CIVIL 369: “Structural Stability”

School of Architecture, Civil & Environmental Engineering
Civil Engineering Institute
Resilient Steel Structures Laboratory (RESSLab)

Plate Buckling – Part 2

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GC B3 465 (bâtiment GC)

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Objectives of the lecture

- Motivation to study plate stability
- Introduce the theoretical background to estimate plate:
  - Linear elastic buckling loads
  - Post-buckling behavior
- Look into design applications:
  - Section classification
  - Stiffened plates
  - Class 4 cross-section resistance
  - Concentrated loading
  - Post-critical web resistance
  - Shear links in EBFs
  - …

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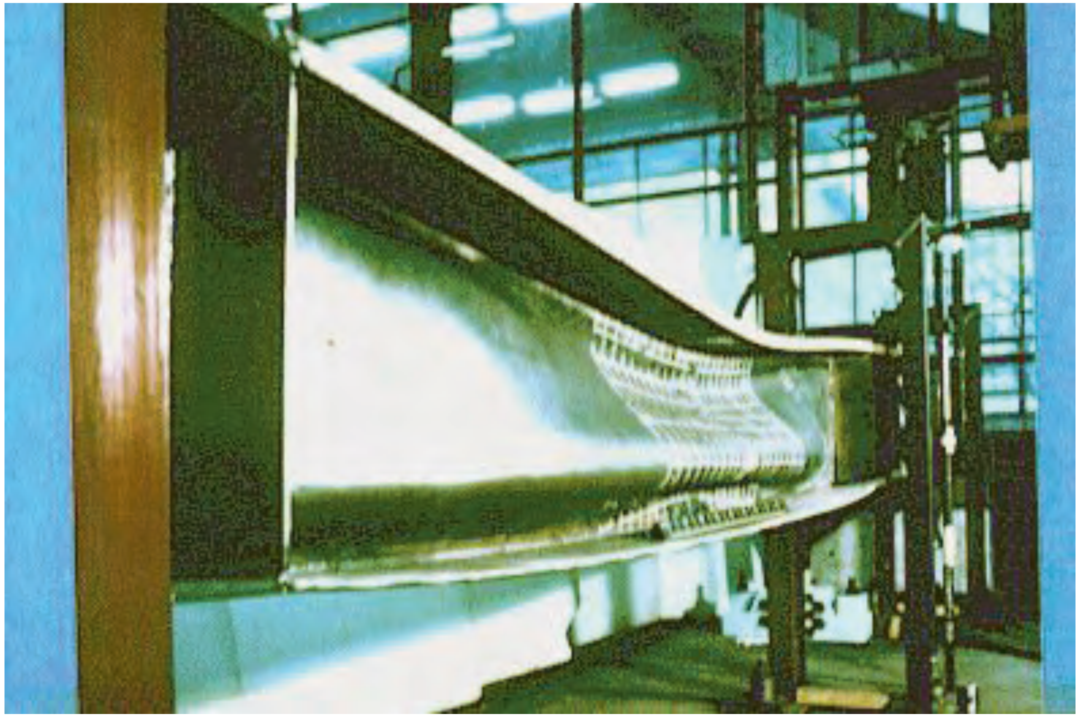
Motivation

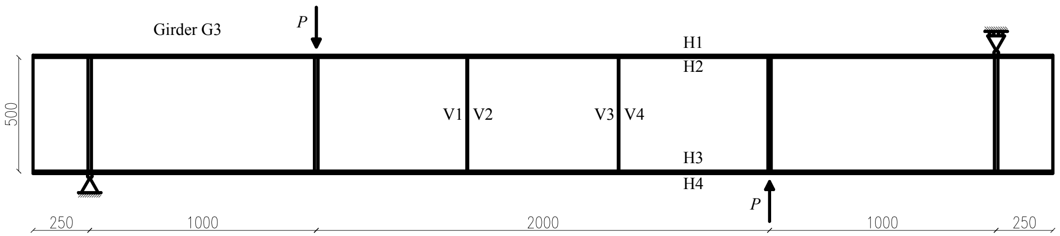
Additional background to part 2

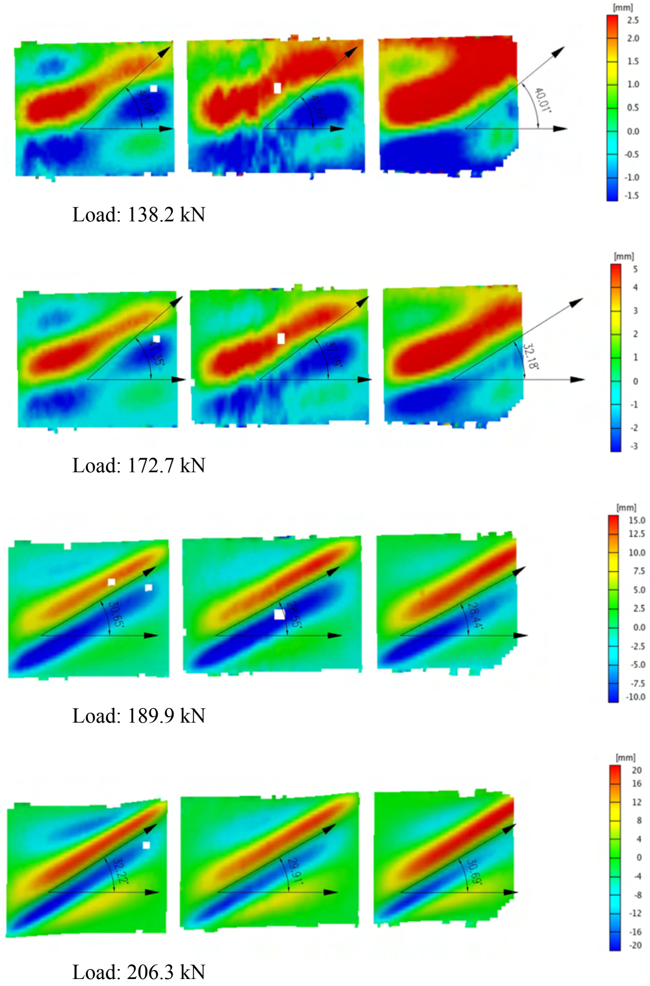


[Hansen, 2006]

