CIVIL 369: “Structural Stability”

School of Architecture, Civil & Environmental Engineering
Civil Engineering Institute
Resilient Steel Structures Laboratory (RESSLab)

Frame Stability

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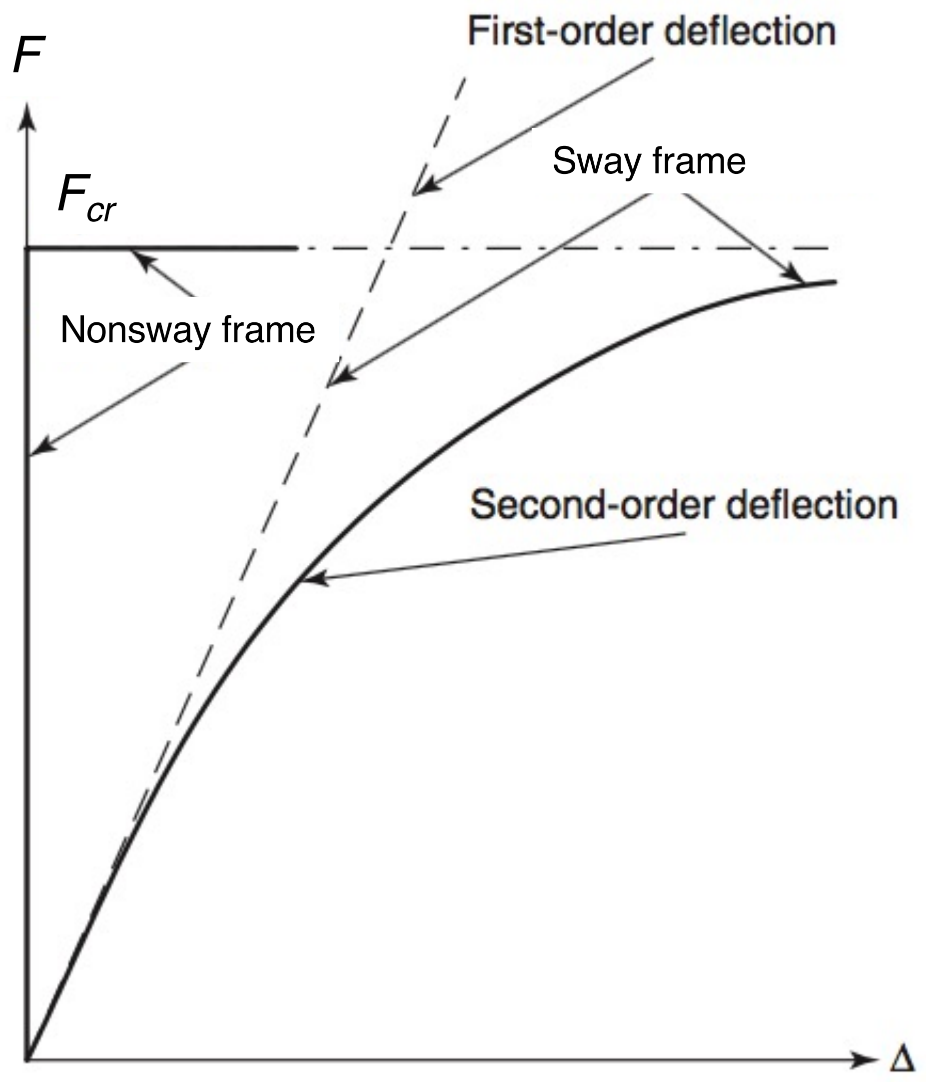
Objectives of this Week's Lecture

To introduce:

- ✧ Frame stability
- ✧ Deformation limits to control second order effects
- ✧ Simplified methods to estimate frame stability in multi-storey frames
- ✧ P-Delta effects
  - ✧ Translation
  - ✧ Torsion
- ✧ Out-of-plumb effects
- ✧ Effect of soil flexibility on second order effects
- ✧ Examples on frame stability (and instability)

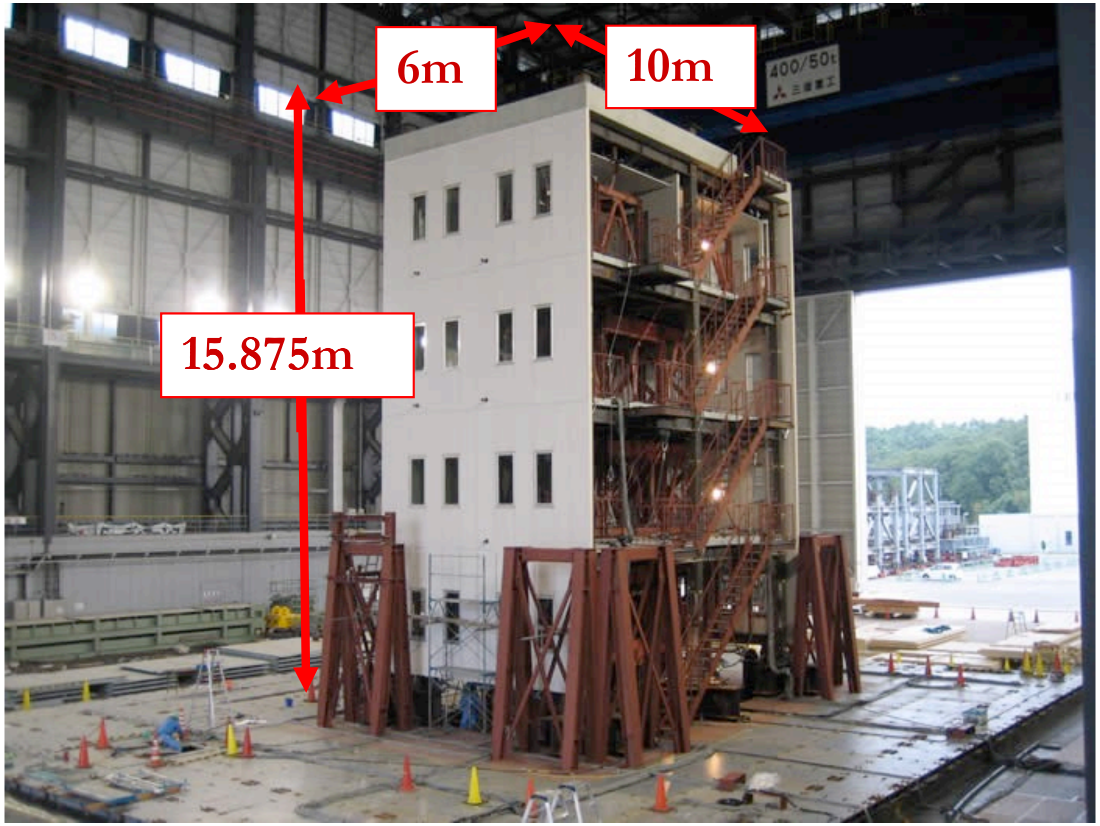
Second Order Versus First-Order Analysis



Second Order Versus First-Order Analysis

Second Order Versus First-Order Analysis

Dynamic Instability due to Earthquake Shaking

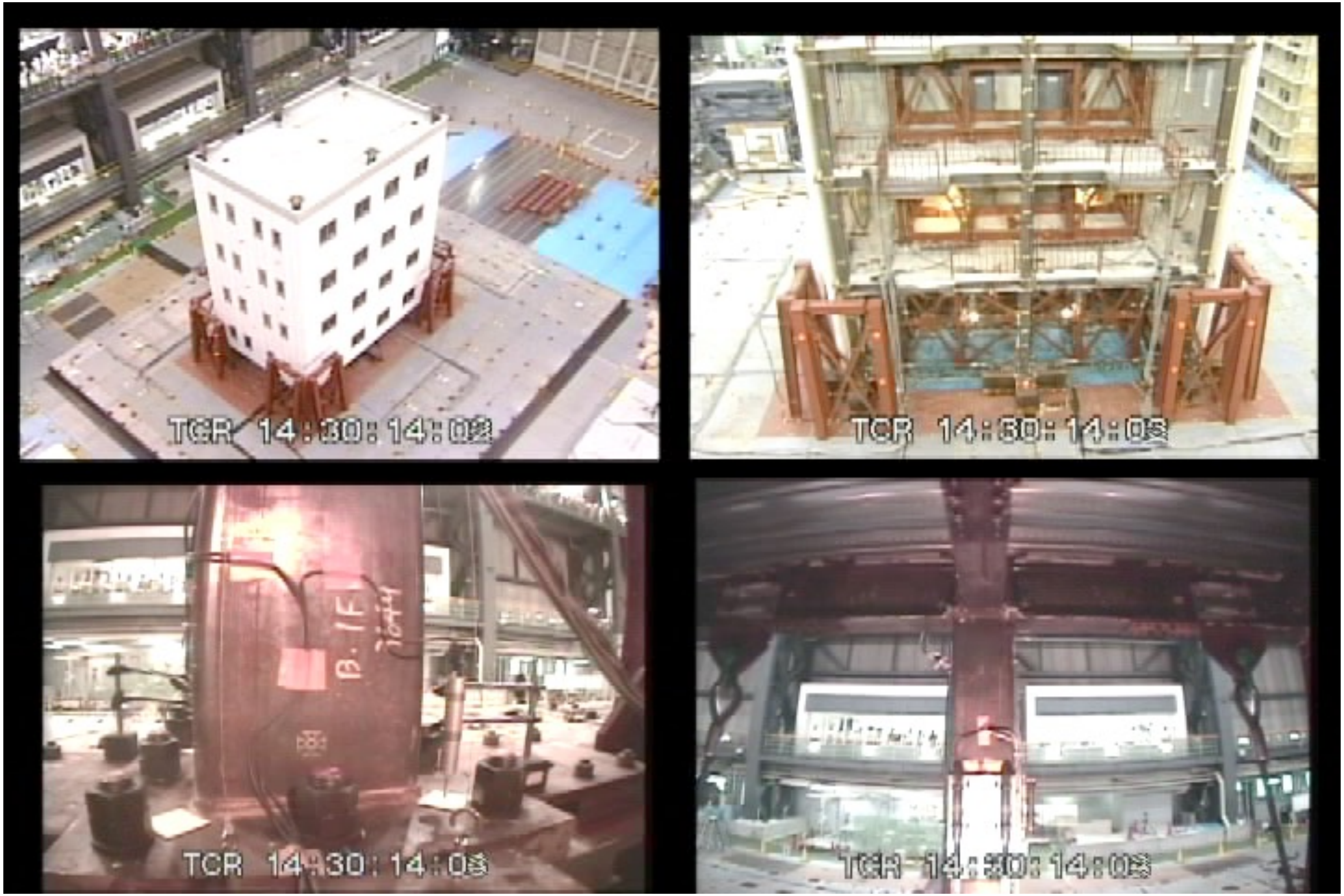


(Photo Source: E-Defense-2007, Suita et al. 2008)

Increasing Scaled Intensity of JR Takatori Earthquake

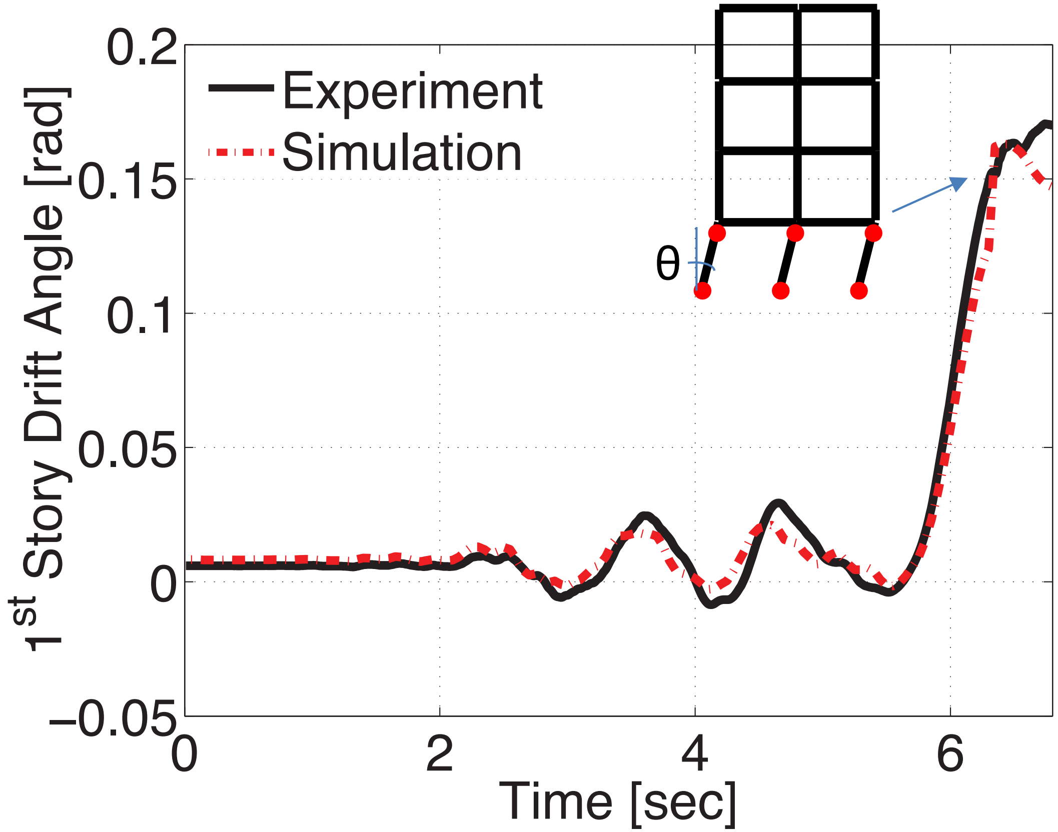
20%, 40%, 60%, 100%

Dynamic Instability due to Earthquake Shaking



EPFL Dynamic Instability due to Earthquake Shaking

Prediction of 1st story collapse mechanism



(Source: Lignos et al. 2013)*

*Lignos, D.G., Hikino, T., Matsuoka, Y., Nakashima, M. (2013). "*Collapse Assessment of Steel Moment Frames based on E-Defense Full-Scale Shake Table Collapse Tests*". ASCE, Journal of Structural Engineering, Vol. 139(1), doi: 10.1061/(ASCE)ST.1943-541X.0000608.

EPFL Collapses due to P-Delta Effects during Earthquakes

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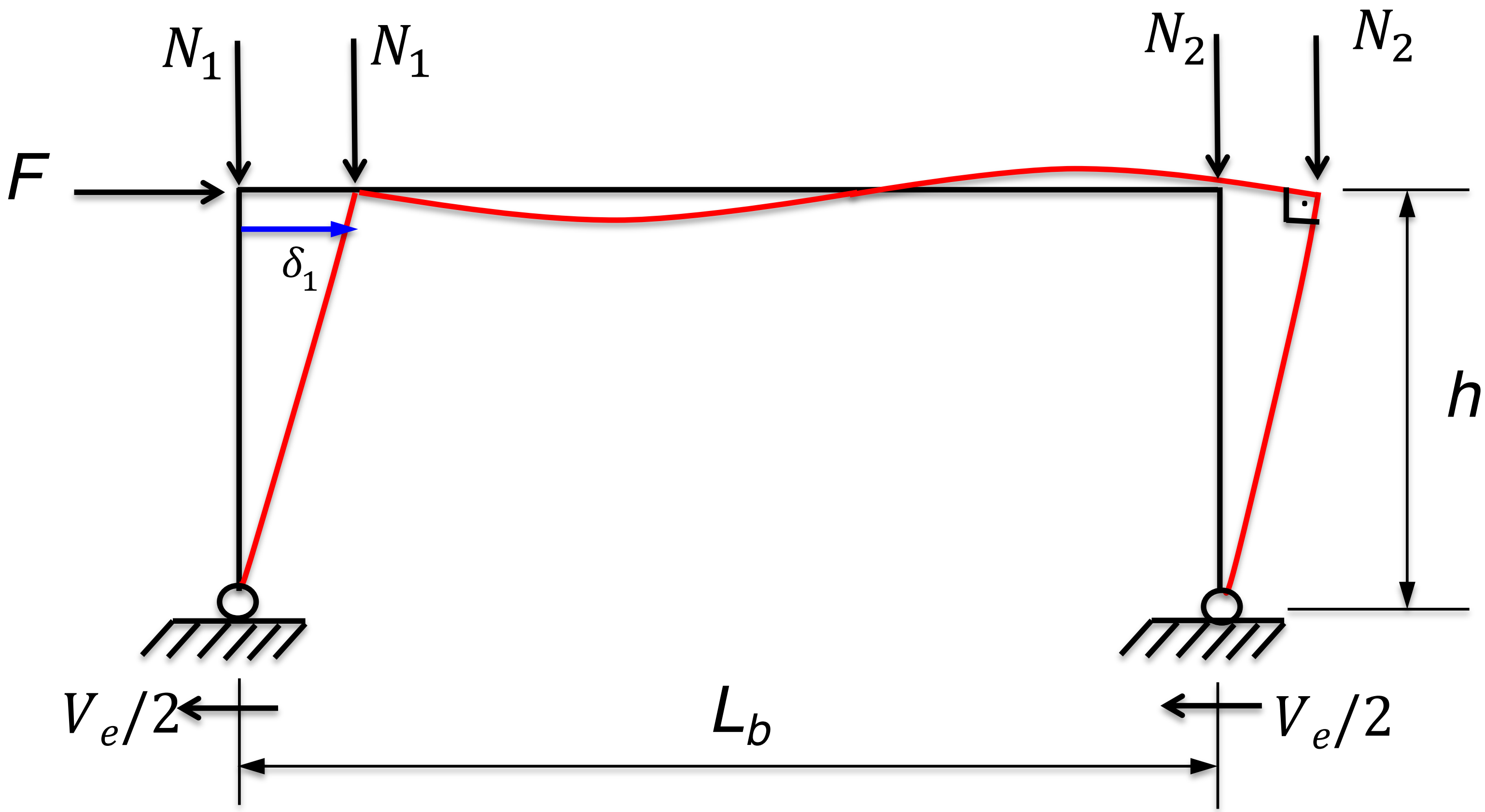
Stability (Second Order) Effects



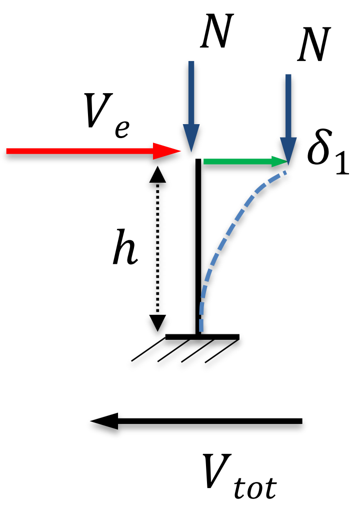
$$V_e = F$$

$$V_{tot} = V_e + V_{P-\Delta}$$

$$V_{tot} = F + \frac{(N_1 + N_2) \cdot \delta_1}{h}$$

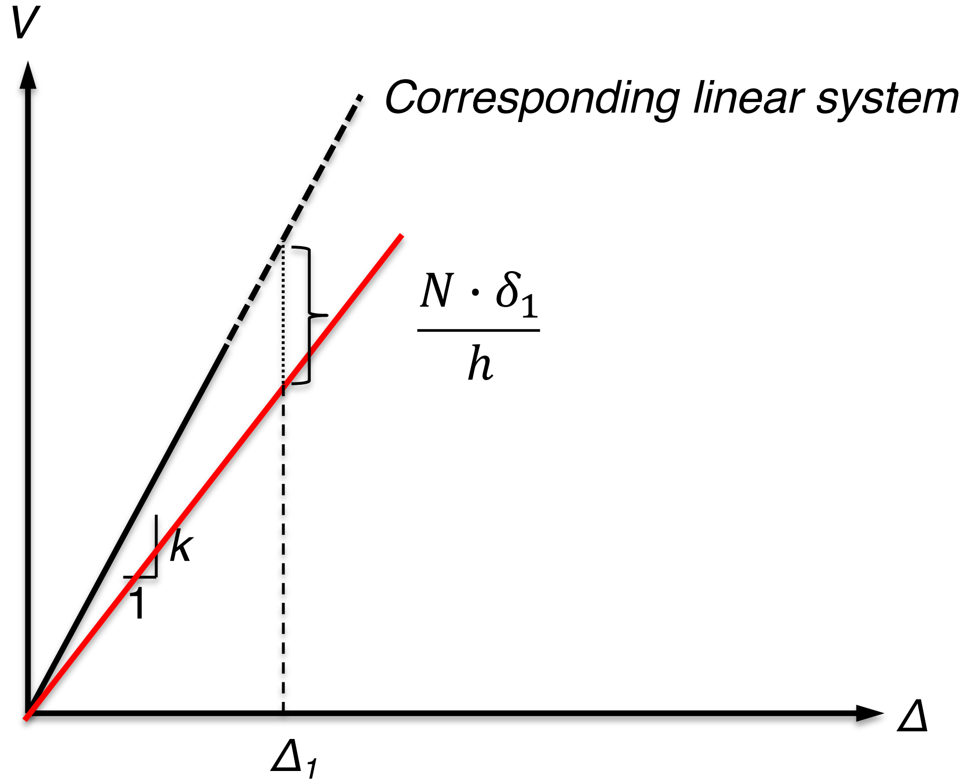
Second-Order Effects Due to Gravity Loads

-Storey Shear Forces Including P-Delta effects

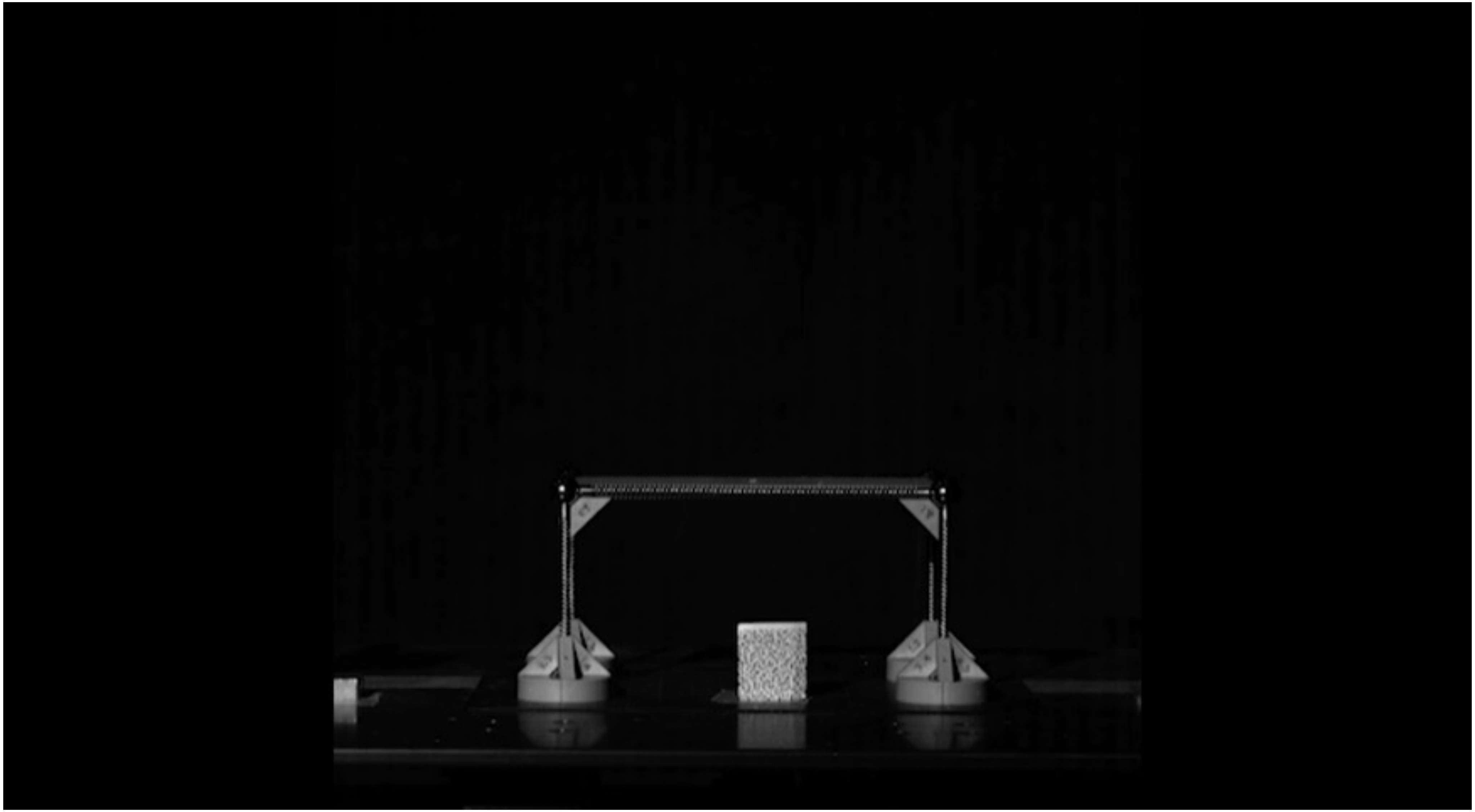


$$V_{tot} = V_e + V_{P-\Delta}$$

$$V_{tot} = V_e + \frac{N \cdot \delta_1}{h}$$

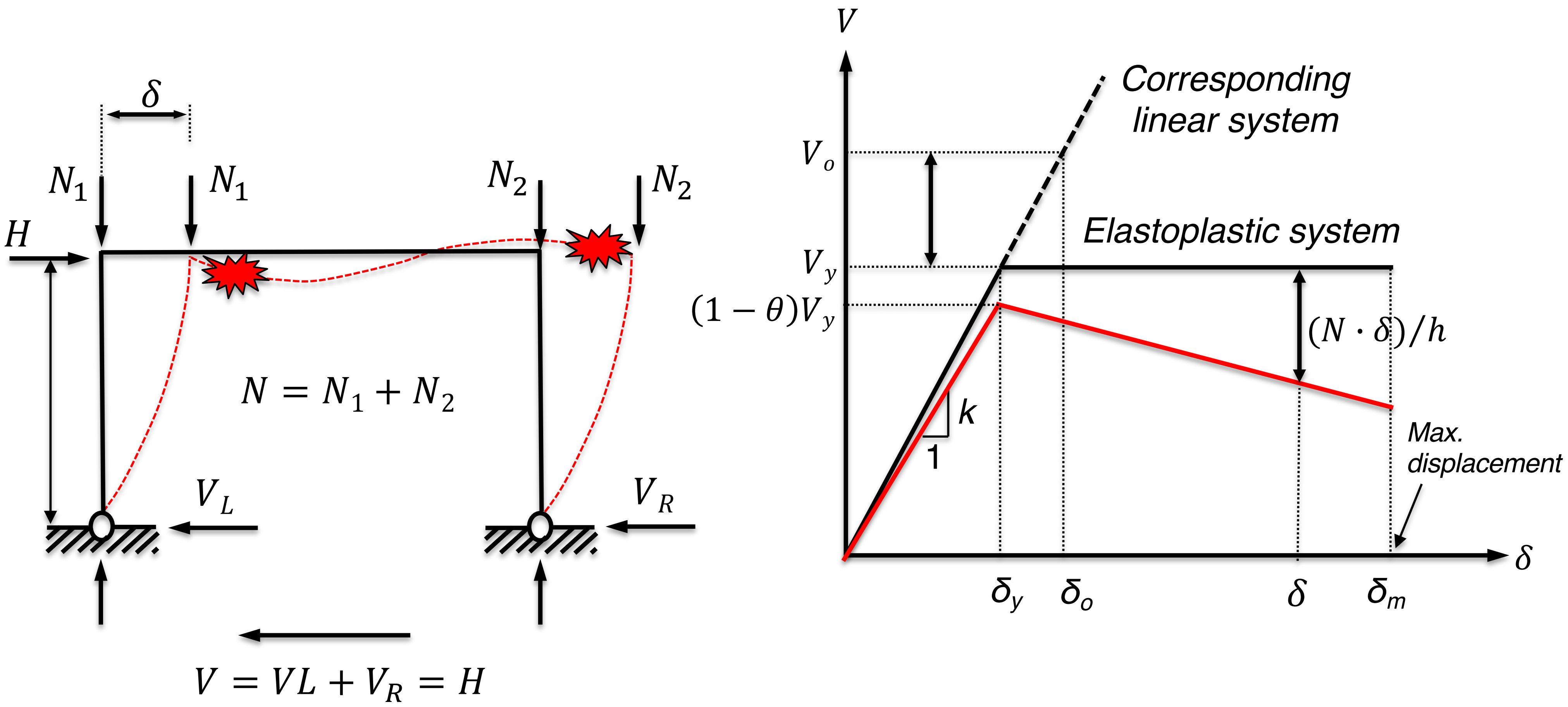


EPFL Electric Shake Table-Collapse Demonstration



Guel, Sousa & Lignos 2017

Material Nonlinearity and P-Delta Effects



EPFL Ways to Handle Frame Stability

- Lateral and torsional deformations must be limited to minimize second order effects.
- A fundamental (and obvious) reason is to prevent structural collapse.
- In terms of serviceability:

a. Deflections must be maintained at a sufficiently low level to allow proper functioning of nonstructural components, elevators, doors.
b. Avoid distressing the structure.
c. Prevent excessive cracking and consequent loss of stiffness
d. Avoid redistribution of loads to non-load-bearing partitions, cladding, or glazing.
e. The structure must be sufficiently stiff to prevent dynamic motions becoming large enough to cause discomfort to occupants, or affect sensitive equipment (e.g., in hospitals, industrial facilities).

