FACULTE ENVIRONNEMENT NATUREL, ARCHITECTURAL ET CONSTRUIT
ENAC
INSTITUT D'INGÉNIERIE CIVILE IIC
Laboratoire des Structures Métalliques Résilientes
RESSLAB

Téléphone : +41 21 693 24 27
Fax : +41 21 693 28 68
E-mail : dimitrios.lignos@epfl.ch
Site web : http://resslab.epfl.ch
Address: EPFL ENAC IIC RESSLAB
GC B3 485, Station 18,
CH-1015, Lausanne

Exercise #4: Interaction Curves

The single storey steel industrial building shown in Figure 1 consists of I-shape S355 steel columns ($f_y = 355MPa, E = 200GPa$), which are connected to a roof truss system at the top. The cross-section sizes are shown in Figure 1. The roof truss has a flexural stiffness that is significantly greater than that of the columns in the direction where the columns bend about their strong axis (assumed as fixed at the base). The roof truss has negligible flexural stiffness in the direction where the columns are bending about their weak axis (assumed as pinned at the base). In the two perimeter frames in the East-West (EW) loading direction, X-braces made of 75mm diameter circular steel rods made of S355 ($f_y = 355MPa, E = 200GPa$), are installed in three bays to create a non-sway frame as shown in Figure 1. The steel rods have a negligible compressive resistance (i.e., tension only bracing system). Axial struts are used to brace laterally the columns in the EW loading direction as shown in Figure 1. The total dead load acting on the roof of the structure is equal to 5.0kPa.

1. Compute the buckling resistance of the HEB500 columns in the perimeter moment-resisting frames. Show explicitly your assumptions and calculations.
2. Sketch the first order bending and shear diagrams only for the columns of the perimeter moment-resisting frames in the North-South loading direction due to the lateral load, F_1.
3. Compute the bending and shear diagrams only for the columns of the perimeter moment-resisting frames in the North-South (NS) loading direction due to the lateral load, F_1, by considering the influence of compressive axial loads on the steel columns. In this case, you should to think how the frames with X-braces may affect the HEB500 steel columns.
4. Check the interaction of compressive axial load and bending moment demands for the HEB500 column in one of the perimeter moment-resisting frames. In your calculations, assume that lateral torsional buckling is not critical for these members.