Motivation

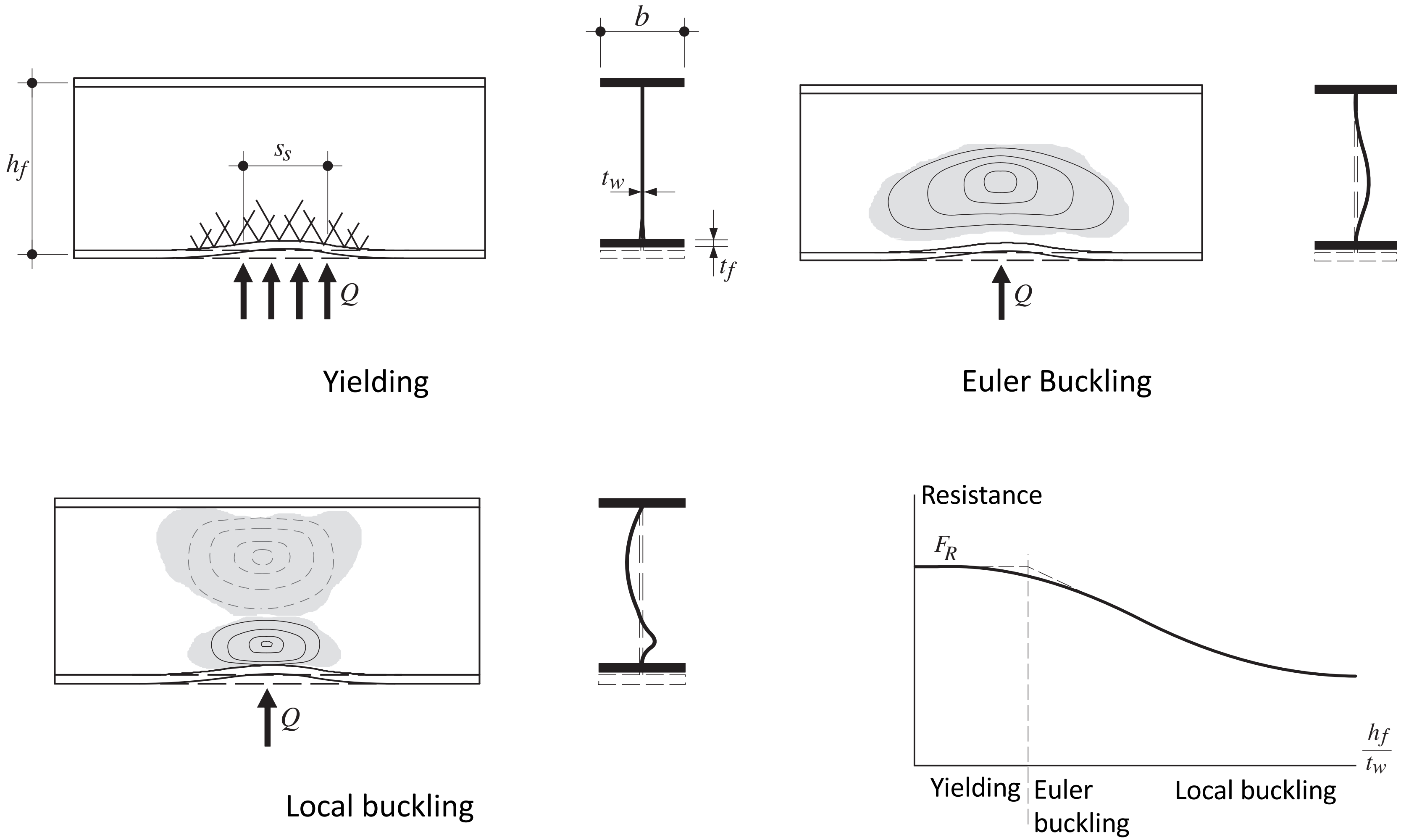
Additional background to part 2





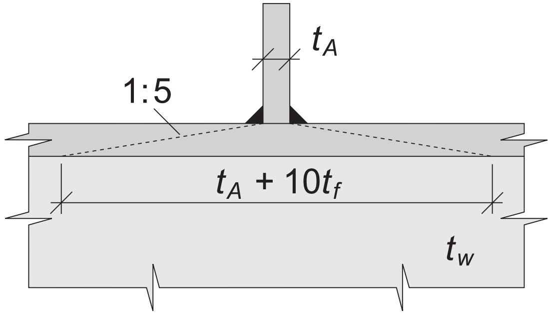
[Hansen, 2006]

Applications
Concentrated loading without stiffeners – Failure modes

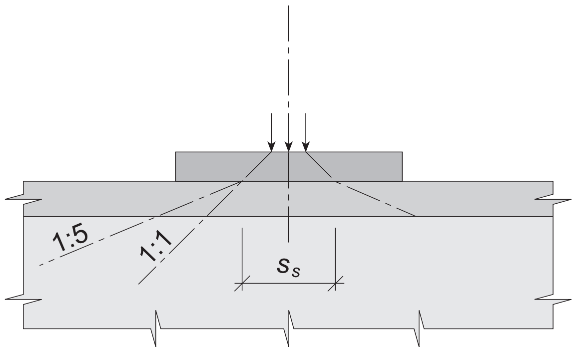


TGC 12 - [Lebet and Hirt, 2009]

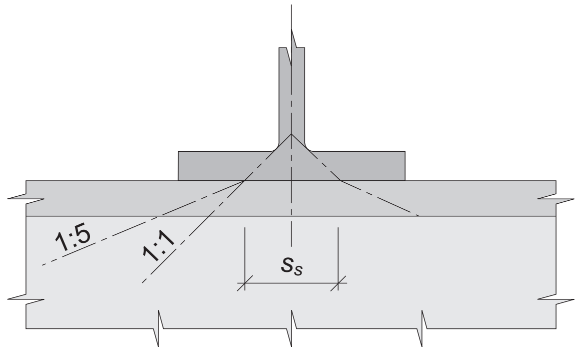
Applications
Concentrated loading without stiffeners – Diffusion widths



Attachment plate



Transfer plate

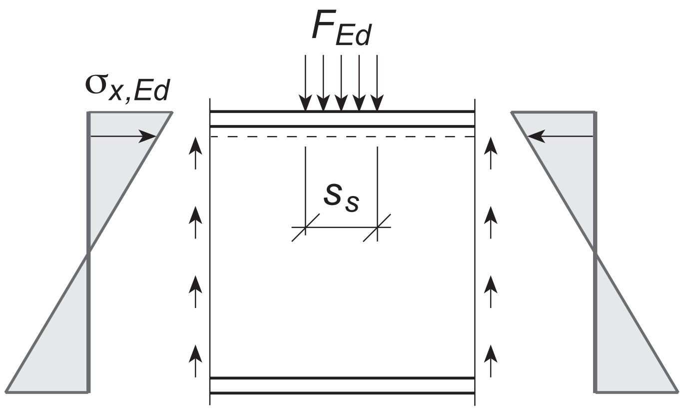


Intersecting element

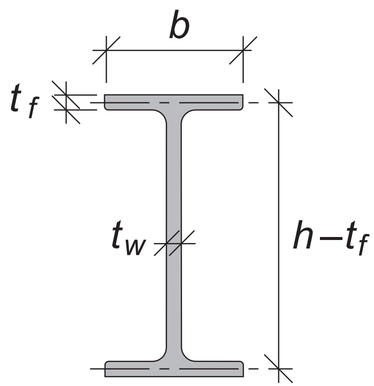
SIA263 - [SIA, 2013]

Applications

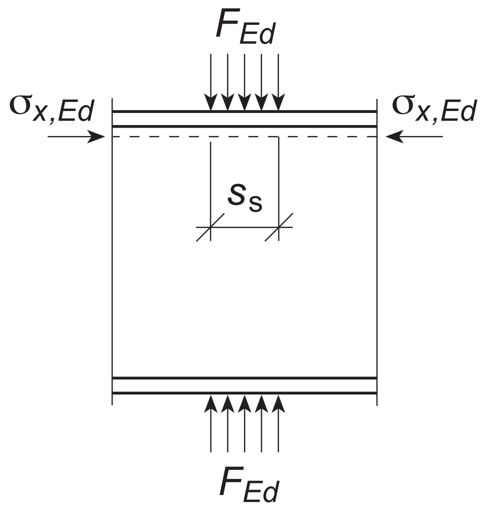
Concentrated loading without stiffeners – Design (SIA263)



One-sided loading



SIA263 - [SIA, 2013]



Two-sided loading

$$F_{Rd} = \frac{1}{\gamma_{M1}} 0.5 t_w^2 f_y \sqrt{\frac{E}{f_y}\frac{t_f}{t_w}} \beta_1 \beta_2 \beta_3 \beta_4 \quad (38)$$

$$F_{Rd} = \frac{1}{\gamma_{M1}} 3 t_w^2 f_y \sqrt{\frac{E}{f_y}\frac{t_f}{t_w}} \beta_1 \beta_3 \beta_4 \quad (39)$$

Applications

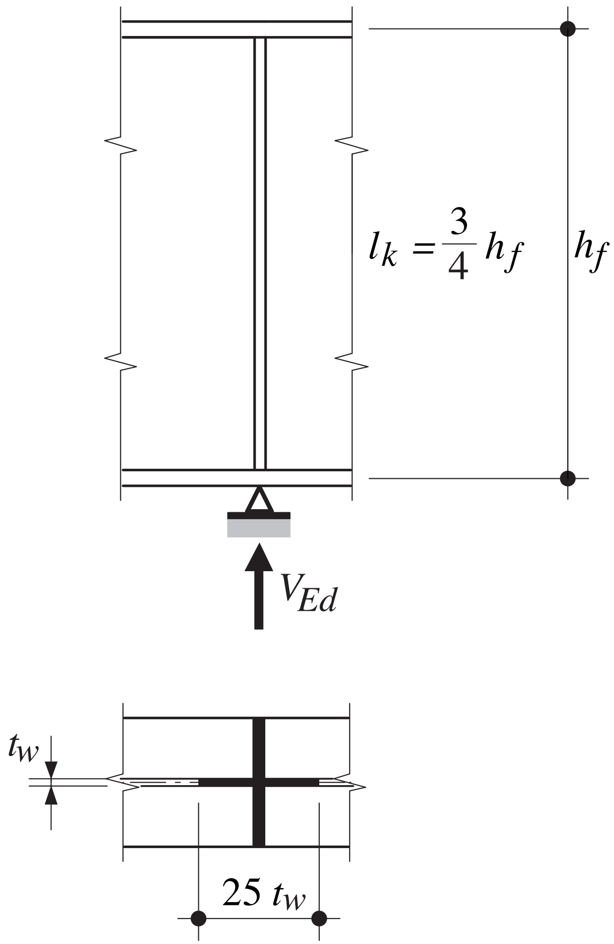
Concentrated loading without stiffeners – Design (SIA263)

With β,

- β_1 - Flange slender coefficient $$\beta_1 = \sqrt[4]{\frac{b/2}{5t_f}} \leq 1.25$$
- β_2 - Web slenderness coefficient $$\beta_2 = \sqrt{\frac{60t_w}{h-t_f}} \geq 1.0$$
- β_3 - Diffusion width of load introduction $$\beta_3 = 1 + \frac{s_s}{h-t_f} \leq 1.5$$
- β_4 - Compressive, in-plane stresses due to global loading $$\beta_4 = 1.5 - \frac{\sigma_{x,Ed}\gamma_{M1}}{f_y} \leq 1.0$$

