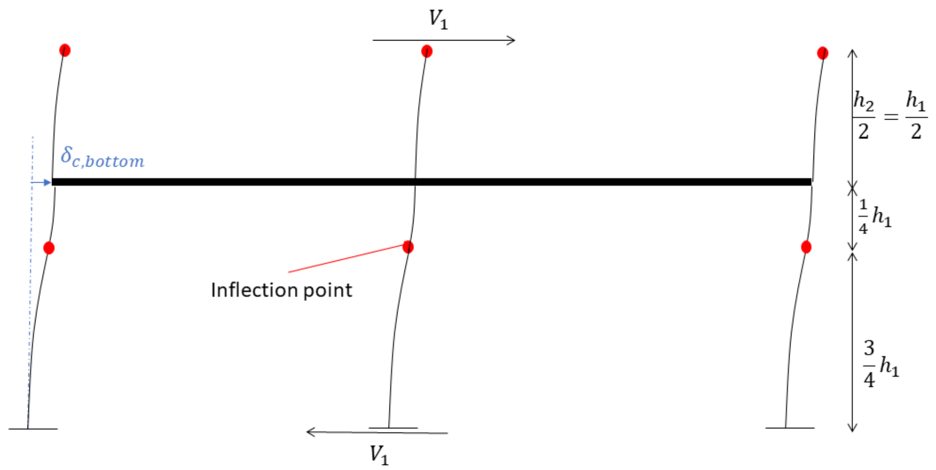
$$\delta_{beam} = \theta \cdot \frac{h_i}{2} = \frac{M_b}{K_b} \cdot \frac{h_i}{2} = \frac{V_i \cdot h_i}{\frac{3E}{\frac{L_i}{2}} \cdot \sum_{k=1}^{N-1} I_{g,k}} \cdot \frac{h_i}{2} \tag{3}$$

-Total deflection of storey i:

$$\delta_{tot,i} = \frac{2V_i \cdot h_i^3}{24E \cdot \sum_{j=1}^{N} I_{c,j}} + \frac{V_i \cdot h_i^2 \cdot L_i}{12E \cdot \sum_{k=1}^{N-1} I_{g,k}} = \frac{V_i \cdot h_i^2}{12E} \cdot \left(\frac{1}{G_i} + \frac{1}{C_i}\right) \quad (4)$$

Q2)

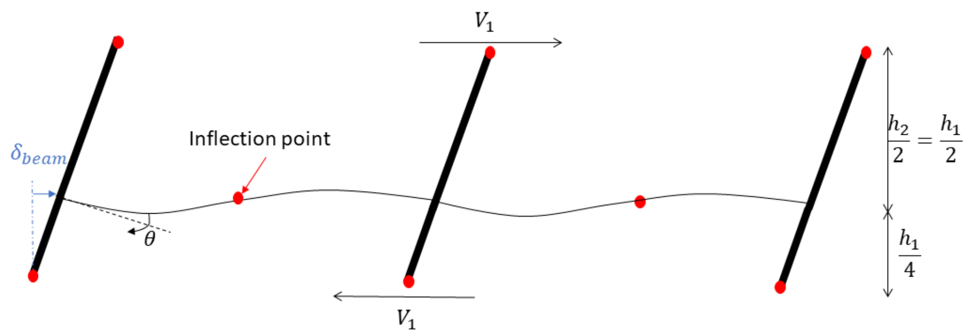
-Deflection due to bending in the columns (assume $I_g = \infty$):



$$\delta_{c,bottom} = \frac{V_1}{\frac{3E}{\left(3 \cdot \frac{h_1}{4}\right)^3} \cdot \sum_{j=1}^{N} I_{c,j}} + \frac{V_1}{\frac{3E}{\left(\frac{h_1}{4}\right)^3} \cdot \sum_{j=1}^{N} I_{c,j}}$$

$$= \frac{7 \cdot V_1 \cdot h_1^3}{48 \cdot E \cdot \sum_{j=1}^{N} I_{c,j}} \quad (5)$$

-Deflection due to bending in the girders (assume $I_c = \infty$):



$$\delta_b = \theta \cdot \frac{h_1}{4} = \frac{M_b}{K_b} \cdot \frac{h_1}{4} = \frac{V_1 \cdot \left(\frac{h_1}{4} + \frac{h_1}{2}\right)}{\frac{3E}{\frac{L_1}{2}} \cdot \sum_{k=1}^{N-1} I_{g,k}} \cdot \frac{h_1}{4} \quad (6)$$

