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Exercise #1: "Easy" statics

Consider Figure 1 that depicts a 1-story and 2-bay industrial frame with height "*h*" and beam length "*L*". Assume that the beam connecting columns 1 and 2 and 3 is infinitely rigid since it is a truss. Columns 1 and 3 are pinned to the ground. Column 2 is fixed to the ground. All members are made of steel S355 (f_y=355MPa). Assume that the material is elastic. Assume that h=6m and L=8m in this case. Ignore the vertical load P acting on the structure for this exercise.

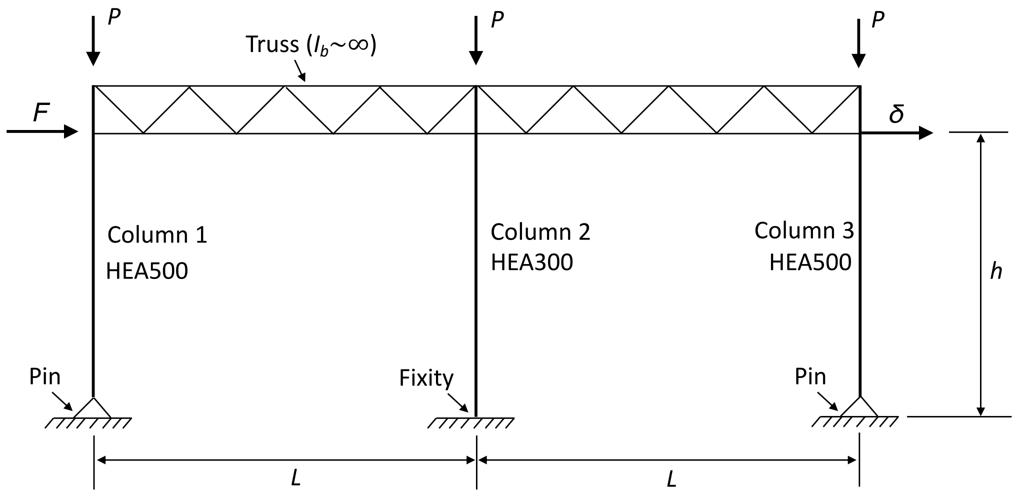
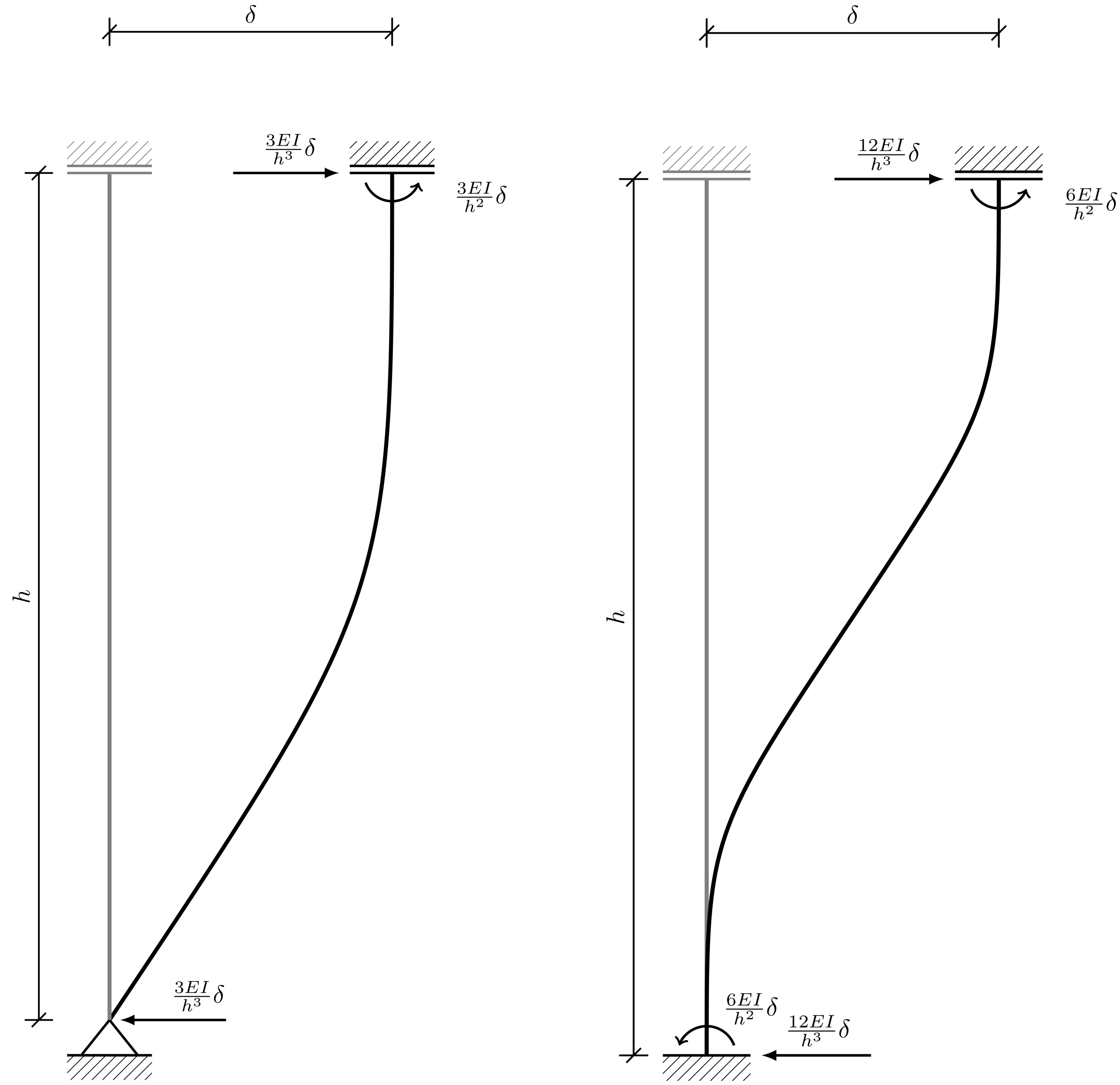