Applications

Concentrated loading with stiffeners – Design (SIA263)



TGC 12 - [Lebet and Hirt, 2009]

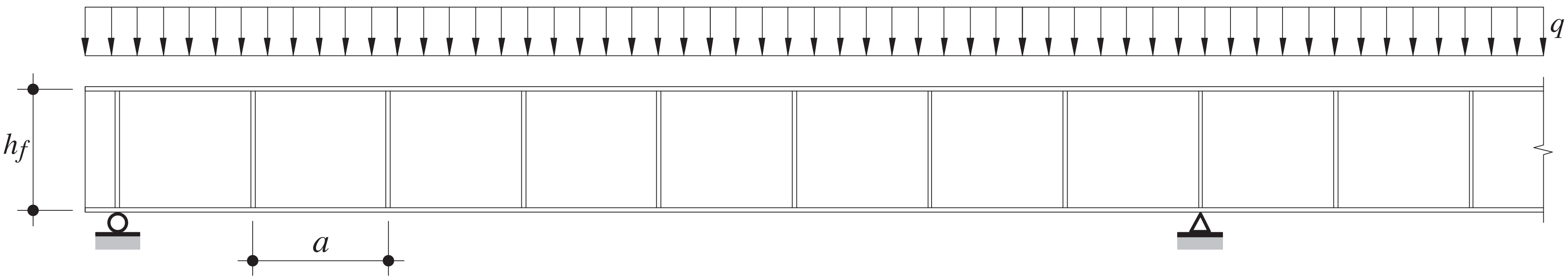
In cases where stiffeners are used for load introduction, SIA263 asks that two things are checked:

1. Effective widths of the stiffeners comply with SIA263 requirements (Eq. 23 – Part 1 of lecture; §4.5.3 – SIA263);

2. Column buckling resistance of an equivalent section is checked with a cruciform cross-section and a buckling length of 75% the distance between flange center-lines

Applications

Shear resistance of web panels



TGC 12 - [Lebet and Hirt, 2009]

EPFL Applications
Shear resistance of web panels

From Part 1 of this lecture series we briefly mentioned how to estimate limit points (linear elastic loss of stability) in cases of shear loading.

Boundary conditions	Normal stresses					Shear stresses (approximate formulas)
	b	σ, $\frac{1}{2}\sigma$	σ, 0	σ, $\frac{1}{2}\sigma$	σ, σ	b, $\alpha = \frac{a}{b}$, a
b	4.00	5.32	7.81	13.40	23.9	$\alpha \geq 1: k = 5.34 + (4.00/\alpha^2)$ $\alpha \leq 1: k = 4.00 + (5.34/\alpha^2)$
	6.97	9.27	13.54	24.5	39.52	$\alpha \geq 1: k = 9.00 + (3.30/\alpha^2)$ $\alpha \leq 1: k = 7.00 + (5.30/\alpha^2)$
	5.41		11.73		39.52	$\alpha \geq 1: k = 7.50 + (4.00/\alpha^2)$ $\alpha \leq 1: k = 6.50 + (5.00/\alpha^2)$
	5.41		9.54		23.94	
	1.28		5.91			
	1.28		1.608		2.134	
	0.426		1.702			
	0.426		0.567		0.851	

TGC10, §12 - [Hirt et al., 2011]

However, much like uniaxial loading, there can be significant post-buckling reserve capacity in plates under shear loading. In general shear capacity can expressed as decomposition of two terms:

$$V_R = V_{cr} + V_\sigma \quad (40)$$

where, V_{cr} is the limit point resistance, and V_σ the post-buckling resistance

EPFL Applications
Shear resistance of web panels

Also like uniaxial plate loading, the shear capacity in Eq. 40 should be assessed considering the **relative weight of the yield stress with respect to the critical shear stress**. For example, if the critical shear stress is 10 times higher that of the yield stress, we can be sure that shear capacity is dominated by inelastic loading of the plate and not its stability.

To this end, we can draw a parallel to the reduced slenderness of Eq. 18, and define resistance as a function of,

$$\bar{\lambda}_w = \sqrt{\frac{\tau_y}{\tau_{cr}}} \quad (41) \qquad \text{with, } \tau_y = \frac{f_y}{\sqrt{3}} \quad \text{and} \quad \tau_{cr} = k(\alpha) \cdot \frac{\pi^2 E}{12(1-\nu^2)\left(\frac{h_f}{t_w}\right)^2}$$

Applications

Shear resistance of web panels

The TGC 12 [Lebet and Hirt, 2009] recommends dividing shear capacity into three cases:

- Case 1 : $\bar{\lambda}_w \leq 0.9$

$$V_R = \tau_y A_w \quad \text{with, } A_w = h_f t_w$$

- Case 2 : $0.9 \leq \bar{\lambda}_w \leq 1.12$

$$V_{cr} = \tau_{cr,red} A_w, \quad \text{with } \tau_{cr,red} = \sqrt{0.8 \tau_y \tau_{cr}}$$

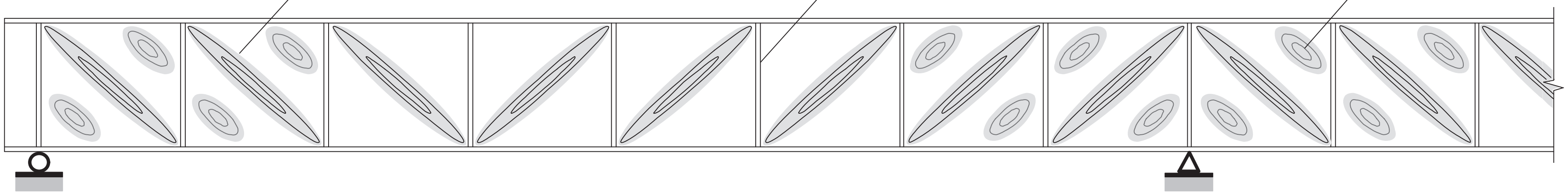
- Case 3 : $\bar{\lambda}_w \geq 1.12$

$$V_{cr} = \tau_{cr} A_w$$

Note: The SIA263 has similar, but slightly different, limits for these cases – cf. Annex F

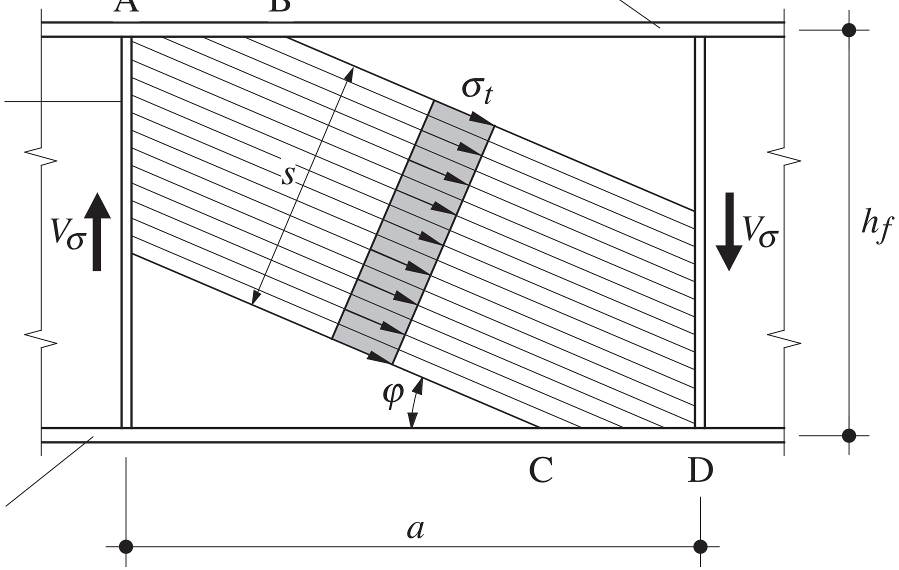
EPFL Applications

Shear resistance of web panels – post-buckling



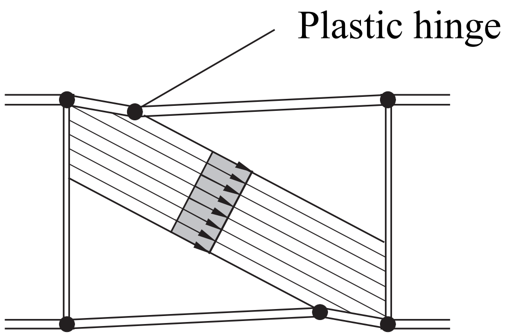
TGC 12 - [Lebet and Hirt, 2009]

[Hansen, 2006]