EPFL Ways to Handle Frame Stability

- Simplest parameter that affords an estimate of the lateral stiffness of a building is the drift.
- Design "storey drift ratios" that have been used range from 0.1% to 0.5% for serviceability.
- Generally, lower values are used for hotels and apartment buildings to prevent noise and movement.
- For collapse prevention, design storey drift ratios are usually much larger (2% or 2.5%)

EPFL Storey Drift Limits in Our Codes

- **Seismic Design (For Damage Limitation Seismic Action)**
  - SIA-263 none
  - EC8 – Part 1-1
    - 0.50% for buildings having non-structural elements of brittle materials attached to the structure
    - 0.75% for buildings having ductile non-structural elements
    - 1.00% for buildings having non-structural elements fixed in a way so as not to interfere with structural deformations

Storey Drift Limits in Our Codes

- **Seismic Design (For Ultimate Limit State)**
  - SIA-263 - none
  - EC8 – Part 1-1 – none (New revision will have drift limits)
  - ASCE 7-16 (USA)

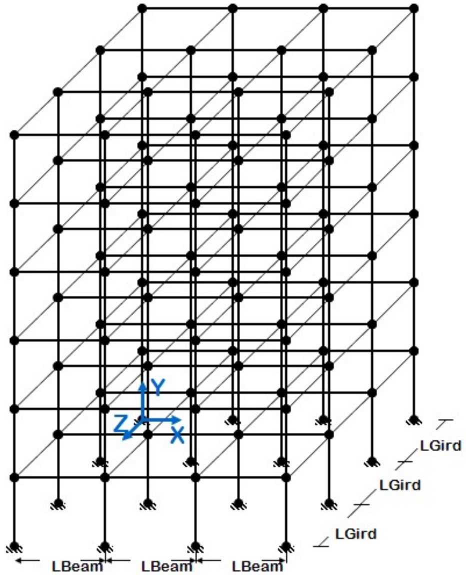
Structure	Risk Category		
	I or II	**III**	**IV**
Structures, other than masonry shear wall structures, four stories or less above the base as defined in Section 11.2, with interior walls, partitions, ceilings, and exterior wall systems that have been designed to accommodate the story drifts	$0.025h_{sx}{}^{c}$	$0.020h_{sx}$	$0.015h_{sx}$
Masonry cantilever shear wall structures[d]	$0.010h_{sx}$	$0.010h_{sx}$	$0.010h_{sx}$
Other masonry shear wall structures	$0.007h_{sx}$	$0.007h_{sx}$	$0.007h_{sx}$
All other structures	$0.020h_{sx}$	$0.015h_{sx}$	$0.010h_{sx}$

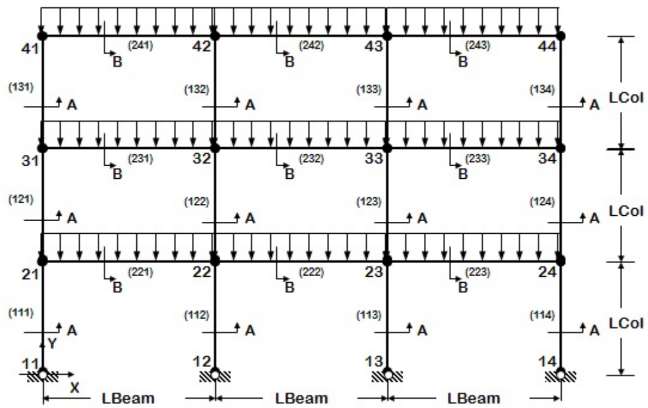
  - NBCC 2015 (Canada)

Structure	Risk Category		
	Normal	**Post-disaster**	**High-importance**
All buildings	$0.025h_{sx}{}^{c}$	$0.020h_{sx}$	$0.010h_{sx}$

Modeling Techniques for Frame Structures To Quickly Assess Frame Stability

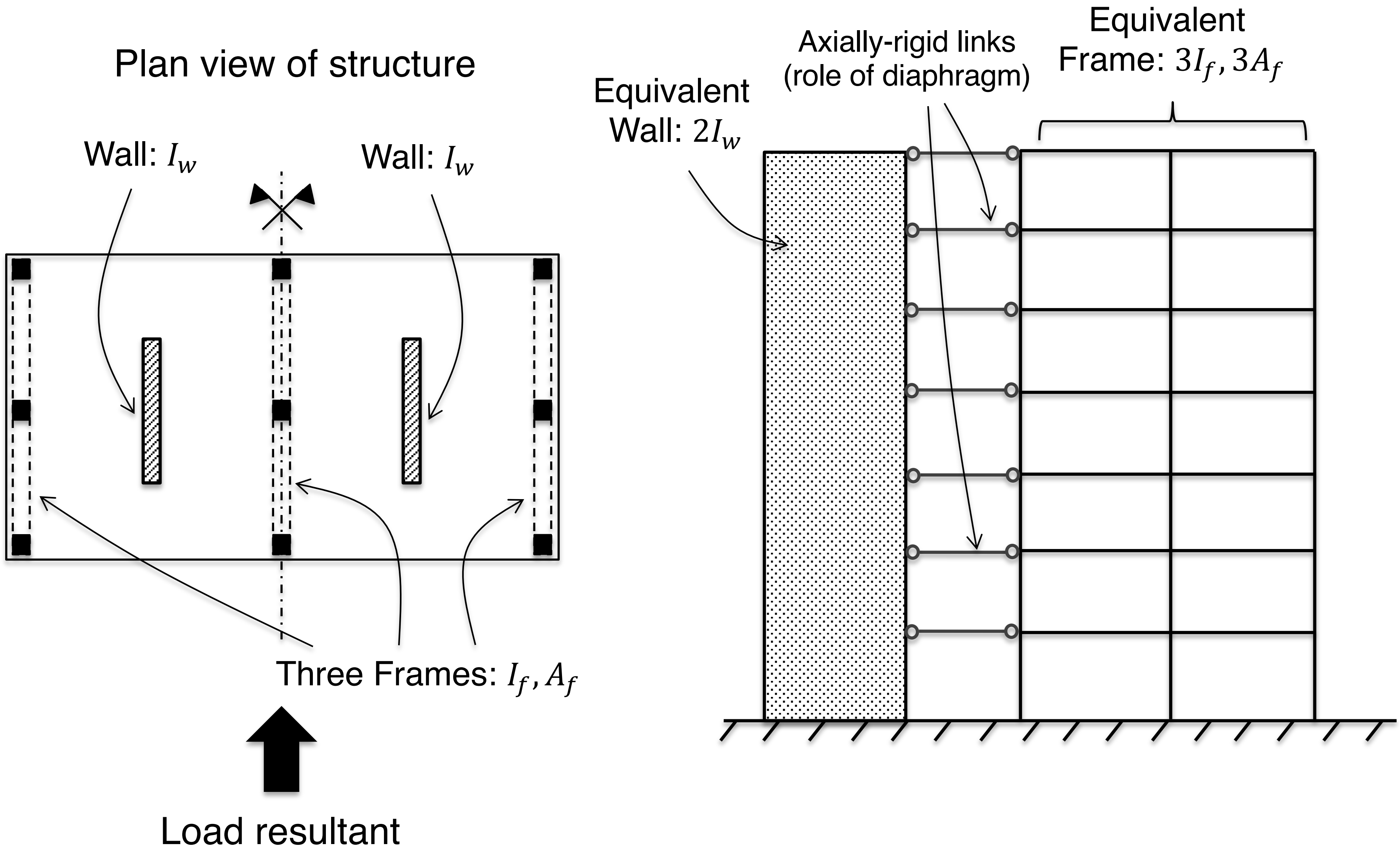
Frame Structures



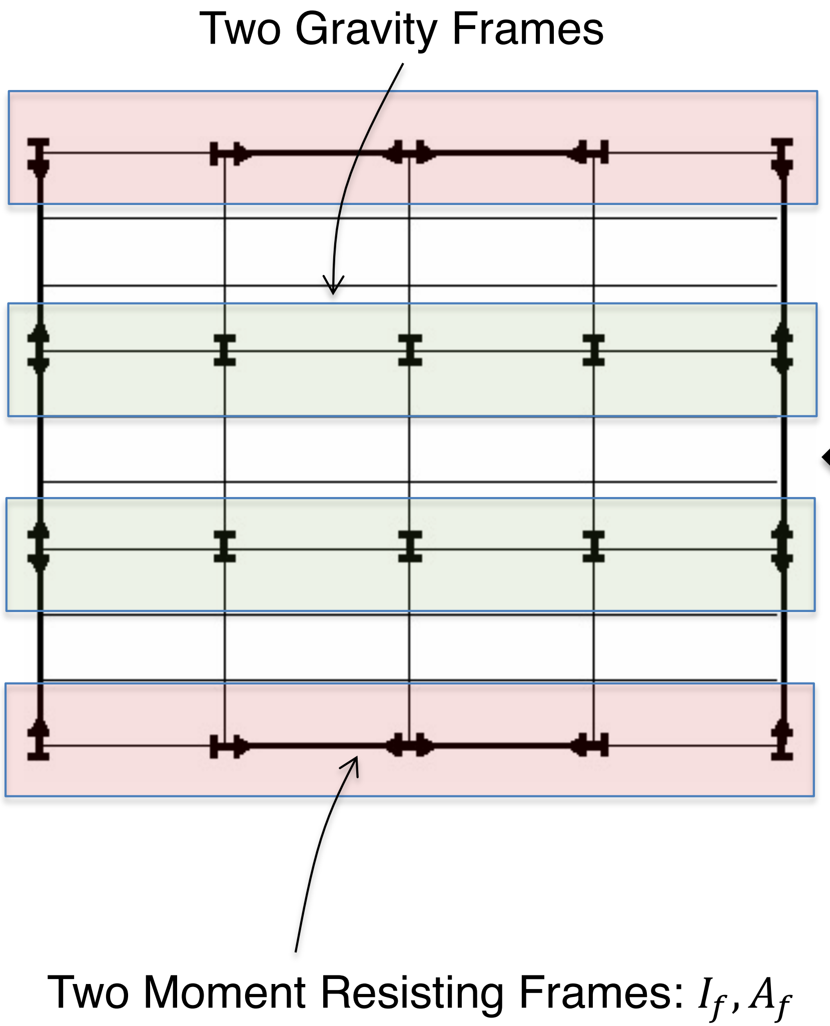


Source: OpenSees Examples

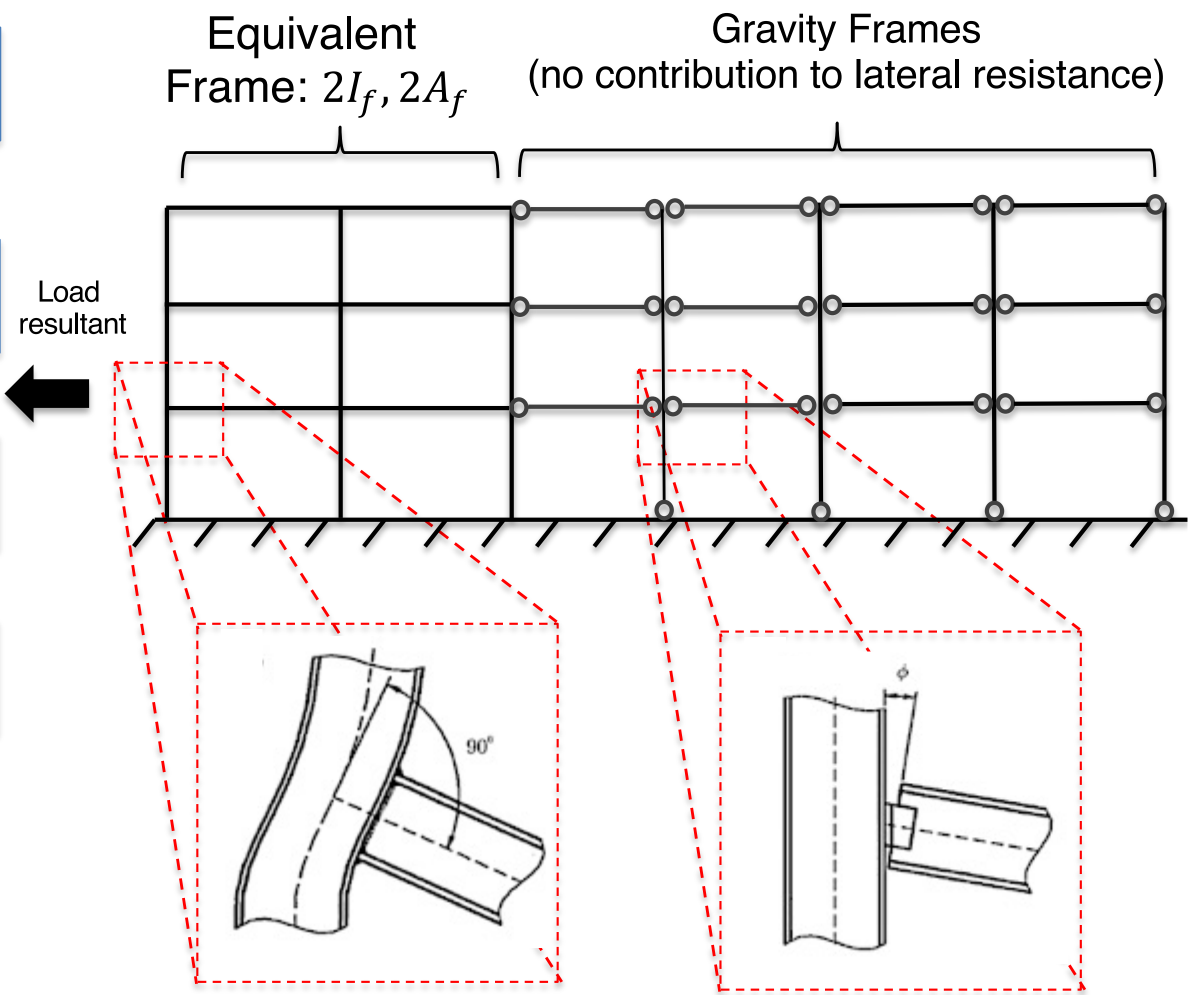
Simplified Modeling Techniques to Assess Stability in Multi-Storey Structures



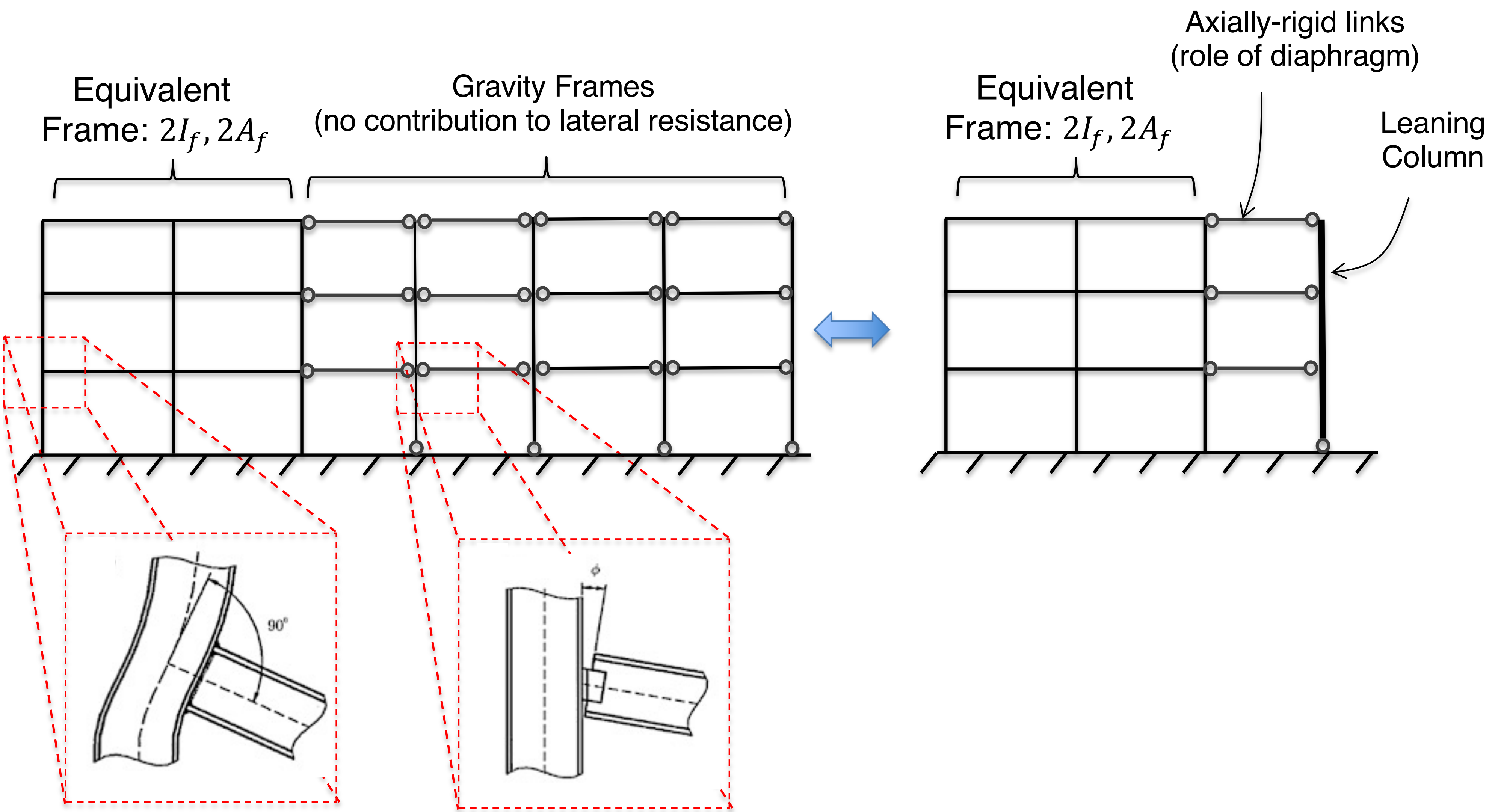
Simplified Modeling Techniques to Assess Stability in Multi-Storey Structures



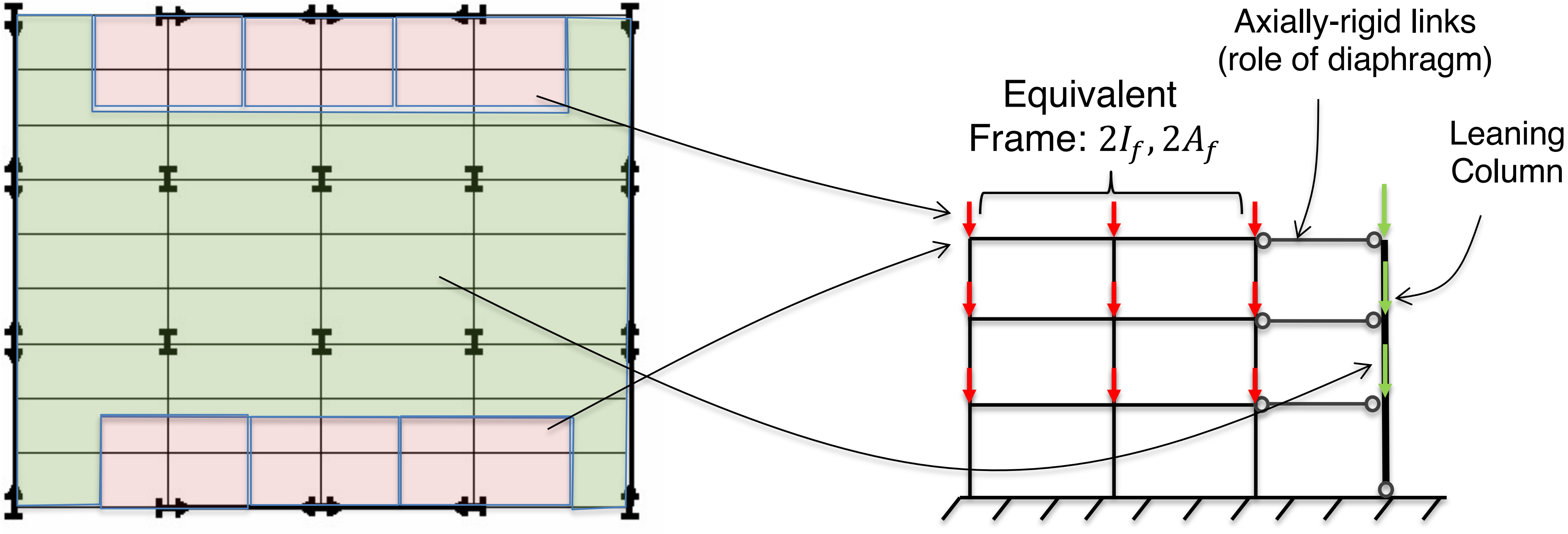
(Source Lignos et al. 2011)



Simplified Modeling Techniques to Assess Stability in Multi-Storey Structures



The Leaning Column for Consideration of P-Delta Effects



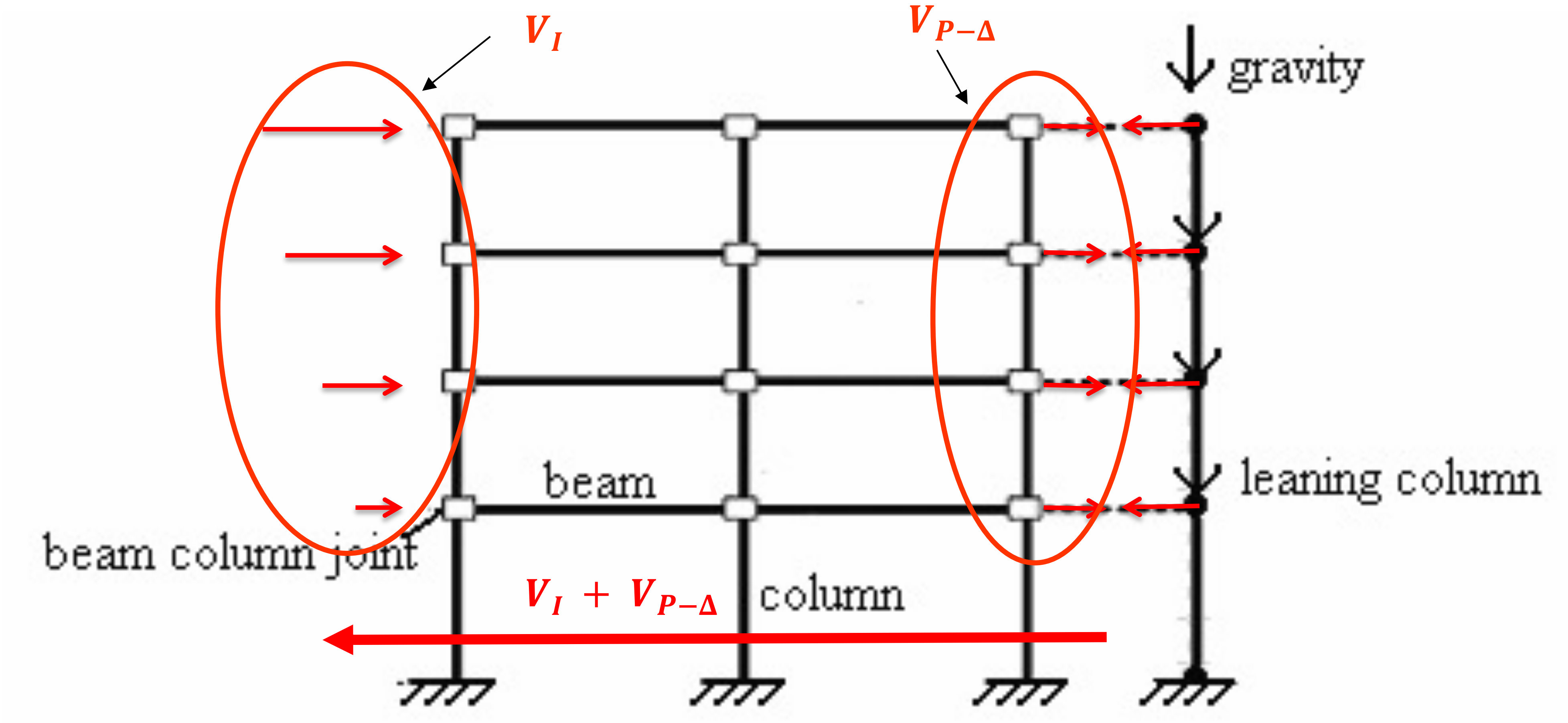
(Source Lignos 2008)

P-Delta Representation with “Leaning Column”

- P-Delta load directly to the lateral load resisting system (LLRS) should be incorporated by placing tributary gravity loads on the LLRS.
  - In frames, these P-Delta effects will cause an increase or decrease in the column axial load and shear force, but not in storey shear force.
- Gravity loads tributary “indirectly” to LLRS are transferred from the gravity system to the LLRS through floor diaphragms. The gravity loads should be placed on a “leaning column” connected to LLRS by links that represent diaphragm rigidity.

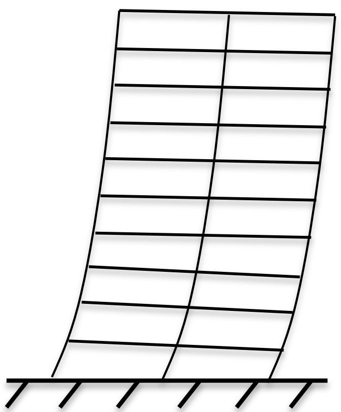
→These P-Delta effects will cause an increase in storey shear force demands.

What Story Shear Force Counts?

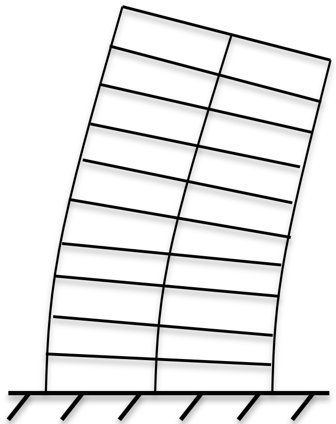


- V_I **is not relevant for design**
- $V_I + V_{P\text{-}\Delta}$ **is the relevant storey shear force demand for design**

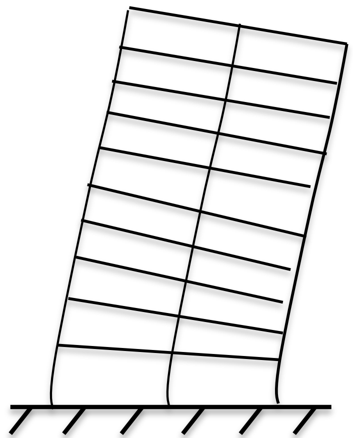
Potential Modes of Overall Buckling in Taller Buildings



Shear mode



Flexural mode



Mixed mode

Overall Buckling Analysis of Frames: Shear Mode

- This mode of buckling occurs in moment resisting frames as a result of storey sway associated with double bending of the columns and girders. Any effects of axial deformations of the columns are neglected in this approximate method. In this case, deflections due to second order effects, δ_i^*, are as follows:

$$\delta_i^* = \frac{1}{1 - \left(\frac{N_i \cdot \delta_i}{h_i \cdot V_i}\right)} \cdot \delta_i^{elastic}$$

- In which, the suffix, i referring to storey i, δ_i is the first-order storey drift caused by the external shear V_i, N_i is the total gravity loading carried by the columns in the storey, and h_i is the storey height.

Overall Buckling Analysis of Frames: Shear Mode

- From EERI Student Earthquake Competitions

Overall Buckling Analysis of Frames: Shear Mode

- E-Defense Structural Collapse - 18 Storey Rigid Frame

Source: E-Defense 2013 collapse tests
Figure courtesy of Prof. D. Lignos

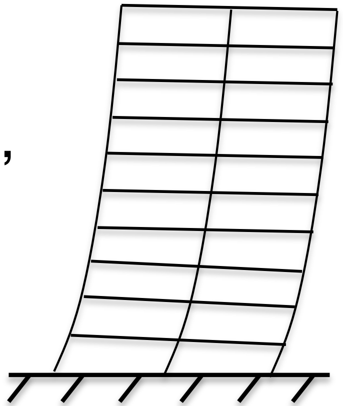
Overall Buckling Analysis of Frames: Shear Mode

- The loss of stability is indicated approximately by a zero denominator,

$$\frac{N_i \cdot \delta_i}{h_i \cdot V_i} = 1 \qquad \delta_i^* \rightarrow \infty$$

- Therefore, the critical load for storey i in the shear mode is,

$$N_{i,cr} = \frac{V_i \cdot h_i}{\delta_i}$$

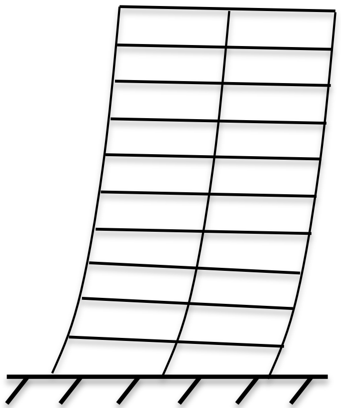


Shear mode

Overall Buckling Analysis of Frames: Shear Mode

- We can prove (with easy statics) by assuming that the point of inflection of columns at a typical storey is at mid-height that the storey drift δ_i is for a concentrated load at the top of the structure,

$$\delta_i = \frac{V_i \cdot h_i^2}{12 \cdot E} \cdot \left(\frac{1}{C_i} + \frac{1}{G_i}\right)$$



- In which,

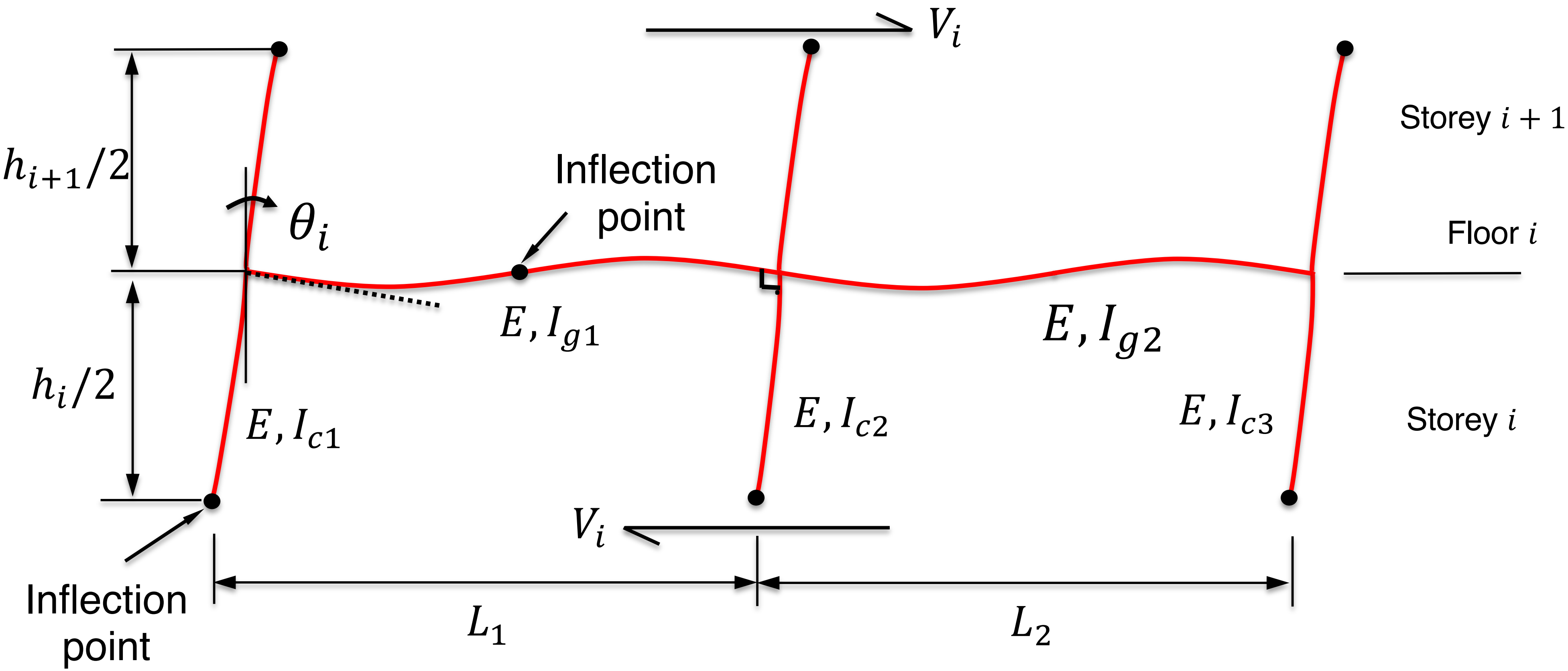
$$C_i = \sum_{j=1}^{N} \left(\frac{I_c}{h}\right)_{i,j}$$

The summation is carried out over N columns, j with inertias, I_c, and height, h, at storey i

$$G_i = \sum_{k=1}^{N-1} \left(\frac{I_g}{L}\right)_{i,k}$$

The summation is carried out over all $(N-1)$ girders, k with inertias I_g and length at floor i

Overall Buckling Analysis of Frames: Shear Mode



- The lateral stiffness, K_i, of storey i may be written as follows,

$$K_i = \frac{V_i}{\delta_i} = \frac{12 \cdot E}{h_i^2 \cdot \left(\frac{1}{C_i} + \frac{1}{G_i}\right)} \qquad C_i = \sum_{j=1}^{N} \left(\frac{I_c}{h}\right)_{i,j} \qquad G_i = \sum_{k=1}^{N-1} \left(\frac{I_g}{L}\right)_{i,k}$$

Overall Buckling Analysis of Frames: Shear Mode

- Therefore, the following expression may be used for the critical load in a typical storey i entirely in terms of the storey member's dimensions and geometric properties for a concentrated load at the top of the structure,

$$N_{i,cr} = \frac{12 \cdot E}{h_i \cdot \left(\frac{1}{C_i} + \frac{1}{G_i}\right)}$$

- Special consideration to the first storey of a frame with rigid base:

Deflection:

$$\delta_1 = \frac{V_i \cdot h_i^2}{12 \cdot E} \cdot \frac{\left(\frac{1}{G_1} + \frac{2}{3 \cdot G_i}\right)}{\left(1 + \frac{C_1}{6 \cdot G_1}\right)}$$

Buckling load:

$$N_{1,cr} = \frac{12 \cdot E \cdot [1 + (C_1/6G_1)]}{h_1 \cdot \left(\frac{1}{C_1} + \frac{2}{3 \cdot G_1}\right)}$$

- Special consideration to the first storey of a frame with pin base:

Deflection:

$$\delta_1 = \frac{V_i \cdot h_i^2}{12 \cdot E} \cdot \left(\frac{3}{2 \cdot G_1} + \frac{4}{C_1}\right)$$

Buckling load:

$$N_{1,cr} = \frac{12 \cdot E}{h_1 \cdot \left(\frac{4}{C_1} + \frac{3}{2 \cdot G_1}\right)}$$

Shear Mode: Some Remarks

- The typical proportioning of member sizes in multi-storey rigid frames is such that girder flexure is the major cause of drift, with column flexure a close second.
- Increasing the girder stiffness is usually the most effective and economical way of correcting excessive drift.
- An estimate of the modified girder sizes required at level i to correct the drift in that storey can be obtained by the following expression,

$$\sum_{j=1}^{N-1} \left(I_g/L\right)_{i,j} = \frac{V_i \cdot h_i}{12 \cdot E \cdot \left[\left(\frac{\delta_i}{h_i}\right) - \frac{V_i \cdot h_i}{12 \cdot E \cdot \sum_{j=1}^{N} (I_c/h)_{i,j}}\right]}$$

Assign the value of the allowable storey drift

Shear Mode: Some Remarks for a Good Designer

- If the frame is unusually proportioned so that column flexure contributes a major part of the drift (e.g., parking garages),

$$\sum_{j=1}^{N}(I_c/h)_{i,j} = \frac{V_i \cdot h_i}{12 \cdot E \cdot \left[\left(\frac{\delta_i}{h_i}\right) - \frac{V_i \cdot h_i}{12 \cdot E \cdot \sum_{j=1}^{N-1}(I_g/h)_{i,j}}\right]}$$

Assign the value of the allowable storey drift

- A relatively simple check on whether girders or columns should be adjusted first may be used. At each joint across the floor levels above and below the story whose drift is critical, the value of a parameter,

$$\psi = \frac{I_c}{h} \Big/ \sum \frac{I_g}{L}$$

$\sum \frac{I_g}{L}$: Refers to the girders connecting into the joint

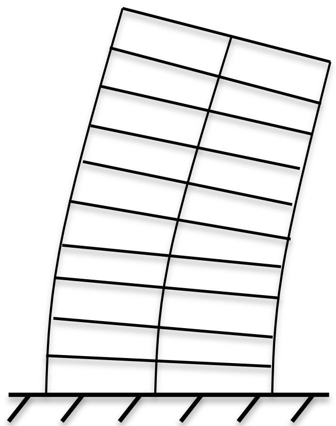
If $\psi \gg 0.5$, adjust the girder sizes

If $\psi \ll 0.5$, adjust the column sizes

If $\psi \approx 0.5$, adjust both column and girder sizes

Overall Buckling Analysis of Frames: Flexural Mode

- This mode presumes that the entire structure buckles as a flexural cantilever by axial deformations of the columns. The greater the slenderness of a structure, the more vulnerable it becomes to instability in the flexural mode as opposed to the shear mode.
- The buckling load is a function of the moment of inertia of the "cantilever", which is taken as the second moment of the column sectional areas about their common centroid.