NOTE for Figure 1: F_1=1800kN and F_2=15,000kN

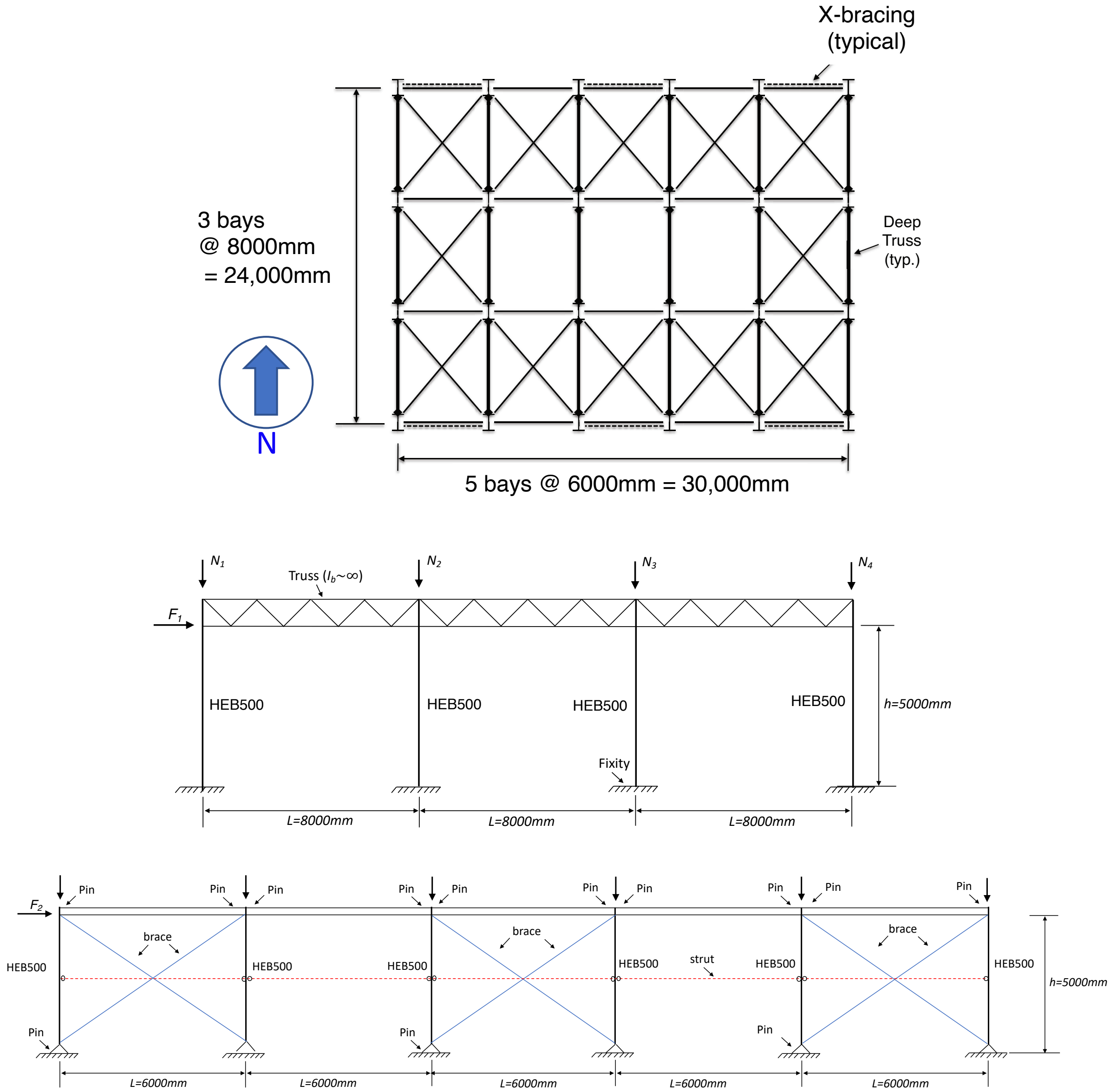
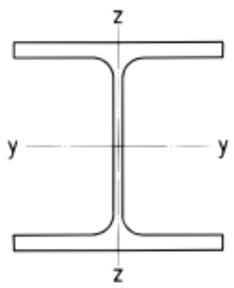