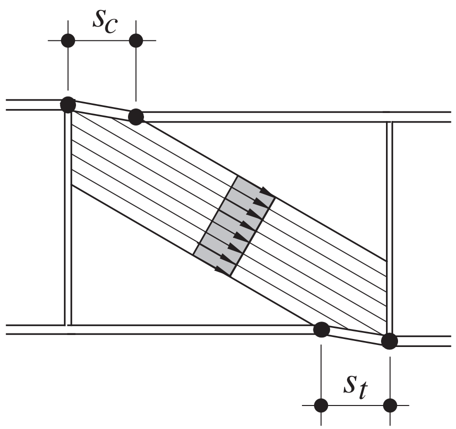


TGC 12 - [Lebet and Hirt, 2009]

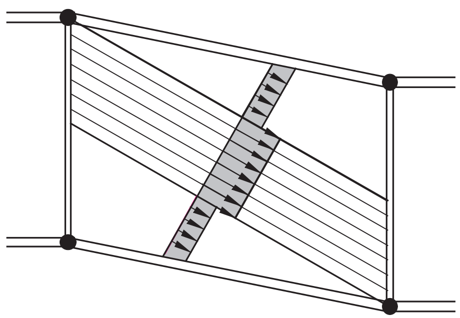
Applications
Shear resistance of web panels – post-buckling



Prague-Cardiff model
[Rockey and Skaloud, 1972]



Cardiff model
[Rockey et al., 1978]



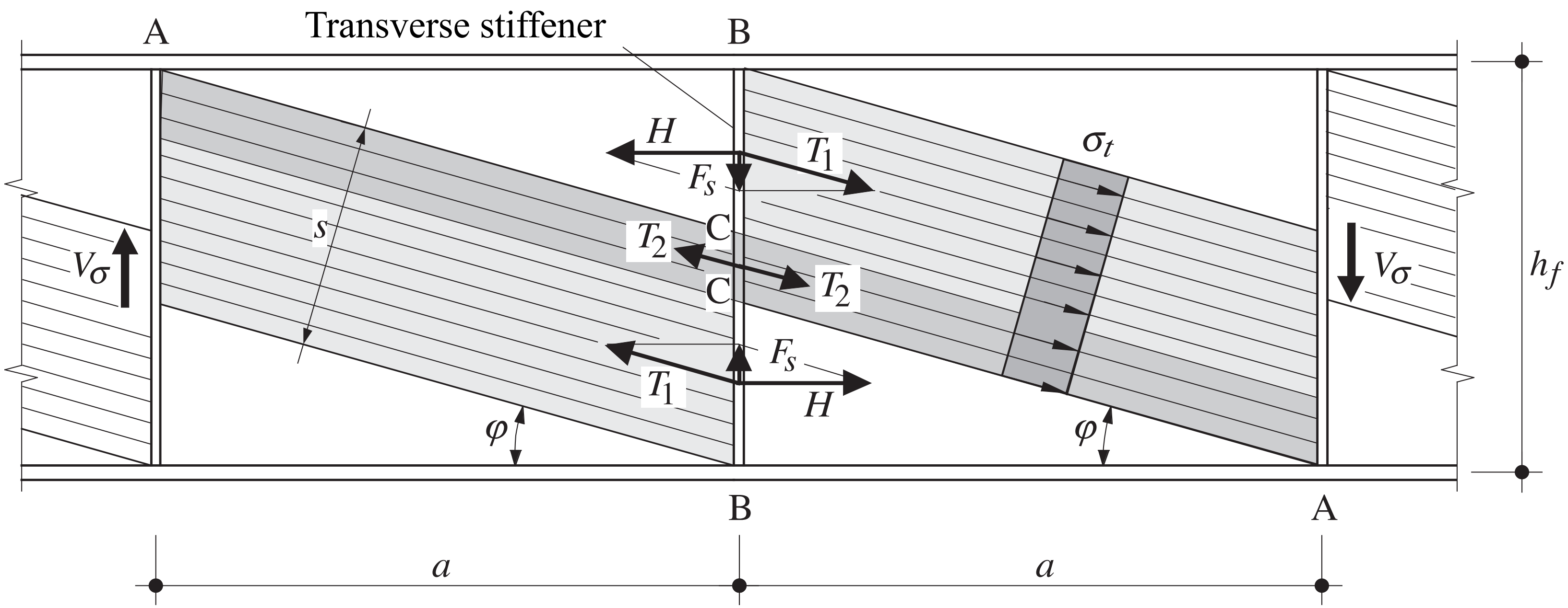
Ostapenko-Chern model
[Ostapenko and Chern, 1969]

TGC 12 - [Lebet and Hirt, 2009]

Applications

Shear resistance of web panels – post-buckling

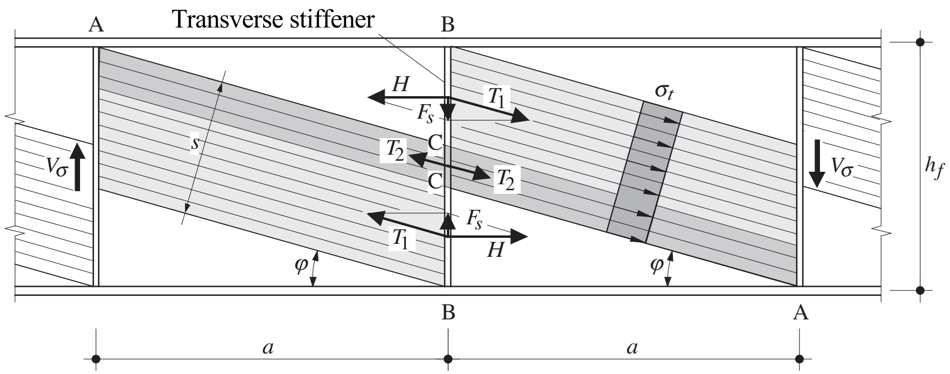
The Basler [1969] model – implemented in SIA263



TGC 12 - [Lebet and Hirt, 2009]

Applications
Shear resistance of web panels – post-buckling

The Basler [1969] model

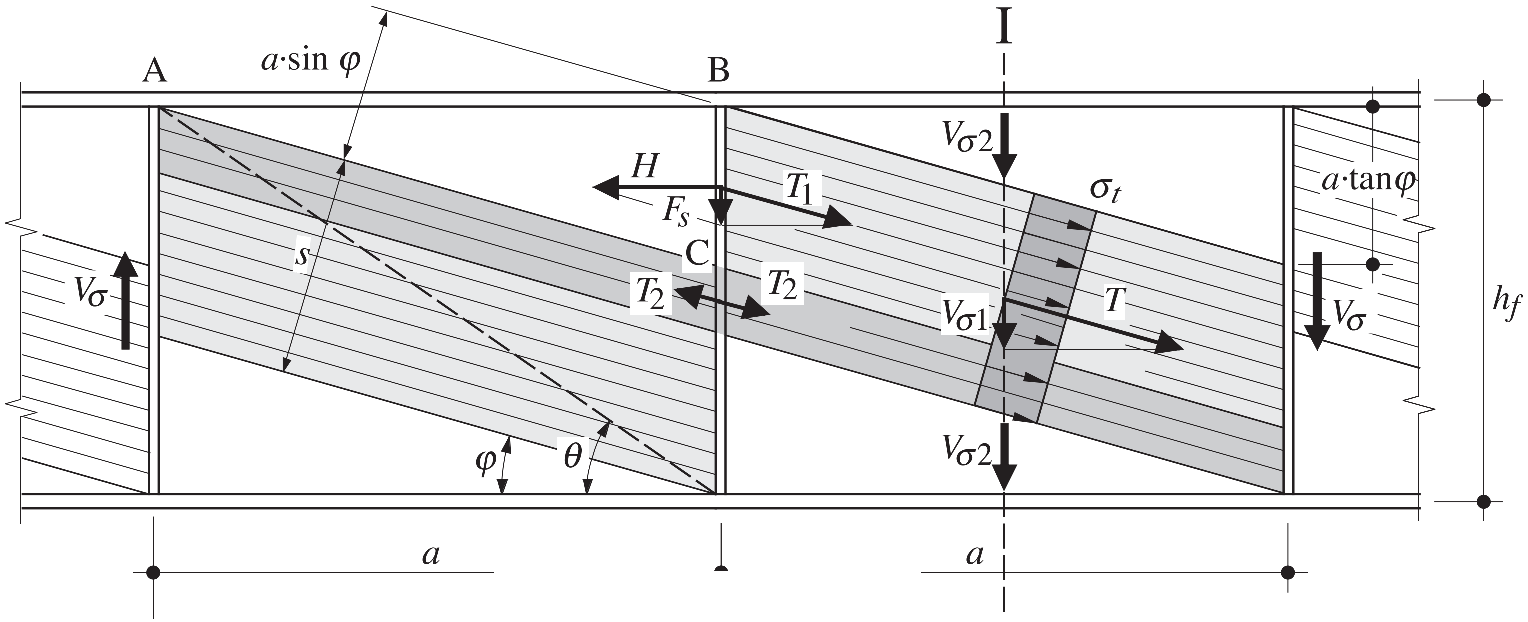


TGC 12 - [Lebet and Hirt, 2009]