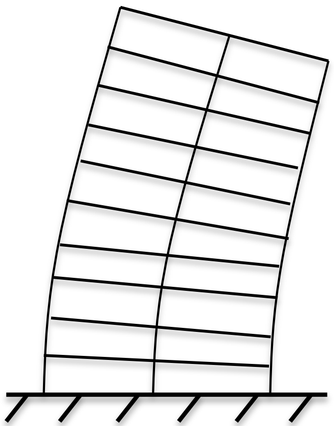


Overall Buckling Analysis of Frames: Flexural Mode

- Assuming this moment of inertia to vary in the frame from I_o at the base to $I_o \cdot (1 - \beta)$ at the top, in order to allow for the reduction in the sizes of the columns un the height,

$$N_{1,cr} = \frac{7.83 \cdot E \cdot I_o}{H^2} \cdot (1 - 0.2974 \cdot \beta)$$

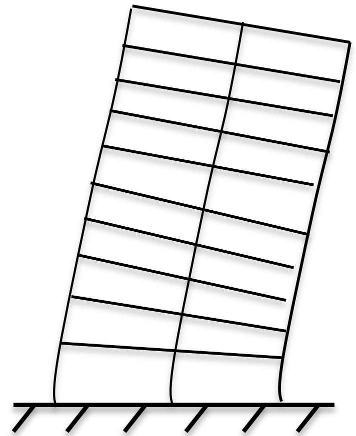
- Where $N_{1,cr}$ is the critical total gravity load on the structure and H is the total height of the structure.



Overall Buckling Analysis of Frames: Combined Shear and Flexural Modes

- For cases in which a combination of shear and flexural modes may contribute to buckling, an analogy is drawn with the case of the buckling of a vertical cantilever with a gravity load at its top, for which the following solution exists,

$$\frac{1}{N_{cr}} = \frac{1}{N_{cr,f}} + \frac{1}{N_{cr,s}}$$



Mixed mode

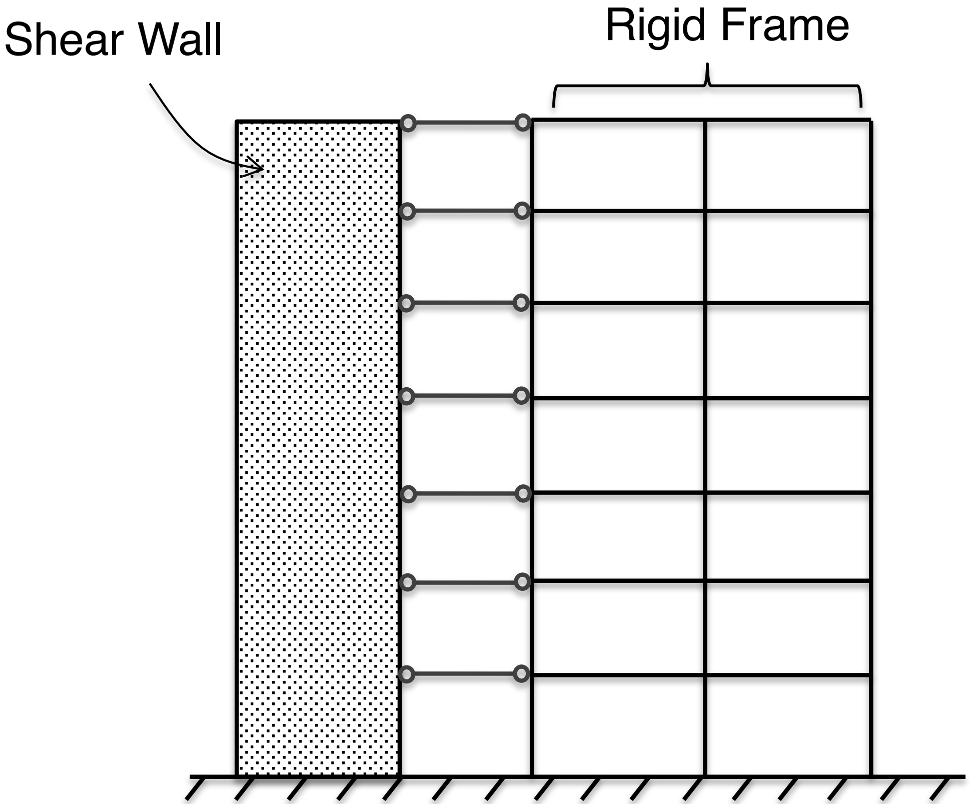
- Where N_{cr}, $N_{cr,f}$ and $N_{cr,s}$ are the critical loads for the combined, flexural and shear modes of buckling, respectively.
- This very approximate approach is suggested as being useful for the preliminary stages of design and for assessing the importance of the flexural mode relative to the usual dominant shear mode of buckling.

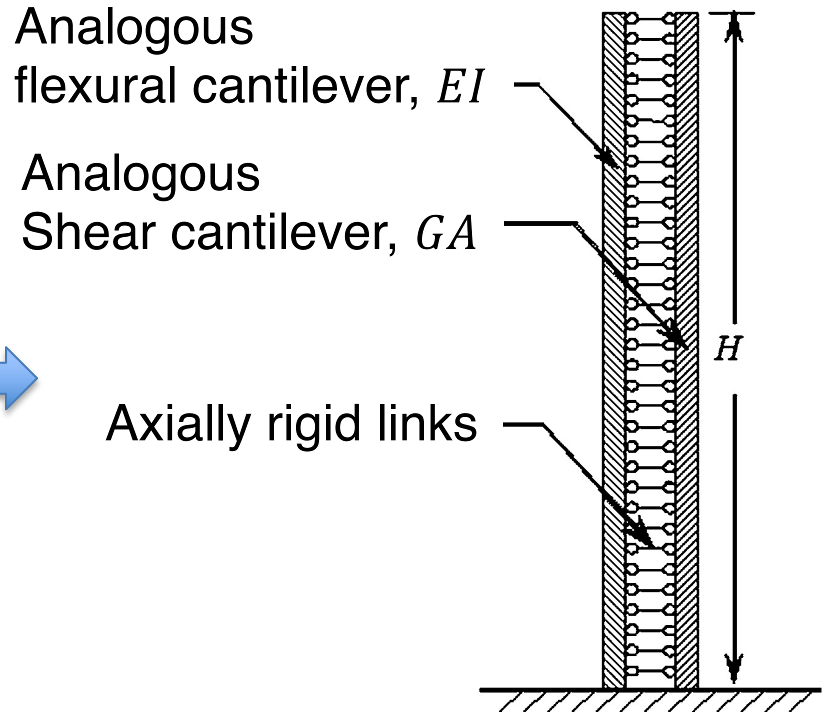
Overall Buckling Analysis of Frames: Mixted Mode

- From EERI Student Earthquake Competitions

Overall Buckling Analysis of Wall-Frames

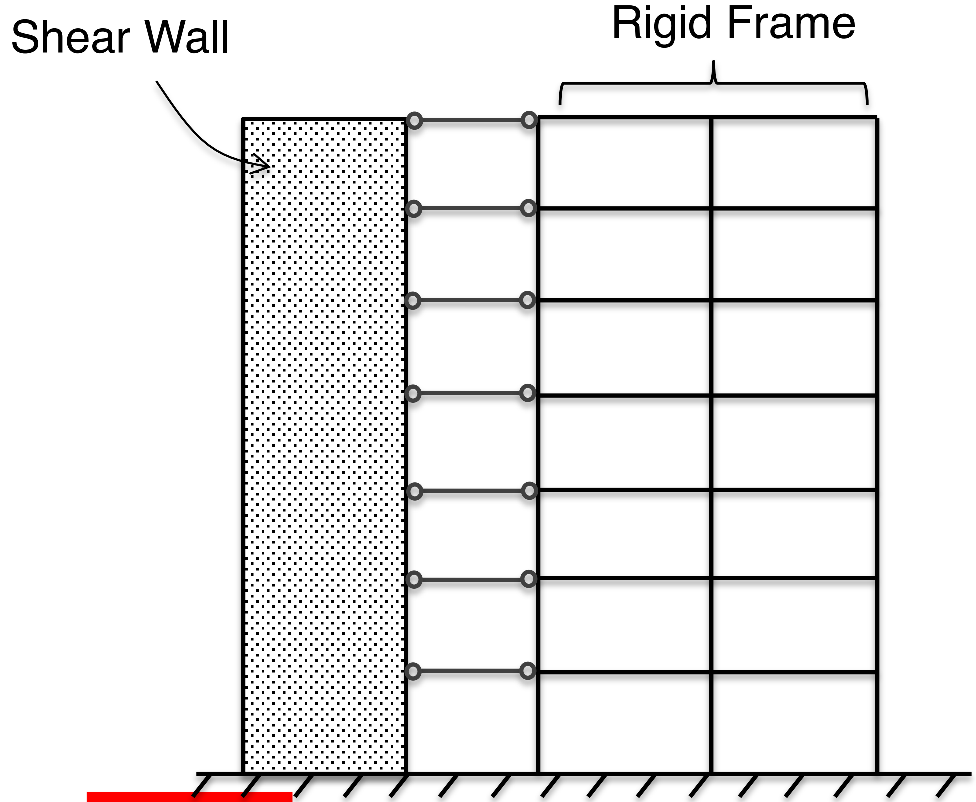
A more rigorous analysis for plan-symmetric, uniform wall-frame structures provides solutions for the buckling loads of frame structures at one extreme, shear wall structures at the other, and any combination of shear walls and frames in between.

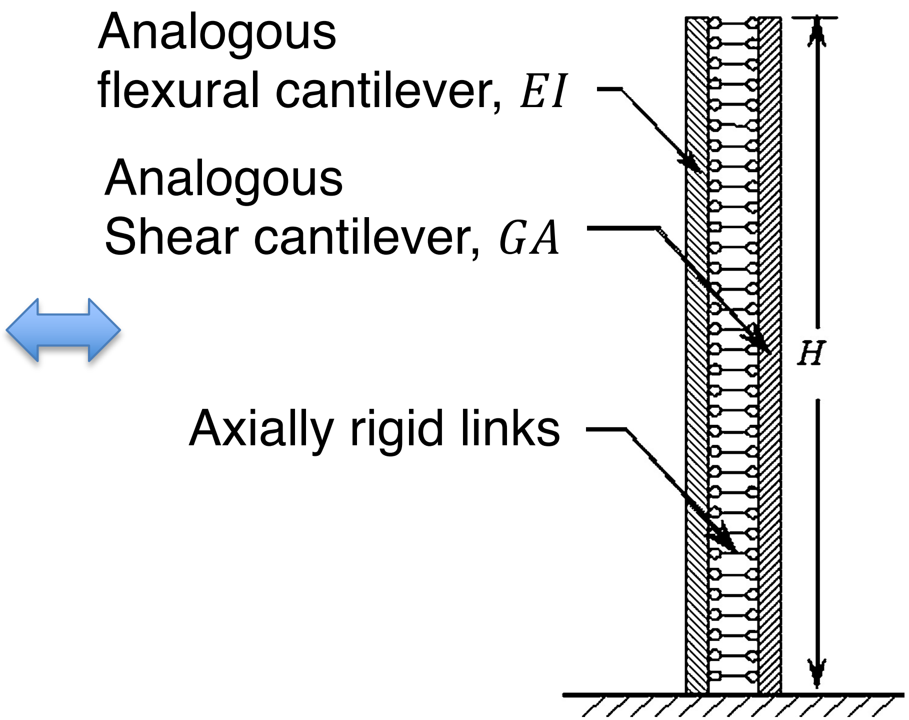




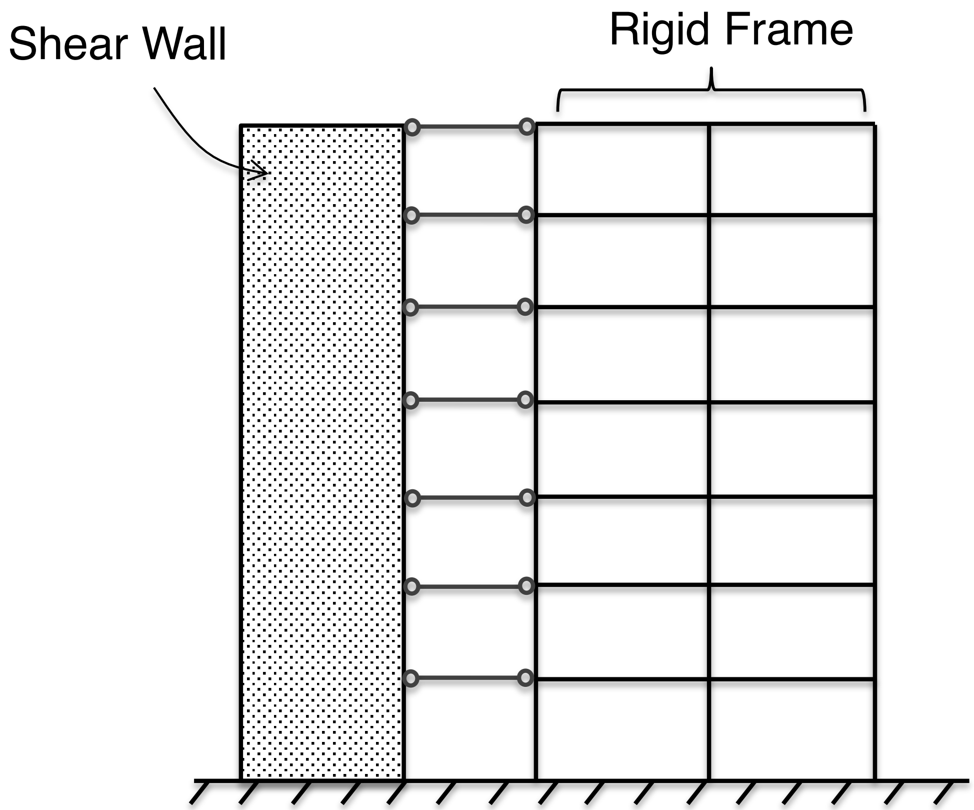
Overall Buckling Analysis of Wall-Frames

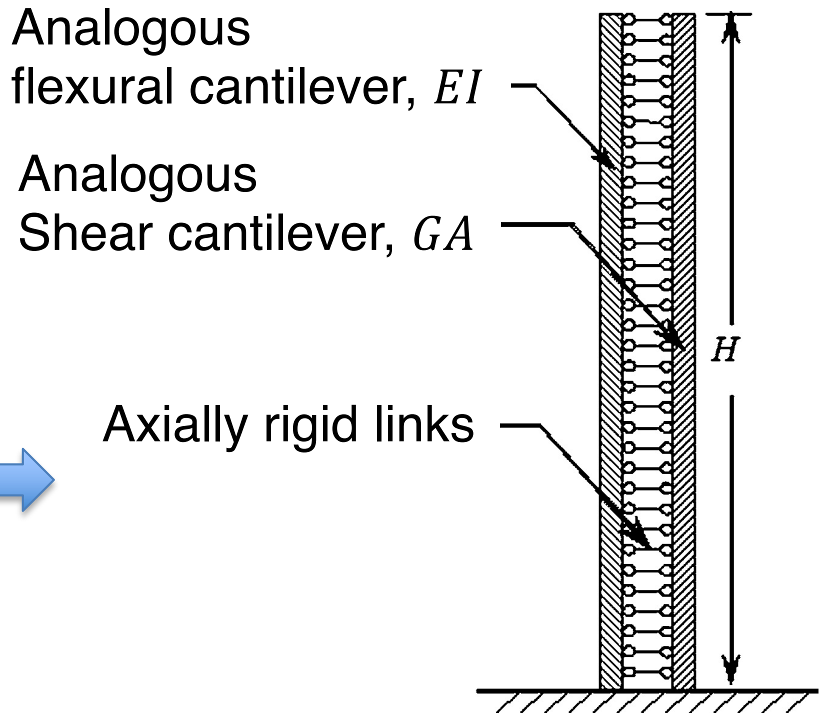
The method assumes the properties of the structure to be uniform and the applied gravity loading to be distributed uniformly throughout the height (see Fig. to the right). Representing the walls collectively by a flexural cantilever, the frames by a shear cantilever, and their connections by a stiff linking medium distributed uniformly over the height.





Overall Buckling Analysis of Wall-Frames

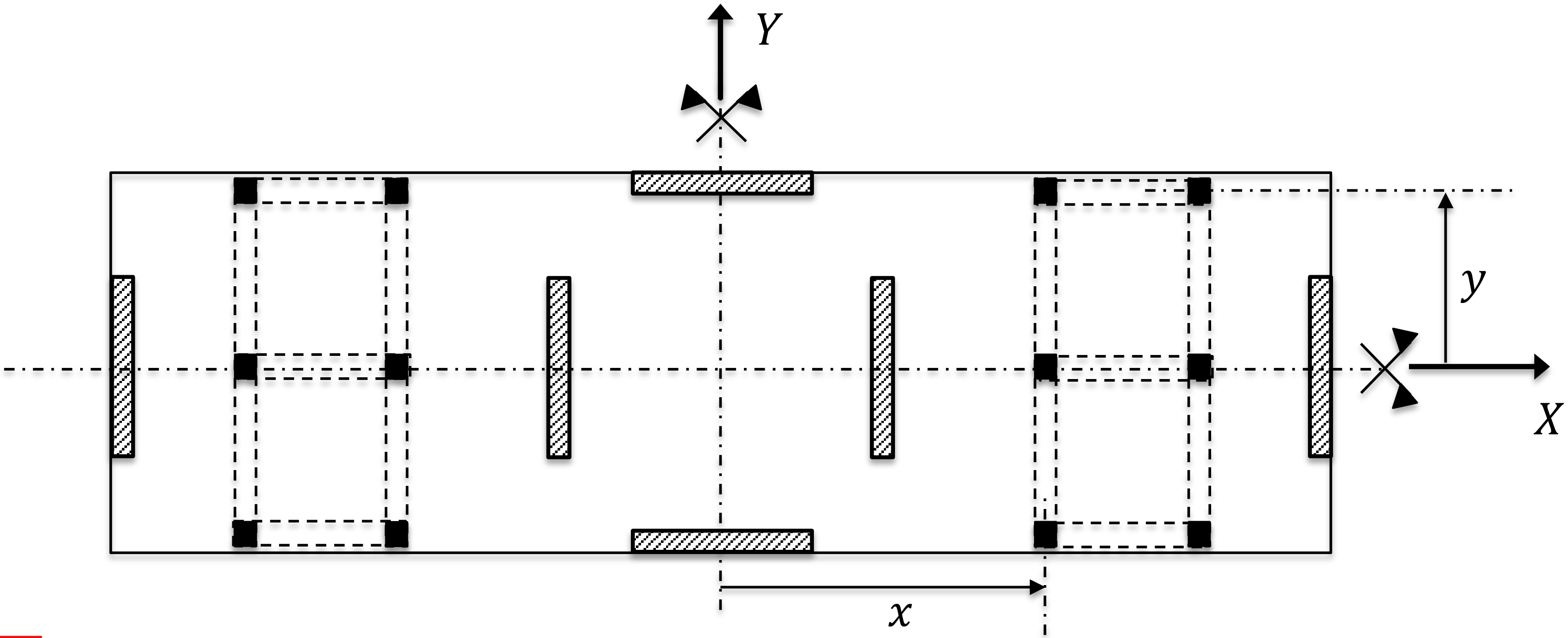




$$\frac{m}{EI}\cdot\frac{\vartheta^2 u(x,t)}{\vartheta t^2}+\frac{1}{H^4}\cdot\frac{\vartheta^4 u(x,t)}{\vartheta x^4}-\frac{1}{H^2}\cdot\left(\frac{GA}{EI}\right)\cdot\frac{\vartheta^2 u(x,t)}{\vartheta x^2}=F_{ext}$$

Overall Buckling Analysis of Wall-Frames
-Analytical Method

- The differential equation for equilibrium was formulated and solved to determine the critical buckling load. Solutions of this equation were obtained for a wide practical range of frame-to-wall relative stiffnesses GA/EI (see later on Slide 47) that can be directly used.
- Consider the doubly symmetric structure, In which the walls and rigid frames are aligned with the principal X and Y axes.



Overall Buckling Analysis of Wall-Frames
-Analytical Method

- The total flexural rigidities, $(EI)_t$ of the walls in the X and Y directions using, respectively,

$$(EI)_{t,x} = \sum (EI)_x \qquad (EI)_{t,y} = \sum (EI)_y$$

- The total shear rigidities, $(GA)_t$ of the frames in the X and Y directions using, respectively,

$$(GA)_{t,x} = \sum (GA)_x \qquad (GA)_{t,y} = \sum (GA)_y$$

- Where the shear rigidity of an individual frame is obtained for a typical storey i from,

$$(GA)_i = \frac{12 \cdot E}{h_i \cdot [(1/C) + (1/G)]_i}$$

Overall Buckling Analysis of Wall-Frames -Analytical Method

- Determine the torsional rigidities, $(EI_w)_t$ for the walls and $(GK)_t$ for the frames.

$$(EI_w)_t = \sum EI_x \cdot y^2 + \sum EI_y \cdot x^2$$

- For the frames:

$$(GK)_t = \sum GA_x \cdot y^2 + \sum GA_y \cdot x^2$$

- In which, x is the distance from a wall or frame aligned in the Y direction to the center of twist, and y is the corresponding distance of a wall or frame aligned in the X direction (see **Slide 42**).

Overall Buckling Analysis of Wall-Frames -Analytical Method

- Since torsional buckling is influenced not only by the plan distribution of the structural components but also by that of the gravity loading, a weight distribution parameter is required and is defined by,

$$R = \frac{\sum pr^2}{\sum p}$$

- In which, the floor loading is represented as a set of point loads p at distances, r, from the center of rotation.
- The transverse and torsional stiffnesses obtained from the previous equations are then used to obtain the following transverse and torsional characteristic parameters,

$$(aH)_x = H \cdot \sqrt{\frac{(GA)_{tx}}{(EI)_{tx}}} \qquad (aH)_y = H \cdot \sqrt{\frac{(GA)_{ty}}{(EI)_{ty}}} \qquad (aH)_\theta = H \cdot \sqrt{\frac{(GK)_t}{(EI_w)_t}}$$

Overall Buckling Analysis of Wall-Frames -Analytical Method

- The three parameters (aH) in the previous slide are used to find the corresponding coefficients, s_x, s_y, s_θ that enable the calculation of the critical loads.

The critical load for transverse buckling at storey 1 is given by,

$$N_{1,cr,x} = \frac{s_x (EI)_{t,x}}{H^2} \qquad N_{1,cr,y} = \frac{s_y (EI)_{t,y}}{H^2}$$

The critical load for torsional buckling is given by,

$$N_{1,cr,\theta} = \frac{s_\theta (EI_w)_t}{R \cdot H^2}$$

These critical loads will be shown to be useful also for evaluating an amplification factor to give an estimate of the P-Delta effects.

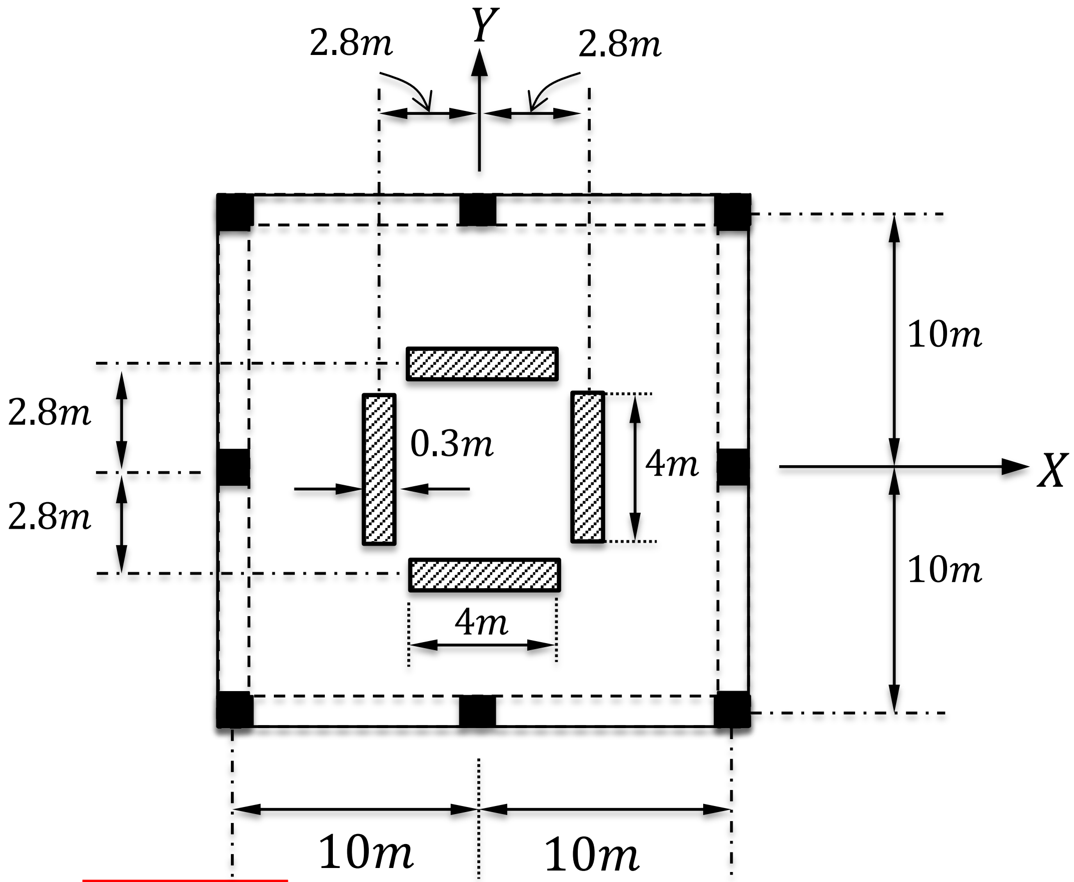
Coefficients s_x, s_y, s_θ for Wall-Frame Instability

aH	s_x, s_y, s_θ	aH	s_x, s_y, s_θ	aH	s_x, s_y, s_θ
0·00	7·84	3·40	36·4	6·80	97·0
0·10	7·90	3·50	37·8	6·90	99·2
0·20	7·97	3·60	39·3	7·00	101·4
0·30	8·14	3·70	40·8	7·10	103·6
0·40	8·33	3·80	42·3	7·20	105·8
0·50	8·61	3·90	43·8	7·30	108·1
0·60	8·94	4·00	45·3	7·40	110·4
0·70	9·28	4·10	46·9	7·50	112·7
0·80	9·74	4·20	48·5	7·60	115·0
0·90	10·3	4·30	50·1	7·70	117·4
1·00	10·8	4·40	51·7	7·80	119·7
1·10	11·4	4·50	53·3	7·90	122·1
1·20	12·1	4·60	55·0	8·00	124·6
1·30	12·8	4·70	56·7	8·10	127·0
1·40	13·5	4·80	58·4	8·20	129·5
1·50	14·3	4·90	60·1	8·30	132·0
1·60	15·2	5·00	61·8	8·40	134·5
1·70	16·1	5·10	63·6	8·50	137·1
1·80	17·0	5·20	65·4	8·60	139·6
1·90	18·0	5·30	67·2	8·70	142·2
2·00	19·0	5·40	69·0	8·80	144·8
2·10	20·0	5·50	70·9	8·90	147·5
2·20	21·1	5·60	72·8	9·00	150·2
2·30	22·2	5·70	74·7	9·10	152·8
2·40	23·4	5·80	76·6	9·20	155·6
2·50	24·6	5·90	78·5	9·30	158·3
2·60	25·8	6·00	80·5	9·40	161·1
2·70	27·0	6·10	82·5	9·50	163·8
2·80	28·3	6·20	84·5	9·60	166·7
2·90	29·6	6·30	86·5	9·70	169·5
3·00	30·9	6·40	88·6	9·80	172·3
3·10	32·2	6·50	90·6	9·90	175·2
3·20	33·6	6·60	92·7	10·00	178·1
3·30	35·0	6·70	94·9		

(Source Rosman 1974)

Example: Stability of Wall-Frame Structure

The doubly symmetric plan of a 20—storey, 80-m-high, reinforced concrete building consisting of shear walls and rigid frames. It is required to determine the magnitudes of the gravity loading that would cause lateral buckling and torsional buckling of the structure.



