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Exercise #11 –Dynamic Method of Equilibrium

The steel braced frame shown in Figure 1a carries a mass, m and is subjected to a lateral load F as shown in the figure. The lateral stability of the braced frame is only provided by two steel braces with a length, L and area, A that are not connected at their center (see Figure 1a). The buckling resistance of the two braces is P_{cr} and assume P_{cr}=EA/(cL) with c>1 m^{-1} (a constant) such that $P_{cr} < P_y = f_y A$ as shown in Figure 2. The braced frame can lose its lateral stability when it is subjected to the lateral force F. This is caused by combined brace buckling and yielding as shown in Figure 1b. The expected brace behavior in tension (negative loading) and compression (positive loading) is shown in Figure 2.

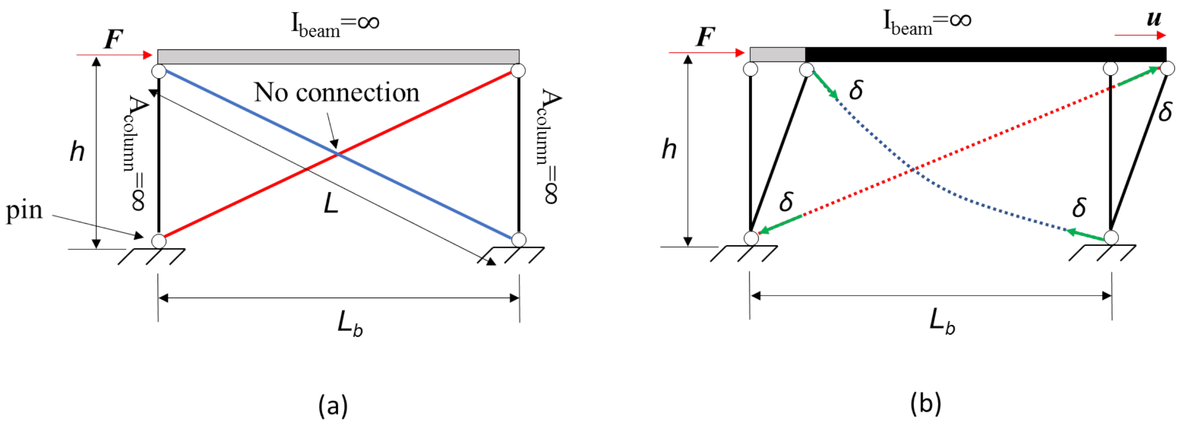

Figure 1. Steel braced frame under lateral loading

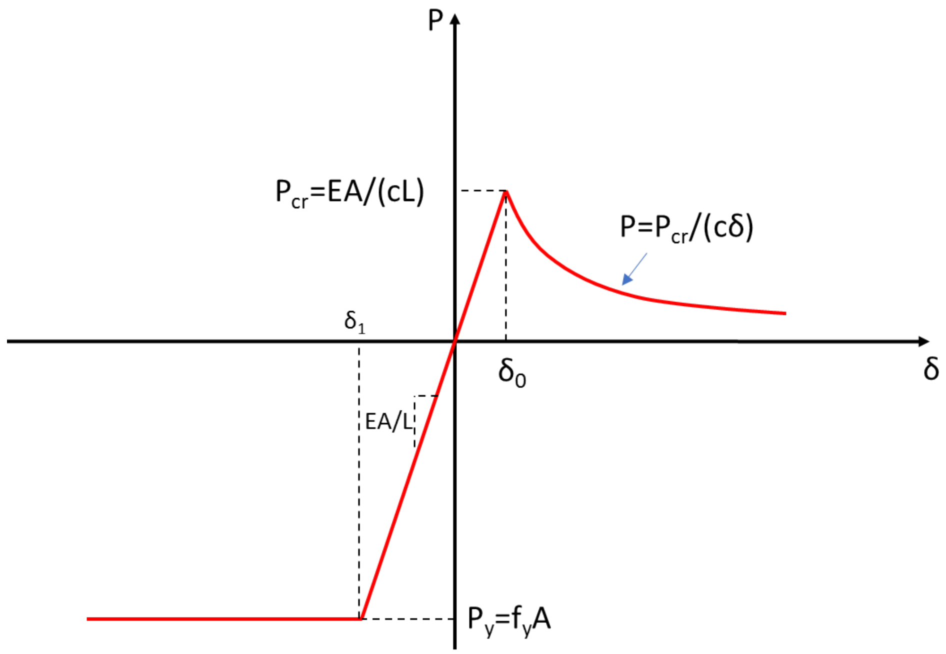

Figure 2. Brace behavior in tension and compression

Compute the following:

1. The lateral stiffness K_{total} of the braced frame before any lateral load application. Explain if the frame is stable or not based on the dynamic approach of stability (computation of radian frequency ω is sufficient in this case).
