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Exercise #2: Moment curvature analysis & Plastic Collapse Mechanisms

Problem 1

The beam profile shown in Figure 1 is a HEB 500 and it is subjected to flexure. It is fabricated from a S355 steel (f_y=355MPa). The simplified stress-strain relation of this material is Figure 1. Compute the following:

1. The yield resistance, M_y (first yield)
2. The flexural resistance once the two flanges of the cross-section are fully plastified
3. The plastic flexural resistance of the cross-section, M_{pl}
4. Using the three points above draw (in scale if possible) the moment curvature diagram of the steel beam. Assume in your calculations that E=200GPa

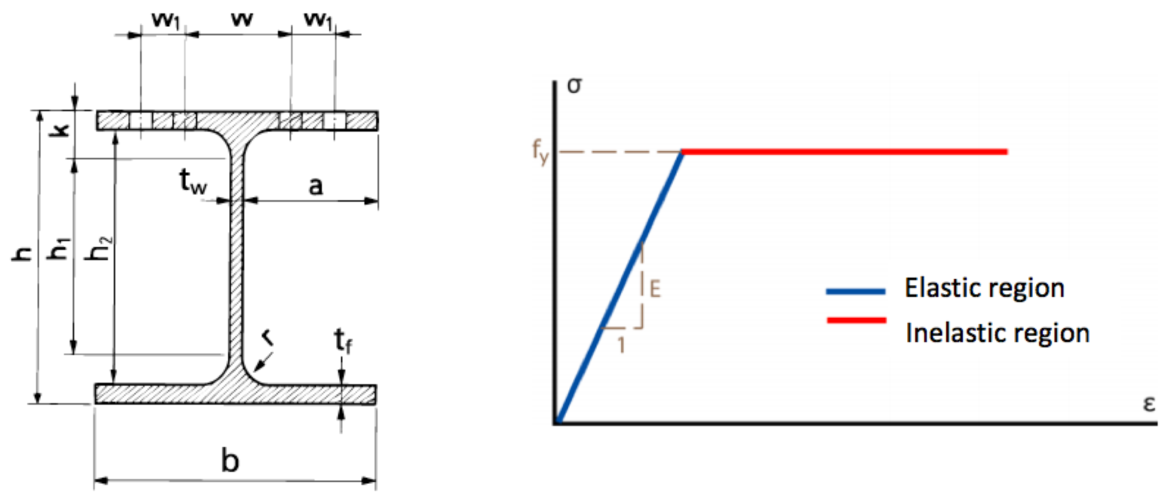


Figure 1. Cross-section and simplified stress-strain curve

Problem 2

The moment resisting frame shown in Figure 2 has an infinitely rigid and strong beam. The steel columns are made of HEB500 and S355 steel. The moment resisting frame experiences an earthquake that causes an inertia force F as shown in the figure. Compute the following:

1. The required force, F_1 that the frame can sustain such that the first plastic hinge can form in one of the two columns.
2. What is the corresponding lateral deflection δ_1 that corresponds to F_1?
3. What would be the incremental load F_2 to develop a full plastic collapse mechanism in the moment resisting frame?
4. What would be the corresponding deflection of the frame once a full collapse mechanism is developed?

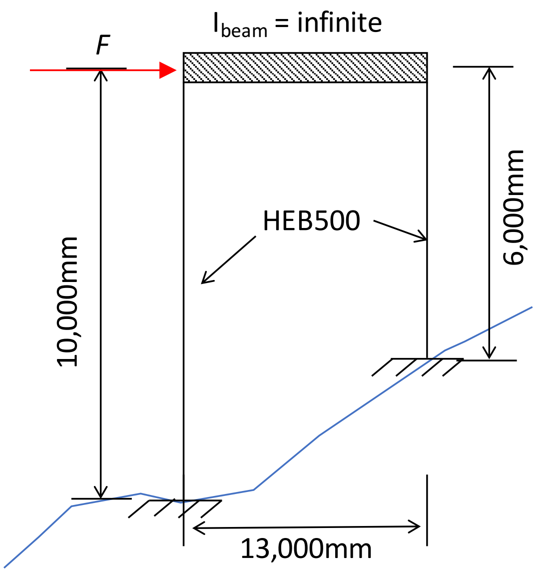


Figure 2. Moment resisting frame subjected to inertia force F

ANNEX

Breitflanschträger HEB — *Profilés à larges ailes* HEB — HEB

Anstelle des nicht mehr gewalzten Profils HEB 1100 können HL-Profile verwendet werden, siehe Seiten 40/41.