20 stories @ 4m = 80m

Columns: 0.4 x 0.4 m

Girders: 0.3 x 0.6 m

E = 2.5 x 10^7 kN/m^2

Dead load + live load = 10 kN/m^2

Member Properties

- Inertia of a single wall about its strong axis $= \frac{0.3 \cdot 4^3}{12} = 1.6m^4$
- Inertia of a single column $= \frac{0.4 \cdot 0.4^3}{12} = 0.002m^4$
- Inertia of a girder $= \frac{0.3 \cdot 0.6^3}{12} = 0.005m^4$
- Modulus of elasticity $E = 2.5 \cdot 10^7 kN/m^2$

Translational Parameters

Because the structure is symmetric and identical in plan about its X and Y axes, only one direction of transverse buckling will be assessed. Considering X-direction buckling, assume the two walls and two frames aligned in the X-direction resist buckling, with a negligible contribution from the Y-direction components.

For the walls:

$$(EI)_{tx}(2\,walls) = 2 \cdot (2.5 \cdot 10^7) \cdot 1.6 = 8.0 \cdot 10^7 kNm^2$$

Translational Parameters

For the frames:

$$(GA)_{tx}(2\ frames) = 2 \cdot \frac{12 \cdot E}{h \cdot [(1/C) + (1/G)]}$$

Where,

$$C = \sum \frac{I_c}{h} = \frac{3 \cdot 0.002}{4} = 0.0015 \qquad G = \sum \frac{I_g}{L} = \frac{2 \cdot 0.005}{10} = 0.001$$

$$(GA)_{tx}(2\ frames) = 2 \cdot \frac{12 \cdot 2.5 \cdot 10^7}{4 \cdot [(1/0.0015) + (1/0.001)]} = 90000kN$$

Then,

$$(aH)_x = H \cdot \sqrt{\frac{(GA)_{tx}}{(EI)_{tx}}} = 80 \cdot \sqrt{\frac{9.0 \cdot 10^4}{8.0 \cdot 10^7}} = 2.68$$

Torsional Parameters

Torsional buckling will be resisted by the four walls and four frames acting in their planes and rotating about the center of the structure

For the walls:

$$(EI_w)_t = 2 \cdot (EI)_x \cdot y^2 + 2 \cdot (EI)_y \cdot x^2$$

$$= 2.5 \cdot 10^7 \cdot (2 \cdot 1.6 \cdot 2.8^2 + 2 \cdot 1.6 \cdot 2.8^2)$$

$$= 1.25 \cdot 10^9 kNm^4$$

For the frames:

$$(GJ)_t = 2 \cdot (GA)_x \cdot y^2 + 2 \cdot (GA)_y \cdot x^2$$

$$= 90000 \cdot 10^2 + 90000 \cdot 10^2 = 1.8 \cdot 10^7 kNm^2$$

Then,

$$(aH)_\theta = H \cdot \sqrt{\frac{(GK)_t}{(EI_w)_t}} = 80 \cdot \sqrt{\frac{1.8 \cdot 10^7}{1.25 \cdot 10^9}} = 9.6$$

Weight Distribution Parameter

Dividing the floor plan at typical level into $25.4\,x\,4m$ regions, each carrying $160kN$ gravity load, and taking the distance from the center of each region to the center of the structure as *r*.

$$\sum pr^2 = 256000kNm^2 \qquad \sum p = 25 \cdot 160k = 4000kN$$

Hence,

$$R = \frac{\sum pr^2}{\sum p} = \frac{256000}{4000} = 64m^2$$

For the gravity load to cause lateral buckling,

For $(aH)_x$=2.68, s_x=26.8

$$N_{1,cr,x} = \frac{s_x (EI)_{t,x}}{H^2} = \frac{26.8 \cdot 8.0 \cdot 10^7}{80^2} = 33.5 \cdot 10^4 kN$$

Because of symmetry, the critical load for lateral buckling in the Y direction will be identical.

Weight Distribution Parameter

For the gravity load to cause torsional buckling:

For $(aH)_\theta$=9.6, s_θ=166.7

$$N_{1,cr,\theta} = \frac{s_\theta (EI_w)_t}{R \cdot H^2} = \frac{166.7 \cdot 1.25 \cdot 10^9}{64 \cdot 80^2} = 50.9 \cdot 10^4 kN$$

The actual maximum value of the total loading over 20 stories is,

$$N_1 = 20 \cdot 4000 = 8.0 \cdot 10^4 kN < \min\{N_{1,cr,x}, N_{1,cr,\theta}\}$$

Which leaves adequate margins of safety against overall buckling in both the translational and torsional modes.

Second-Order Effects of Gravity Loading

- In an extreme case of lateral flexibility combined with exceptionally heavy gravity loading, the additional forces from the P-Delta effect might cause the strength of some members to be exceeded with the possible consequence of collapse. Or, the additional P-Delta external moment may exceed the internal moments that the structure is capable of mobilizing by drift, in which case the structure would collapse through instability.

- Torsional P-Delta mode is possible and should be assessed in addition to translational P-Delta effect.

Second-Order Effects of Gravity Loading

- The torsional mode occurs when a building twists, and its walls and frames displace at each floor about some center of rotation. As a result the gravity loading, which is distributed over the building, is vertically misaligned with the axes of the resting elements causing, in effect, an additional torque. The building responds by twisting more until the additional internal resisting torque and the external P-Delta torque are in equilibrium.

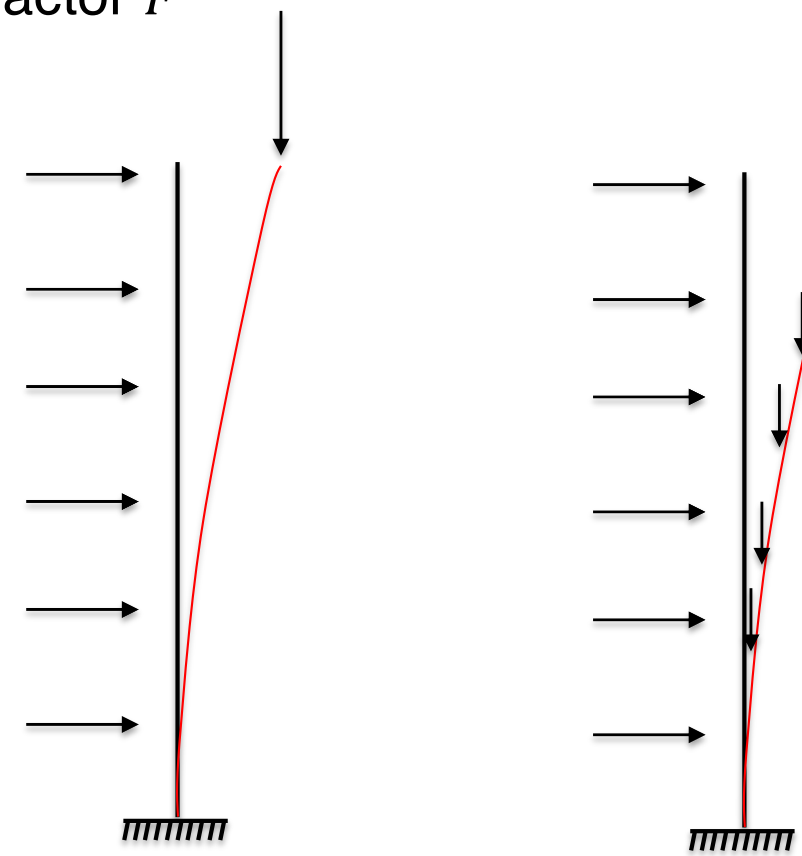
- Since the P-Delta torque and the torsional resistance of the structure depend on the plan locations of the gravity loading and of the walls and frames, these locations must be included in the parameters of a stability analysis. The more widely dispersed the vertical bents are from the center of rotation, the more effective they are in resisting torsion and the P-Delta torsional effects.

Methods of P-Delta Analysis

- A very approximate method in which a constant amplification factor is applied to all the results of a first-order analysis.
- An iterative method in which the gravity loads are applied to the laterally deflected structure.
- A direct method for rigid frame structures in which iterations are avoided by making a direct second-order adjustment of the displacements and moments.
- Structural modelling so that a stiffness matrix analysis incorporates both the first-order and second-order effects (…come to CIVIL-449: Nonlinear Analysis of Structures for this).

Amplification Factor P-Delta Analysis

It has been shown for a vertical cantilever displaced laterally by a uniformly distributed horizontal load that the addition of a concentrated vertical load P at the free end of the cantilever increases the horizontal displacements by an amplification factor F



Amplification Factor P-Delta Analysis

- This amplification is as follows (we saw this in Week #4):

$$F = \frac{1}{1 - (N/N_{cr})}$$

- The final displacement $\Delta^* = F \cdot \Delta = \frac{1}{1-(N/N_{cr})} \cdot \Delta$

- Since the amplification factor is a constant over the height of the structure subjected to load N, the increase in deflection is proposal to the initial displacements at all levels.
- Extending the amplification factor method to a tall building structure in which the gravity loading is distributed throughout the height, N is replaced by N_1, the total gravity load, and N_{cr} becomes $N_{1,cr}$, the overall buckling load, so that the equation for the total drift is taken as,

$$\Delta^* = F \cdot \Delta = \frac{1}{1-(N_1/N_{1,cr})} \cdot \Delta$$

Amplification Factor P-Delta Analysis

- The P-Delta effect causes an increase not only in drift but also in internal moments. Therefore, an initial set of moments M in a structure, calculated by a first-order analysis, would be increased by second-order effects to a set of final moments,

$$M^* = \frac{1}{1 - (N_1/N_{1,cr})} \cdot M$$

- To assess the torsional P-Delta effects on the structure, the same procedure can be used with a torsional amplification factor applied to the forces and displacements caused by torque. The value of $N_{1,cr}$ to be used for torsion should be determined from $N_{1,cr,\theta}$

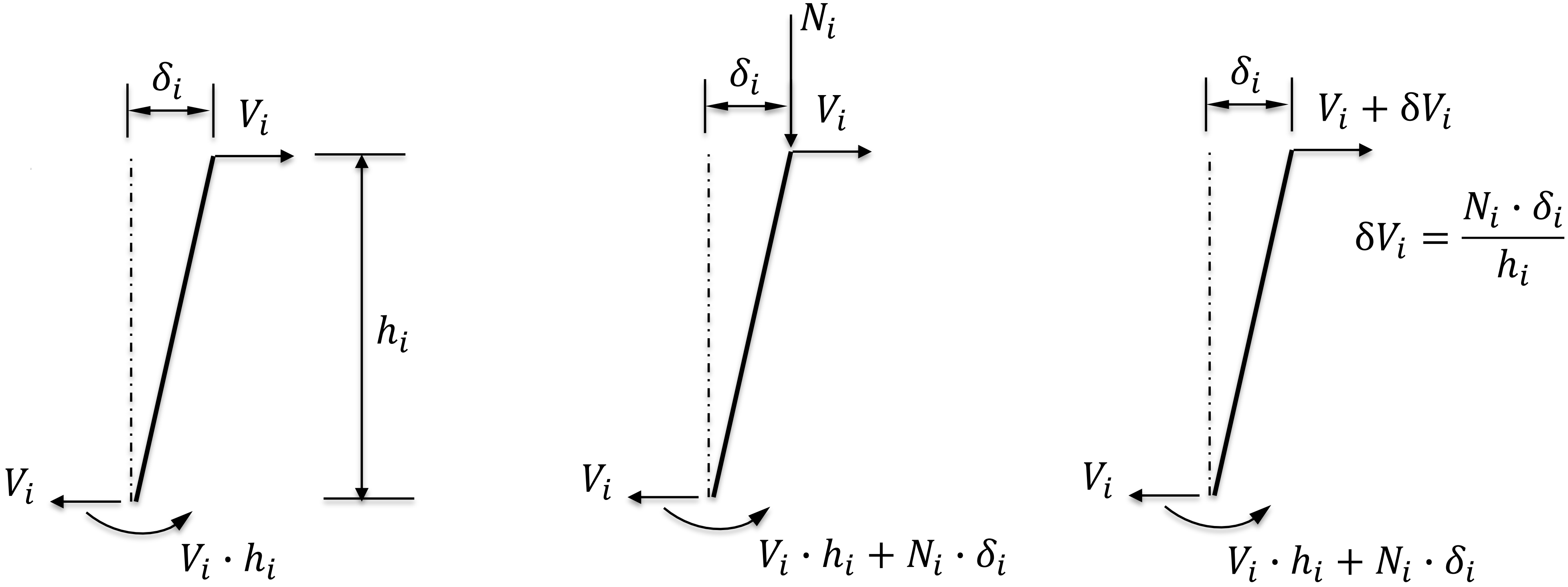
Iterative P-Delta Analysis

- The accuracy of the amplification factor method diminishes for flexible structures or heavy gravity loading.

- In the iterative second-order method, an initial first-order analysis of the structure is made with the external horizontal loading. The resulting horizontal deflections are then used in conjunction with the gravity loading to compute at each floor level an equivalent increment of horizontal load.

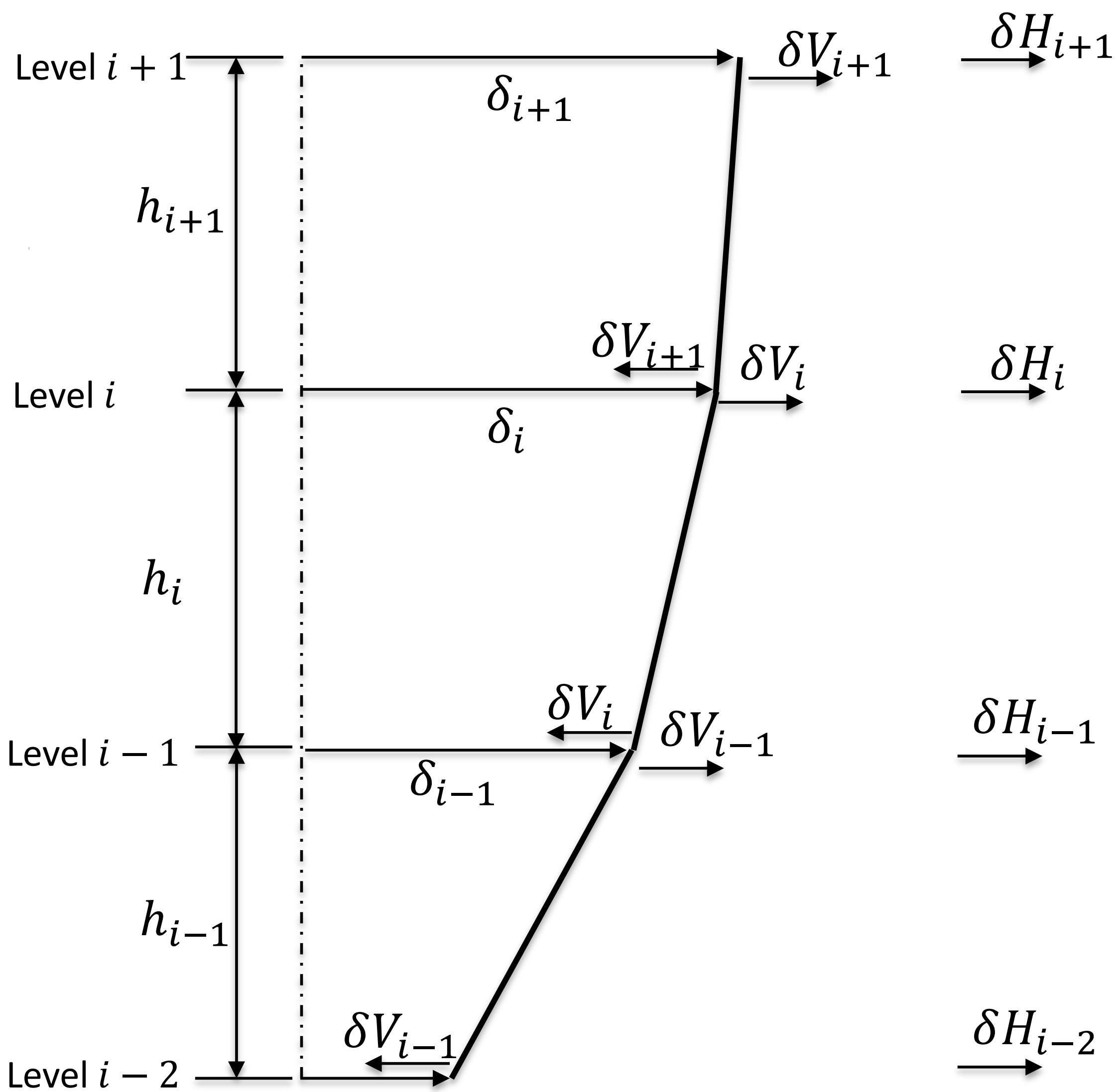
- The increment is added to the initial horizontal load and the analysis is repeated.

Iterative P-Delta Analysis



Iterative P-Delta Analysis

- Consider now the resultant effect of the shear increments in successive stories,



Increment of shear:

$$\delta V_i = \frac{N_i \cdot (\delta_i - \delta_{i-1})}{h_i}$$

Resultant additional increment of horizontal load to be applied at floor level i,

$$\delta H_i = \delta V_i - \delta V_{i+1}$$

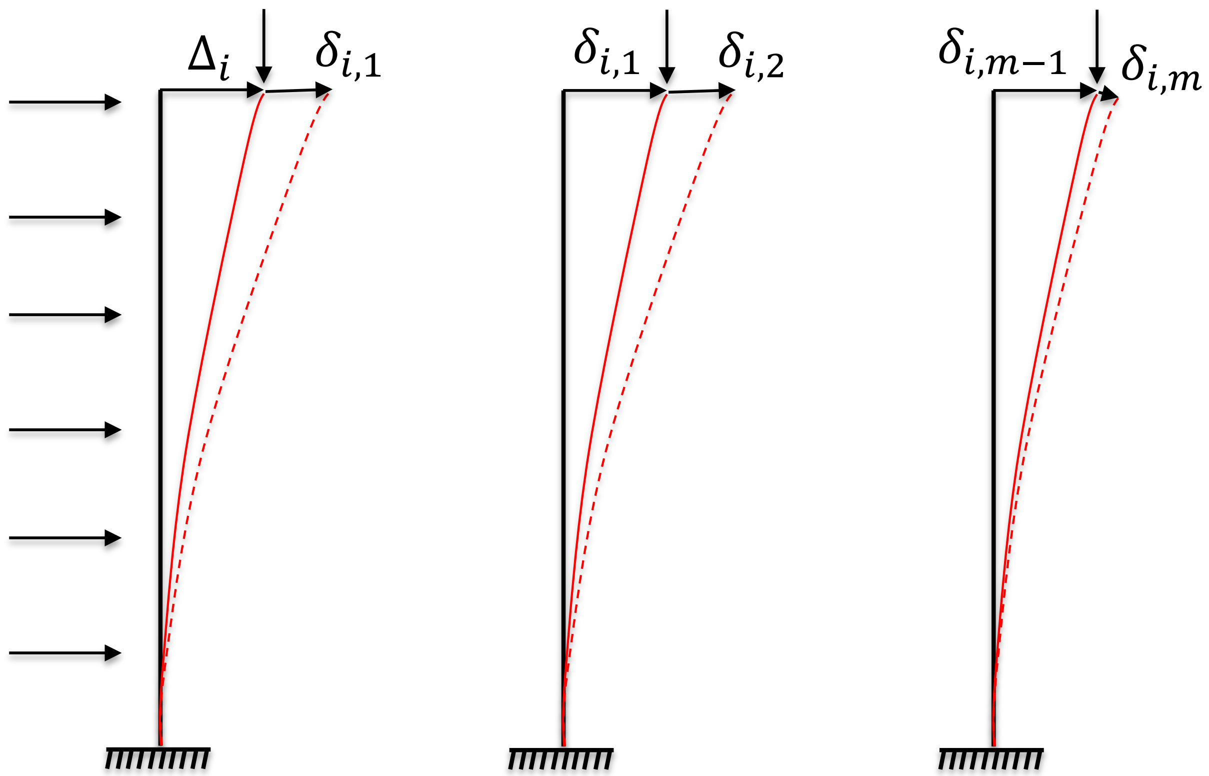
Iterative Gravity Load P-Delta Analysis

- In the previous method, the requirement of having to repeatedly evaluate the increments of horizontal load at many floors can be tedious.
- This is avoided with the iterative (and more realistic) gravity load method of P-Delta analysis.
- After a first-order horizontal load analysis of the structure, the gravity loads are applied to the unloaded structure deflected by the first-order values of drift, Δ_i, to obtain an increment of drift $\delta_{i,1}$.
- The gravity loads are then applied to the structure deflected by the increments $\delta_{i,1}$ to obtain another increment in drift $\delta_{i,2}$.
- The procedure is repeated until the additional drift increment $\delta_{i,m}$ is negligible. Therefore, the final drift at storey i, including the P-Delta effect, is,

$$\Delta_i^* = \Delta_i + \delta_{i,1} + \delta_{i,2} + \cdots + \delta_{i,m}$$

Iterative Gravity Load P-Delta Analysis

- The iterations are required because when the vertical loads are applied, they are not being applied to the final deflected shape.



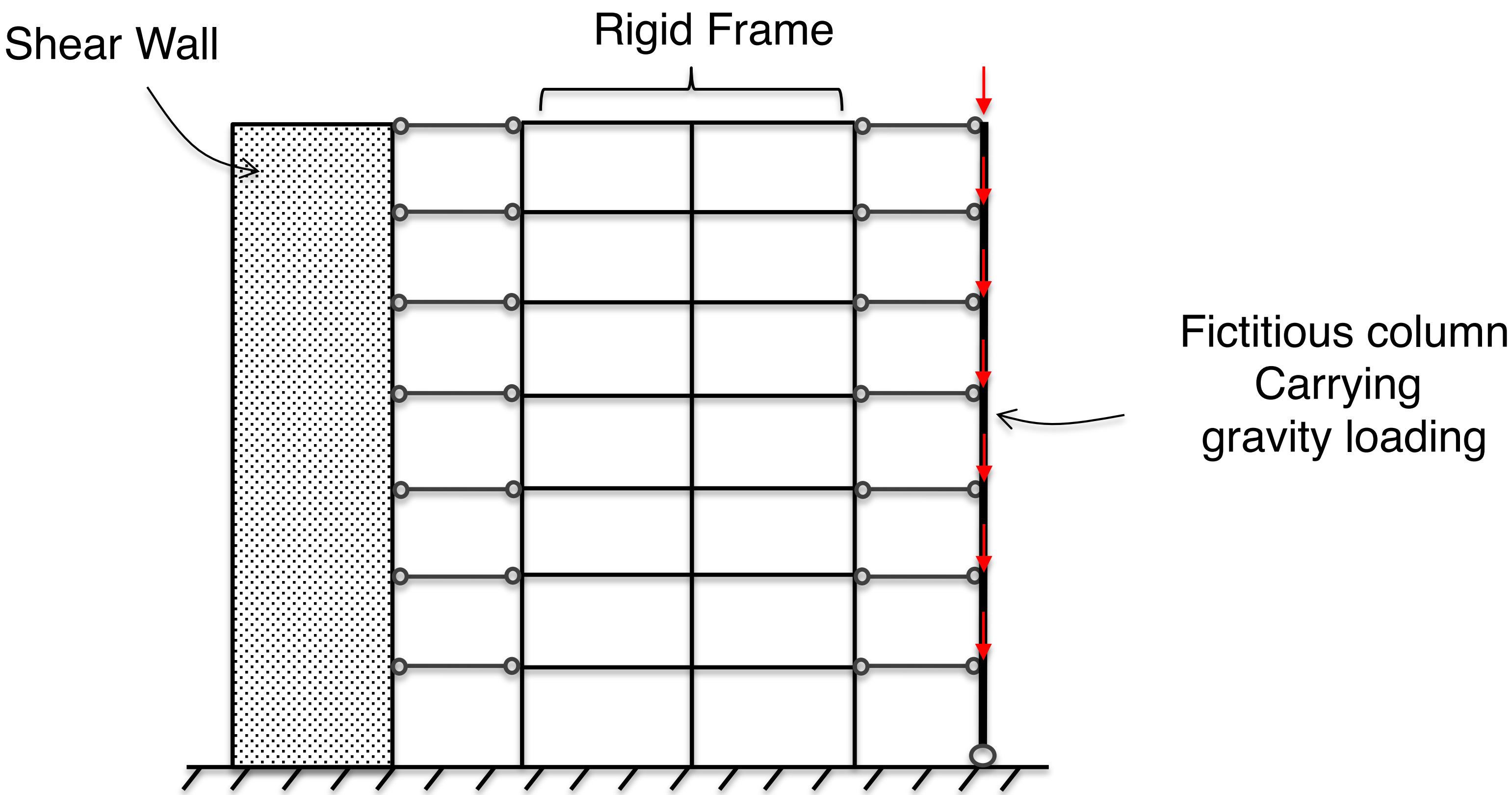
Iterative Gravity Load P-Delta Analysis

- The iterations are required because when the vertical loads are applied, they are not being applied to the final deflected shape. The final moment at storey, i including P-Delta effects is:

$$M_i^* = M_i + \delta M_{i,1} + \delta M_{i,2} + \cdots + \delta M_{i,m}$$

Iterative Gravity Load P-Delta Analysis

- In practice, the method can be simplified by adding a full-height, axially rigid fictitious column (leaning column) with a flexural stiffness equivalent to zero and connecting it to the structure by axially rigid links.



Direct P-Delta Analysis

- The iterative analysis described before can be reduced for rigid frame structures to a first-order analysis plus a direct second-order adjustment.
- From the first-order analysis, using horizontal loading only, the shear stiffness of storey, i of a rigid frame structure can be expressed as,

$$K_{s,i} = \frac{V_i}{\delta_i}$$

- The P-Delta effect at the final deflected state can now be represented by the initial shear V_i and increment δV_i, to give an effective total shear,

$$V_i^* = V_i + \delta V_i = V_i + \frac{N_i \cdot \delta_i^*}{h_i}$$

- Consequently, the final drift in storey i, is as follows,

$$\delta_i^* = \left[V_i + \frac{N_i \cdot \delta_i^*}{h_i} \right] / K_{s,i}$$

Direct P-Delta Analysis

- That is,

$$\delta_i^* = \left[V_i + \frac{N_i \cdot \delta_i^*}{h_i}\right] / \left[\frac{V_i}{\delta_i}\right]$$

- Then,

$$\delta_i^* = \frac{1}{1 - \left(\frac{N_i \cdot \delta_i}{V_i \cdot h_i}\right)} \cdot \delta_i$$

Stability coefficient, θ

- The corresponding moment due to second order effects should be,

$$M_i^* = \frac{1}{1 - \left(\frac{N_i \cdot \delta_i}{V_i \cdot h_i}\right)} \cdot M_i$$

Approximating Second-Order Effects in Frames

Second-order effects (P-Δ effects) need not be taken into account, if the following condition is fulfilled in all stories:

$$\theta = \frac{N_i \cdot \delta_i}{V_i \cdot h_i} \leq 0.10$$

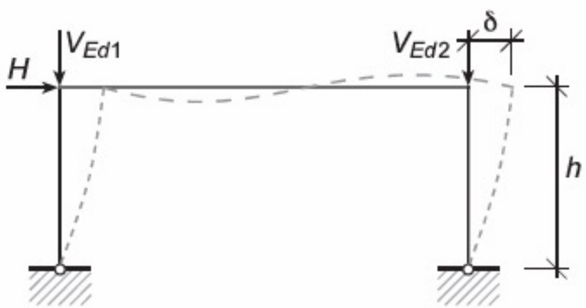
- θ_i Storey stability coefficient
- N_i Total gravity load above the storey considered in the lateral load situation (wind and/or seismic)
- V_i Storey shear based on first-order analysis
- h_i storey height
- δ_i storey displacement based on first-order analysis

Approximating Second-Order Effects in Frames

$0.10 \leq \theta \leq 0.20$	The second order effects may be approximately taken into account by multiplying the relevant seismic action effects by $1/(1-\theta)$
$0.20 \leq \theta \leq 0.30$	The second order effects should be considered by conducting nonlinear static analysis explicitly. This is crucial in taller structures
$\theta > 0.30$	Not permitted (the structure should be re-designed and stiffened)

Second-Order Effects in Frames – SIA 263 (Section 4.2.4)

Figure 6: Rigidité latérale des cadres

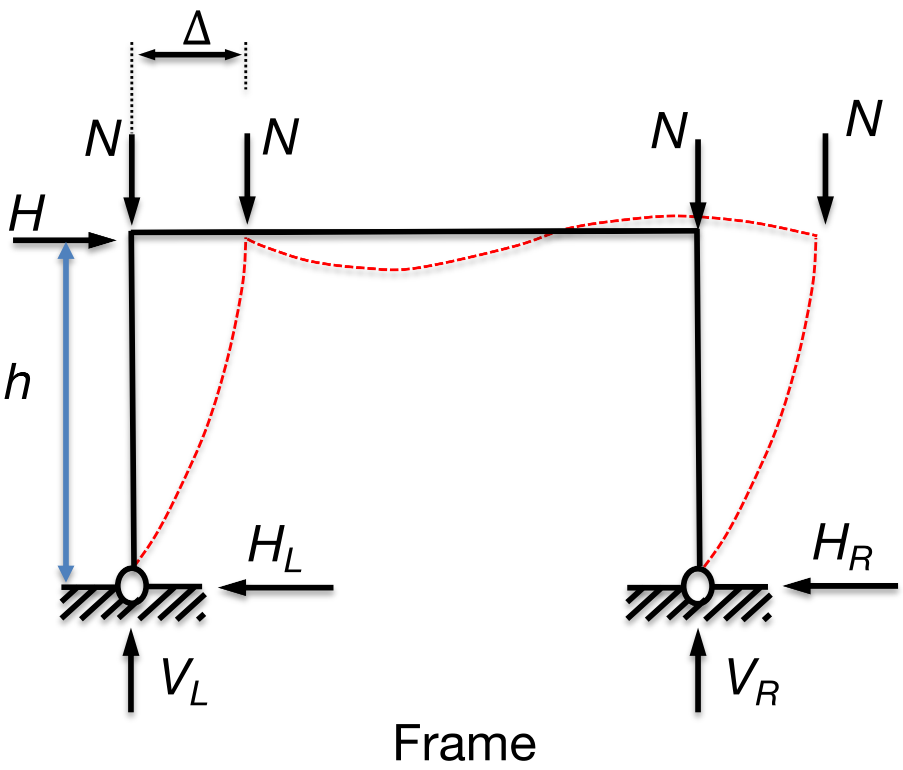


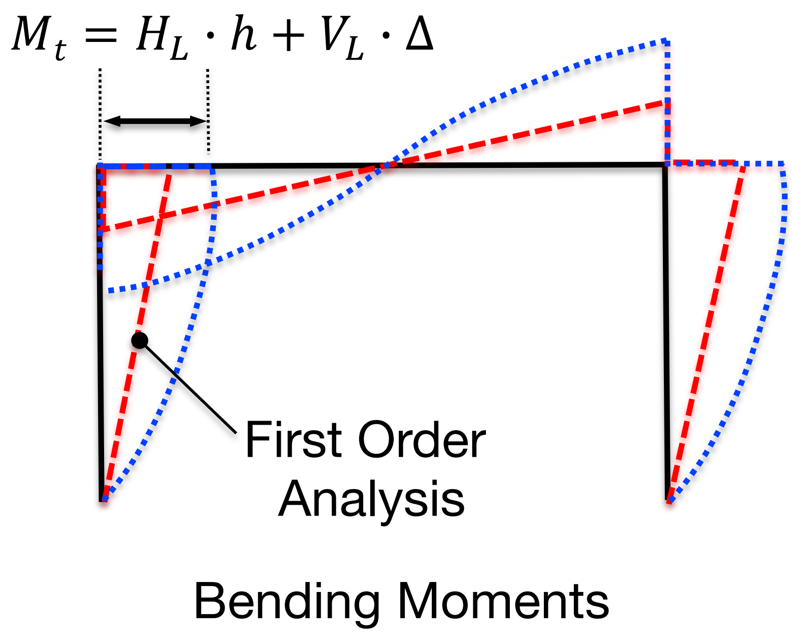
$$\left(\frac{\delta}{h}\right)\left(\frac{\Sigma V_{Edi}}{H}\right) \leq 0{,}1 \qquad (4)$$

δ	déplacement horizontal du cadre dû à la force horizontale H
h	hauteur du cadre
H	force horizontale agissant sur le cadre (à choix)
ΣV_{Edi}	valeur de calcul de la force verticale d'ensemble agissant sur le cadre (y compris les efforts normaux agissant sur les poteaux pendulaires stabilisés par le cadre).