-Total deflection:

$$\delta_{tot,1} = \frac{7 \cdot V_1 \cdot h_1^3}{48E \cdot \sum_{j=1}^{N} I_{c,j}} + \frac{V_1 \cdot h_1^2 \cdot L_1}{32E \cdot \sum_{k=1}^{N-1} I_{g,k}} = \frac{V_1 \cdot h_1^2}{12E} \cdot \left(\frac{3}{8G} + \frac{7}{4C}\right)_1 \quad (7)$$

Q3)

$$\frac{\delta_{tot,i}}{h_i} = \frac{V_i \cdot h_i}{12 \cdot E} \cdot \left(\frac{1}{\sum_{k=1}^{N-1} \left(\frac{I_{g,k}}{L_i} \right)} + \frac{1}{\sum_{j=1}^{N} (I_{c,j} / h_i)} \right) \quad (8)$$

For example, for the third storey ($i = 3$):

$$0.02 = \frac{1000 \cdot 3500}{12 \cdot 200} \cdot \left(\frac{1}{3 \cdot \left(\frac{\frac{I_g}{1.5}}{3500} \right)} + \frac{1}{2 \cdot \frac{I_g}{8000}} \right)$$

After some basic mathematical operations, the equation becomes:

$$0.02 = \frac{8385416.7}{I_g}$$

Therefore,

$$I_g = 4.19 \cdot 10^8 mm^4$$

This corresponds to the inertia of an HEB 360.

-For the first storey, the size of the girders is obtained using Equation 7:

$$\frac{\delta_{tot,1}}{h_i} = \frac{V_1 \cdot h_1}{12E} \cdot \left(\frac{3}{8 \cdot \sum_{k=1}^{N-1} \left(\frac{I_{g,k}}{L_i} \right)} + \frac{7}{4 \cdot \sum_{j=1}^{N} (I_{c,j} / h_i)} \right) \quad (9)$$

The numerical application gives:

$$0.02 = \frac{1000 \cdot 3500}{12 \cdot 200} \cdot \left(\frac{3}{8 \cdot 2 \cdot \frac{I_g}{8000}} + \frac{7}{4 \cdot \left(3 \cdot \frac{\frac{I_g}{1.5}}{3500} \right)} \right)$$

After some basic mathematical operations, the equation becomes:

$$0.02 = \frac{6653645.8}{I_g}$$

Therefore,

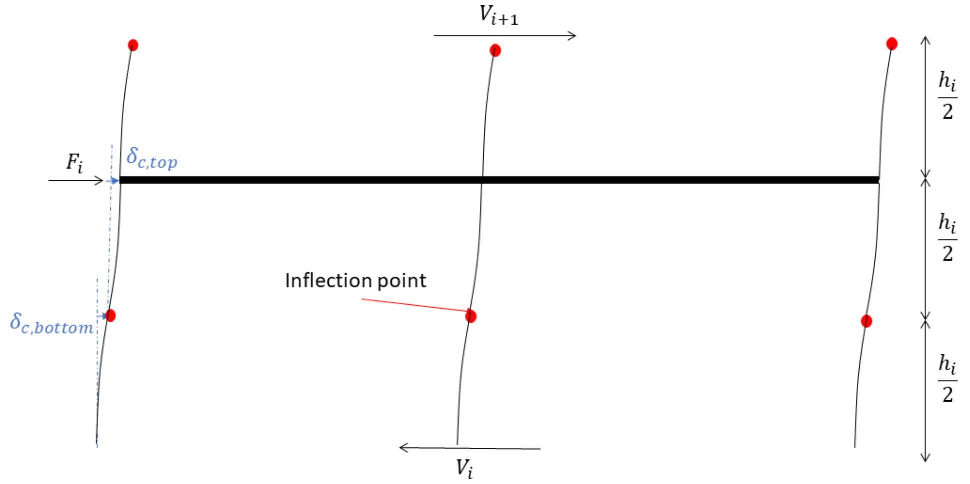
$$I_g = 3.33 \cdot 10^8 mm^4$$

This corresponds to the inertia of an HEB 340

Solution to Problem 2 (Uniform wind load)

Q1) Using the easy statics approach we can derive the deflections at the top of each storey. Note that because we have uniform wind load, the storey shear from one storey to the other increases by F_i. The deflections will be calculated based on the deflections due to bending in the columns of the storey of interest plus the deflections due to bending in the girders above the storey of interest.