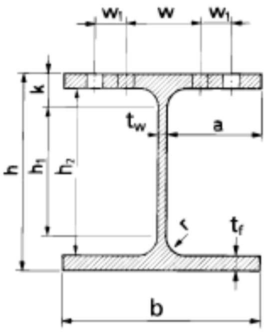
w_1 mit $Ø_{max}$ nur für versetzte Schrauben.

Walztoleranzen siehe Seite 116

Au lieu du profilé HEB 1100 qui n'est plus laminé, on utilisera des profilés HL (voir pages 40/41).

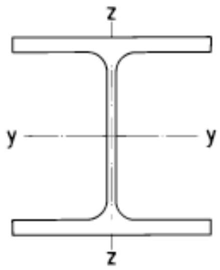
w_1 avec $Ø_{max}$ seulement pour boulons décalés.

Tolérances de laminage voir page 116



HEB	m	Profilmasse *Dimensions de la section*					Konstruktionsmasse *Dimensions de construction*							Oberfläche *Surface*		HEB
	kg/m	h mm	b mm	t_w mm	t_f mm	r mm	h_1 mm	k mm	a mm	h_2 mm	w mm	w_1 mm	$Ø_{max}$	U_m m^2/m	U_t m^2/t	
100	20,4	100	100	6	10	12	56	22	47	80	56		M12	0,567	27,8	100
120	26,7	120	120	6,5	11	12	74	23	56	98	66		M16	0,686	25,7	120
140	33,7	140	140	7	12	12	92	24	66	116	76		M20	0,805	23,9	140
160	42,6	160	160	8	13	15	104	28	76	134	86		M20	0,918	21,5	160
180	51,2	180	180	8,5	14	15	122	29	85	152	100		M24	1,04	20,3	180
200	61,3	200	200	9	15	18	134	33	95	170	110		M24	1,15	18,8	200
220	71,5	220	220	9,5	16	18	152	34	105	188	120		M24	1,27	17,8	220
240	83,2	240	240	10	17	21	164	38	115	206	96	35	M24	1,38	16,6	240
260	93,0	260	260	10	17,5	24	176	42	125	225	106	40	M24	1,50	16,1	260
280	103	280	280	10,5	18	24	196	42	134	244	110	45	M24	1,62	15,7	280
300	117	300	300	11	19	27	208	46	144	262	120	45	M27	1,73	14,8	300
320	127	320	300	11,5	20,5	27	224	48	144	279	120	45	M27	1,77	13,9	320
340	134	340	300	12	21,5	27	242	49	144	297	120	45	M27	1,81	13,5	340
360	142	360	300	12,5	22,5	27	260	50	143	315	120	45	M27	1,85	13,0	360
400	155	400	300	13,5	24	27	298	51	143	352	120	45	M27	1,93	12,4	400
450	171	450	300	14	26	27	344	53	143	398	120	45	M27	2,03	11,9	450
500	187	500	300	14,5	28	27	390	55	142	444	120	45	M27	2,12	11,3	500
550	199	550	300	15	29	27	438	56	142	492	120	45	M27	2,22	11,2	550
600	212	600	300	15,5	30	27	486	57	142	540	120	45	M27	2,32	11,0	600
650	225	650	300	16	31	27	534	58	142	588	120	45	M27	2,42	10,8	650
700	241	700	300	17	32	27	582	59	141	636	126	45	M27	2,52	10,5	700
800	262	800	300	17,5	33	30	674	63	141	734	130	40	M27	2,71	10,4	800
900	291	900	300	18,5	35	30	770	65	140	830	130	40	M27	2,91	10,0	900
1000	314	1000	300	19	36	30	868	66	140	928	130	40	M27	3,11	9,9	1000