(Source SIA 263)

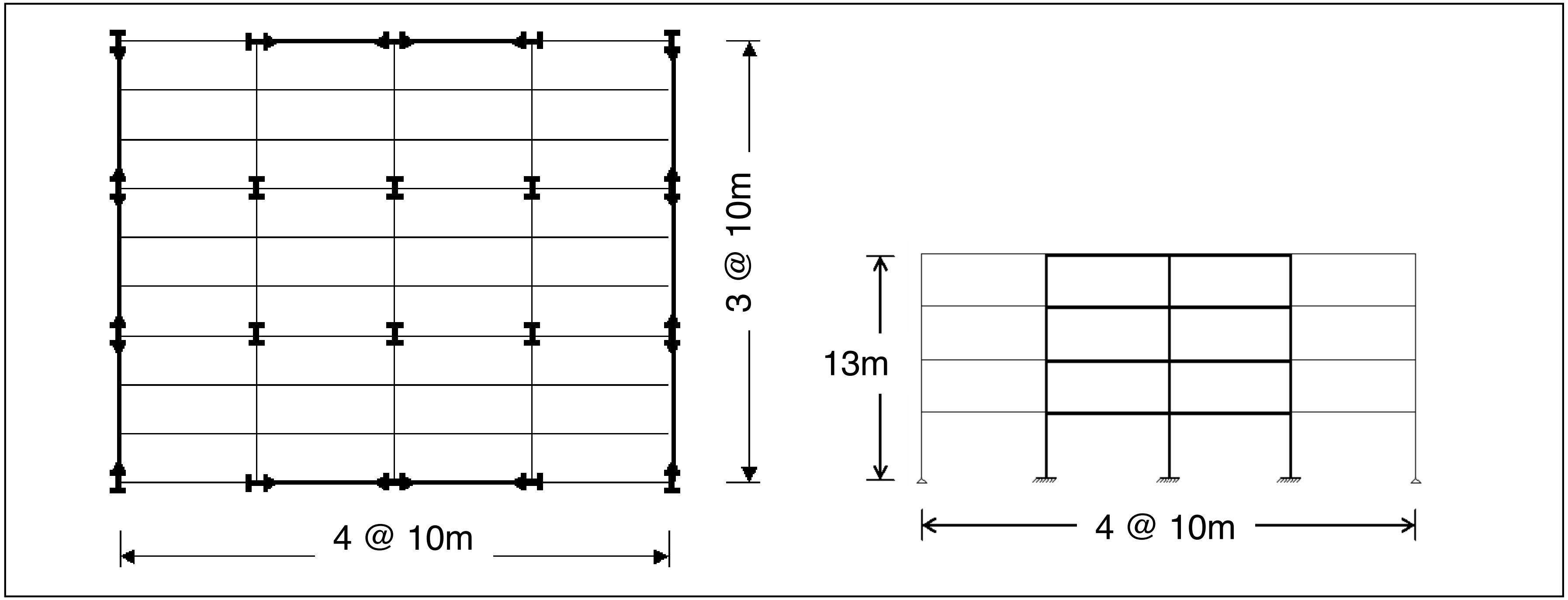
Second Order Effects





✧ P*Δ: Additional moment (couple) due to the axial force acting through the relative transverse displacement of member ends

Approximating Second Order Effects in Frames for Overall Stability Safety

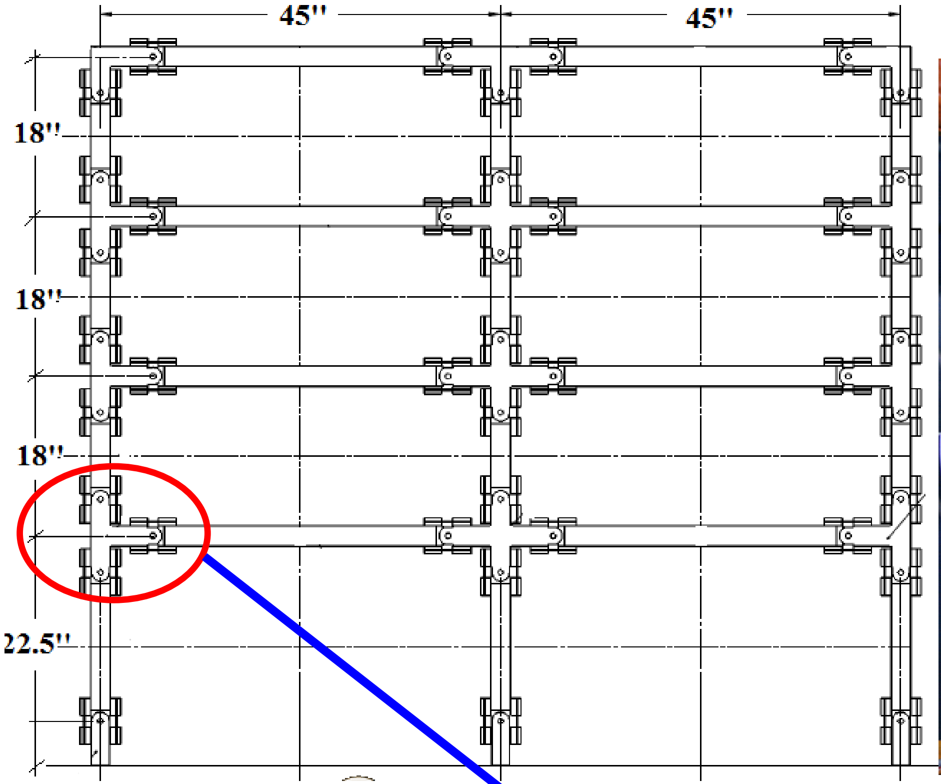


- Design Codes: IBC-2003, AISC-2005
- Beams with RBS are designed based on FEMA-350
- Design Area: Los Angeles
- T_1=1.32sec, $(\theta = 0.12)$

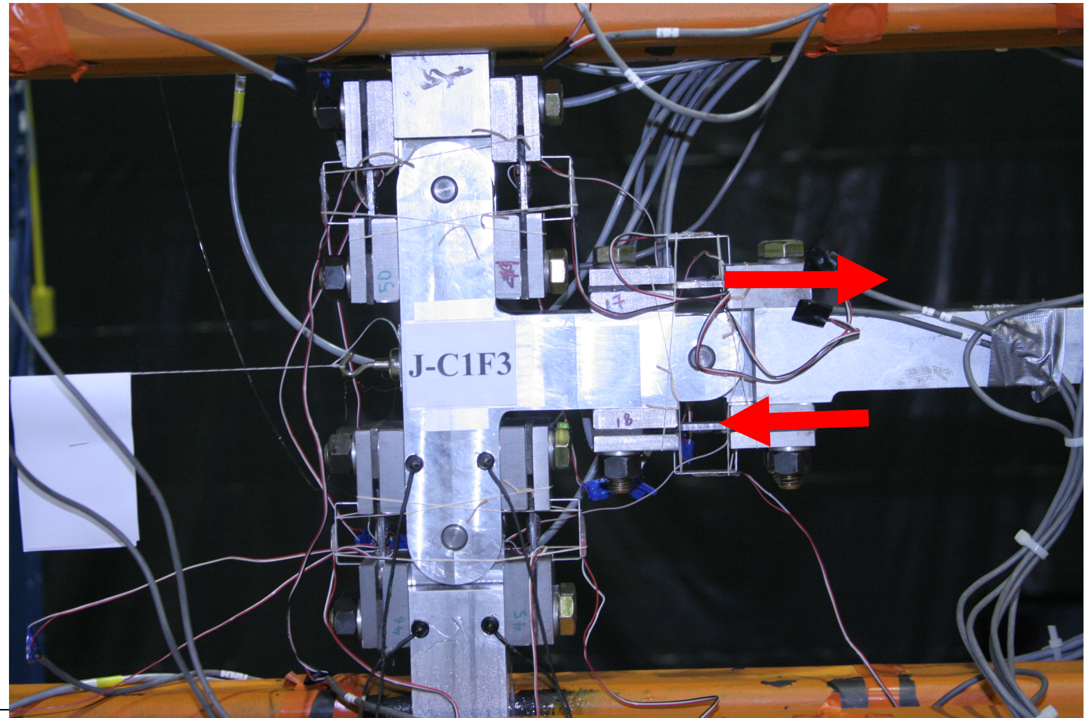
(Source Lignos et al. 2011)

Approximating Second Order Effects in Frames for Overall Stability Safety

1/8 Scale model

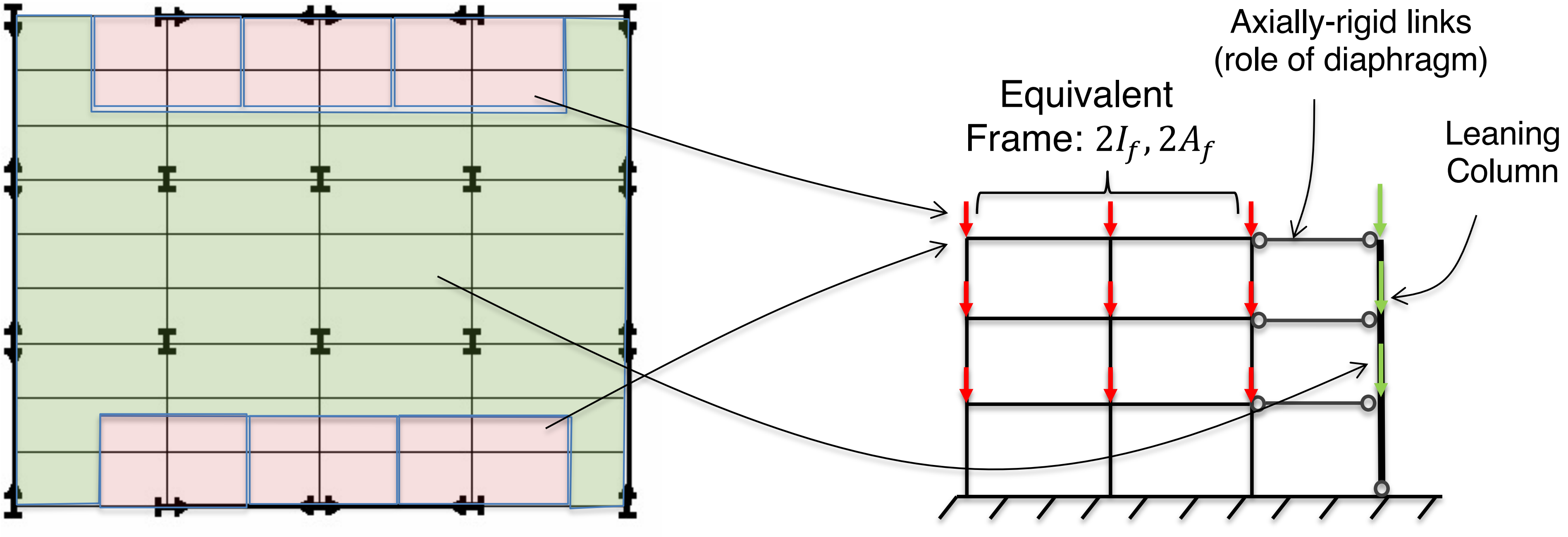


Typical plastic hinge location of Test Model



(Source Lignos et al. 2011)

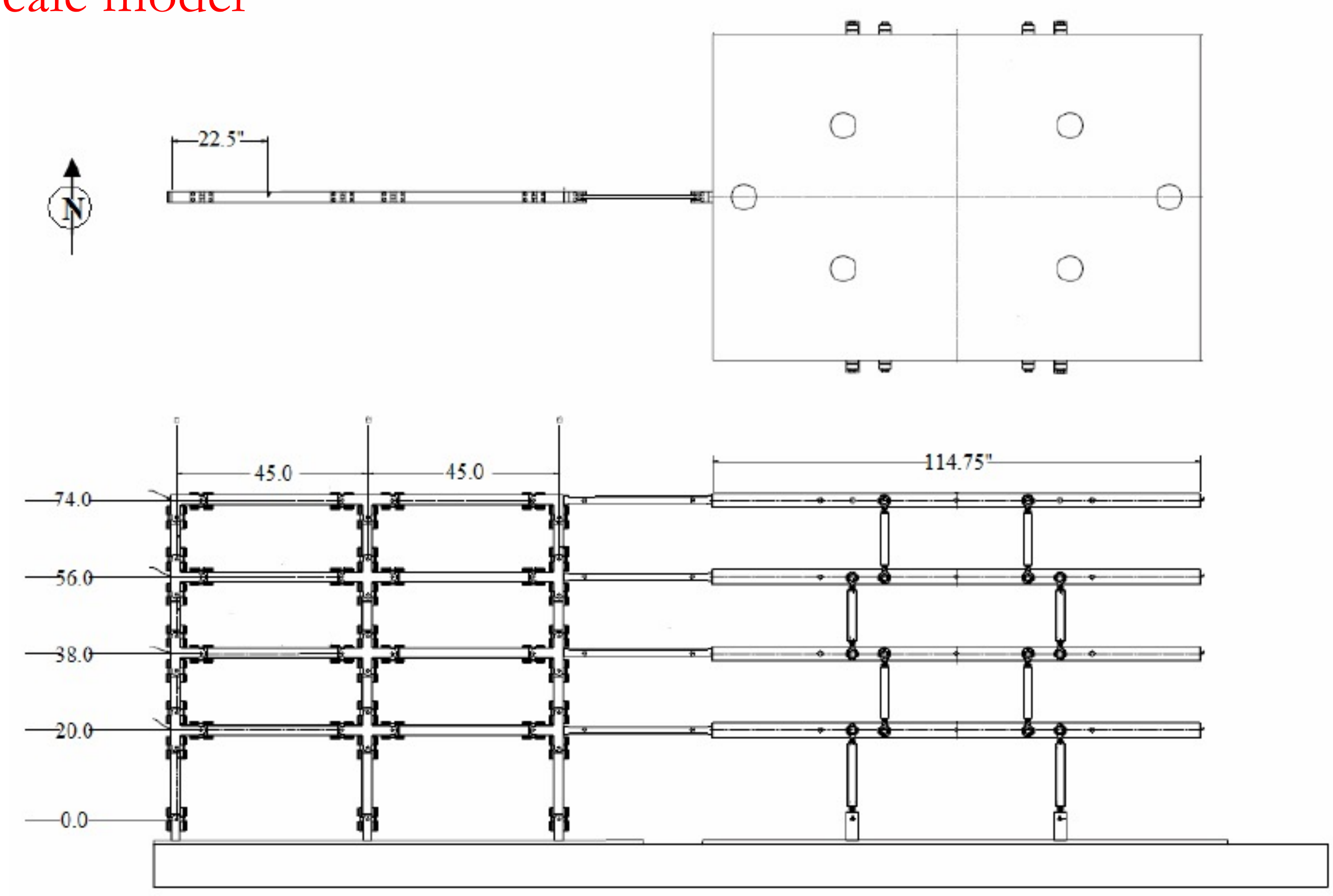
Steel Moment Resisting Frame with Leaning Column



(Source Lignos et al. 2011)

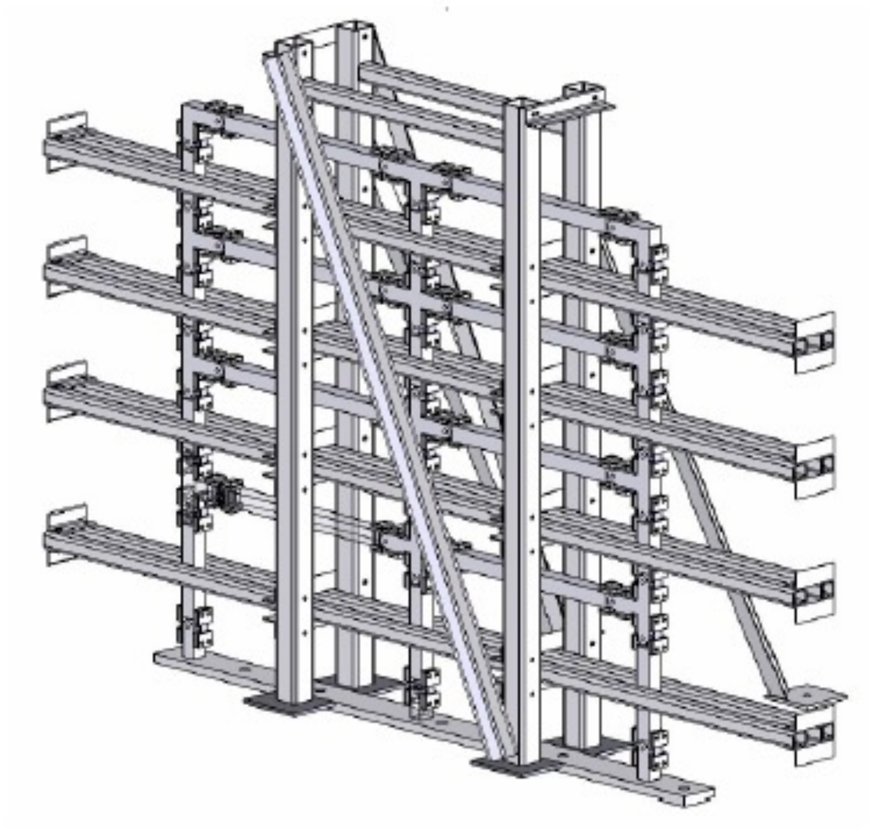
Scale Models for Shaking Table Collapse Tests

1/8 Scale model

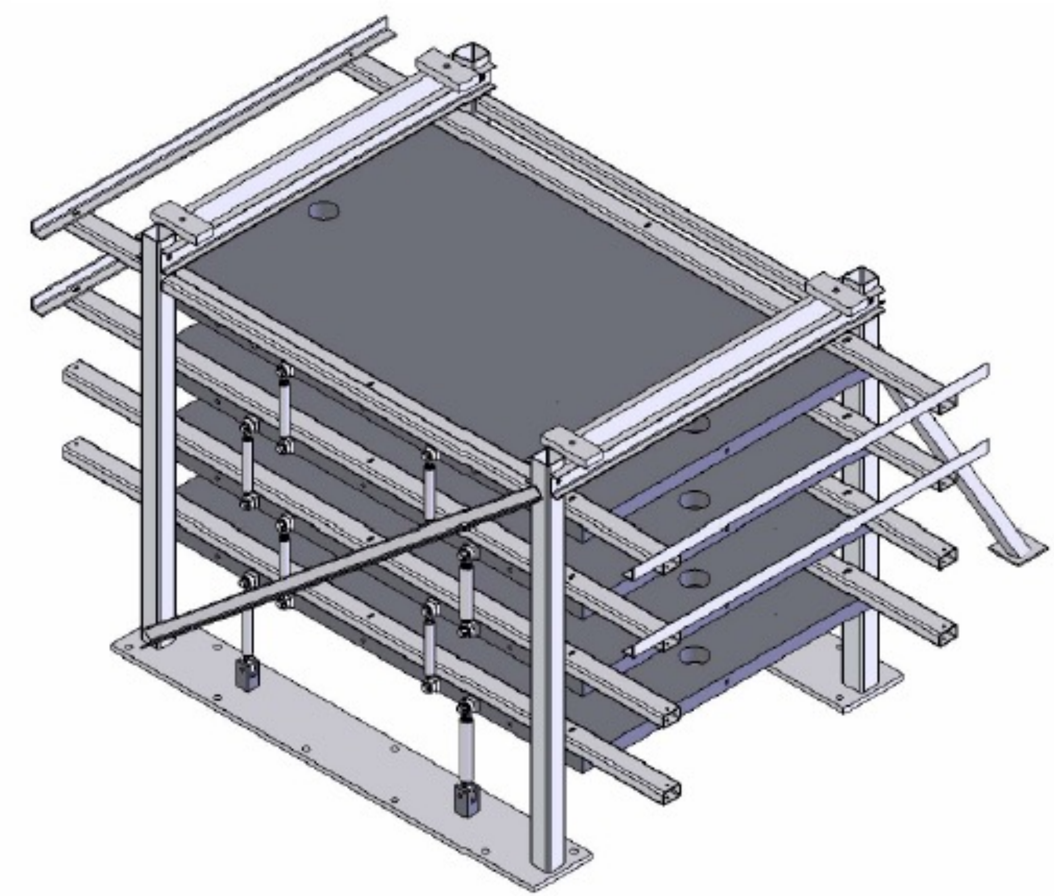


(Source Lignos et al. 2011)

EPFL Shake Table Collapse Tests



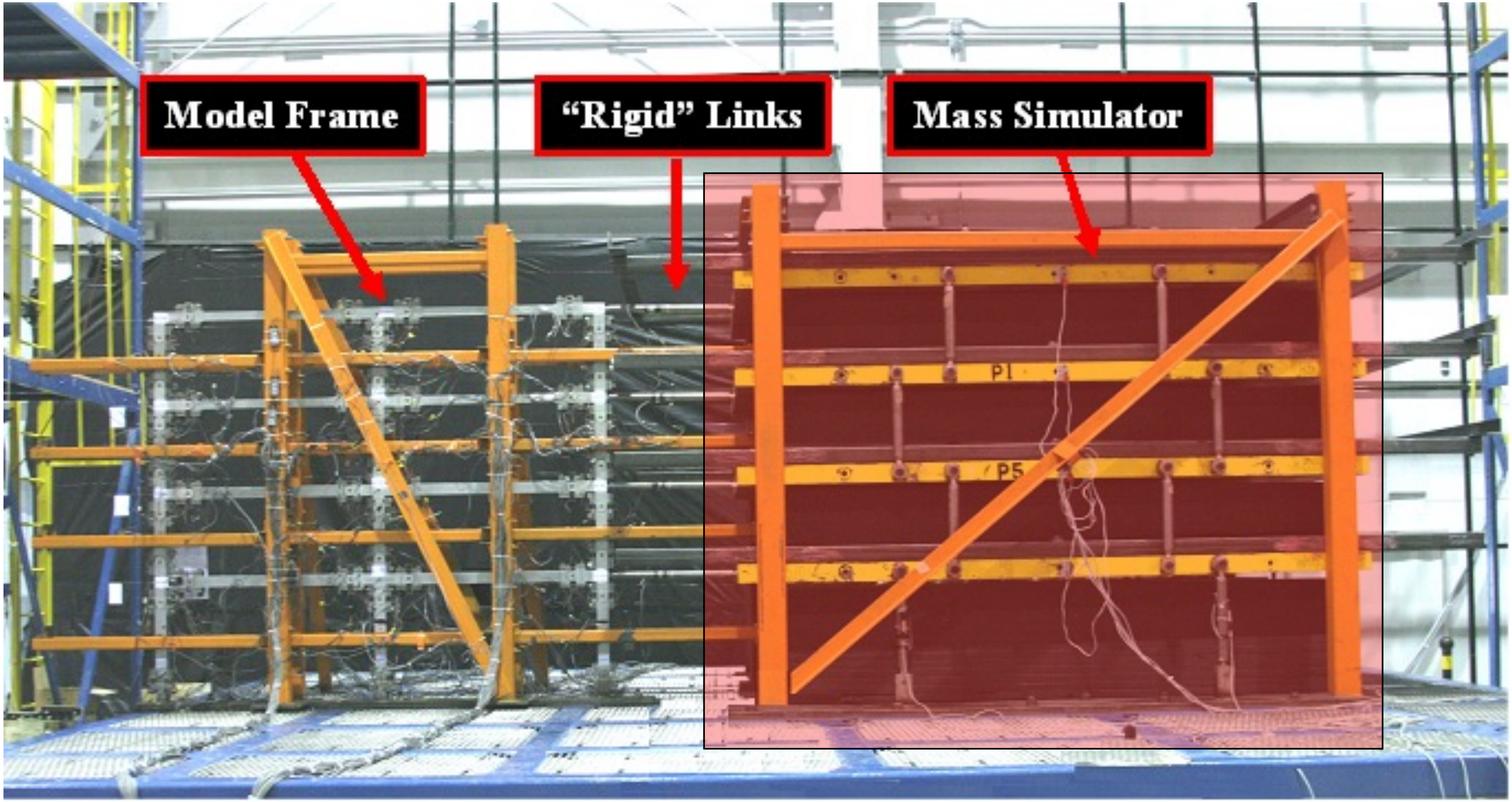
Lateral Support for Model Frame



Lateral Support for Leaning Column (Mass Simulator)

(Source Lignos 2008, CAD in SolidWorks)

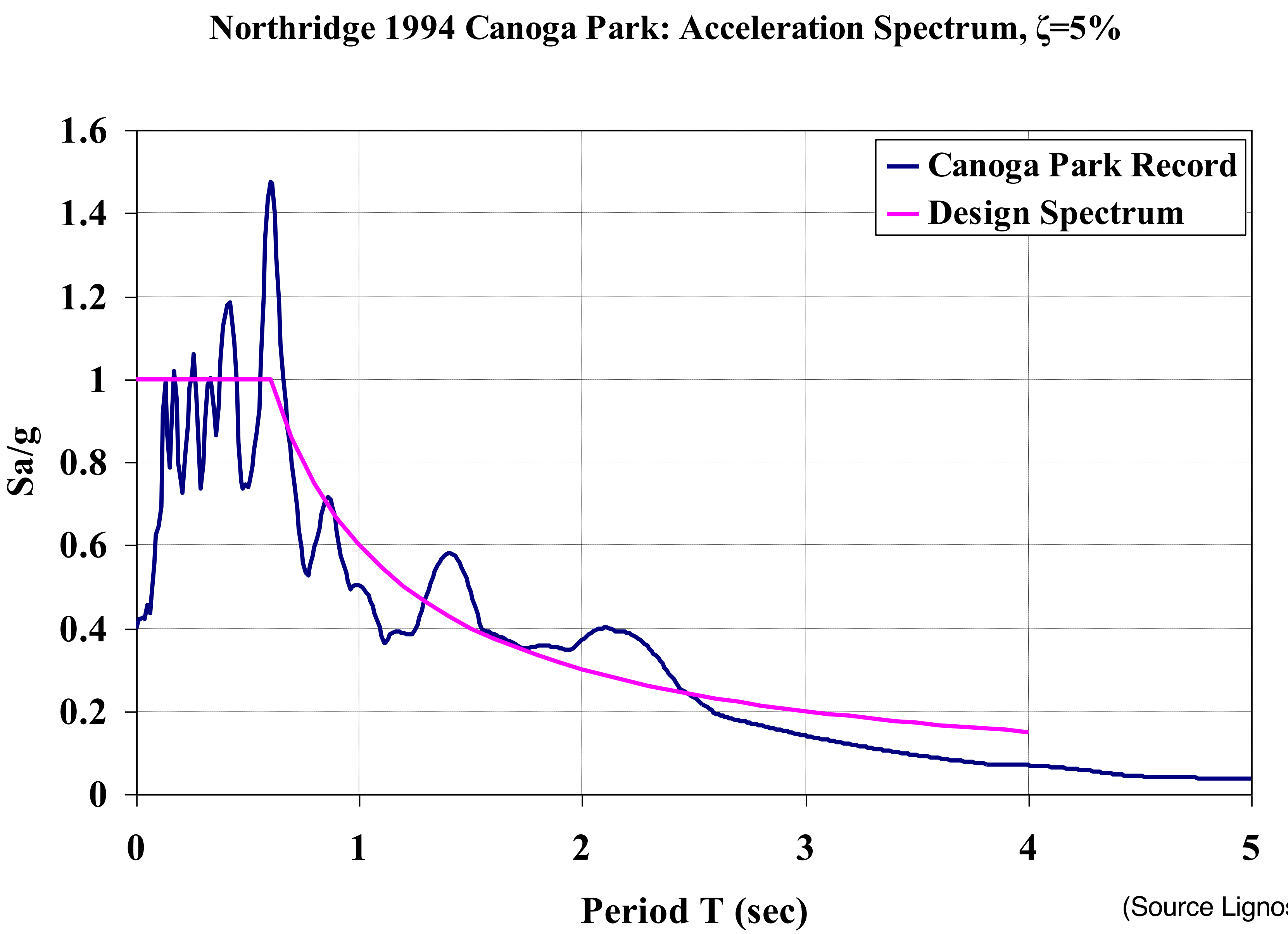
Two Shaking Table Tests



(Source Lignos et al. 2011)

Leaning Column: Carried 20tons to simulate P-Delta Effects

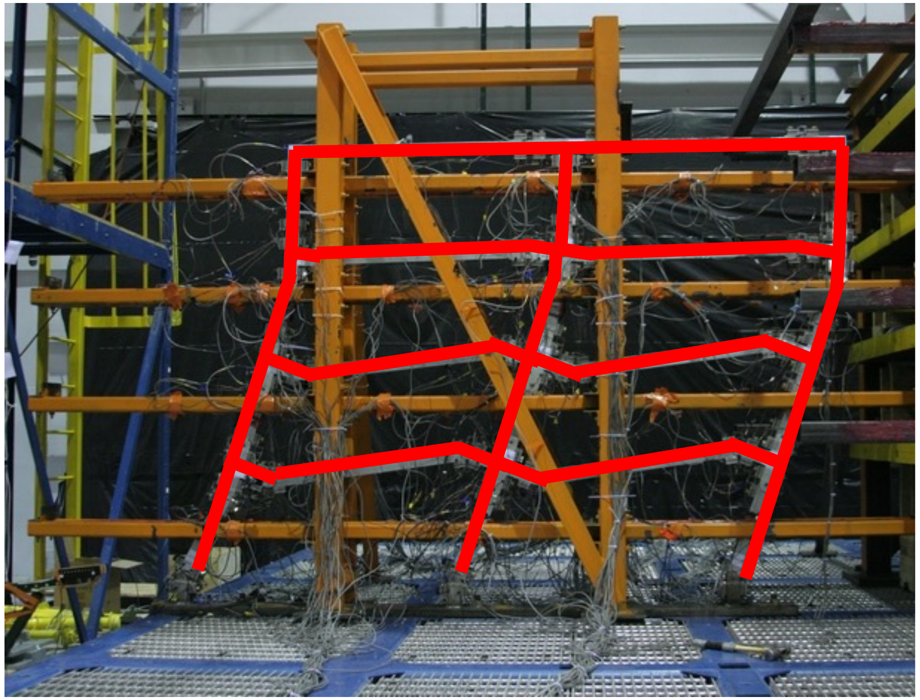
Ground Motion for Shake Table Test



(Source Lignos et al. 2011)