Assumptions underlying the model:

1. Post-buckling behavior of beams follows a Prat Truss;
2. Prat truss diagonals (tension) effectively load rigid transverse stiffeners that perform as the struts of the truss. This condition is often coined as "**anchoring**" the diagonals. This depends on the stiffness of adjacent panels – cf. triangle ABC;
3. Flexural contributions of flanges and transverse stiffeners are ignored

Applications
Shear resistance of web panels – post-buckling



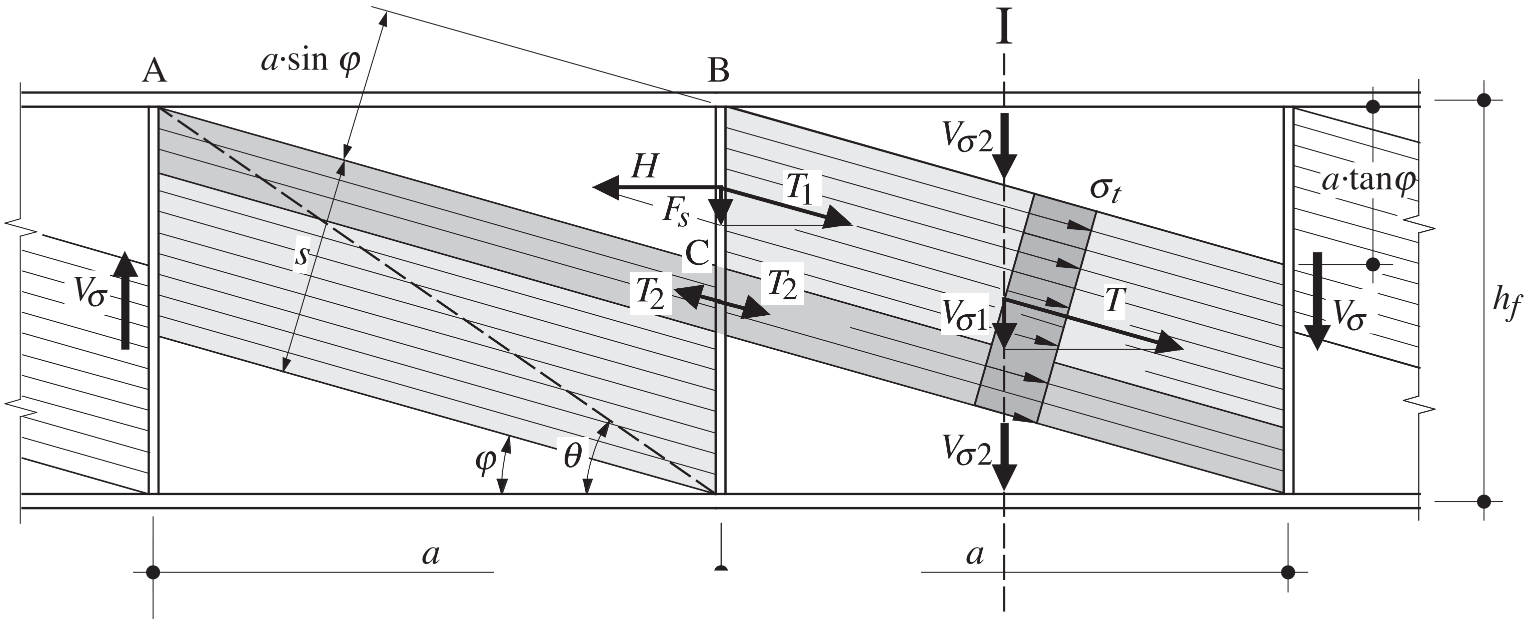
TGC 12 - [Lebet and Hirt, 2009]

$$V_\sigma = V_{\sigma 1} + 2V_{\sigma 2} \quad (42)$$

$$V_{\sigma 1} = T \cdot \sin\varphi = \sigma_t \cdot t_w \cdot s \cdot \sin\varphi =$$

$$= \sigma_t \cdot t_w \cdot \left(h_f \cos\varphi - a \sin\varphi\right) \cdot \sin\varphi \quad (43)$$

Applications
Shear resistance of web panels – post-buckling



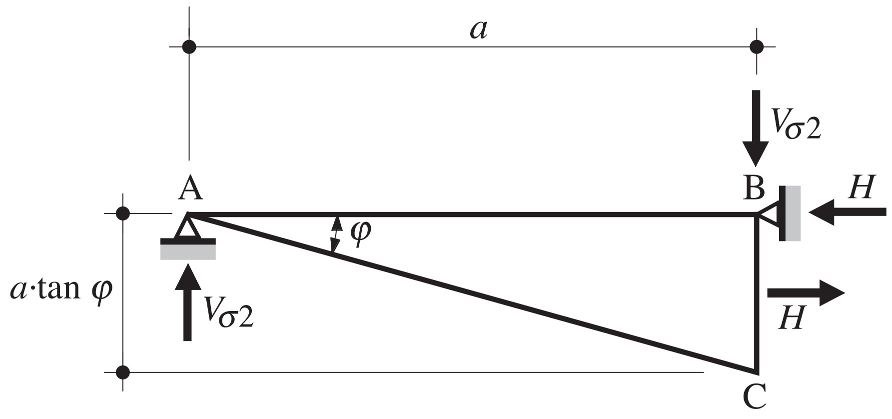
TGC 12 - [Lebet and Hirt, 2009]

Inclination φ computed at the point the $V_{\sigma 1}$ is maximal:

$$\frac{\partial V_{\sigma 1}}{\partial \varphi} = 0 \;\Rightarrow \tan 2\varphi = \frac{h_f}{a} = \frac{1}{\alpha} = \tan\theta \Rightarrow \varphi = \frac{\theta}{2} \quad (44)$$

Applications
Shear resistance of web panels – post-buckling

Triangle ABC in shear panel



TGC 12 - [Lebet and Hirt, 2009]

Moment Equilibrium at point B

$$a \cdot V_{\sigma 2} = \frac{a}{2} \tan\varphi \, H \quad \Leftrightarrow$$

$$\Leftrightarrow V_{\sigma 2} = \frac{H}{2} \tan\varphi$$

Since,

$$H = \sigma_t \cdot t_w \cdot a \cdot \sin\varphi \cos\varphi$$

Then,

$$V_{\sigma 2} = \frac{\sigma_t \cdot t_w \cdot a \cdot (\sin\varphi)^2}{2} \quad (45)$$

Taking Eq. 43 and Eq. 45 and substituting in Eq. 42,

$$V_\sigma = \sigma_t \cdot t_w \cdot \left(h_f \cos\varphi \cdot \sin\varphi - a(\sin\varphi)^2\right) + 2 \cdot \frac{\sigma_t \cdot t_w \cdot a \cdot (\sin\varphi)^2}{2} =$$

$$= \sigma_t \cdot t_w \cdot h_f \left(\cos\varphi \cdot \sin\varphi - \frac{a}{h_f}(\sin\varphi)^2 + -\frac{a}{h_f}(\sin\varphi)^2\right) =$$

$$= \sigma_t \cdot t_w \cdot h_f \cdot \cos\varphi \cdot \sin\varphi = \frac{\sigma_t \cdot t_w \cdot h_f}{2} \sin 2\varphi = \frac{\sigma_t \cdot t_w \cdot h_f}{2} \sin\theta =$$

$$= \frac{\sigma_t \cdot t_w \cdot h_f}{2} \cdot \frac{h_f}{\sqrt{h_f^2 + a^2}} = \frac{\sigma_t \cdot t_w \cdot h_f}{2} \cdot \frac{1}{\sqrt{1 + \alpha^2}} = \frac{\sigma_t \cdot A_w}{2} \cdot \frac{1}{\sqrt{1 + \alpha^2}} \quad (46)$$

Applications

Shear resistance of web panels – post-buckling

The only variable left to define is the stress in the diagonal σ_t. This stress can be computed by considering that,

1. The web panel is already loaded with a pure shear component equal to τ_{cr} along the xy-plan of the web panel;
2. We are searching for an increment in panel loading that will lead to yielding of the panel at stress f_y;
3. Rotating all stress along the diagonal,

$$\sigma_u = \tau_{cr,xy} \sin 2\varphi + \sigma_t \qquad \sigma_v = -\tau_{cr,xy} \sin 2\varphi \qquad \tau_{uv} = \tau_{cr,xy} \cos 2\varphi$$