-Deflection due to bending in the columns (assume $I_g = \infty$):



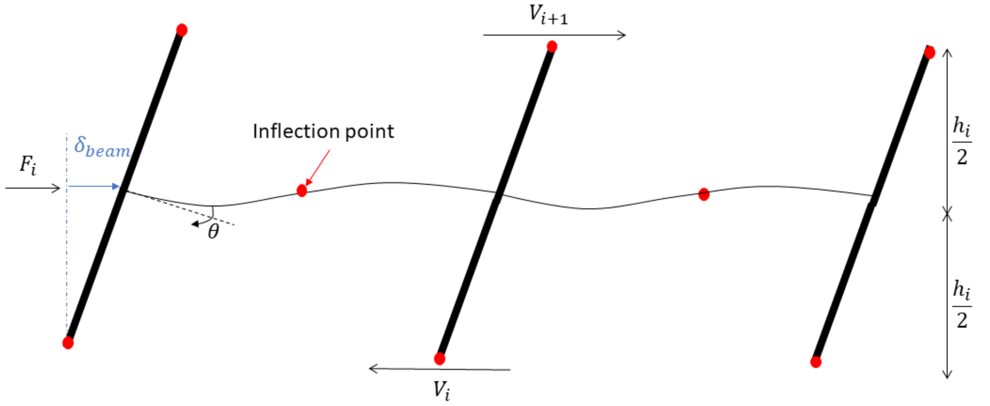
For the bottom part of the column:

$$\delta_{c,bottom} = \frac{V_i}{\frac{3E}{\left(\frac{h_i}{2}\right)^3} \cdot \sum_{j=1}^{N} I_{c,j}} \tag{10}$$

For the top part of the column:

$$\delta_{c,top} = \frac{V_i}{\frac{3E}{\left(\frac{h_i}{2}\right)^3} \cdot \sum_{j=1}^{N} I_{c,j}} \tag{11}$$

-Deflection due to bending in the girders (assume $I_c = \infty$):



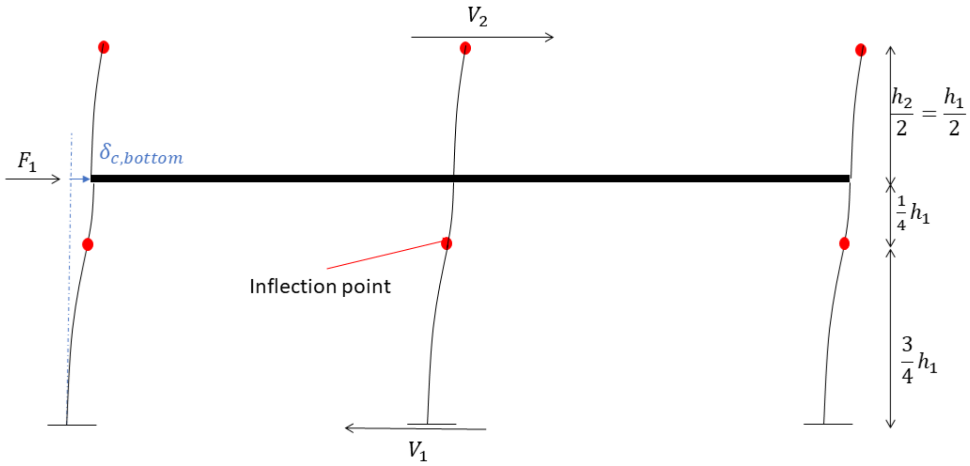
$$\delta_{beam} = \theta \cdot \frac{h_i}{2} = \frac{M_b}{K_b} \cdot \frac{h_i}{2} = \frac{V_i \cdot \frac{h_i}{2} + V_{i+1} \cdot \frac{h_i}{2}}{\frac{3E}{\frac{L_i}{2}} \cdot \sum_{k=1}^{N-1} I_{g,k}} \cdot \frac{h_i}{2} \tag{12}$$

-Total deflection of storey i:

$$\delta_{tot,i} = \frac{2V_i \cdot h_i^3}{24E \cdot \sum_{j=1}^{N} I_{c,j}} + \frac{(V_i + V_{i+1}) \cdot h_i^2 \cdot L_i}{24E \cdot \sum_{k=1}^{N-1} I_{g,k}} = \frac{h_i^2}{24E} \cdot \left(\frac{2 \cdot V_i}{C_i} + \frac{(V_i + V_{i+1})}{G_i}\right) \tag{13}$$

Q2)

-Deflection due to bending in the columns (assume $I_g = \infty$):