Observations – Collapse Mechanisms

Frame # 1



Frame # 2

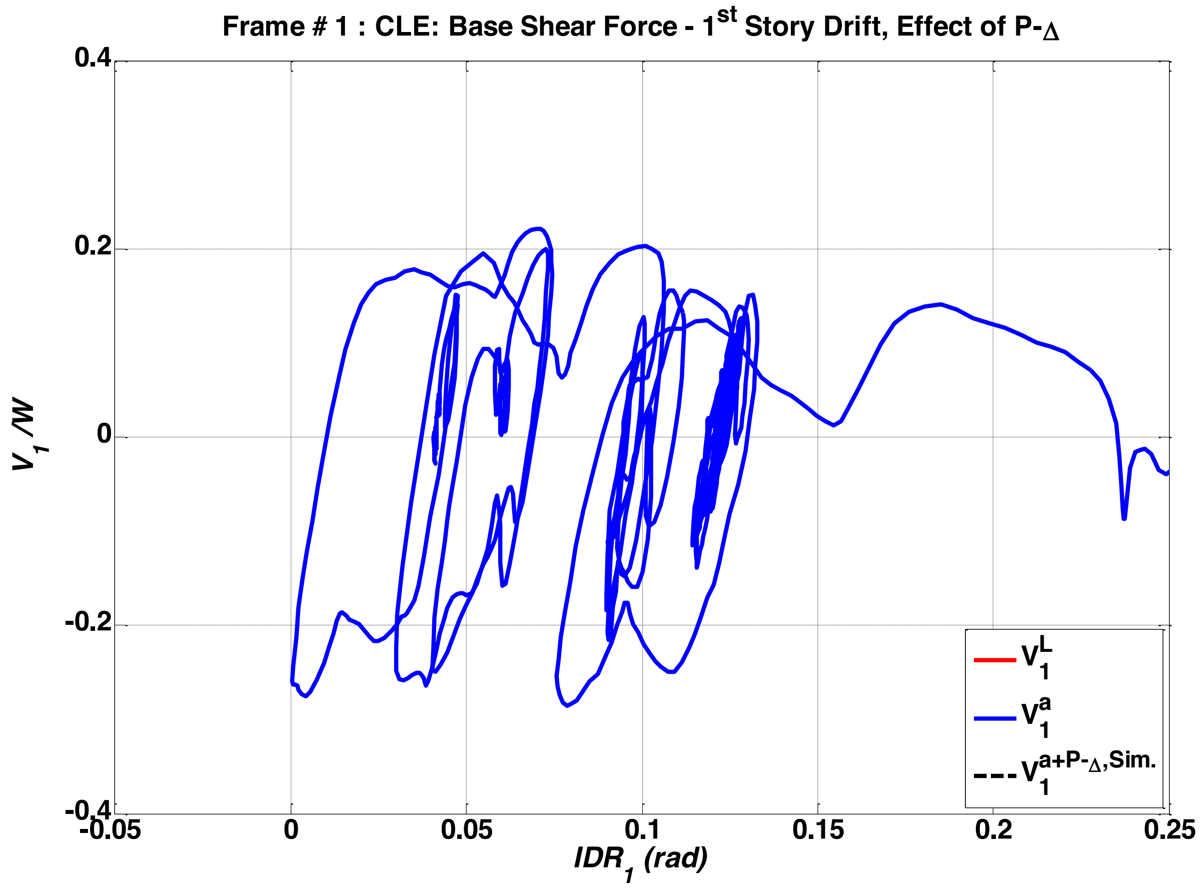
(Source Lignos et al. 2011)

Experimental Demonstration

Lignos et al. 2011

Base Shear vs 1st Storey Drift

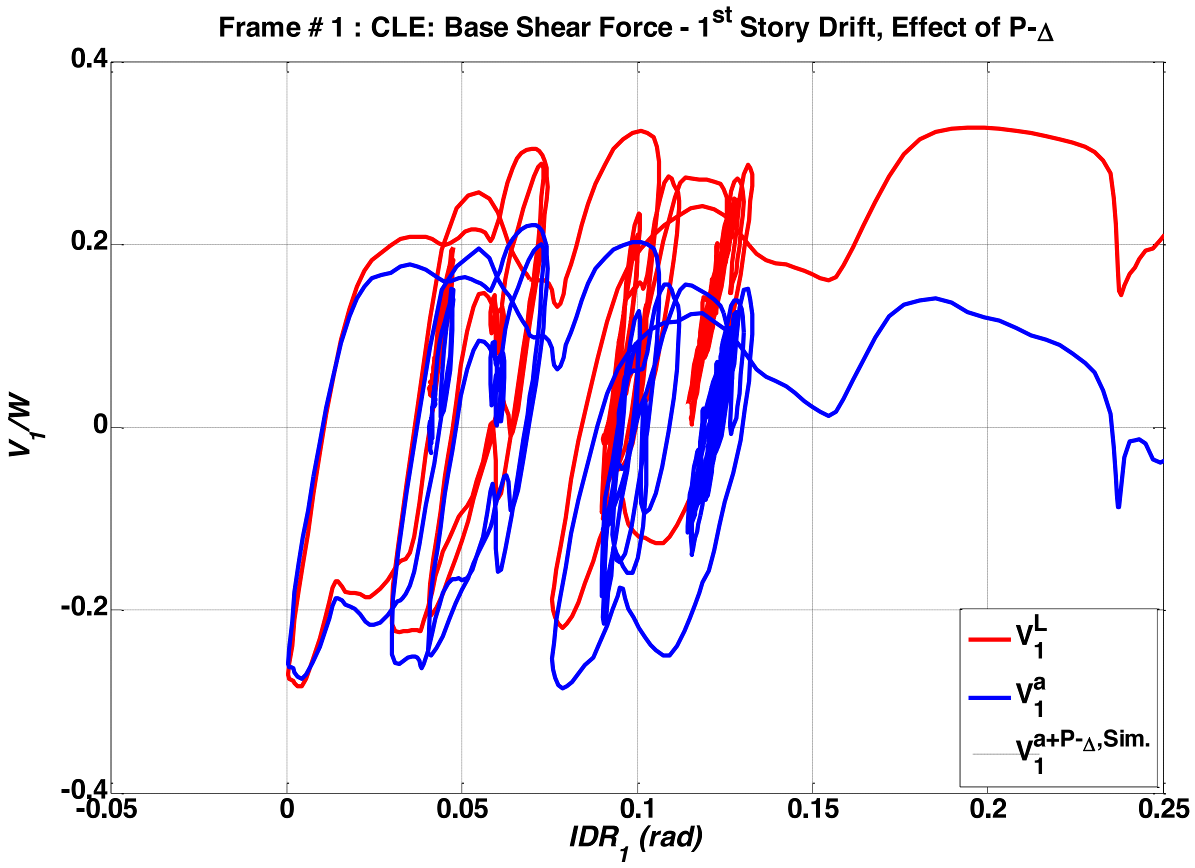
- Inertia Forces



(Source Lignos et al. 2011)

Base Shear vs 1st Storey Drift

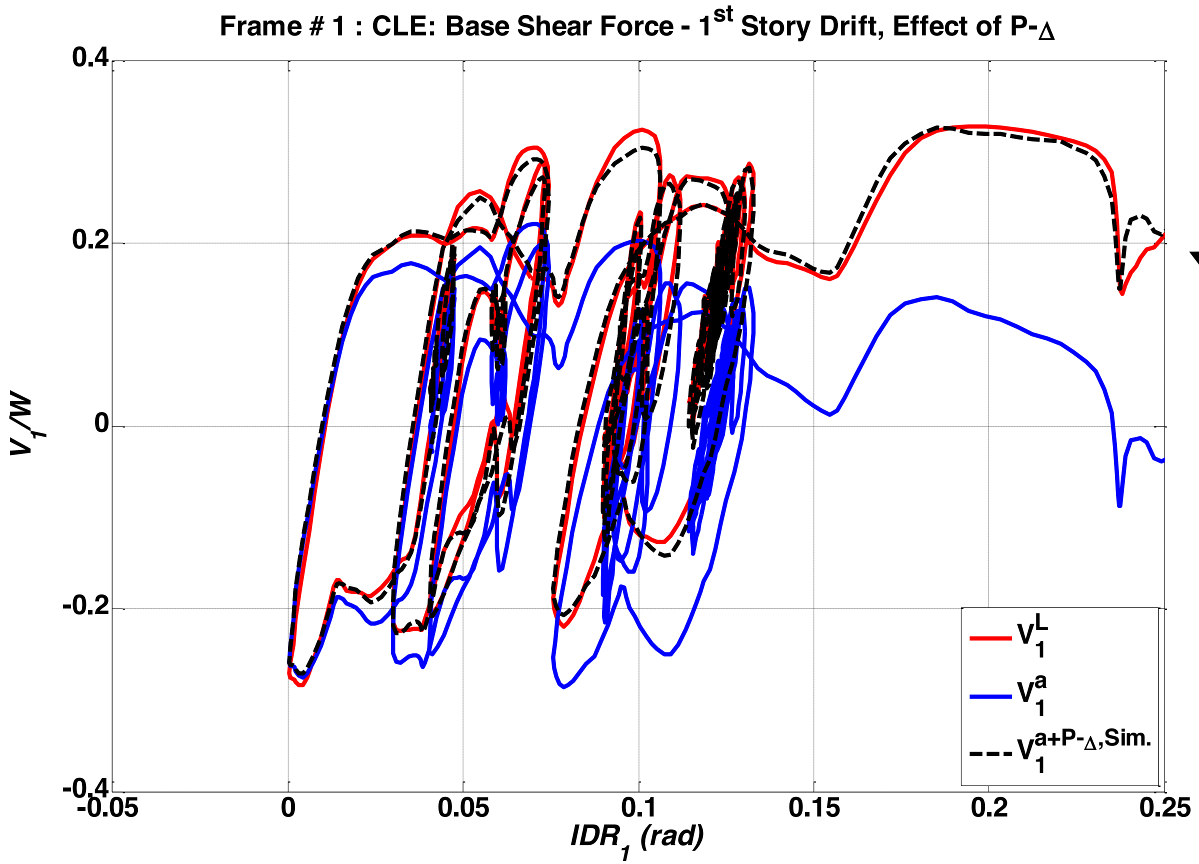
- P-Delta effect can be quantified through collapse



(Source Lignos et al. 2011)

Base Shear vs 1st Storey Drift

•P-Delta effect can be quantified through collapse



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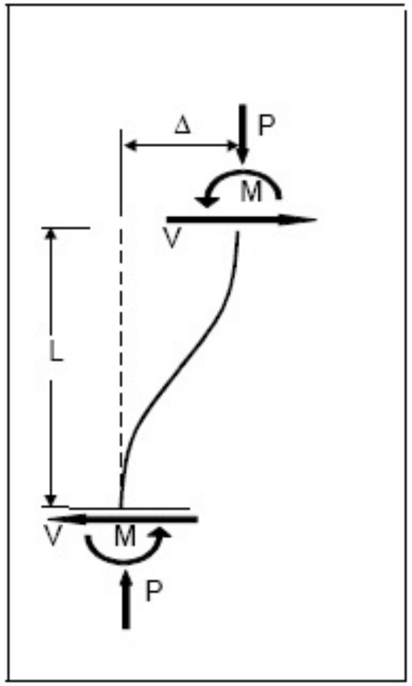
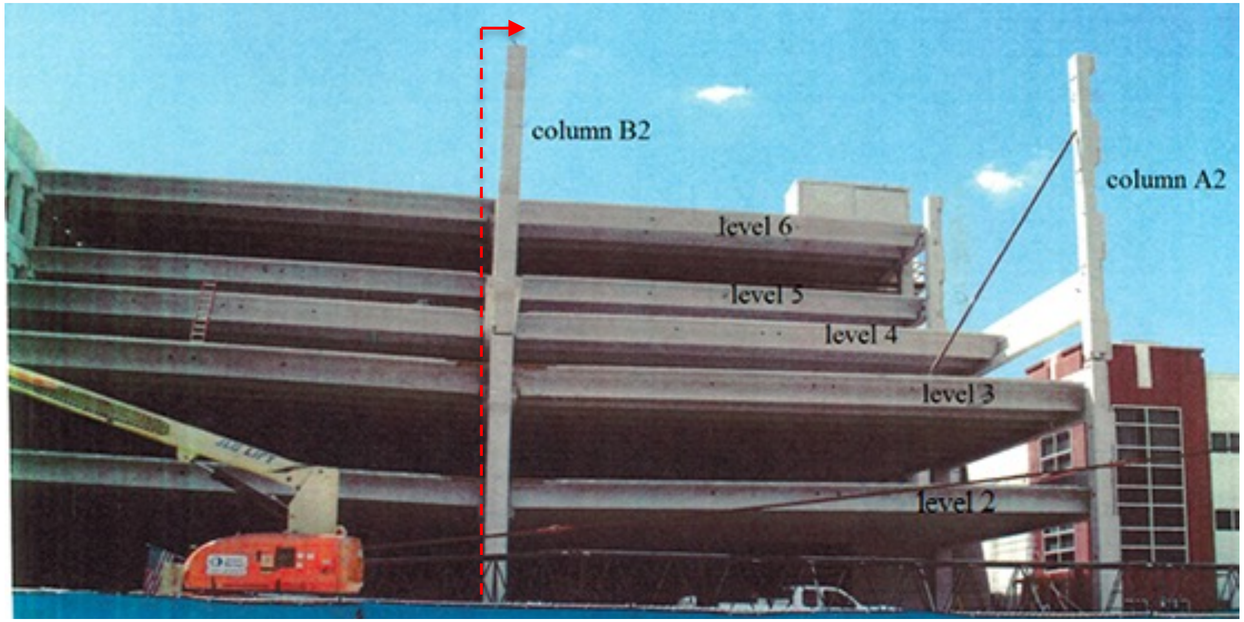
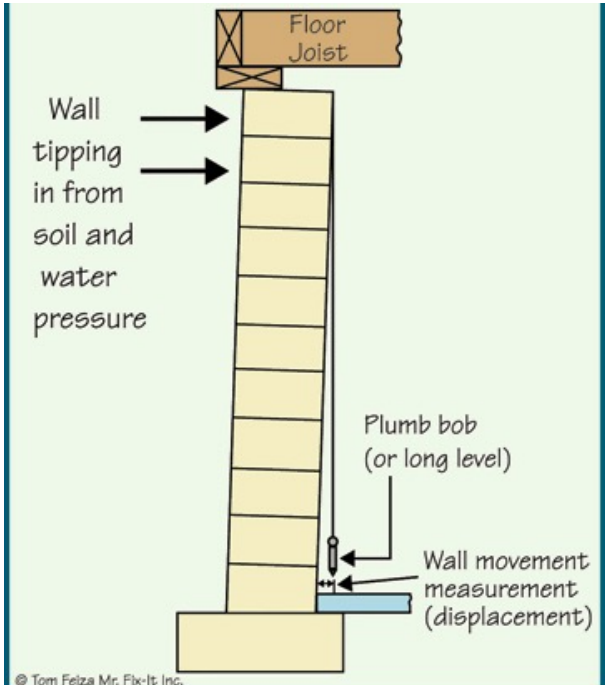


(Source Lignos et al. 2011)

Out-of-Plumb Effects

- When walls or columns are constructed out-of-plumb, the gravity loads acting on the vertical misalignment cause drift and moment P-Delta effects.
- The normally allowed erection tolerances restrict the out-of-plumb effects.



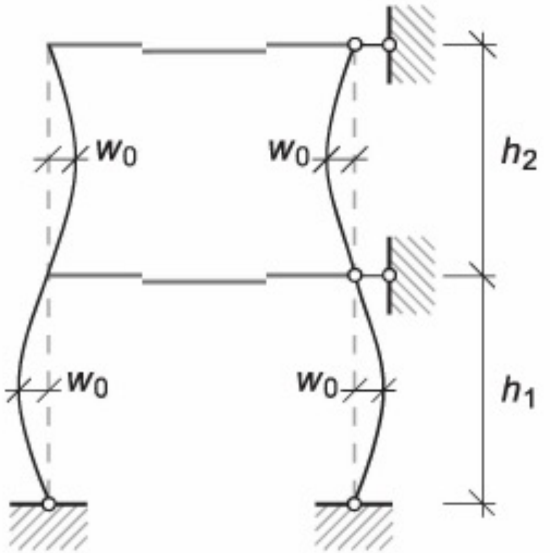


Out-of-Plumb Effects

- When the usual P-Delta effects are small, the check whether the out-of-plumb P-Delta effects are larger.
- If they are larger, and of significance, the out-of-plumb P-Delta effects should then be used in designing the structure.
- The out-of-plumb effects can be accounted for by analyzing the structure for equivalent lateral loads δH_i as in the iterative method. The first values of δH_i should be obtained by using the out-of-plumb displacements, based on the allowable tolerances.
- The erection tolerances used to estimate δ_i vary between Codes of Practice but $h/1000$ is a typical value.

Out-of-Plumb Effects (SIA-263, Section 4.2.3.2)

Figure 3: Imperfections équivalentes à introduire pour le calcul de barres droites comprimées dans les cadres tenus latéralement (défaut de rectitude initial avec flèche w_0 au milieu de la barre)



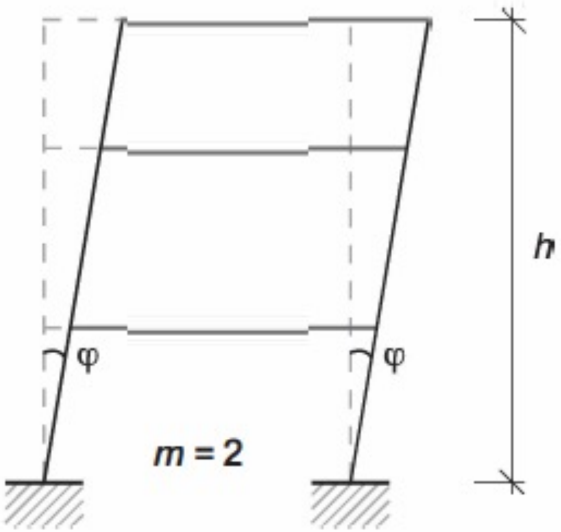
Type de barre	Flèche w_0 de la déformée initiale	
Barres simples avec la courbe de flambage déterminée par leur section, selon la figure 7	EE	EP
a	$L/300$	$L/250$
b	$L/250$	$L/200$
c	$L/200$	$L/150$
d	$L/150$	$L/100$

L longueur de barre.

(Source SIA 263)

Out-of-Plumb Effects (SIA-263, Section 4.2.3.3)

Figure 4: Inclinaison initiale d'un cadre



$$\varphi = \frac{\alpha_h \cdot \alpha_m}{200} \qquad (2)$$

avec $\alpha_h = \frac{2}{\sqrt{h}}$ $\qquad \frac{2}{3} \leq \alpha_h \leq 1{,}0$

$\alpha_m = \sqrt{0{,}5\left(1 + \frac{1}{m}\right)}$ $\qquad \alpha_m \leq 1{,}0$

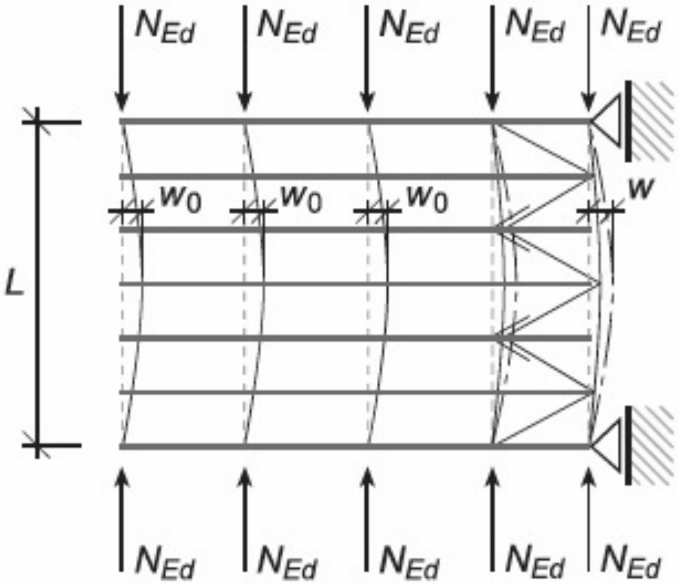
m nombre de poteaux dans le plan du cadre

h hauteur du cadre (en mètres)

(Source SIA 263)

Out-of-Plumb Effects (SIA-263, Section 4.2.3.5)

Figure 5: Défaut de rectitude initial (flèche w_0) des éléments à stabiliser



$$w_0 = \frac{L}{500}\alpha_m \quad (3)$$

avec $\alpha_m = \sqrt{0{,}5\left(1 + \frac{1}{m}\right)}$ $\quad \alpha_m \le 1{,}0$

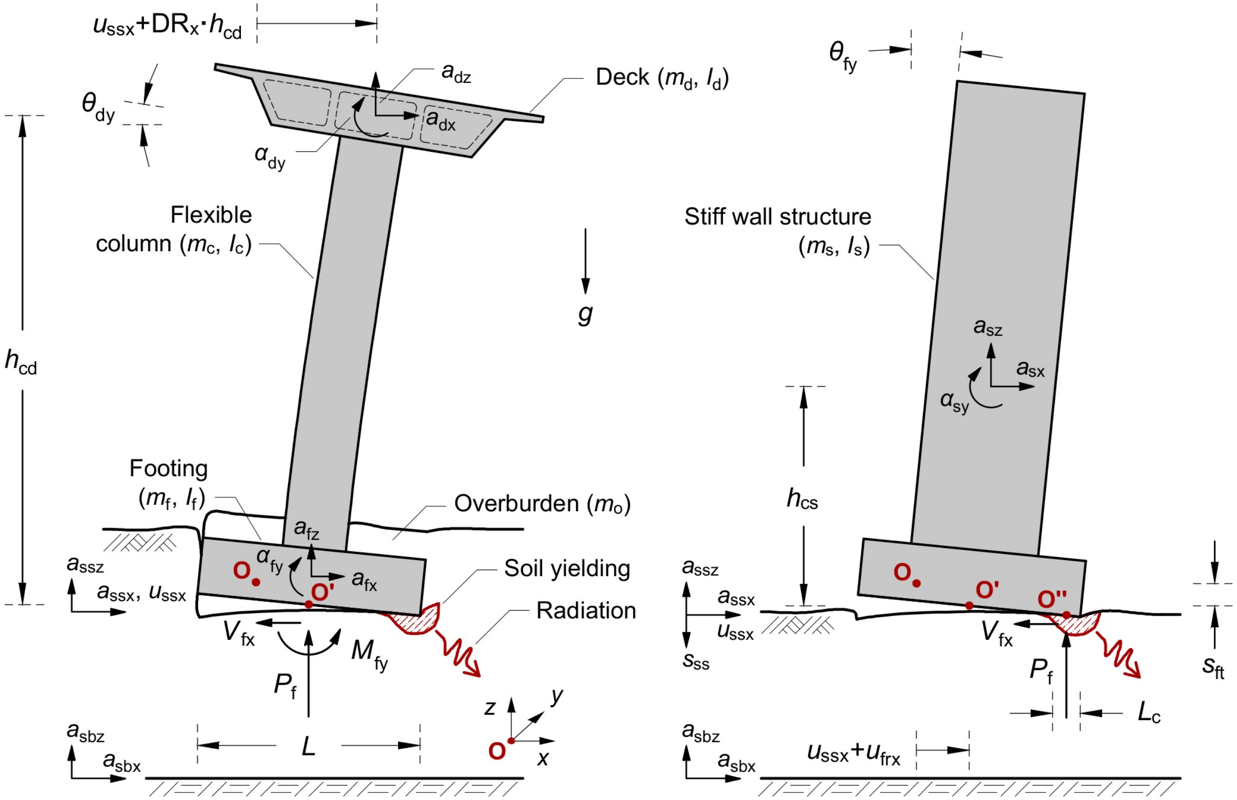
w flèche additionnelle de l'élément stabilisateur

m nombre d'éléments à stabiliser (par ex. m = 3 dans le cas représenté par la figure 5)

(Source SIA 263)

Effect of Foundation Rotation

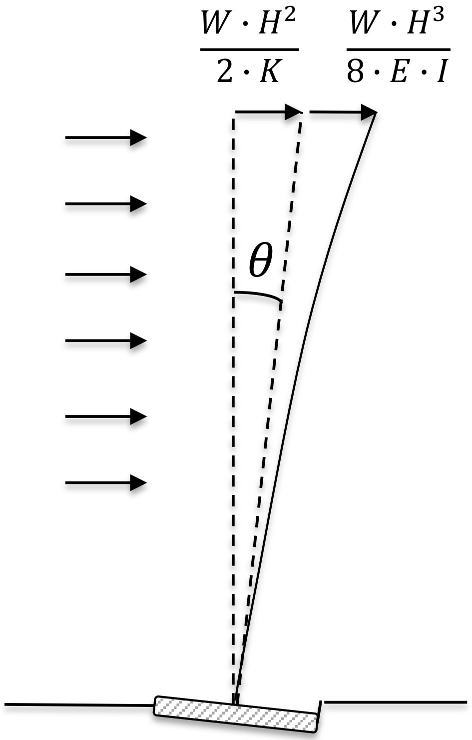
- A flexible foundation will affect the overall stability of a building by reducing the effective stiffness of the vertical cantilever structure.
- It will also increase the deflections from horizontal loading and hence increase the P-Delta effect.



(Source Gavras et al. 2020)

Effect of Foundation Rotation

- The top deflection of a uniform flexural cantilever of height H is subjected to uniformly distributed total load W is given by,



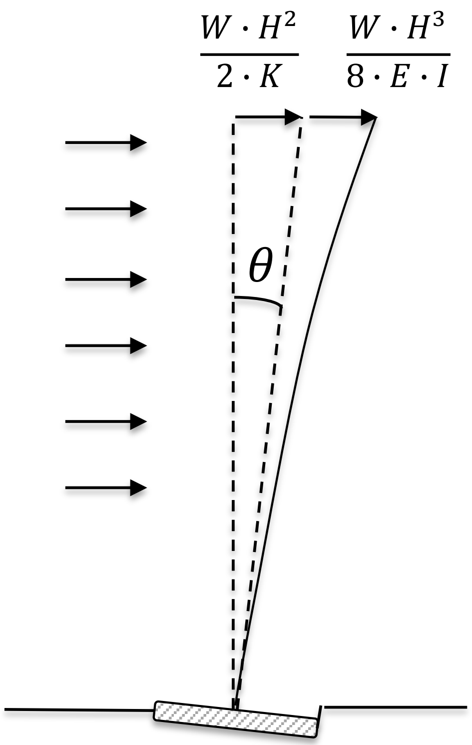
$$\Delta_T = \frac{W \cdot H^3}{8 \cdot E \cdot I}$$

Assume a foundation rotational rigidity, defined as the rotation per unit moment, is K, then the top deflection is increased to,

$$\Delta_T = \frac{W \cdot H^3}{8 \cdot E \cdot I} + \frac{W \cdot H^2}{2 \cdot K}$$

Effect of Foundation Rotation

- Rewriting the previous equation,



$$\Delta_T = \frac{W \cdot H^3}{2 \cdot E \cdot I}\left[\frac{1}{4} + \frac{E \cdot I}{K \cdot H}\right]$$

$$\Delta_T = \frac{W \cdot H^3}{2K \cdot H}\left[\frac{1}{4} \cdot \frac{K \cdot H}{E \cdot I} + 1\right]$$

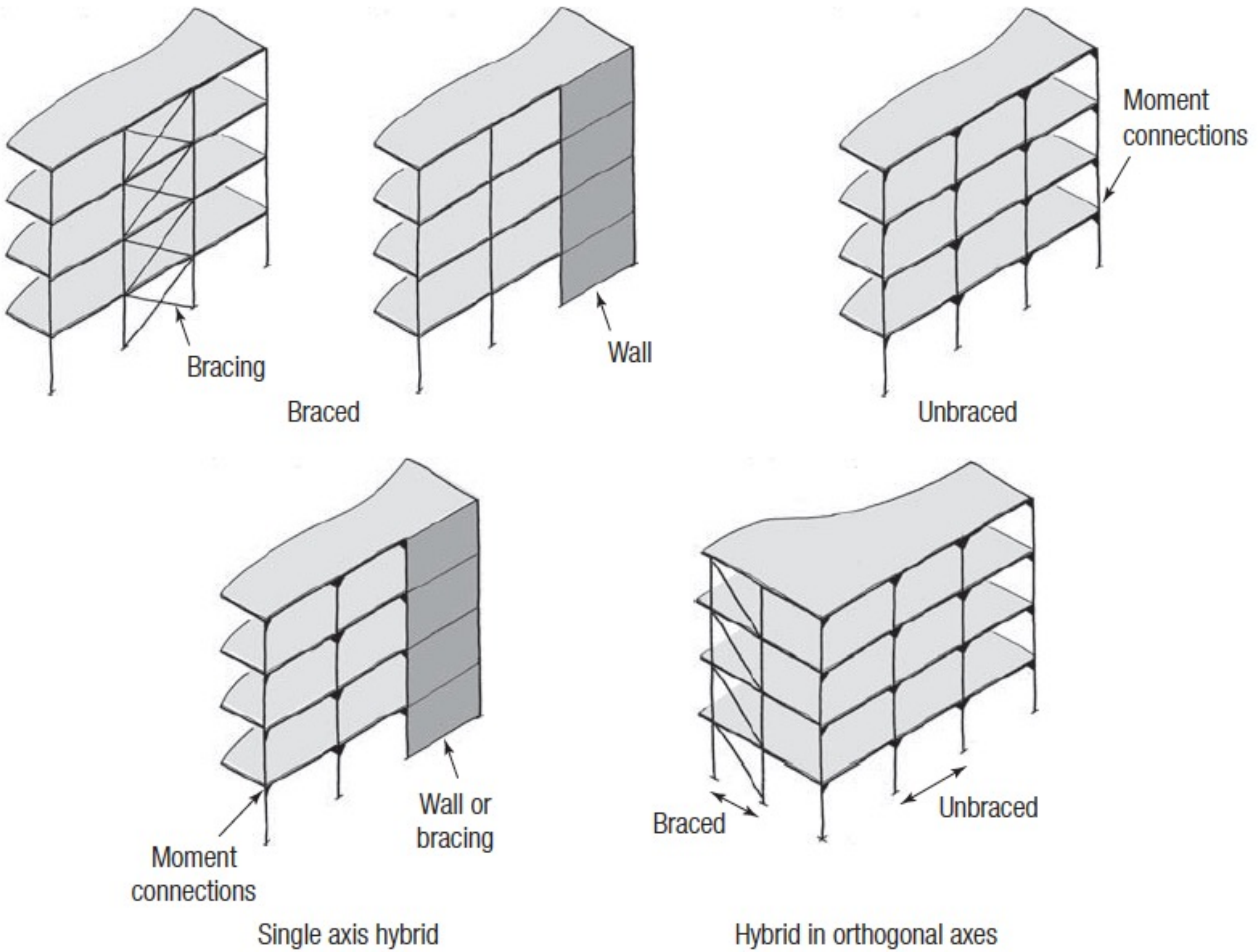
Assume $\mu = \frac{K \cdot H}{E \cdot I}$

$$\rightarrow \Delta_T = \frac{W \cdot H^3}{8 \cdot E \cdot I}\left[\frac{\mu + 4}{\mu}\right]$$

Therefore, the critical load may be approximated,

$$N_{cr} = \frac{\mu}{\mu + 4} \cdot N_{1,cr}^{fixed\ base}$$

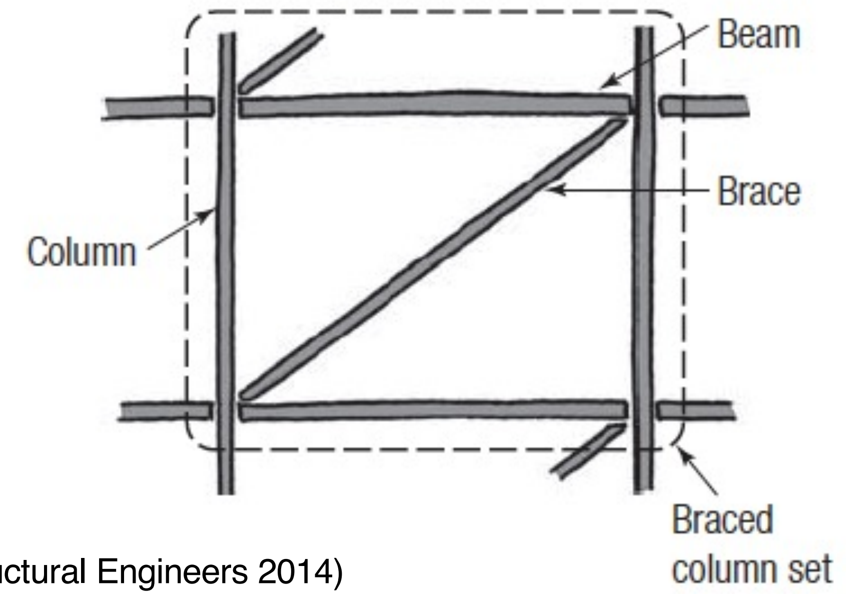
Braced Vertical Stability Systems



(Image source: The Institution of Structural Engineers 2014)

Braced Vertical Stability Systems

- A "braced" structure is one in which defined systems (elements or assemblies) are assumed to contribute resistance to the overall lateral stability of a structure, while other elements specifically do not.
- The resisting systems are typically multiple orders of magnitude stiffer than the general frame. This is a form that facilitates simple (pinned) frame construction in the exteme and allows general frame elements (e.g., braces, walls) to be considered restrained at storey levels.
- Framed bracing (or bracing) is often most material-efficient method of providing lateral stability.