2. Calculate the lateral displacement u_0 at which the brace in compression buckles. Based on the dynamic approach of stability (computation of radian frequency ω is sufficient in this case), is the braced frame stable at this displacement? Explain your answer.
3. Calculate the displacement $u > u_0$ at which the braced frame loses completely its lateral stability. Explain your answer based on the dynamic approach shown in class (computation of radian frequency ω is sufficient in this case).
4. Suggest a solution to increase the critical load, P_{cr} such that the lateral stability of the braced frame is less of an issue.

Solution

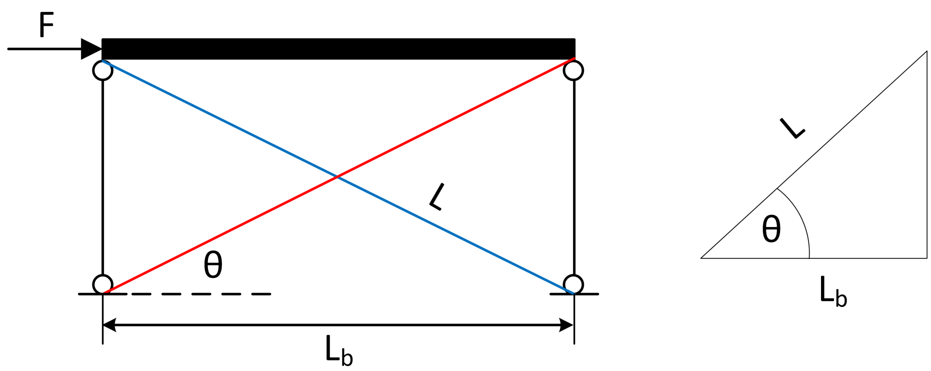

Figure 3: Problem 3 definitions.

Part 1:

For the braced system, the relation between the brace and the frame is

$$K_{total} = K_{ten} + K_{comp} \tag{1}$$

When no loading is imposed, the stiffness in tension and compression will be $K_{brace} = EA/L$. The relation between the brace stiffness and the frame stiffness is $K_{frame} = K_{brace} * \cos^2\theta$. Taking $\cos\theta = L_b/L$ as shown in Figure 5, K_{total} is then

$$K_{total} = \frac{EA}{L} * \left(\frac{L_b}{L}\right)^2 + \frac{EA}{L} * \left(\frac{L_b}{L}\right)^2 = \frac{2EAL_b^2}{L^3} \tag{2}$$

The dynamic stability is based on the natural frequency, $\omega = \sqrt{k/m}$. Since the mass is positive, and the total frame stiffness is positive from Equation (2), $\omega > 0$. The system is stable since the condition for instability is that $\omega < 0$ or imaginary.