HEB — Breitflanschträger HEB — *Profilés à larges ailes* HEB



$A_v = A - 2bt_f + (t_w + 2r)\, t_f$

$A_w = (h - t_f) \cdot t_w$

$S_y = ½\, W_{ply}$

$S_z = ½\, W_{plz}$

$$W_{ely} = \frac{I_y}{h/2} \qquad \overline{W}_y = \frac{I_y}{(h - t_f)/2} \qquad W_{elz} = \frac{I_z}{b/2}$$

Maximale Lagerlängen /
Longueurs maximales en stock:
h ≤ 180   18 m
h ≥ 200   24 m

EURONORM 53 – 62, DIN 1025/2

Andere Bezeichnungen / *Autres désignations*: DIN, IPB

* Auch in S355J0 oder S355J2 ab Schweizer Lager erhältlich

* *Livrable en S355J0 ou S355J2 du stock suisse*

HEB	m	Statische Werte / *Valeurs statiques*													
	kg/m	A mm²	A_v mm²	A_w mm²	I_y mm⁴	W_{ely} mm³	$\overline{W}_y$ mm³	W_{ply} mm³	i_y mm	I_z mm⁴	W_{elz} mm³	W_{plz} mm³	i_z mm	$K = I_x$ mm⁴
					x 10⁶	x 10³	x 10³	x 10³		x 10⁶	x 10³	x 10³		x 10⁶
100*	**20,4**	2600	904	540	4,50	89,9	100	104	41,6	1,67	33,5	51,4	25,3	0,0931
120*	**26,7**	3400	1096	708	8,64	144	158	165	50,4	3,18	52,9	81,0	30,6	0,139
140*	**33,7**	4300	1308	896	15,1	216	236	245	59,3	5,50	78,5	120	35,8	0,202
160*	**42,6**	5430	1759	1180	24,9	311	339	354	67,8	8,89	111	170	40,5	0,312
180*	**51,2**	6530	2024	1410	38,3	426	461	481	76,6	13,6	151	231	45,7	0,422
200*	**61,3**	7810	2483	1660	57,0	570	616	643	85,4	20,0	200	306	50,7	0,596
220*	**71,5**	9100	2792	1940	80,9	736	793	827	94,3	28,4	258	394	55,9	0,770
240*	**83,2**	10600	3323	2230	112,6	938	1010	1050	103	39,2	327	498	60,8	1,04
260*	**93,0**	11800	3759	2420	149,2	1150	1230	1280	112	51,3	395	602	65,8	1,26
280*	**103**	13100	4109	2750	192,7	1380	1470	1530	121	65,9	471	718	70,9	1,45
300*	**117**	14900	4743	3090	251,7	1680	1790	1870	130	85,6	571	870	75,8	1,87
320*	**127**	16100	5177	3440	308,2	1930	2060	2150	138	92,4	616	939	75,7	2,29
340*	**134**	17100	5609	3820	366,6	2160	2300	2410	146	96,9	646	986	75,3	2,62
360*	**142**	18100	6060	4220	431,9	2400	2560	2680	155	101	676	1030	74,9	2,98
400*	**155**	19800	6998	5080	576,8	2880	3070	3230	171	108	721	1100	74,0	3,61
450*	**171**	21800	7966	5940	798,9	3550	3770	3980	191	117	781	1200	73,3	4,49
500	**187**	23900	8982	6840	1072	4290	4540	4820	212	126	842	1290	72,7	5,50
550	**199**	25400	10010	7820	1367	4970	5250	5590	232	131	872	1340	71,7	6,12
600	**212**	27000	11080	8840	1710	5700	6000	6420	252	135	902	1390	70,8	6,80
650	**225**	28600	12200	9900	2106	6480	6800	7320	271	140	932	1440	69,9	7,52
700	**241**	30600	13710	11400	2569	7340	7690	8330	290	144	963	1490	68,7	8,42
800	**262**	33400	16180	13400	3591	8980	9360	10230	328	149	994	1550	66,8	9,62
900	291	37100	18880	16000	4941	10980	11400	12580	365	158	1050	1660	65,3	11,5
1000	314	40000	21250	18300	6447	12890	13400	14860	401	163	1090	1720	63,8	12,7