(Image source: The Institution of Structural Engineers 2014)

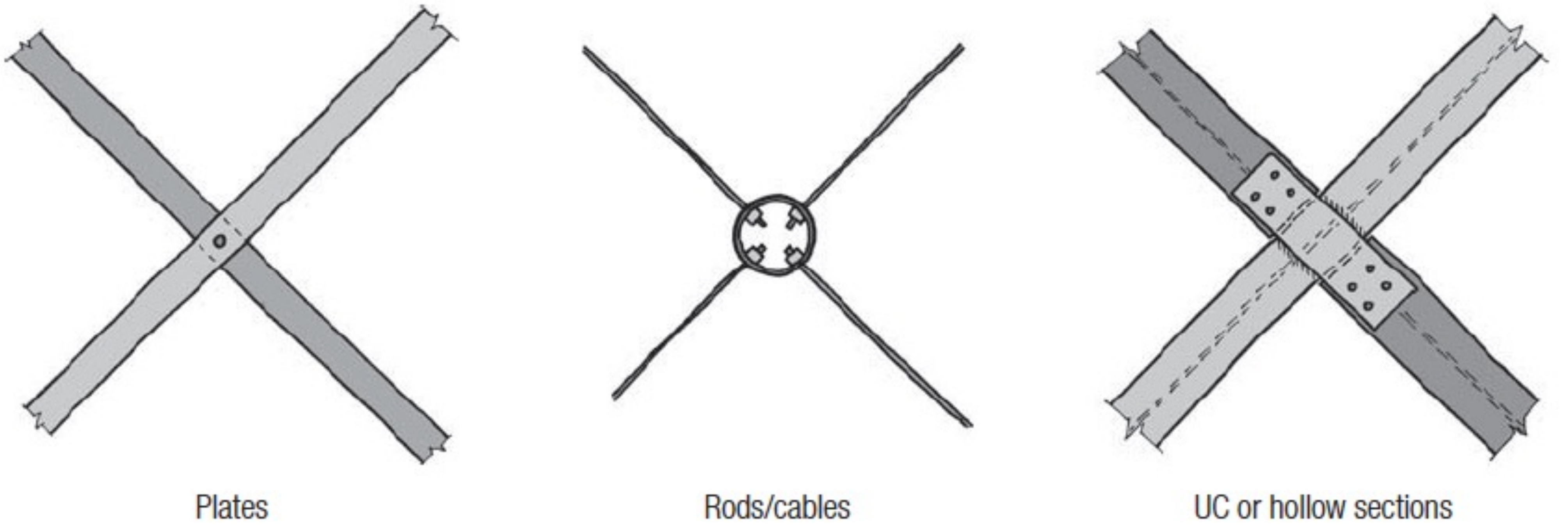
Braced Vertical Stability Systems

(Image source: The Institution of Structural Engineers 2014)

Braced Vertical Stability Systems

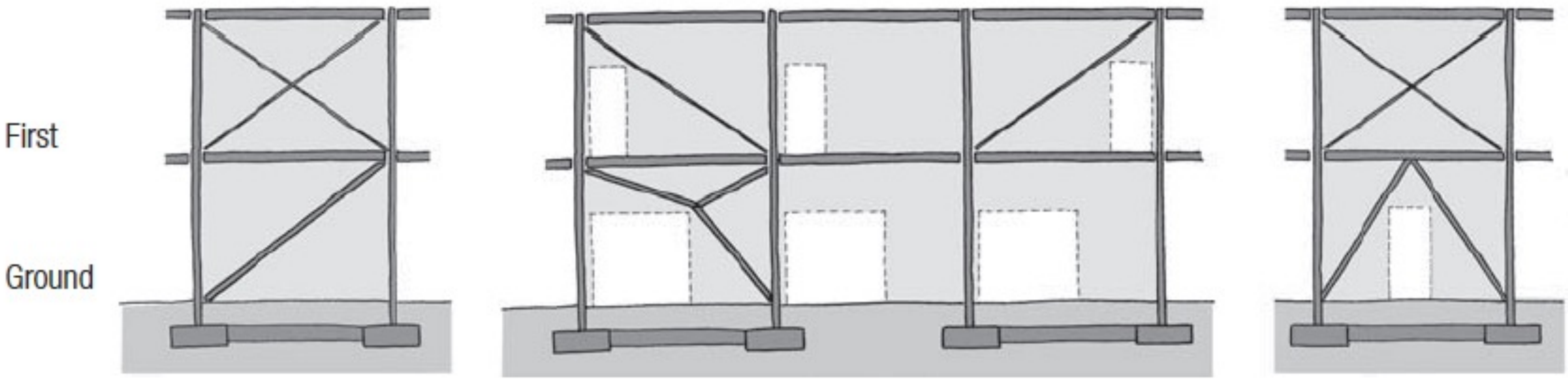
(Image source: The Institution of Structural Engineers 2014)

Braced Vertical Stability Systems



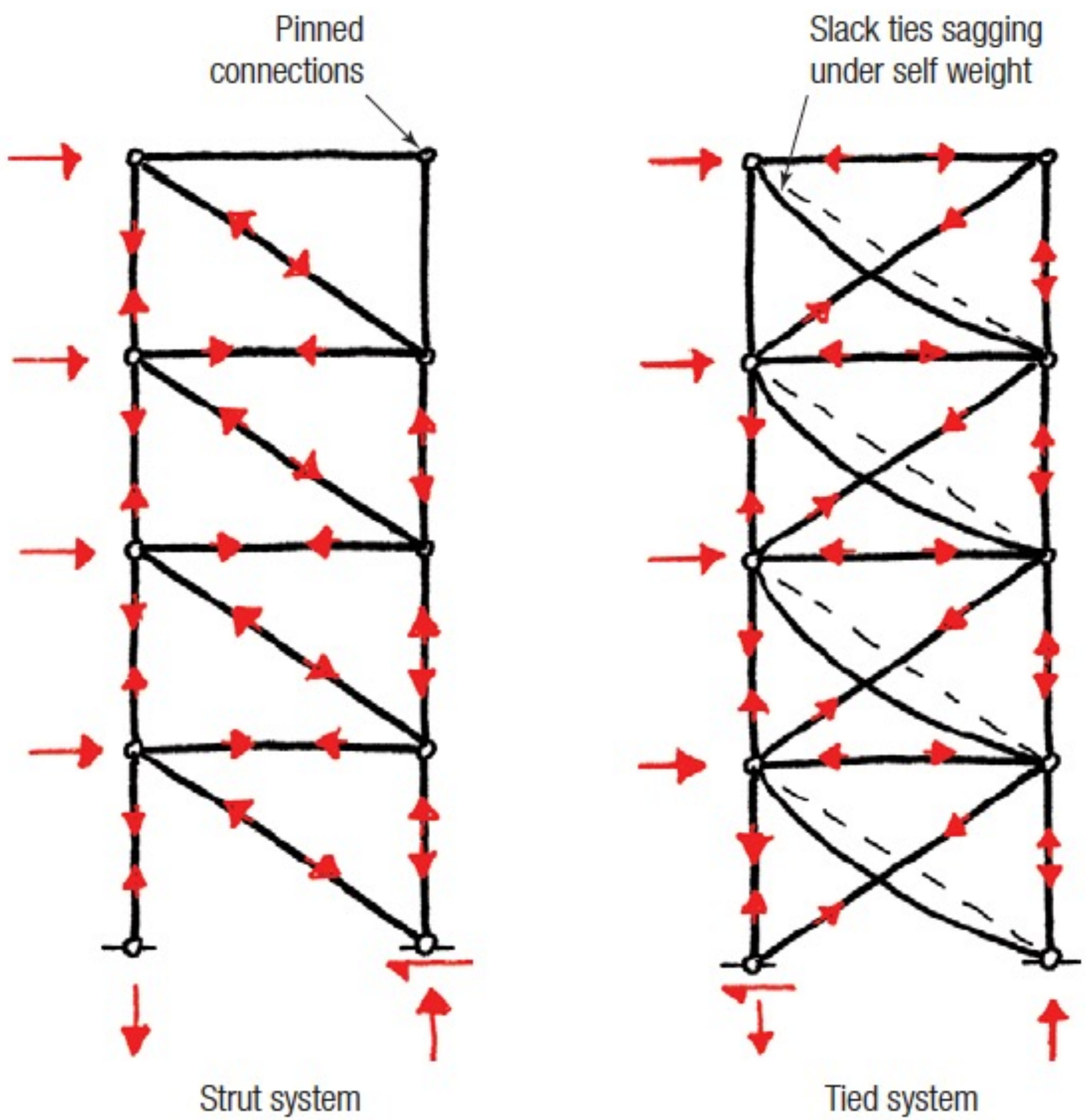
(Image source: The Institution of Structural Engineers 2014)

Vertical Bracing Configurations



(Image source: The Institution of Structural Engineers 2014)

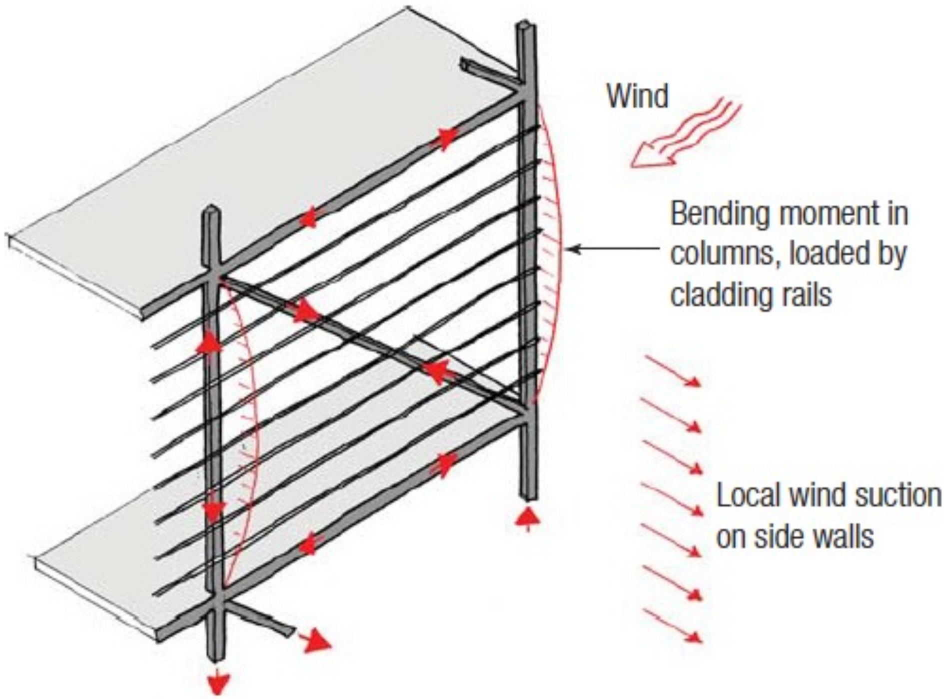
Braced Frames – Behaviour of Bracing



(Image source: The Institution of Structural Engineers 2014)

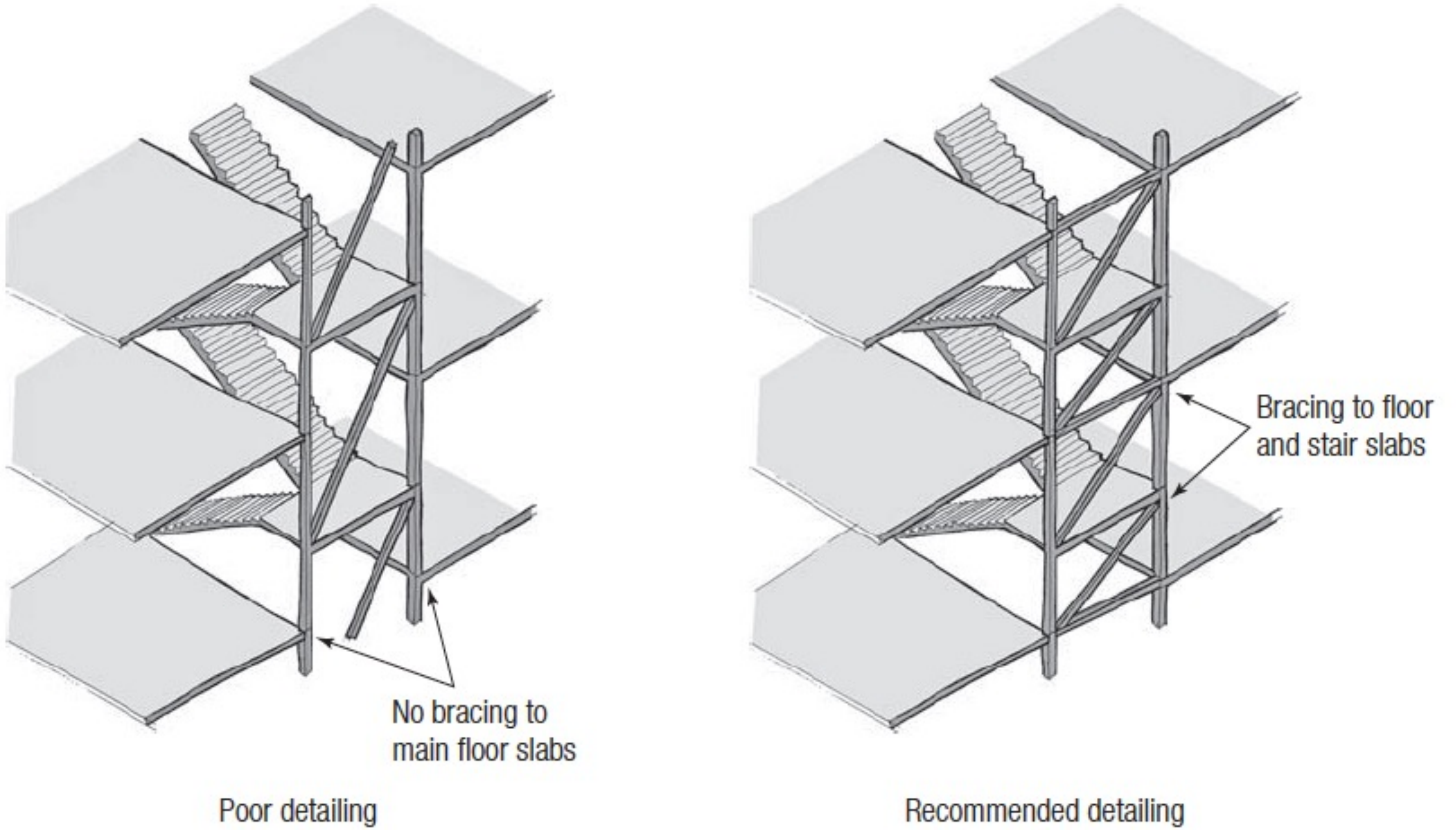
Braced Frames – Behaviour of Bracing

- Bracing is most efficient where diagonal elements are inclined between 35° and 50° to the horizontal. This ensures relatively modest element forces and compact connection details.
- Narrow bracing systems with steeply inclined diagonal elements have less flexural stiffness, increased column forces and will increase the sway sensitivity.



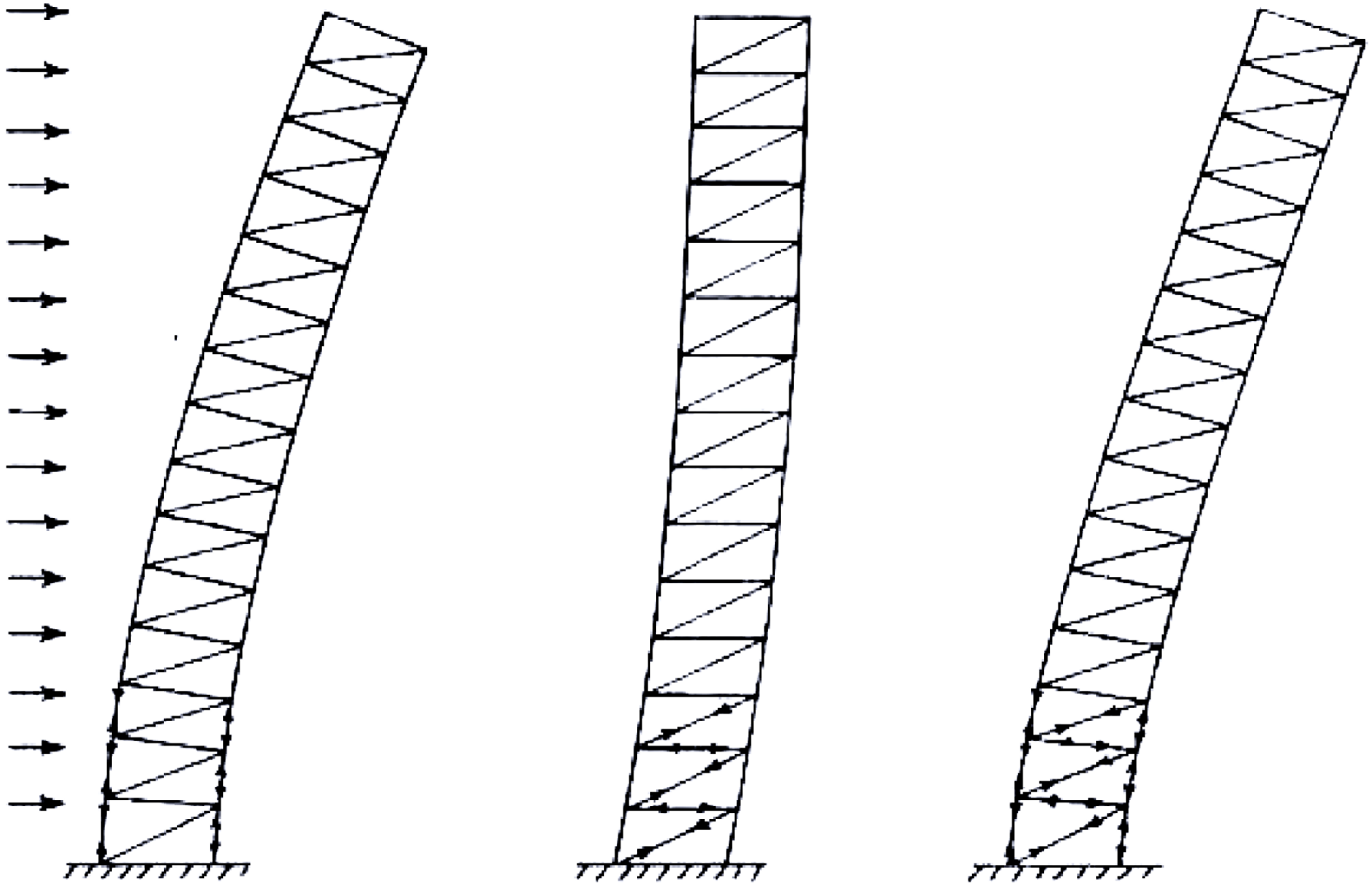
(Image source: The Institution of Structural Engineers 2014)

Braced Frames – Behaviour of Bracing



(Image source: The Institution of Structural Engineers 2014)

Braced Frames – Behaviour of Braced Bent



Flexural mode Shear mode Mixed mode

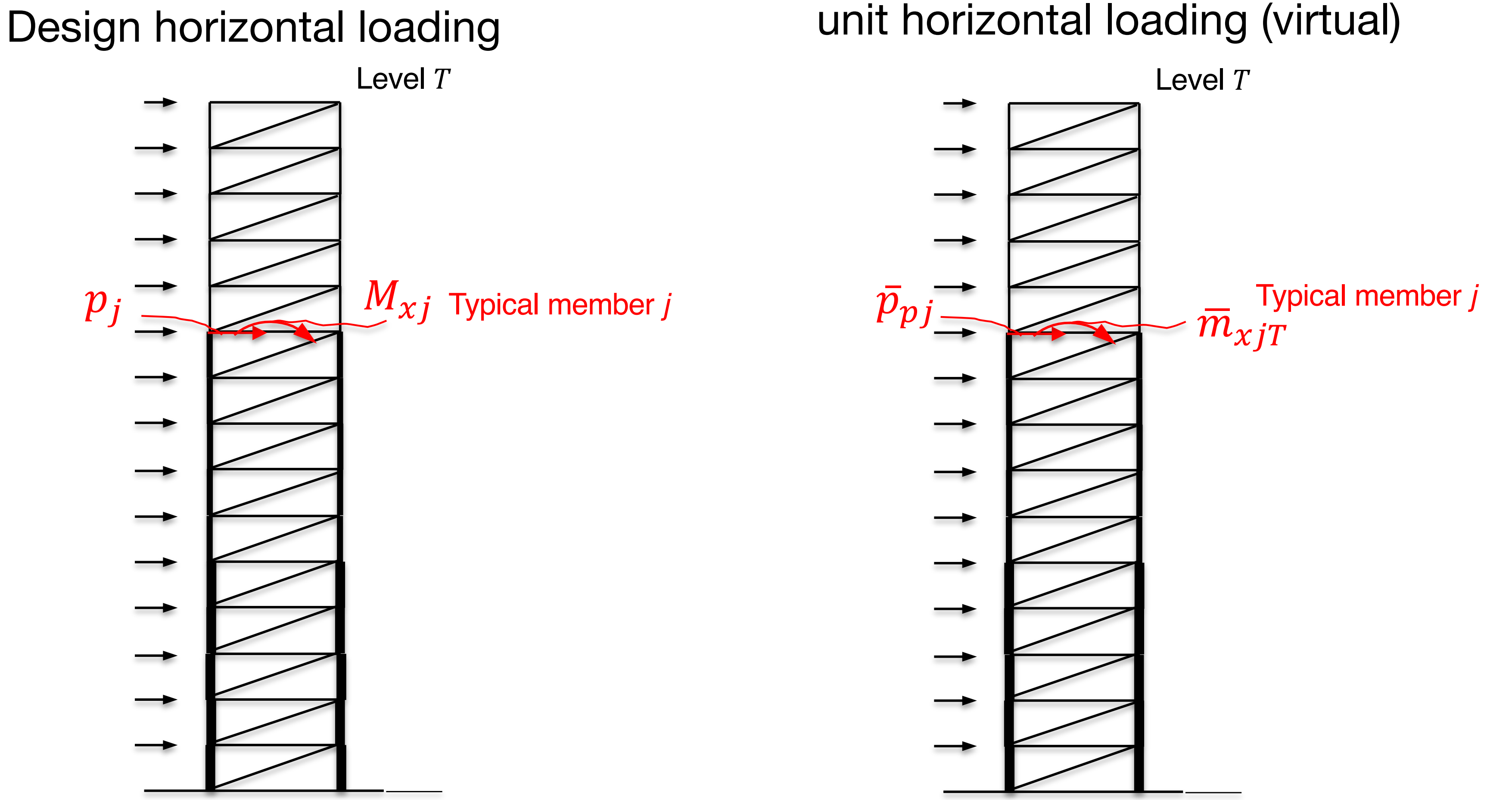
(Image source: Tall Building Structures, Analysis and Design)

EPFL Braced Frames – Drift Analysis

- It is important to appreciate the relative influence of the flexural and shear mode contributions, due to the column axial deformations and to the diagonal and girder deformations, respectively.
- In low-rise frames with bracings, the shear mode displacements are the most significant. These largely determine the lateral stiffness of the structure.
- In mid- to high-rise structures, however, the higher axial forces and deformations in the columns, and the accumulation of their effects over a greater height, cause the flexural component of displacement to become dominant.
- In braced bays with single diagonal bracing and a height-to-width ratio of 8, the total drift may be typically 60-70% attributable to the flexural component, with the remainder due to the shear component.
- The storey drift ratio is more strongly influenced by the flexural component of deflection. This is because the inclination of the structure caused by the flexural component accumulates up the structure, while the storey shear component diminishes towards the top.

Braced Frames – Drift Analysis

- **Virtual Work Drift Analysis:** In this method a force analysis of the structure subjected to the design horizontal loading is first made to determine the axial force N_j in each member j, as well as the bending moment M_{xj} at sections X along those members subjected to bending.



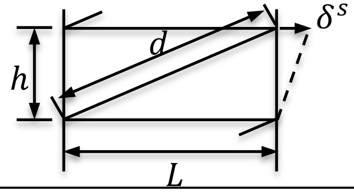
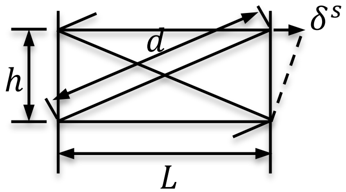
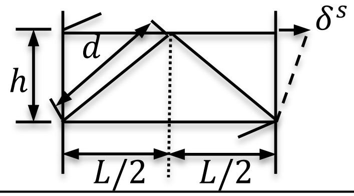
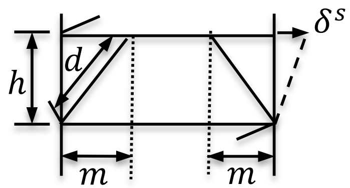
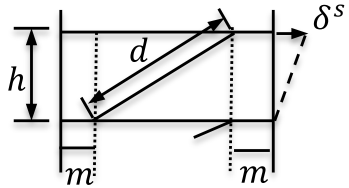
Braced Frames – Drift Analysis

- A second force analysis is then made with the structure subjected to only a unit imaginary or dummy horizontal load at the level T whose drift is required to give the axial force $\bar{p}_{pT}$, and moment $\bar{m}_{xjT}$ at section X in the bending members. The resulting horizontal deflection at n is then given by,

$$\Delta_T = \sum \bar{p}_{pT} \cdot \left(\frac{N \cdot L}{E \cdot A}\right)_j + \sum \int_0^{L_j} \bar{m}_{xjT} \cdot \left(\frac{M_x}{EI}\right)_j dx$$

- In which, L_j, A_j, I_j are the length, sectional area and moment of inertia, respectively, for each member j, and E is the elastic modulus. The first summation refers to all members subjected to axial loading, while the second refers to only those members subjected to bending, if any.
- If the drift is required at another level, n, of the structure, another unit load analysis will have to be made, but with the unit load applied only at level, n.
- The result values $\bar{p}_{pn}$, and moment $\bar{m}_{xjn}$ will be substituted in the above equation to give the drift.