Figure 1. Single-storey industrial building – plan and elevation views

HEB

Breitflanschträger HEB — *Profilés à larges ailes* HEB



$$A_v = A - 2bt_f + (t_w + 2r)\, t_f$$

$$A_w = (h - t_f) \cdot t_w \qquad W_{ely} = \frac{I_y}{h/2}$$

$$S_y = \tfrac{1}{2} W_{ply}$$

$$S_z = \tfrac{1}{2} W_{plz} \qquad \overline{W}_y = \frac{I_y}{(h - t_f)/2}$$

$$W_{elz} = \frac{I_z}{b/2}$$

*Auch in S355J0 oder S355J2 ab Schweizer Lager erhältlich

Maximale Lagerlängen /
Longueurs maximales en stock:
h ≤ 180 18 m
h ≥ 200 24 m

EURONORM 53 – 62, DIN 1025/2

Andere Bezeichnungen / *Autres désignations*: DIN, IPB

* *Livrable en S355J0 ou S355J2 du stock suisse*

HEB	m	Statische Werte / *Valeurs statiques*												
	kg/m	A mm²	A_v mm²	A_w mm²	I_y mm⁴	W_{ely} mm³	$\overline{W}_y$ mm³	W_{ply} mm³	i_y mm	I_z mm⁴	W_{elz} mm³	W_{plz} mm³	i_z mm	$K = I_x$ mm⁴
					x 10⁶	**x 10³**	**x 10³**	**x 10³**		**x 10⁶**	**x 10³**	**x 10³**		**x 10⁶**
100*	**20,4**	2600	904	540	4,50	89,9	100	104	41,6	1,67	33,5	51,4	25,3	0,0931
120*	**26,7**	3400	1096	708	8,64	144	158	165	50,4	3,18	52,9	81,0	30,6	0,139
140*	**33,7**	4300	1308	896	15,1	216	236	245	59,3	5,50	78,5	120	35,8	0,202
160*	**42,6**	5430	1759	1180	24,9	311	339	354	67,8	8,89	111	170	40,5	0,312
180*	**51,2**	6530	2024	1410	38,3	426	461	481	76,6	13,6	151	231	45,7	0,422
200*	**61,3**	7810	2483	1660	57,0	570	616	643	85,4	20,0	200	306	50,7	0,596
220*	**71,5**	9100	2792	1940	80,9	736	793	827	94,3	28,4	258	394	55,9	0,770
240*	**83,2**	10600	3323	2230	112,6	938	1010	1050	103	39,2	327	498	60,8	1,04
260*	**93,0**	11800	3759	2420	149,2	1150	1230	1280	112	51,3	395	602	65,8	1,26
280*	**103**	13100	4109	2750	192,7	1380	1470	1530	121	65,9	471	718	70,9	1,45
300*	**117**	14900	4743	3090	251,7	1680	1790	1870	130	85,6	571	870	75,8	1,87
320*	**127**	16100	5177	3440	308,2	1930	2060	2150	138	92,4	616	939	75,7	2,29
340*	**134**	17100	5609	3820	366,6	2160	2300	2410	146	96,9	646	986	75,3	2,62
360*	**142**	18100	6060	4220	431,9	2400	2560	2680	155	101	676	1030	74,9	2,98
400*	**155**	19800	6998	5080	576,8	2880	3070	3230	171	108	721	1100	74,0	3,61
450*	**171**	21800	7966	5940	798,9	3550	3770	3980	191	117	781	1200	73,3	4,49
500	**187**	23900	8982	6840	1072	4290	4540	4820	212	126	842	1290	72,7	5,50
550	**199**	25400	10010	7820	1367	4970	5250	5590	232	131	872	1340	71,7	6,12
600	**212**	27000	11080	8840	1710	5700	6000	6420	252	135	902	1390	70,8	6,80
650	**225**	28600	12200	9900	2106	6480	6800	7320	271	140	932	1440	69,9	7,52
700	**241**	30600	13710	11400	2569	7340	7690	8330	290	144	963	1490	68,7	8,42
800	**262**	33400	16180	13400	3591	8980	9360	10230	328	149	994	1550	66,8	9,62
900	291	37100	18880	16000	4941	10980	11400	12580	365	158	1050	1660	65,3	11,5
1000	314	40000	21250	18300	6447	12890	13400	14860	401	163	1090	1720	63,8	12,7

Breitflanschträger HEB — *Profilés à larges ailes* HEB — HEB

Anstelle des nicht mehr gewalzten Profils HEB 1100 können HL-Profile verwendet werden, siehe Seiten 40/41.

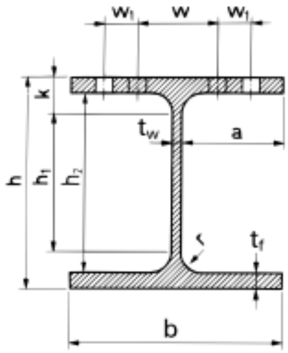
Au lieu du profilé HEB 1100 qui n'est plus laminé, on utilisera des profilés HL (voir pages 40/41).

w_1 mit $\varnothing_{max}$ nur für versetzte Schrauben.

w_1 avec $\varnothing_{max}$ seulement pour boulons décalés.