Then, we can use the von Mises criterion,

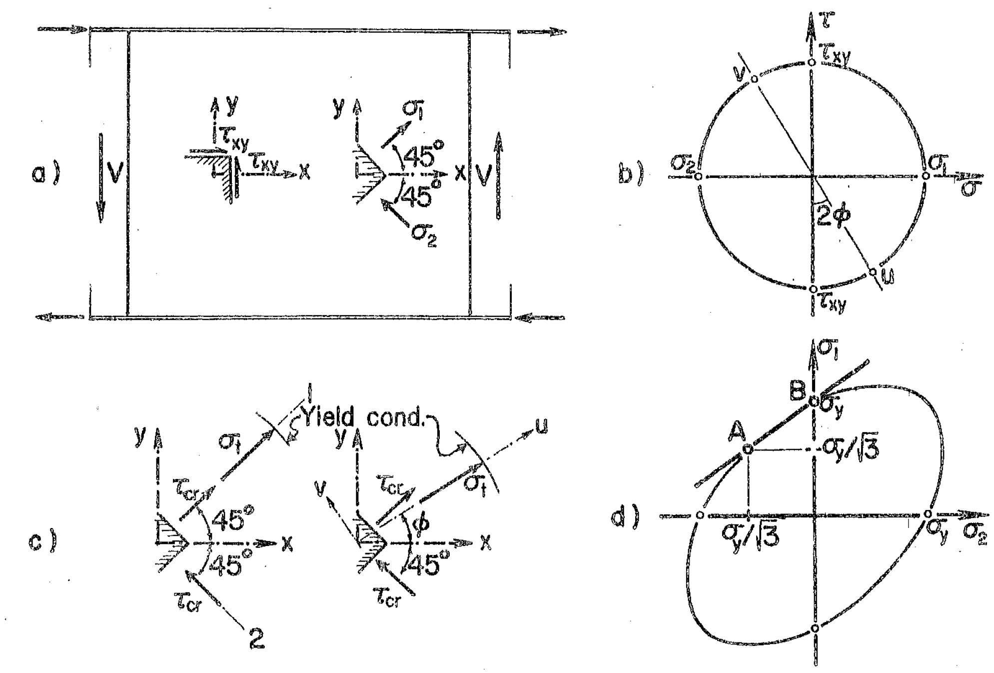
$$(\sigma_u - \sigma_v)^2 + 3\tau_{uv}^2 = f_y^2$$

Applications

Shear resistance of web panels – post-buckling

σ_t can then be expressed as,

$$\sigma_t = \sqrt{f_y^2 - \tau_{cr}^2\left(3 + \left(\frac{3}{2}\sin 2\varphi\right)^2\right)} - \frac{3}{2}\tau_{cr}\sin 2\varphi \quad (47)$$



[Basler, 1969]

To simplify the Eq. 47, Basler [1969] proposes to:

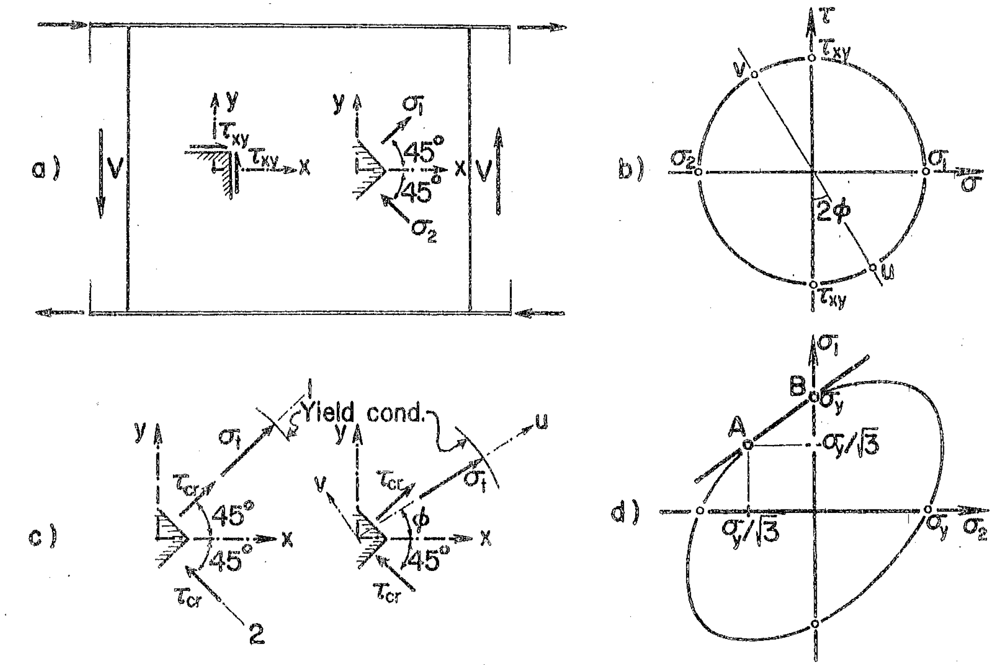
1. Consider only cases where τ_{cr} contributes maximally to the diagonal tie (i.e. $2\varphi = 45°$);
2. Simplify the quadratic nature of Von Mises criterion to a linear relation in principal stress space

Applications

Shear resistance of web panels – post-buckling

σ_t can be expressed as,

$$\sigma_t = \sqrt{f_y^2 - \tau_{cr}^2\left(3 + \left(\frac{3}{2}\sin 2\varphi\right)^2\right)} - \frac{3}{2}\tau_{cr}\sin 2\varphi \quad (47)$$



[Basler, 1969]

Eq. 47 then becomes,

$$\sigma_t = f_y\left(1 - \frac{\tau_{cr}}{\tau_y}\right) =$$

$$= \sqrt{3}(\tau_y - \tau_{cr}) \ (48)$$

Applications

Shear resistance of web panels – post-buckling

And so finally, assuming that prerequisites in slide 18 for the Basler model are fullfilled, the post-buckling shear resistance can be computed as:

$$V_R = V_{cr} + V_\sigma \ (40)$$

$$V_R = \tau_{cr} A_w + \frac{\sqrt{3}}{2} \cdot \frac{(\tau_y - \tau_{cr})}{\sqrt{1+\alpha^2}} \cdot A_w =$$

$$= \left(\tau_{cr} + \frac{\sqrt{3}}{2} \cdot \frac{(\tau_y - \tau_{cr})}{\sqrt{1+\alpha^2}} \right) \cdot A_w =$$

$= \tau_R \ A_w$ (49) and so for design, $V_{Rd} = \dfrac{\tau_R}{\gamma_{M1}} A_w \ (50)$

Applications

Shear resistance of web panels – post-buckling

Having derived the post buckling resistance for cases where the **prerequisites** in slide 18 are fullfilled, it is interesting to discuss **cases where they are not**; more specifically, cases where «**anchoring**» of the tension diagonals is not possible.

Anchoring of the tension diagonals fundamentally depends of the stiffness of adjacent panels to load effectively the transverse stiffeners. This can be called into question when there are:

1. Flexible web panels **at the beam extremities**. In this case the TGC 12 [Lebet and Hirt, 2009] suggest limiting the resistance to;

$$V_R = 0.9\sqrt{\tau_y \tau_{cr}} \cdot A_w \ (51)$$

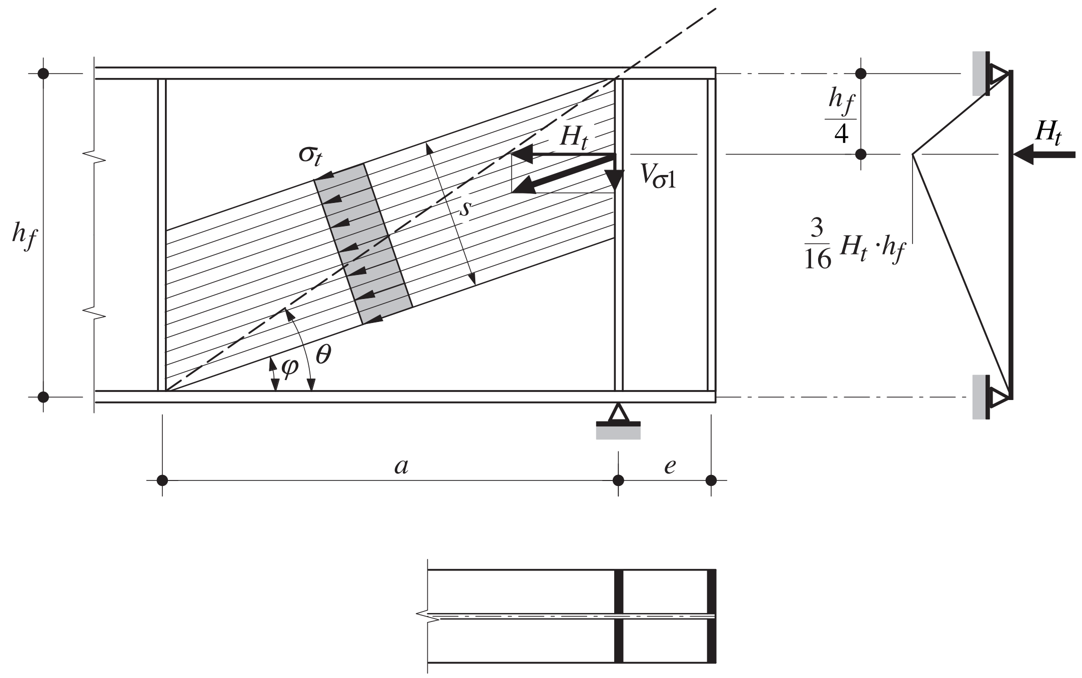
2. Flexible intermediate stiffeners;

3. No transverse stiffeners throughout the length of the beam, except at the extremities (anchored ends). Here, the TGC 12 recommends (based on experimental data of [Höglund, 1993]) that Eq. 49 be used for $\alpha = 3$.