Figure 1. 1 story, 2-bay industrial frame

1. Compute the lateral stiffness, K_{total} of the frame. For stiffness computations assume that flexural deformations dominate the lateral deformation of the steel MRF (i.e., shear and axial deformations can be neglected); ignore the effect of the gravity load on the shear resistance for this computation.
2. Compute the expected lateral displacement, δ of the frame due to the horizontal force F = 800kN.
3. Compute the shear forces and bending moments of the steel MRF (use of "Easy Statics" shown in class).

Solution



Q1: The truss is a horizontal rigid boundary that assures the same horizontal displacement δ in all columns. The equivalent lateral stiffness by force F can be taken from the sum of all lateral forces of the columns before any plastic hinge occurs:

$$F = K_{total}\delta = \frac{3EI^{HEA500}}{h^3}\delta + \frac{12EI^{HEA300}}{h^3}\delta + \frac{3EI^{HEA500}}{h^3}\delta =$$

$$= \left(\frac{3EI^{HEA500}}{h^3} + \frac{12EI^{HEA300}}{h^3} + \frac{3EI^{HEA500}}{h^3}\right)\delta \Rightarrow$$

$$\Rightarrow K_{total} = \frac{200e6}{6^3}(2 * 3 * 869.7e6 * 10^{-12} + 12 * 182.6e6 * 10^{-12}) = 6860.55\ kN/m$$

Q2: for F=800kN, the corresponding lateral deflection is $\delta = \frac{800}{6860.55} = 0.12m = 120mm$

Q3: The shear force in columns 1 and 3 is computed as follows:

$$V_{1,3} = \frac{K_{1,3}}{K_{total}} \cdot F = \frac{2415.84}{6860.55} \cdot 800 = 281.7\ kN$$

The inflection point (point of zero moment) is at the pined end of the columns, and the moment diagram is linear, so the moment at the top of the columns is computed using:

$$M_{1,3} = V_{1,3} \cdot h = 281.7 \cdot 6 = 1690\ kNm$$

The shear force in column 2 is computed using:

$$V_2 = \frac{K_2}{K_{total}} \cdot F = \frac{2028.89}{6860.55} \cdot 800 = 236.6\ kN$$