Walztoleranzen siehe Seite 116

Tolérances de laminage voir page 116



HEB	m	Profilmasse *Dimensions de la section*					Konstruktionsmasse *Dimensions de construction*							Oberfläche *Surface*		HEB
	kg/m	h mm	b mm	t_w mm	t_f mm	r mm	h_1 mm	k mm	a mm	h_2 mm	w mm	w_1 mm	$\varnothing_{max}$	U_m m²/m	U_t m²/t	
100	**20,4**	100	100	6	10	12	56	22	47	80	56		M12	0,567	27,8	**100**
120	**26,7**	120	120	6,5	11	12	74	23	56	98	66		M16	0,686	25,7	**120**
140	**33,7**	140	140	7	12	12	92	24	66	116	76		M20	0,805	23,9	**140**
160	**42,6**	160	160	8	13	15	104	28	76	134	86		M20	0,918	21,5	**160**
180	**51,2**	180	180	8,5	14	15	122	29	85	152	100		M24	1,04	20,3	**180**
200	**61,3**	200	200	9	15	18	134	33	95	170	110		M24	1,15	18,8	**200**
220	**71,5**	220	220	9,5	16	18	152	34	105	188	120		M24	1,27	17,8	**220**
240	**83,2**	240	240	10	17	21	164	38	115	206	96	35	M24	1,38	16,6	**240**
260	**93,0**	260	260	10	17,5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	M24	1,50	16,1	**260**
280	**103**	280	280	10,5	18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	M24	1,62	15,7	**280**
300	**117**	300	300	11	19	27	208	46	144	262	120	45	M27	1,73	14,8	**300**
320	**127**	320	300	11,5	20,5	27	224	48	144	279	120	45	M27	1,77	13,9	**320**
340	**134**	340	300	12	21,5	27	242	49	144	297	120	45	M27	1,81	13,5	**340**
360	**142**	360	300	12,5	22,5	27	260	50	143	315	120	45	M27	1,85	13,0	**360**
400	**155**	400	300	13,5	24	27	298	51	143	352	120	45	M27	1,93	12,4	**400**
450	**171**	450	300	14	26	27	344	53	143	398	120	45	M27	2,03	11,9	**450**
500	**187**	500	300	14,5	28	27	390	55	142	444	120	45	M27	2,12	11,3	**500**
550	**199**	550	300	15	29	27	438	56	142	492	120	45	M27	2,22	11,2	**550**
600	**212**	600	300	15,5	30	27	486	57	142	540	120	45	M27	2,32	11,0	**600**
650	**225**	650	300	16	31	27	534	58	142	588	120	45	M27	2,42	10,8	**650**
700	**241**	700	300	17	32	27	582	59	141	636	126	45	M27	2,52	10,5	**700**
800	**262**	800	300	17,5	33	30	674	63	141	734	130	40	M27	2,71	10,4	**800**
900	291	900	300	18,5	35	30	770	65	140	830	130	40	M27	2,91	10,0	900
1000	314	1000	300	19	36	30	868	66	140	928	130	40	M27	3,11	9,9	1000

Solution

1. The procedure follows the approach described in the SIA 263, section 4.5.1

Interior column

-Strong axis boundary conditions: Fixed-Fixed:

$$L_{effective} = 0.5 \cdot h \tag{1}$$

Euler critical buckling load:

$$N_{cr,y} = \frac{\pi^2 E I_y}{(0.5h)^2} = \frac{\pi^2 \cdot 200 \cdot 1072 \cdot 10^6}{(0.5 \cdot 5000)^2} = 338567kN \tag{2}$$

$$\sigma_{cr,ky} = \frac{N_{cr,y}}{A} = \frac{338567 \cdot 1000}{23900} = 14166MPa \tag{3}$$

Slenderness coefficient:

$$\overline{\lambda_{ky}} = \sqrt{\frac{f_y}{\sigma_{cr,ky}}} = \sqrt{\frac{355}{14166}} = 0.158 \tag{4}$$