Applications

Shear resistance of web panels – post-buckling

To ensure that we don't have flexibility at the extremities, we have to employ some special detailing:



TGC 12 - [Lebet and Hirt, 2009]

$$H_t = \sigma_t t_w s \cos \varphi$$

From Eq. 48 and given that $2\varphi = \theta$,

$$H_t = \sqrt{3}\left(\tau_y - \tau_{cr}\right) t_w \cdot \frac{h_f}{2}$$

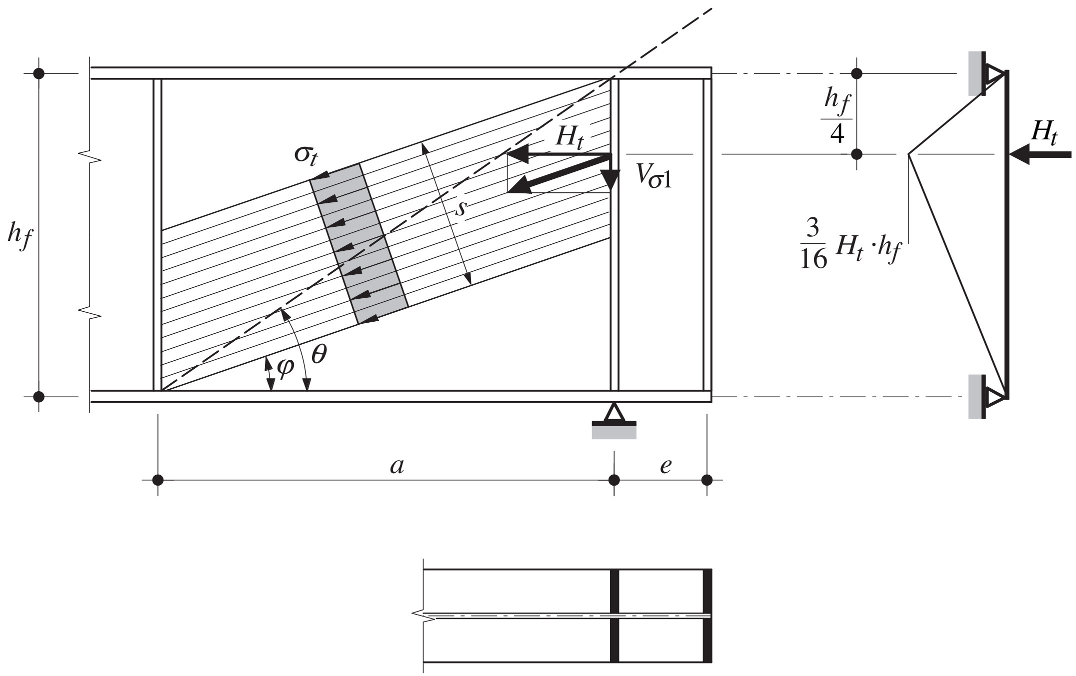
Using Eq. 49, it can also be shown that,

$$H_t = (\tau_R - \tau_{cr})\sqrt{1 + \alpha^2} A_w \ (52)$$

Applications

Shear resistance of web panels – post-buckling

To ensure that we don't have flexibility at the extremities, we have to employ some special detailing:



TGC 12 - [Lebet and Hirt, 2009]

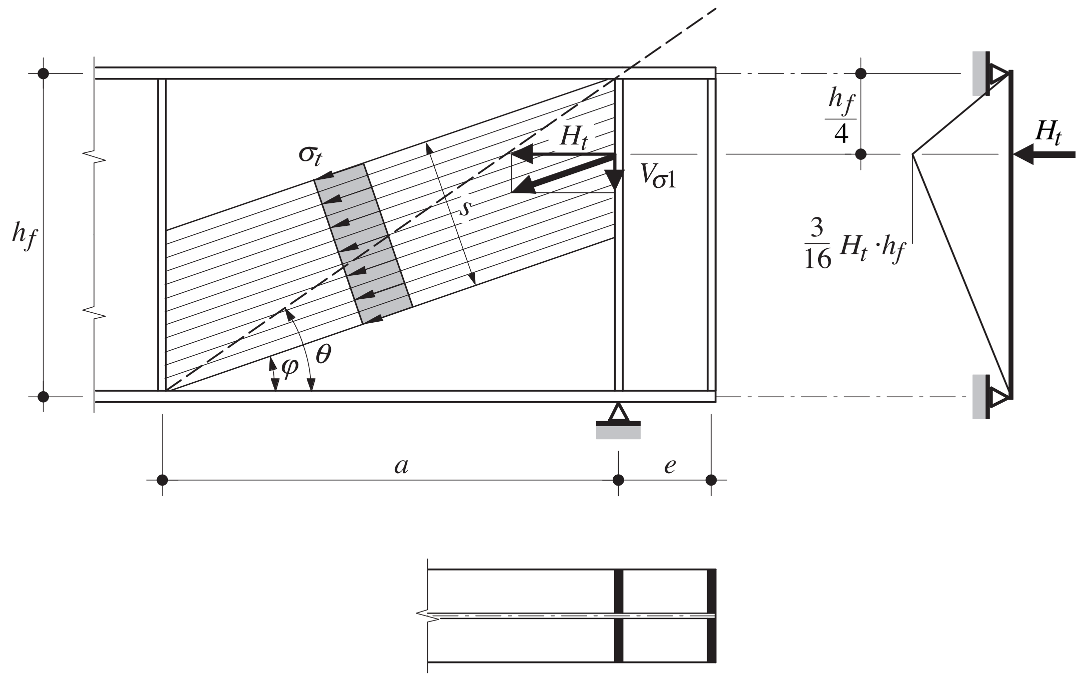
If in the limit we have a flexible end-stiffener, then from Eq. 51 we can take $\tau_{cr} = 0.9\sqrt{\tau_y \tau_{cr}}$, and so Eq. 52 can become,

$$H_t = \sqrt{3}\left(\tau_y - 0.9\sqrt{\tau_y \tau_{cr}}\right) t_w \cdot \frac{h_f}{2} \quad (53)$$

Which is the SIA 263 definition.

Applications
Shear resistance of web panels – post-buckling

To ensure that we don't have flexibility at the extremities, we have to employ some special detailing:



TGC 12 - [Lebet and Hirt, 2009]

And so, to conclude we need to design the end member to resist:

1. In flexure

$$M_{Rd} = \frac{3}{16} h_f \frac{H_t}{\gamma_{M1}} \quad (54)$$

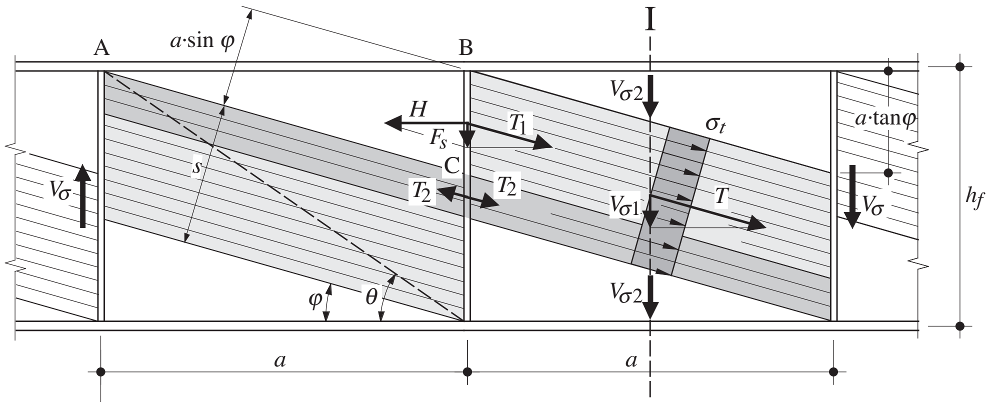
2. And locally for the concentrated load, taking into account what was discussed in slide 10, For the total support reaction V_{Ed}