Problem 1 - Solution

$$\varepsilon_y = \frac{f_y}{E} = \frac{355}{200000} = 0.001775$$

1. The yield strength, M_y at first yield for the cross-section can be computed as follows with reference to the figure below:

$$M_y = W_{el} f_y = 4290x10^3 mm^3 x355N/mm^2 \sim 1523kN - m$$

The corresponding curvature is as follows:

$$\varphi_y = \frac{\varepsilon_y}{\frac{h}{2}} = \frac{0.001775}{500/2} = 7.1x10^{-6}$$

2. Referring to the figure below, the flexural strength of the beam once both flanges fully plastified is computed as follows:

Resultant force on the flange:

$$F_1 = f_y \cdot t_f \cdot b = 355N/mm^2 x28mmx300mm = 2982kN$$

Resultant force on the web:

$$F_2 = \frac{1}{2} \cdot f_y \cdot t_w \cdot \frac{h_2}{2} = \frac{1}{2} \cdot 355N/mm^2 x14.5mmx444/2mm = 571.4kN$$

The corresponding flexural strength (without considering the radius cut) can be computed as follows:

$$M_{flange_{pl}} = \left(2 \cdot \frac{h - t_f}{2} \cdot F_1\right) + 2 \cdot \left(2 \cdot \frac{h_2}{2 \cdot 3}\right) \cdot F_2 = 1576kN - m$$

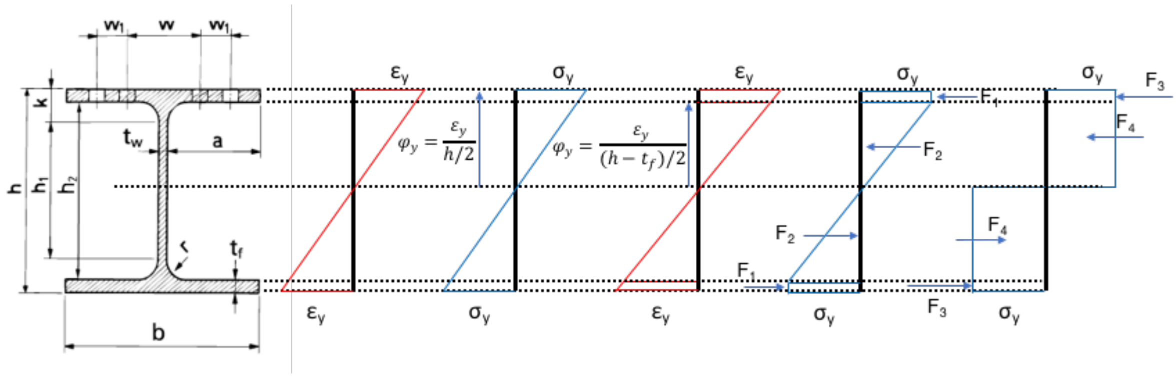
The corresponding curvature at this point is as follows:

$$\varphi_y = \frac{\varepsilon_y}{\frac{h_2}{2}} = \frac{0.001775}{222} = 7.9955x10^{-6}$$

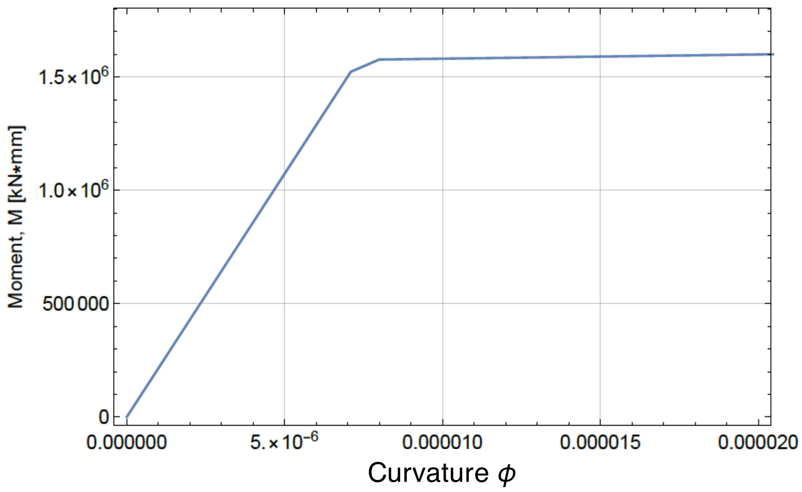
3. The corresponding flexural strength when the full cross-section is plastified is as follows:

$$M_{pl} = W_{ply} \cdot f_y = 4820x10^3 mm^3 x\ 355N/mm^2 = 1711kN - m$$

The curvature in this case becomes infinite



The moment curvature relationship in this case can be plotted as follows



Problem 2 - Solution

1. The horizontal beam is considered as rigid therefore in order to compute the moment diagram in the steel moment frame we first need to see how much shear goes in each column: Column 1 (10,000mm)

Lateral Stiffness:

$$K_1 = \frac{12EI_y}{L_1^3} = \frac{12x200x1072x10^6}{10000^3} = 2.5728kN/mm$$

Column 2 (6,000mm)

Lateral Stiffness:

$$K_2 = \frac{12EI_y}{L_1^3} = \frac{12x200x1072x10^6}{6000^3} = 11.9111kN/mm$$

Therefore the shorter column is stiffer than the longer one as expected considering that they both have the same moment of inertia.

In this case, $V_2 = K_2/(K_1+K_2)$ x F = 11.9111/(11.9111 + 2.5728) F = 0.822F
And $V_1 = K_1/(K_1+K_2)$ x F = 2.5728/(11.9111 + 2.5728) F = 0.178F

Moment in Column 1:

$M_1 = V_1$ x $L_1/2$ = 0.178F x 10,000/2 = 0.890F kN-m

Moment in Column 2:

$M_2 = V_2$ x $L_2/2$ = 0.822F x 6,000/2 = 2.466F kN-m

As expected the moment in column 2 (Shorter of the two) is much larger than the moment in column 1 (longer of the two); therefore, this is the one to be plastified first:

$$M_{pl} = W_{ply} \cdot f_y = 4820x10^3 mm^3 x\ 355N/mm^2 = 1711kN - m$$

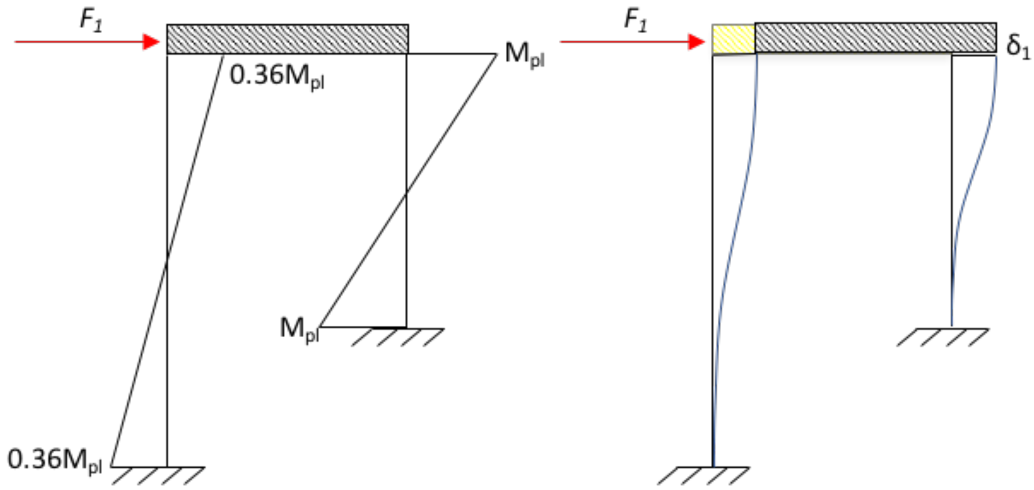
Therefore, $M_2 = M_{pl}$ = 1711 ➔2.466F = 1711 → F = 1711/2.466 = 693.8kN

At this point, the corresponding moment in column 1 is M_1 = 0.890F = 617.5kN-m
= 0.36M_{pl}