The HEB500, has a cross-section aspect ration, $\frac{h}{b} = \frac{500}{300} = 1.7$, and a flange thickness of $t_f = 28mm \leq 40mm$. Therefore, according to Figure 7 of the SIA 263, when considering buckling with respect to the strong axis, the buckling curve *a* is used; hence, the imperfection factor is:

$$\alpha_{k,y} = 0.21 \tag{5}$$

We have:

$$\phi_{k,y} = 0.5\left[1 + \alpha_{k,y}\left(\overline{\lambda_{ky}} - 0.2\right) + \overline{\lambda_{ky}}^2\right] = 0.508 \tag{6}$$

Therefore, the reduction factor is:

$$\chi_{k,y} = \frac{1}{\phi_{k,y} + \sqrt{\phi_{k,y}^2 - \overline{\lambda_{ky}}^2}} = 1.04 \rightarrow \chi_{k,y} = 1 \tag{7}$$

The buckling resistance of the column with respect to its strong axis is:

$$N_{ky,Rd} = \chi_{k,y} \cdot \frac{f_y A}{\gamma_{M1}} = 8080.5kN \tag{8}$$

Th safety factor $\gamma_{M1} = 1.05$.

-Weak axis boundary conditions: Pined-pined

$$L_{effective} = 1 \cdot h \tag{9}$$

Euler critical buckling load:

$$N_{cr,z} = \frac{\pi^2 EI_z}{(h)^2} = \frac{\pi^2 \cdot 200 \cdot 126 \cdot 10^6}{(5000)^2} = 9949kN \tag{10}$$

$$\sigma_{cr,kz} = \frac{N_{cr,z}}{A} = \frac{9949 \cdot 1000}{23900} = 416.3MPa \tag{11}$$

Slenderness coefficient:

$$\overline{\lambda_{kz}} = \sqrt{\frac{f_y}{\sigma_{cr,kz}}} = \sqrt{\frac{355}{416.3}} = 0.923 \tag{12}$$

The HEB500, has an cross-section aspect ratio, $\frac{h}{b} = \frac{500}{300} = 1.7$, and a flange thickness of $t_f = 28mm \leq 40mm$. Therefore, according to Figure 7 of the SIA 263, when considering buckling with respect to the weak axis, the buckling curve *b* is used; hence, the imperfection factor is:

$$\alpha_{k,z} = 0.34 \tag{13}$$

We have:

$$\phi_{k,z} = 0.5\left[1 + \alpha_{k,z}\left(\overline{\lambda_{kz}} - 0.2\right) + \overline{\lambda_{kz}}^2\right] = 1.05 \tag{14}$$

Therefore, the reduction factor is:

$$\chi_{k,z} = \frac{1}{\phi_{k,z} + \sqrt{\phi_{k,z}^2 - \overline{\lambda_{kz}}^2}} = 0.645 \tag{15}$$

The buckling resistance of the column with respect to its weak axis is:

$$N_{kz,Rd} = \chi_{k,z} \cdot \frac{f_y A}{\gamma_{M1}} = 5211N \tag{16}$$

The buckling resistance of the interior column, $N_{k,Rd} = \min\{8080.5, 5211\} = 5211kN$

Exterior column

-Strong axis boundary conditions: Fixed-Fixed:

The corresponding buckling resistance of the column with respect to its strong axis is identical to that of the interior column.

-Weak axis boundary conditions: Pined-pined

Weak axis buckling is restrained at the middle of the column by the axial strut. As such,

$$L_{effective} = 0.5 \cdot h \tag{17}$$

Euler critical buckling load:

$$N_{cr,z} = \frac{\pi^2 EI_z}{(0.5h)^2} = \frac{\pi^2 \cdot 200 \cdot 126 \cdot 10^6}{(0.5 \cdot 5000)^2} = 39794kN \tag{18}$$

$$\sigma_{cr,kz} = \frac{N_{cr,z}}{A} = \frac{39794 \cdot 1000}{23900} = 1665MPa \tag{19}$$

Slenderness coefficient:

$$\overline{\lambda_{kz}} = \sqrt{\frac{f_y}{\sigma_{cr,kz}}} = \sqrt{\frac{355}{1665}} = 0.462 \tag{20}$$