EPFL Braced Frames – Shear Deflection Component

Type of Bracing	Dimensions	Shear Deflection per Storey
Single diagonal		$\delta^s = \frac{V}{E} \cdot \left[\frac{d^3}{L^2 A_d} + \frac{L}{A_g}\right]$
Double diagonal		$\delta^s = \frac{V}{2E} \cdot \left[\frac{d^3}{L^2 A_d}\right]$
V-brace		$\delta^s = \frac{V}{E} \cdot \left[\frac{2d^3}{L^2 A_d} + \frac{L}{4A_g}\right]$
Frame with eccentric bracing		$\delta^s = \frac{V}{E} \cdot \left[\frac{d^3}{2m^2 A_d} + \frac{m}{2A_g} + \frac{h^2 (L-2m)^2}{12 I_g L}\right]$
Offset diagonal		$\delta^s = \frac{V}{E} \cdot \left[\frac{d^3}{(L-2m)^2 A_d} + \frac{(L-2m)}{A_g} + \frac{h^2 m^2}{3 I_g L}\right]$

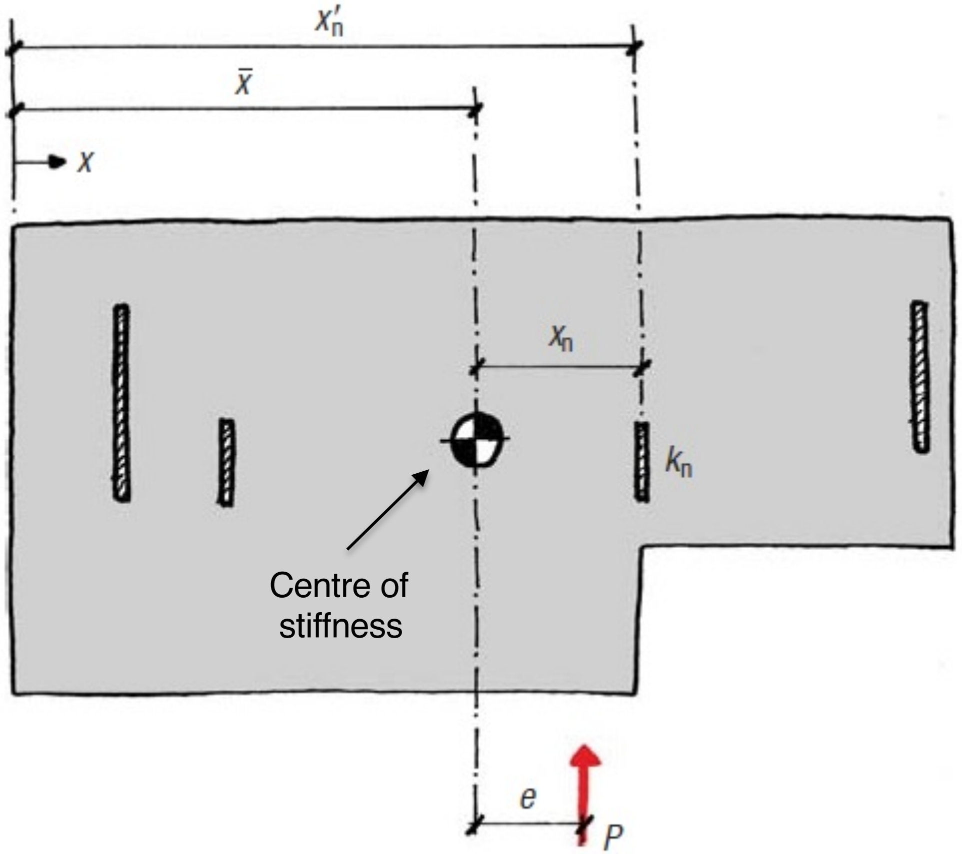
V: storey shear

A_d: sectional area of the diagonal

A_g, I_g: are, respectively, the sectional area and inertia of the upper girder

E: is the elastic modulus

Approximate Analysis Methods for Braced Frames



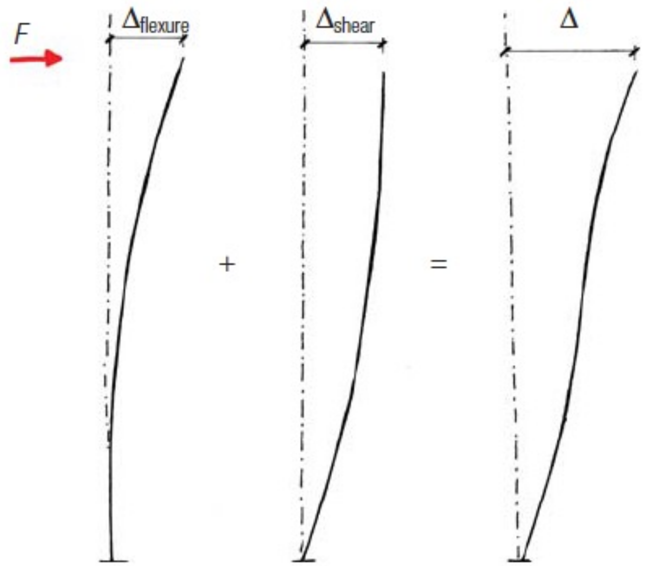
Force acting on the n^{th} bracing frame

$$P_n = \frac{P \cdot k_n}{\sum k_n} \pm \frac{P \cdot e \cdot x_n \cdot k_n}{\sum (x_n^2 \cdot k_n)}$$

(Image source: The Institution of Structural Engineers 2014)

Braced Frames – Evaluating Stiffness

- System stiffness comprises a flexural component and a shear component. The flexural component will tend to dominate in taller, more slender braced column sets while shear will dominate in short, wide systems.
- The stiffness of each stability system can be determined in turn by analyzing the displacement resulting from an arbitrary force F applied to the system in isolation

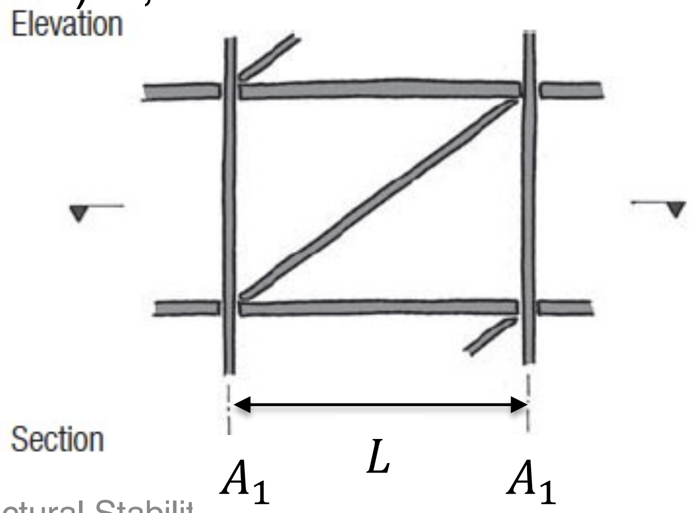


$$k_n = \frac{F}{\Delta} = \frac{F}{\Delta_{flexure} + \Delta_{shear}}$$

Braced Frames – Drift Analysis

- An approximate calculation of the drift can be made by assuming that the flexural mode stiffness is entirely attributable to the axial areas of the columns. This is common in the majority of bracing types.
- A detailed force analysis of the frame is not necessary. Only the external moment and the total shear force at each level are required.
- **Flexural component:** The procedure for obtaining the flexural component of drift is to first calculate the external moment diagram for the structure.
- To compute for the different vertical regions of the bent, the second moment of area I of the column sectional areas about their common centroid the parallel axis theorem (*or Steiner's Theorem*) is used in which, the value for the lower region of the braced bent in the figure (see next slide) is,

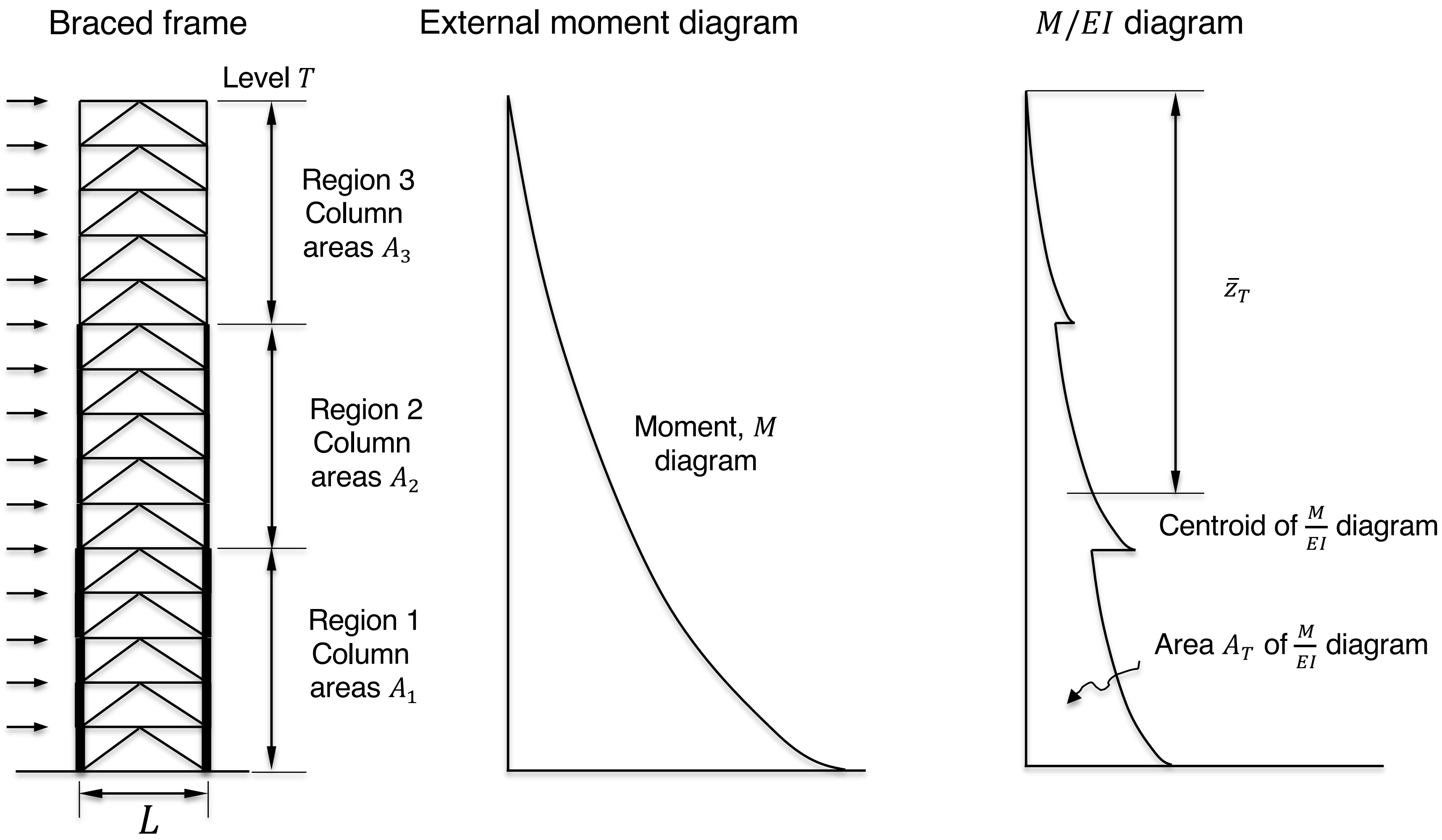
$$I_1 \approx 2A_1 \left(\frac{L}{2}\right)^2 = \frac{A_1 L^2}{2}$$



(Image source: The Institution of Structural Engineers 2014)

Braced Frames – Drift Analysis

- The moment diagram and the values of I are used to construct an M/EI diagram as shown in the next figure.



Braced Frames – Drift Analysis

- The storey drift in sttorey, i, due to flexure is,

$$\delta_{if} = h_i \cdot \theta_{if}$$

- In which, h_i is the height of the storey i and θ_{if} is the inclination of storey i, which is equal to the area under the M/EI curve between the base of the structure and the mid-height of storey i.
- The total drift at floor n, due to flexure, is then,

$$\Delta_{nf} = \sum_{1}^{n} \delta_{if}$$

Braced Frames – Drift Analysis

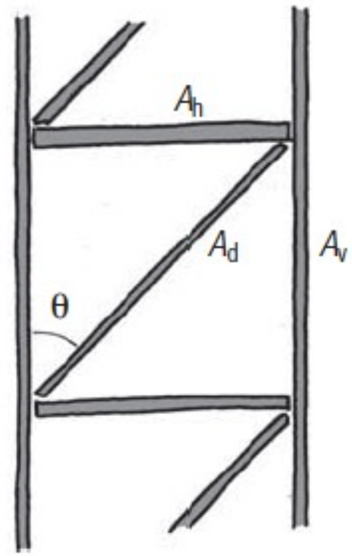
- **Shear component:** the shear component of the storey drift in storey i is a function of the external shear and the properties of the braces and girder in that storey. The shear component of total drift at floor level n,

$$\Delta_{shear}^{(n)} = \sum_{1}^{n} \delta_{shear}^{(i)}$$

- The shear component is proportional to the shear stiffness, GA_s

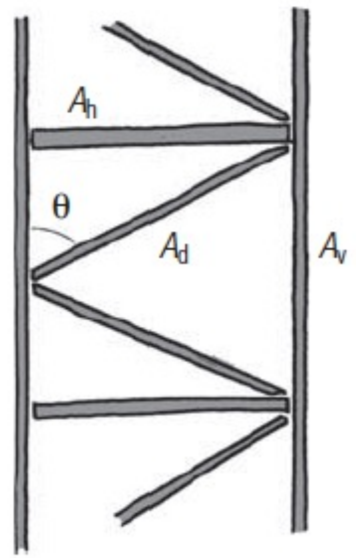
Braced Frames – Drift Analysis

- The shear stiffness, GA_s, can be approximated for single strut bracing systems using the following equation,



$$GA_s \approx \frac{cos\theta}{\left(\frac{1}{\sin^2\theta \cdot EA_d}\right) + \left(\frac{sin\theta}{EA_h}\right)}$$

- For K-brace systems



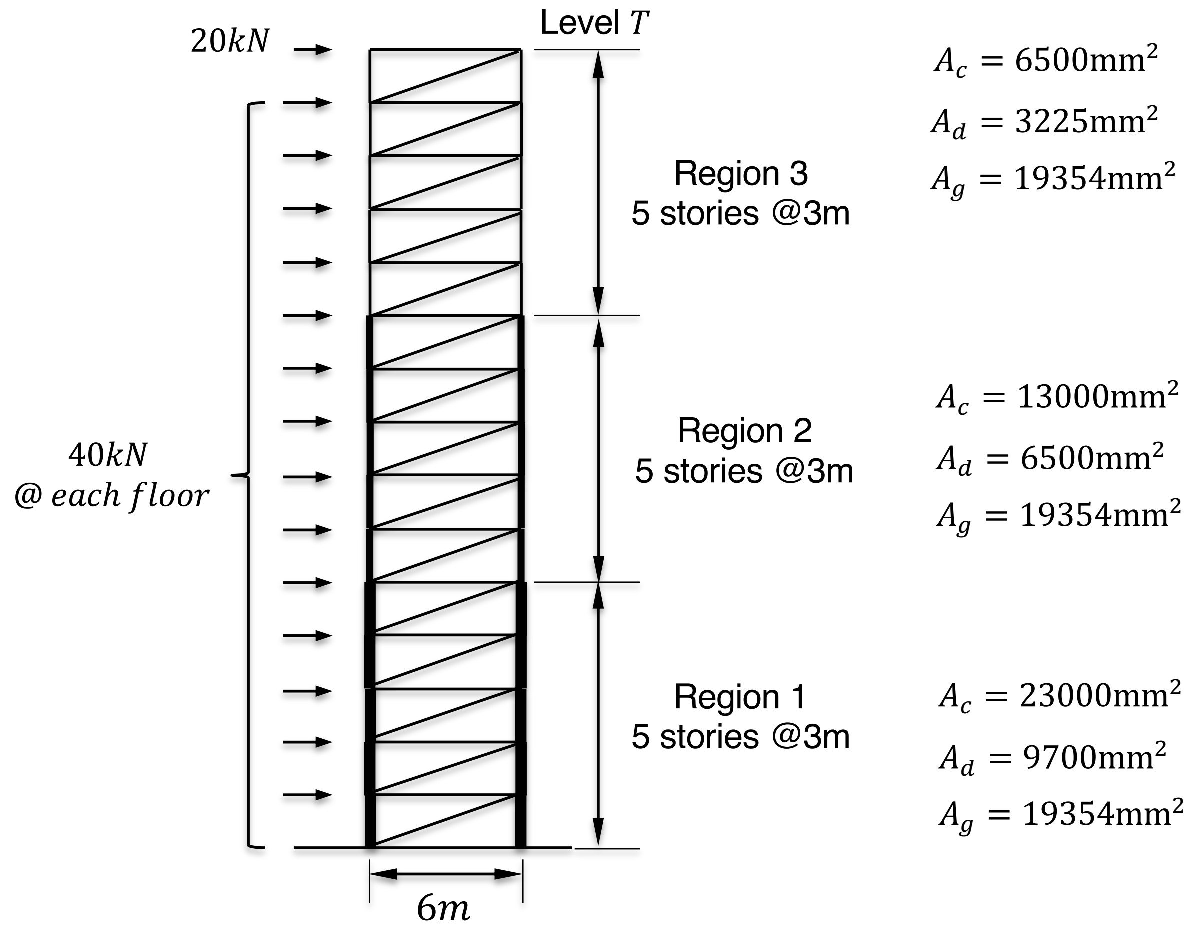
$$GA_s \approx \sin^2\theta \cdot cos\theta \cdot EA_h$$

- Alternatively, the shear deformation table may be used.

(Image source: The Institution of Structural Engineers 2014)

Worked Example for Calculating Drift in Braced Frames by Approximate Method

- A 15-storey single diagonal braced frame consists of three 5-storey regions. It is required to determine the drift at floors 5, 10 and 15 for a uniform wind load of 40 kN per storey (20kN at the top floor). Assume the elastic modulus, $E = 200GPa$.



Worked Example for Calculating Drift in Braced Frames by Approximate Method

- The flexural and shear components of drift will be determined separately, as follows,
- **Flexural component:**

Step 1: Compute the moment of inertia of the column sectional areas about their common centroid for each of the three height regions and record the values (see column 3 in summary)
In the frame under consideration the column areas are equal, therefore, their common centroid is mid-way between the columns,

$$I_c \approx 2 \times A_c \left(\frac{L}{2}\right)^2 = \frac{A_c L^2}{2}$$

As an example, for the lower region, stories 1-5, where, $A_1 = 23000mm^2$

$$I_1 \approx \frac{A_1 L^2}{2} = \frac{23000x6000^2}{2} = 4.14x10^{11}mm^4$$

Worked Example for Calculating Drift in Braced Frames by Approximate Method

Step 2: Compute the value of the external moment M at each mid-storey level (see column 4 in summary table). For example, in storey 12,

$$M = 40x(1.5 + 4.5 + 7.5) + 20x10.5 = 750000kN \cdot mm$$

Step 3: Determine for each the value of hM/EI (see column 5 in summary table). For example, in storey 5 is:

$$\frac{hM}{EI} = \delta\theta_{5,flexure} = \frac{3000x6630000}{4.14x10^{11}xE} = \frac{0.048}{E}$$

Step 4: Determine for each storey i the accumulation of $\delta\theta_{iflexure}$ from storey 1 up to and including storey, i (see column 6 in summary table). For example, the accumulation of $\delta\theta_{i,flexure}$ up to storey 5 is:

$$\sum_{i=1}^{5} h\frac{M}{EI} = \frac{0.0915 + 0.0793 + 0.0680 + 0.0576 + 0.0480}{E} = \frac{0.345}{E}$$

Worked Example for Calculating Drift in Braced Frames by Approximate Method

Step 5: Record the product of h_i and $\theta_{i,flexure}$ (see column 7 in summary table). For example, in storey 5 due to flexure,

$$\delta_{5,flexure} = 3000x\frac{0.0345}{E} = \frac{1034}{E}\ mm$$

Step 6: At each level where the value of the lateral drift is required evaluate the accumulation of the storey drifts, $\delta_{i,flexure}$ from storey 1 up to the considered nth floor, to give the drift $\Delta_{flexure}^{(n)}$ (see column 8 in summary table). For example, at floor 5 is:

$$\Delta_{5,flexure} = \frac{275 + 513 + 717 + 890 + 1034}{E} = \frac{3427}{200} = 17.1mm$$

Worked Example for Calculating Drift in Braced Frames by Approximate Method
-Flexural Component Summary

Storey	Frame Inertia I_i [mm^4]	External Moment M_i [kN-mm]	$\delta\theta_i$ [rad/E]	Storey Inclination θ_{if} [rad/E]	Storey Drift δ_{if} [mm/E]	$\Sigma\delta_{if}$ [mm/E]	$\Delta_{m,flexure}$ [mm]
15	1.17E+11	30000	0.0008	0.635	1905	20330	101.6
14	1.17E+11	150000	0.0038	0.634	1903		
13	1.17E+11	390000	0.0100	0.630	1891		
12	1.17E+11	750000	0.0192	0.620	1861		
11	1.17E+11	1230000	0.0315	0.601	1803		
10	2.34E+11	1830000	0.0235	0.570	1709	10967	54.8
9	2.34E+11	2550000	0.0327	0.546	1638		
8	2.34E+11	3390000	0.0435	0.513	1540		
7	2.34E+11	4350000	0.0558	0.470	1410		
6	2.34E+11	5430000	0.0696	0.414	1243		
5	4.14E+11	6630000	0.0480	0.345	1034	3427	17.1
4	4.14E+11	7950000	0.0576	0.297	890		
3	4.14E+11	9390000	0.0680	0.239	717		
2	4.14E+11	10950000	0.0793	0.171	513		
1	4.14E+11	12630000	0.0915	0.092	275		

Worked Example for Calculating Drift in Braced Frames by Approximate Method

- **Shear component:**

Step 1: Compute the value of the external shear V_i acting in each storey i (see column 2 in summary table).

Step 2: Compute for each storey i the storey drift due to shear, $\delta_{i,shear}$, by substituting the value of the storey shear and member properties into the appropriate formula from Slide 107 (see column 3 in summary table). For example, the shear deflection formula for the single-diagonally braced example frame is:

$$\delta_i^s = \frac{V_i}{E} \cdot \left[\frac{d^3}{L^2 A_d} + \frac{L}{A_g}\right]_{\mathrm{i}}$$

and using this to compute the drift in storey 8 due to shear,

$$\delta_8^s = \frac{300}{200} \cdot \left[\frac{6708.2^3}{6000^2 x 6500} + \frac{6000}{19354}\right]_8 = 2.4\text{mm}$$

Worked Example for Calculating Drift in Braced Frames by Approximate Method

Step 3: Sum the storey drifts due to shear up to and including stories 5, 10 and 15 to obtain the total shear drift at floor levels 5, 10, and 15, (see column 4 in summary table). For example, the drift due to shear at floor 5:

$$\Delta_{shear}^{(5)} = 3.4 + 3.2 + 2.9 + 2.7 + 2.5 = 14.7mm$$

Worked Example for Calculating Drift in Braced Frames by Approximate Method
-Shear Component - Summary

Storey	Shear V_i [kN]	Storey Drift δ_{is} [mm]	$\Delta_{m,shear}$ [mm]
15	20.0	0.3	34.0
14	60.0	0.9	
13	100.0	1.5	
12	140.0	2.0	
11	180.0	2.6	
10	220.0	1.8	26.7
9	260.0	2.1	
8	300.0	2.4	
7	340.0	2.7	
6	380.0	3.0	
5	420.0	2.5	14.7
4	460.0	2.7	
3	500.0	2.9	
2	540.0	3.2	
1	580.0	3.4	

Worked Example for Calculating Drift in Braced Frames by Approximate Method

Total Drift: The total drift at any floor level is the sum of the flexural and shear drifts at that level; for example, the total drift at the top of the 15-storey frame in question is,

$$\Delta_{total}^{(5)} = \Delta_{flexure}^{(5)} + \Delta_{shear}^{(5)} = 17.1 + 14.7 = 31.8mm$$

With a computer analysis program we estimate 28.1mm

Use of Large-Scale Bracing for Frame Stability

Use of megabracing

8 Chifley Sq. Sydney

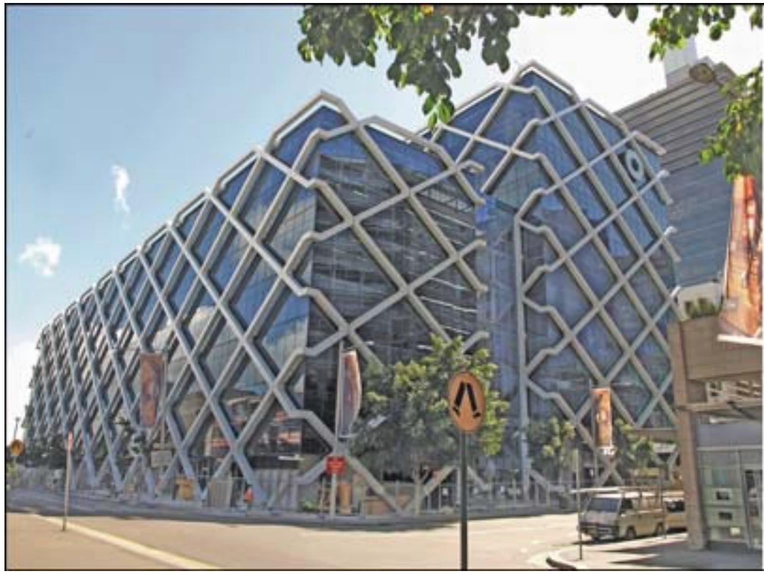
Bank of China, Hong Kong

Neo Bankside, London

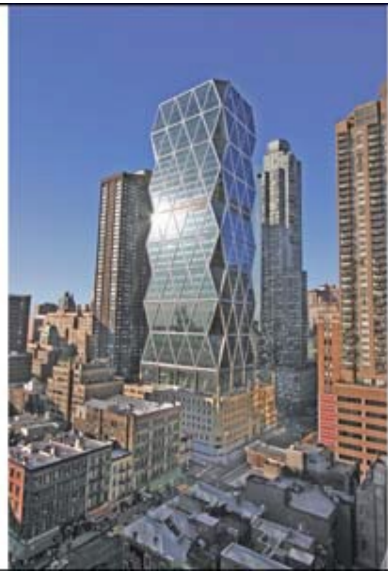
(Image source: The Institution of Structural Engineers 2014)

Use of Large-Scale Bracing for Frame Stability

Use of diagrid structures



1 Shelley St, Sydney



Hearst Tower, New York

Aldar HQ, Abu Dhabi

(Image source: The Institution of Structural Engineers 2014)

Use of Large-Scale Bracing for Frame Stability

Use of space frames

The Water Cube, Beijing

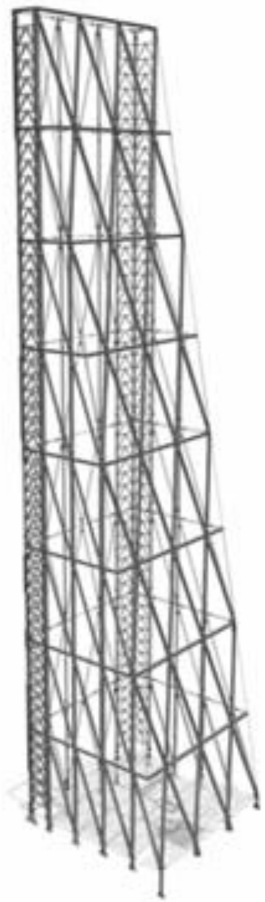
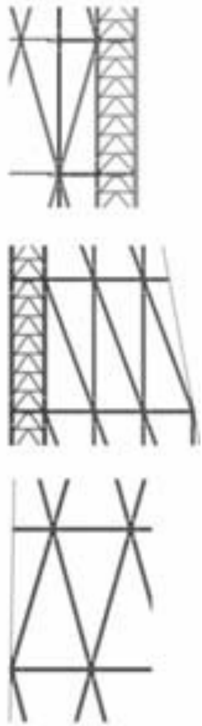
Federation Square, Melbourne

Stansted Airport, London

(Image source: The Institution of Structural Engineers 2014)

Example: The Leadenhall building, London

Lateral and vertical force-carrying frame comprising both diagrid and megabracing systems

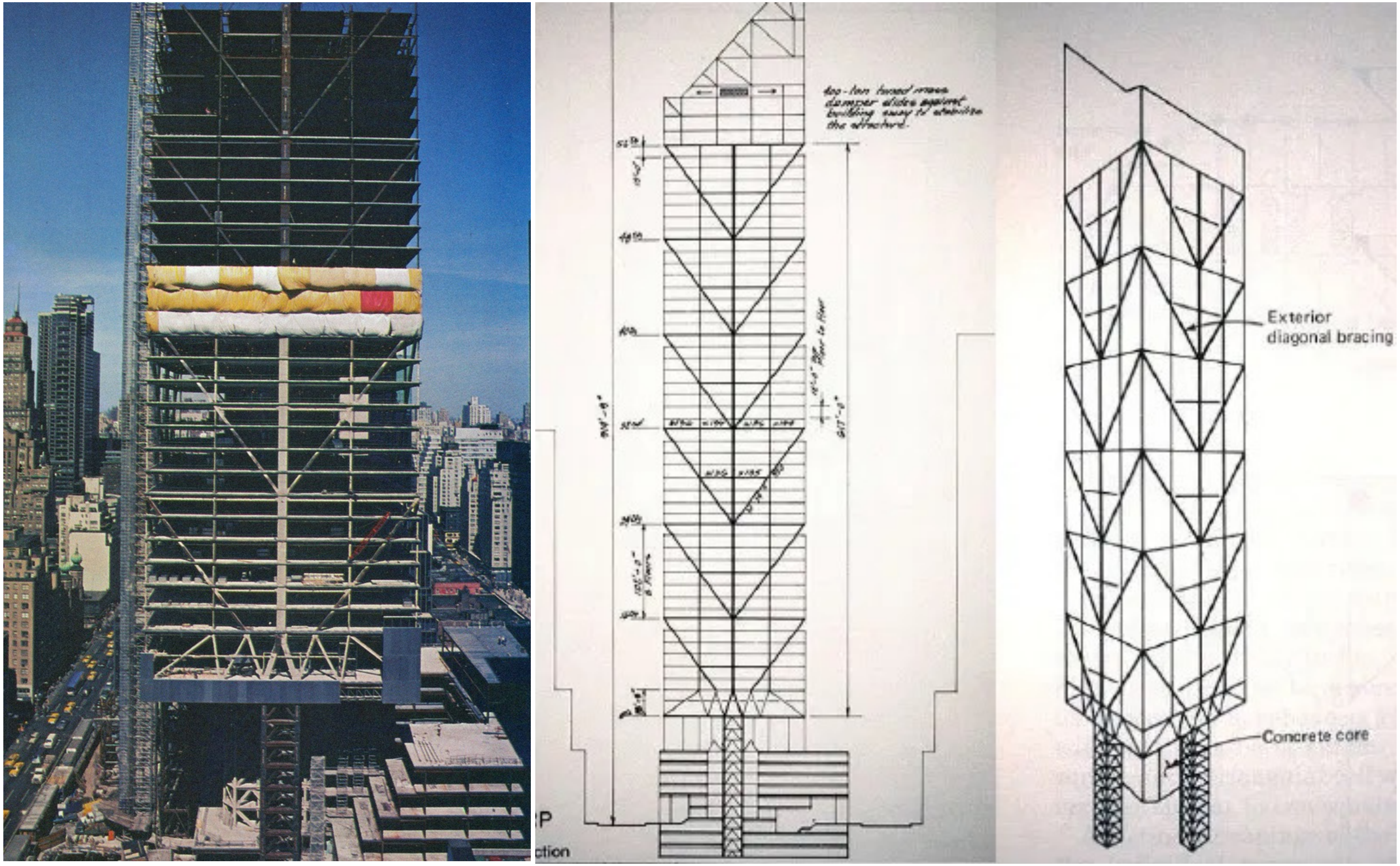
(Image source: The Institution of Structural Engineers 2014)

Use of Large-Scale Bracing – Other Structures

Alcan building, San Francisco

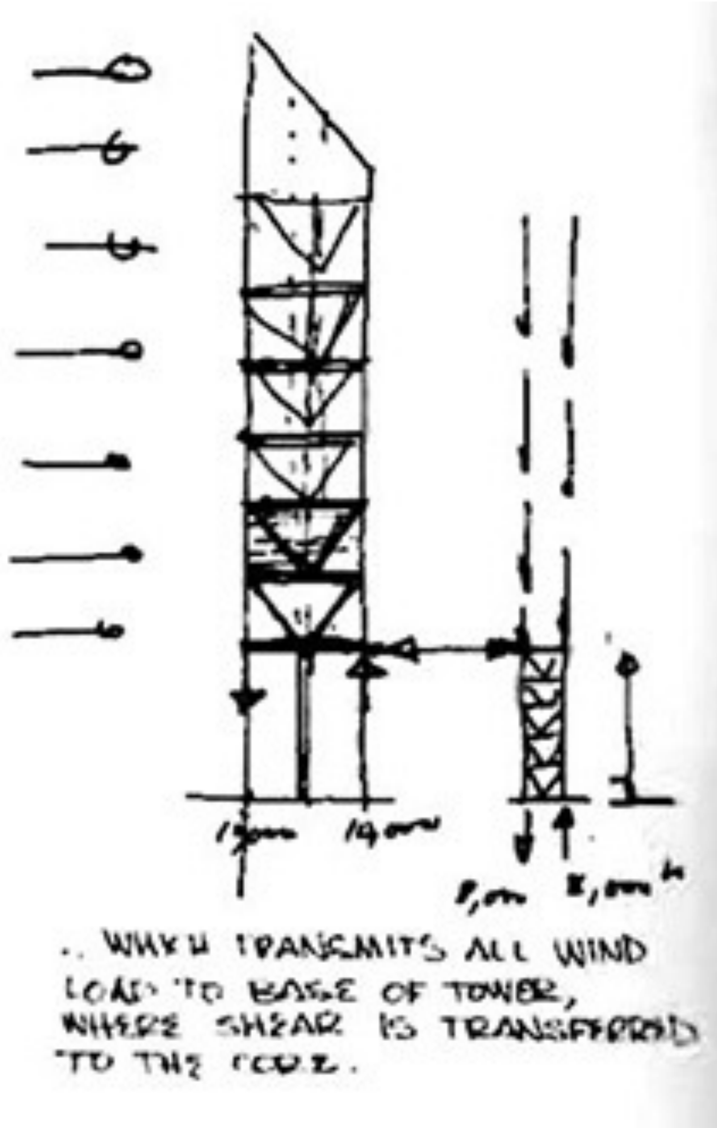
Citicorp building, New York City

Citicorp Building, New York City



(Source: https://misfitsarchitecture.com/2016/01/23/misfits-guide-to-new-york/chevronbracingciticorp/)

Citicorp Building, New York City



(Source: https://www.theaiatrust.com/whitepapers/ethics/study.php)