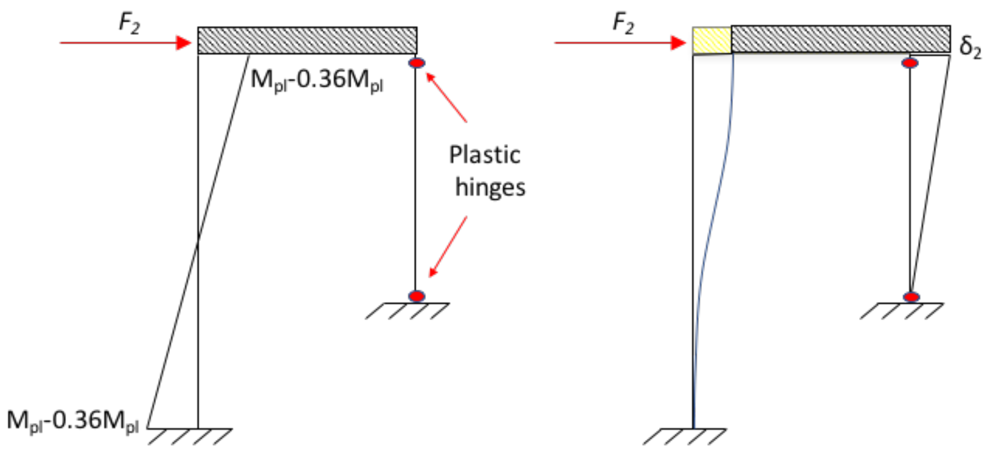
Therefore the moment diagram once the first plastic hinge forms is shown in the figure below.

2. The lateral displacement of the steel moment frame once the first plastic hinge forms can be computed as follows:

$\delta_1 = F_1/K_{tot}$ = 693.8/(11.911 + 2.5728) = 693.8/14.4839 = 47.90mm



3. The incremental load that is needed to develop a second plastic hinge in column 1 can be computed as follows:



additional moment in column 1 due to F_1:

$M_1 = F_2 \times L_1/2 = 5000F_1 = 5F_2$ KN-m

In order for the plastic hinge to develop $M_1 + 617.5 = M_{pl} \rightarrow 5F_2 = 1711\text{-}617.5$

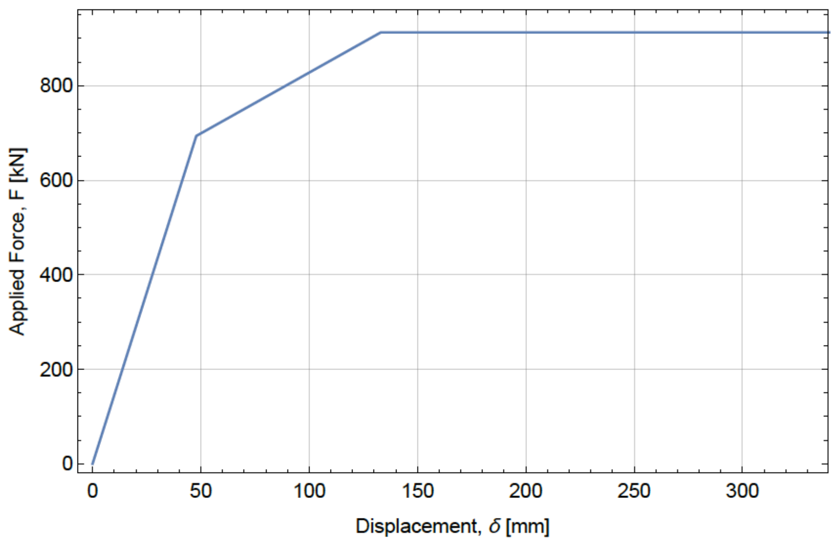
Therefore, $F_2 = 218.7$kN

The corresponding additional displacement at this point will be:

$\delta_2 = F_2/K_1 = 218.7/2.5728 = 85.12$mm (Note that $K_2 = 0$ because we have a plastic hinge in column 2)

The total displacement at this point is $\delta = \delta_1 + \delta_2 \sim 133.0$mm

Therefore, the corresponding force – displacement diagram is as follows:


Applied Force, F [kN]
800
600
400
200
0
0
50
100
150
200
250
300
Displacement, δ [mm]