The HEB500, has a slenderness of $\frac{h}{b} = \frac{500}{300} = 1.7$, and a flange thickness of $t_f = 28mm \leq 40mm$. Therefore, according to Figure 7 of the SIA 263, when considering buckling with respect to the weak axis, the buckling curve b is used; the imperfection factor is:

$$\alpha_{k,z} = 0.34 \tag{21}$$

We have:

$$\phi_{k,z} = 0.5\left[1 + \alpha_{k,z}\left(\overline{\lambda_{kz}} - 0.2\right) + \overline{\lambda_{kz}}^2\right] = 0.651 \tag{22}$$

Therefore, the reduction factor is:

$$\chi_{k,z} = \frac{1}{\phi_{k,z} + \sqrt{\phi_{k,z}^2 - \overline{\lambda_{kz}}^2}} = 0901 \tag{23}$$

The buckling resistance of the exterior column with respect to its weak axis is:

$$N_{kz,Rd} = \chi_{k,z} \cdot \frac{f_y A}{\gamma_{M1}} = 7282kN \tag{24}$$

The buckling resistance of the exterior column, $N_{k,Rd} = \min\{7282kN, 8080.5kN\} = 7282kN$

2.The first order moment and shear diagrams of the columns are as follows:

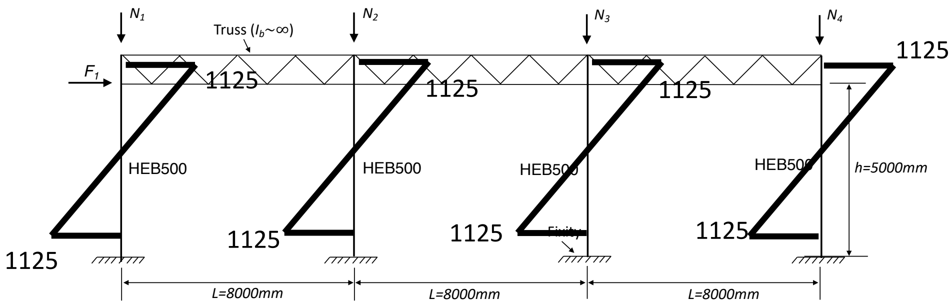


Figure 1 – First order moment diagram [kNm]

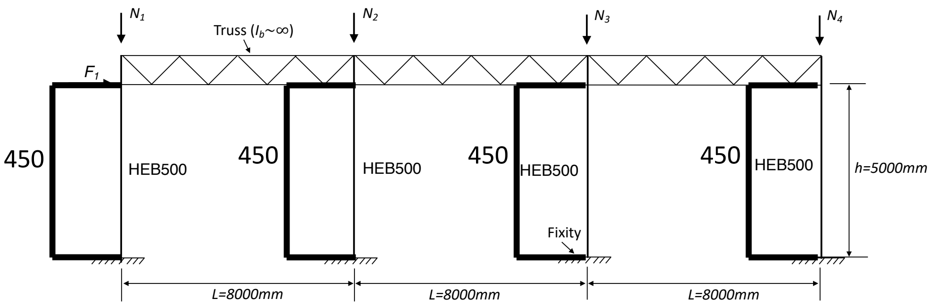


Figure 2 - First order shear diagram [kN]

3. In the EW loading direction, the force F_2 creates a tensile force in the brace. This tensile force will result in an axial load in the exterior column of the perimeter moment-resisting frames in addition to the gravity load. The compressive brace does not provide lateral resistance. Because there are 3 identical frames with bracing,

$$F_b = \frac{F_2}{3} \tag{25}$$

The tensile force in the brace is obtained as follows:

$$T = \frac{F_b}{\cos\alpha} \tag{26}$$

Where α denotes the angle between the brace and the horizontal beam. The axial load in the column due to the tensile force is computed as follows:

$$N_1 = T \cdot \sin\alpha = F_b \cdot \tan\alpha = \frac{F_2}{3} \cdot \frac{h}{l} = \frac{15{,}000 \cdot 5}{3 \cdot 6} = 4167kN \tag{27}$$