Part 2:

First, we need the relation between the lateral displacement and brace elongation (or contraction), $u = \delta/\cos\theta$. The brace buckles at P_{cr}, the brace contraction at this point is δ_0. This can be related to P_{cr} by the stiffness as follows

$$\delta_0 = \frac{P_{cr}}{K_{comp}} = \frac{EA}{cL} * \frac{L}{EA} = \frac{1}{c} \tag{3}$$

To find the stiffness of the frame, the stiffness in the post-buckling region is required. This can be found from the definition of stiffness: $K = \frac{\partial P}{\partial \theta}$. Therefore, the stiffness in the post-buckling region is

$$K_{comp,buckle} = \frac{\partial}{\partial\theta}\left(\frac{P_{cr}}{c} * \delta^{-1}\right) = \frac{-P}{c\delta^2} \tag{4}$$

The stiffness of the frame can now be found for the region after buckling of the brace in compression, but prior yielding of the brace in tension:

$$K_{total} = K_{ten} + K_{comp,buckle} = \left(\frac{EA}{L} + \frac{-P_{cr}}{c\delta^2}\right)\cos^2\theta \quad (5)$$

now subbing for P_{cr},

$$K_{total} = \left(\frac{EA}{L} - \frac{EA}{c^2L\delta^2}\right)\cos^2\theta \quad (6)$$

The condition of instability is based on the natural frequency ω. Investigating the point when the brace buckles, where $\delta = \delta_0 = 1/c$

$$K_{total} = \left(\frac{EA}{L} - \frac{EA}{c^2L\left(\frac{1}{c}\right)^2}\right)\cos^2\theta = 0 \quad (7)$$

At the displacement δ_0, the behavior is unstable since $\omega = 0$. This is because the solution to the ODE will be: $u(t) = c1 + c2*t$, therefore the displacement grows unbounded with respect to time. However, since the compression stiffness tends to zero as δ increases, (i.e., the maximum negative stiffness is obtained at the critical point) stability will be regained after the critical point, as seen in Figure 4. Therefore, the behavior is stable prior to yielding of the tension brace.

Part 3:

Since the tensile stiffness is constant until yielding, the response is stable until the brace yields in tension. At this point the stiffness of the tensile brace is equal to zero, as seen in Figure 2. Therefore, the stiffness of the frame is

$$K_{total} = \left(0 - \frac{EA}{c^2L(\delta_1)^2}\right)\cos^2\theta < 0. \quad (8)$$

Since the stiffness is less than zero, ω becomes imaginary, and the displacements become unbounded with time. This occurs at a brace elongation of δ_1, that corresponds to a frame displacement of

$$u = \delta_1/\cos\theta \quad (9)$$

The full behavior can be seen in the following plot:

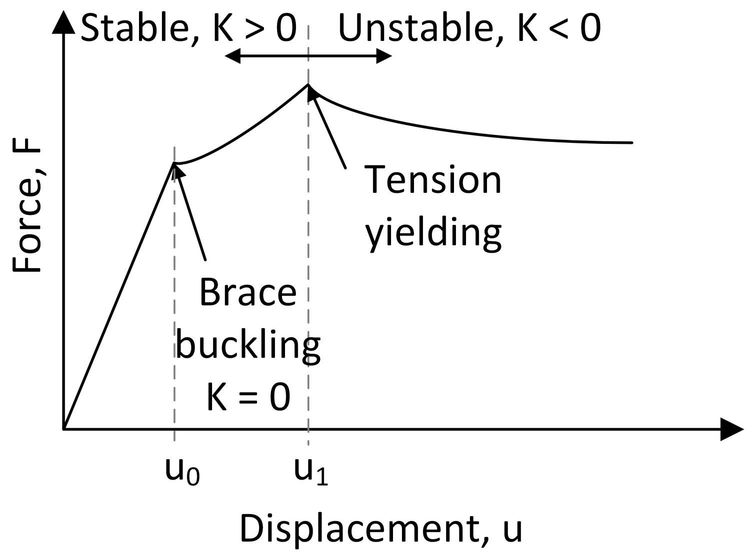

Figure 4: Stiffness plot for the frame

Part 4:

A few suggestions to increase the critical load, P_{cr}:

1. Fix the tensile and compressive brace at the center, such that the effective length is $L=2$
2. Increase the inertia of the brace
3. Increase the material stiffness of the brace (increase E)