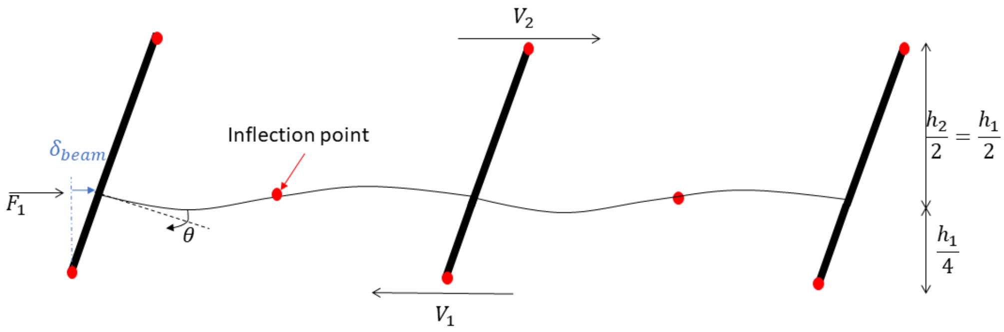


$$\delta_{c,bottom} = \frac{V_1}{\frac{3E}{\left(3 \cdot \frac{h_1}{4}\right)^3} \cdot \sum_{j=1}^{N} I_{c,j}} + \frac{V_1}{\frac{3E}{\left(\frac{h_1}{4}\right)^3} \cdot \sum_{j=1}^{N} I_{c,j}}$$

Therefore,

$$\delta_{c,bottom} = \frac{7 \cdot V_1 \cdot h_1^3}{48 \cdot E \cdot \sum_{j=1}^{N} I_{c,j}} \tag{14}$$

-Deflection due to bending in the girders (assume $I_c = \infty$):



$$\delta_b = \theta \cdot \frac{h_1}{4} = \frac{M_b}{K_b} \cdot \frac{h_1}{4} = \frac{V_1 \cdot \frac{h_1}{4} + V_2 \cdot \frac{h_1}{2}}{\frac{3E}{\frac{L_1}{2}} \cdot \sum_{k=1}^{N-1} I_{g,k}} \cdot \frac{h_1}{4} \qquad (15)$$

-Total deflection:

$$\delta_{tot,1} = \frac{7 \cdot V_1 \cdot h_1^3}{48E \cdot \sum_{j=1}^{N} I_{c,j}} + \frac{(V_1 + 2 \cdot V_2) \cdot h_1^2 \cdot L_1}{96E \cdot \sum_{k=1}^{N-1} I_{g,k}} = \frac{h_1^2}{48E} \cdot \left(\frac{7 \cdot V_1}{C} + \frac{(V_1 + 2 \cdot V_2)}{2G}\right)_1 \qquad (16)$$

Q3)

-For the general case of the storey i, the size of the girders can be found using Equation 13:

$$\frac{\delta_{tot,i}}{h_i} = \frac{h_i}{24E} \cdot \left(\frac{(V_i + V_{i+1})}{\sum_{k=1}^{N-1} \left(\frac{I_{g,k}}{L_i}\right)} + \frac{2 \cdot V_i}{\sum_{j=1}^{N} (I_{c,j} / h_i)} \right)$$

For example, for the third storey ($i = 3$):

$V_3 = 5 \cdot 1000/7 kN, V_4 = 4 \cdot 1000/7\ kN$

$$0.02 = \frac{3500}{24 \cdot 200} \cdot \left(\frac{5 \cdot \frac{1000}{7} + 4 \cdot \frac{1000}{7}}{2 \cdot \frac{I_g}{8000}} + \frac{2 \cdot \left(5 \cdot \frac{1000}{7}\right)}{3 \cdot \left(\frac{\frac{I_g}{1.5}}{3500}\right)} \right)$$

After some basic mathematical operations, the equation becomes:

$$0.02 = \frac{5572916.7}{I_g}$$

Therefore,

$$I_g = 2.79 \cdot 10^8 mm^4$$

This corresponds to the inertia of an HEB 320

-For the first storey, the size of the girders is obtained using Equation 16:

$$\frac{\delta_{tot,1}}{h_1} = \frac{h_1}{48 \cdot E} \cdot \left(\frac{(V_1 + 2 \cdot V_2)}{2 \cdot \sum_{k=1}^{N-1} \left(\frac{I_{g,k}}{L_i}\right)} + \frac{7 \cdot V_1}{\sum_{j=1}^{N} (I_{c,j} / h_i)} \right)_1$$

The numerical application gives:

$V_1 = 1000kN, V_2 = 6 \cdot 1000/7 = 857.1kN$

$$0.02 = \frac{3500}{48 \cdot 200} \cdot \left(\frac{1000 + 2 \cdot 857.1}{2 \cdot \left(2 \cdot \frac{I_g}{8000}\right)} + \frac{7 \cdot 1000}{3 \cdot \left(\frac{\frac{I_g}{1.5}}{3500}\right)} \right)$$

After some basic mathematical operations, the equation becomes:

$$0.02 = \frac{6445250}{I_g}$$

Therefore,

$$I_g = 3.22 \cdot 10^8 mm^4$$

This corresponds to the inertia of an HEB 340