The axial load in the corner column due to the dead load is computed using the area of influence (i.e., tributary area) of the column:

$$N_2 = q_d \cdot A_{influence} = 5{,}0 \cdot 4 \cdot 3 = 60kN \tag{28}$$

The total axial load is $N = N_1 + N_2 = 4167 + 60 = 4227kN$

The amplification factor is computed with:

$$\varphi = \frac{1}{1-\frac{N}{N_{cr,y}}} = \frac{1}{1-\frac{4227}{338567}} \cong 1.01 \tag{29}$$

The second order moment and shear diagrams are obtained by multiplying the ones from Question 2 by the amplification factor φ,

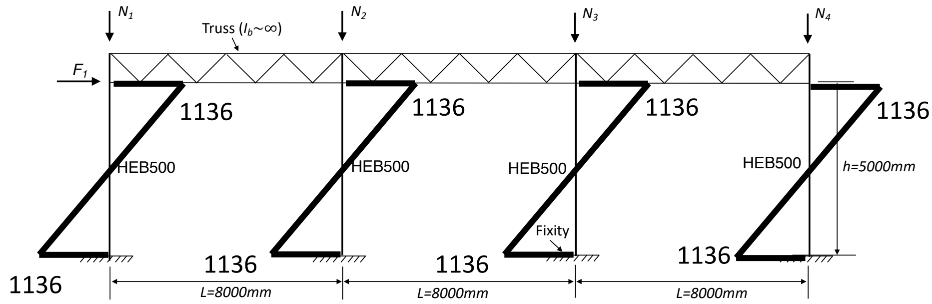


Figure 3 - Moment diagram [kNm]

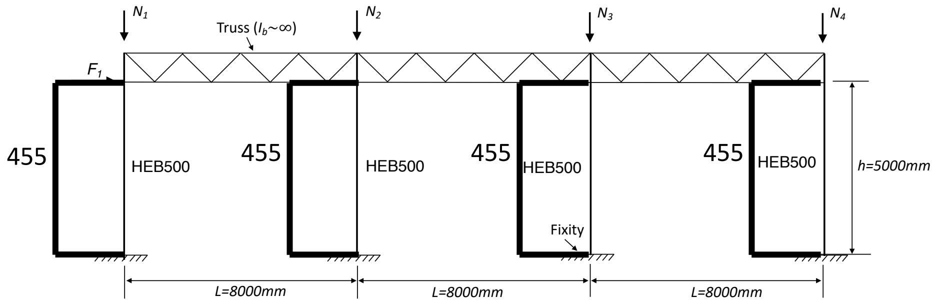


Figure 4 - Shear diagram [kN]

The values of the shear and moment diagrams do not increase by much because the amplification factor is near 1.0. This is not always the case!

4. The check is performed using the following equation:

$$\frac{N_{ed}}{N_{k,Rd}} + \frac{\omega}{1-\frac{N_{ed}}{N_{cr,y}}} \cdot \frac{M_{ed}}{M_{Rd}} \leq 1 \tag{30}$$

With:

$$\omega = 0.6 + 0.4\left(\frac{M_{min}}{M_{max}}\right) \geq 0.4 \tag{31}$$

In our case, $\omega = 0.4$ because $\frac{M_{min}}{M_{max}} = -1$.

Lateral torsional buckling is neglected in this case; therefore, the resisting moment can be computed as follows:

$$M_{Rd} = \frac{f_y \cdot W_{pl,y}}{\gamma_{M1}} = \frac{355 \cdot 4820000}{1.05} = 1629.6kNm \tag{32}$$

Equation 28 becomes:

$$\frac{4227}{7282} + \frac{0.4}{1 - \frac{4227}{338567}} \cdot \frac{1125}{1629.6} = 0.58 + 0.28 \ = 0.86 \ \leq 1 \tag{33}$$

The interaction check is satisfied. Notice though that the axial load coming from the bracing members in the N-S loading direction is much larger than the corresponding compressive axial load due to gravity loading.