For column 2, the inflection point is in the middle of the column, and the moment diagram is linear, so the moments at the ends of the column are computed using:

$$M_2 = V_2 \cdot \frac{h}{2} = 236.6 \cdot \frac{6}{2} = 709.8kNm$$

In order to find the shear and moment diagrams in the truss, the axial load in the columns must be determined first.
The axial load in the middle column is equal to 0 kN.
To find the reaction force in the exterior columns, we do the moment equilibrium with respect to the fixed end:

$$R_1 \cdot L + R_3 \cdot L - F \cdot h + M_2 = 0$$

To satisfy the equilibrium of the vertical forces,

$$R_1 + R_3 = 0$$

And so:

$$2 \cdot R_1 \cdot L = -M_2 + F \cdot h \rightarrow R_1 = -255.6\ kN$$

$$R_3 = 255.6kN$$

By equilibrium at the connection column/truss, the normal force in the column is equal to the shear force in the beam:

$$V_{beam} = 255.6kN$$

The moment diagram in the beam is obtained by integrating the shear diagram: the variation of moment along the beam is equal to the shear force, and the moment diagram is linear, so we have:

$$\frac{M_1 + x}{L} = V_{beam} \rightarrow x = 354.8kNm$$

Where x denotes the moment at the intersection between the column 2 and the beam.

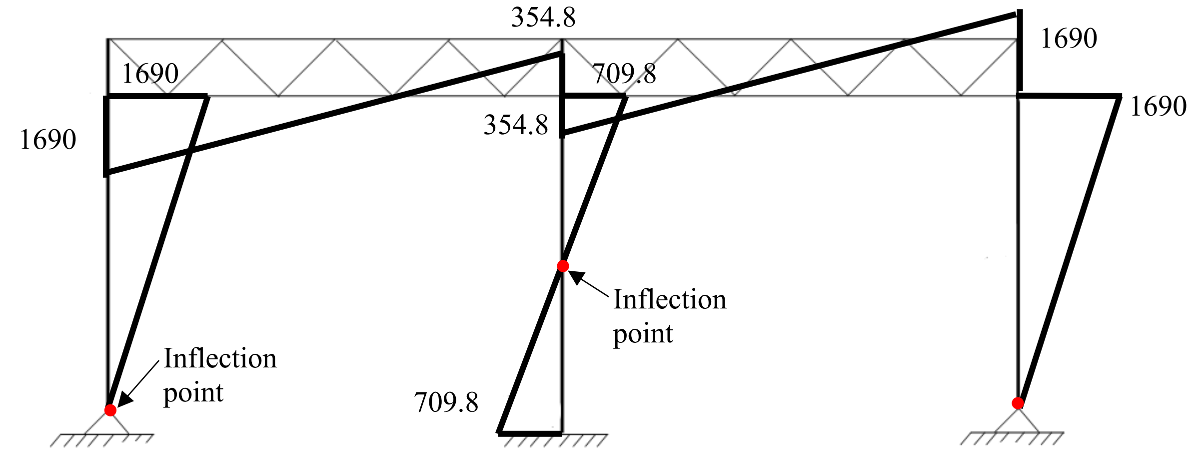


Figure 2 - Moment diagram [kNm]

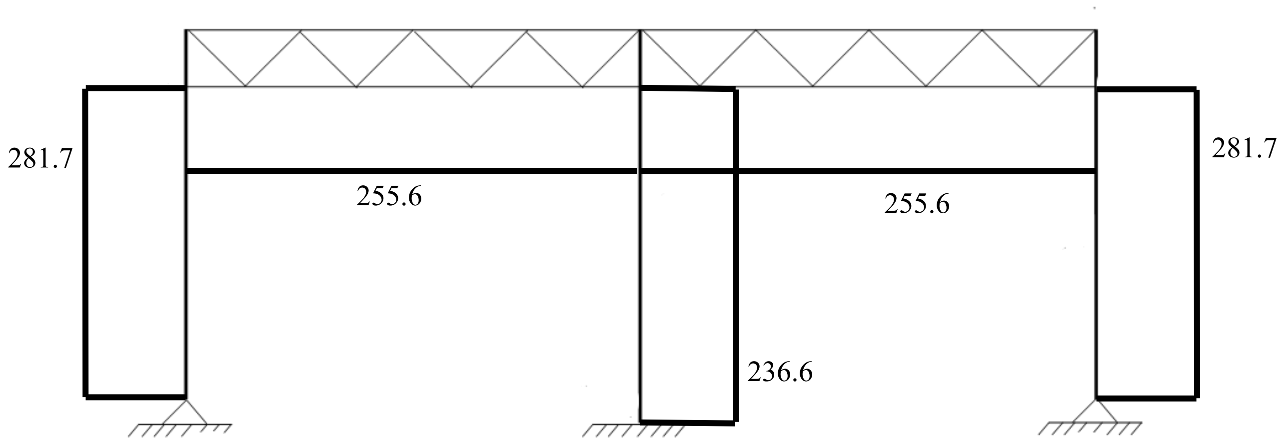


Figure 3 - Shear diagram [kN]