Applications

Shear resistance of web panels – post-buckling

With respect to the intermediate stiffeners, we must ensure that they:

1. Are sufficiently stiff;
2. Can resist the unbalanced component F_s, since at ultimate load the web is fully yielded



TGC 12 - [Lebet and Hirt, 2009]

To ensure sufficiently stiff intermediate transverse stiffeners, the TGC 12 recommends that the stiffener's inertia I_s (as measured from the middle plane of the web), respect the following condition,

$$I_s \geq \left(\frac{h_f}{50}\right)^4 \eta_1^{3/2} \quad (55)$$

with, $\eta_1 = \frac{f_{yw}}{f_{ys}}$, the ratio between web and stiffener yield stresses

Applications
Shear resistance of web panels – post-buckling

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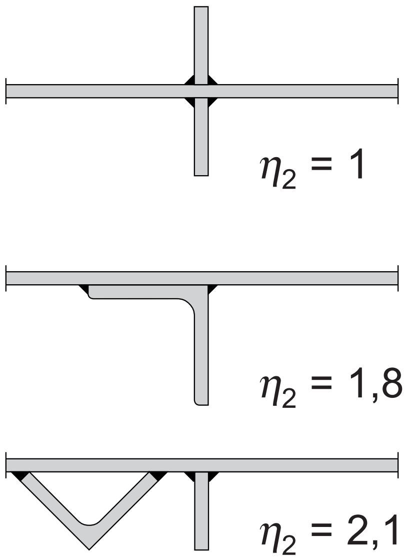


SIA263 - [SIA, 2013]

As for the resistance to the unbalanced load F_s, we must ensure that the stiffener area A_s be,

$$A_s \geq \left(1 - \frac{\tau_{cr}}{\tau_y}\right)\left(\frac{\alpha}{2} - \frac{\alpha^2}{2\sqrt{1+\alpha^2}}\right) A_w \eta_1 \eta_2 \eta_3 \quad (56)$$

with, η_2 - the eccentricity coefficient

$\eta_3 = \frac{V_{Ed}}{V_{Rd}}$ - a demand reduction factor, if the demand does not reach the capacity

Applications
Interaction of Bending Moment and Shear

To take into account the interaction between bending moment and shear we can **decompose** the bending moment resistance into a part carried **by the flanges and** another by **web**. For example, considering the effective properties of a bi-symmetric I-girder, this leads to,

$$M_{Ed} \leq M_{Rd,V} = M_{Rd}^{f} + M_{Rd,V}^{w} = b_{eff} t_f h_f \frac{f_y}{\gamma_{M1}} + W_{eff,web} \left[1 - \left(\frac{V_{Ed}}{V_{Rd}}\right)^2\right] \frac{f_y}{\gamma_{M1}} \quad (57)$$

EBF Link: Behaviour Without Stiffeners

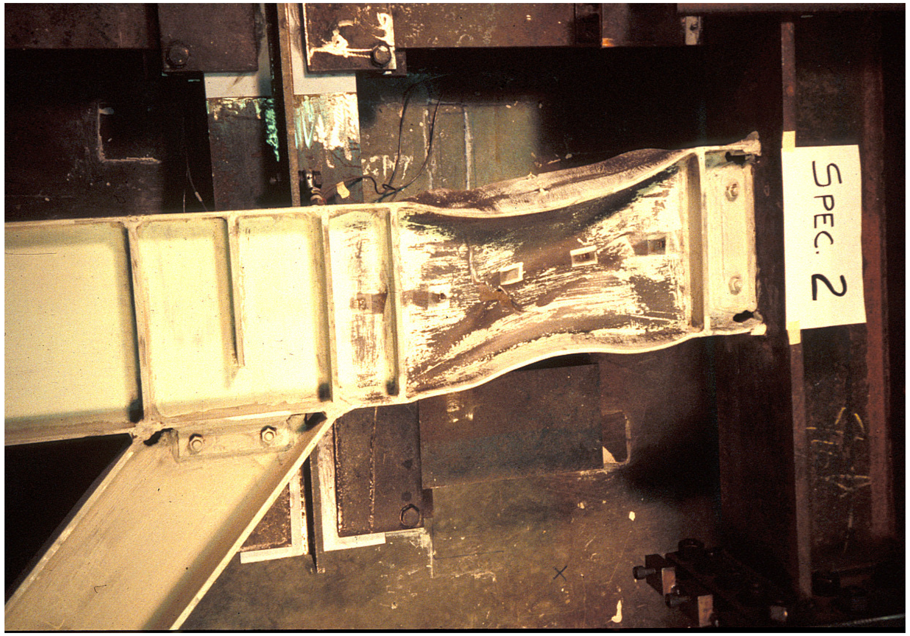
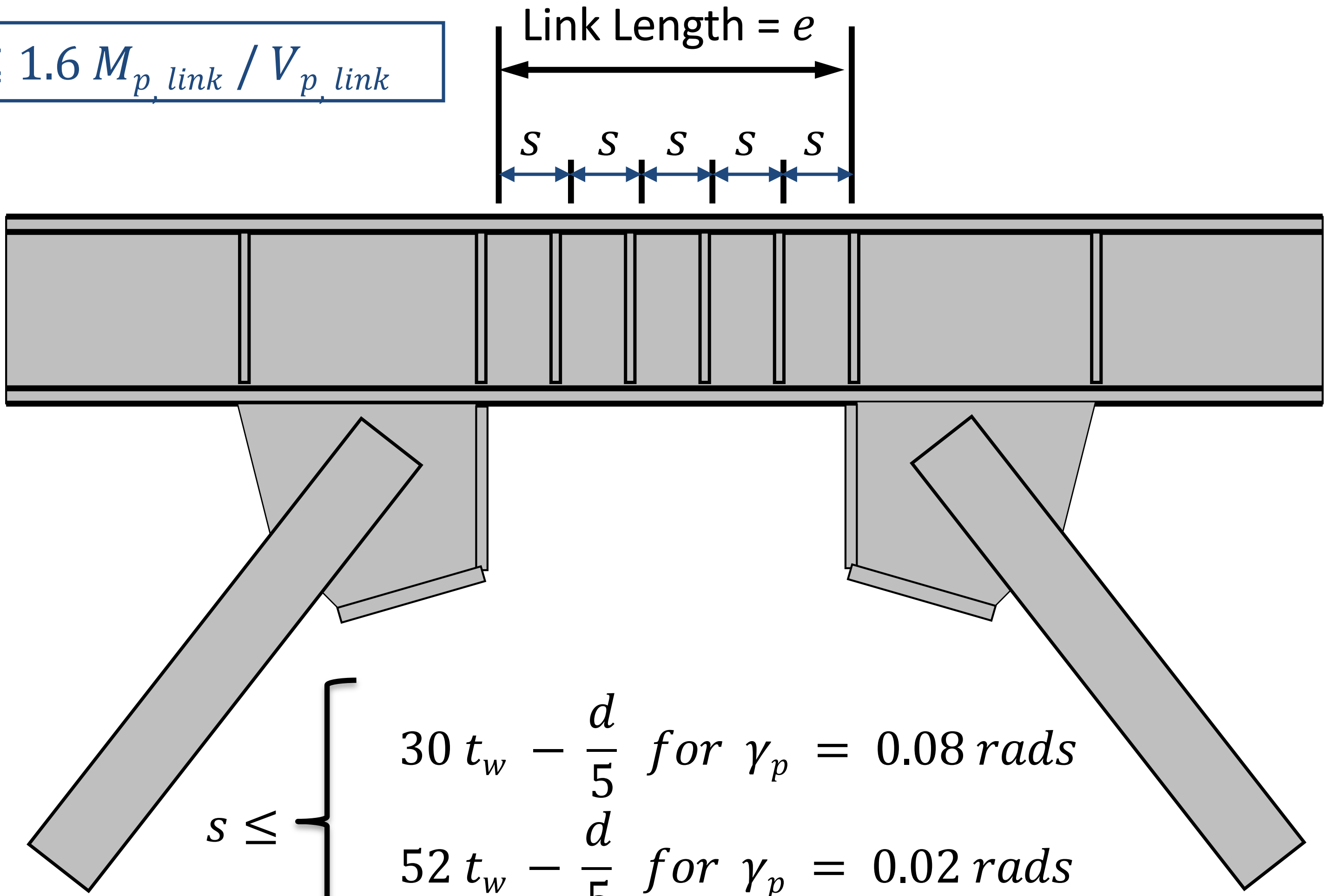


Image courtesy of Prof. Englelhardt

Short Link Stiffeners (Shear Yielding Links)

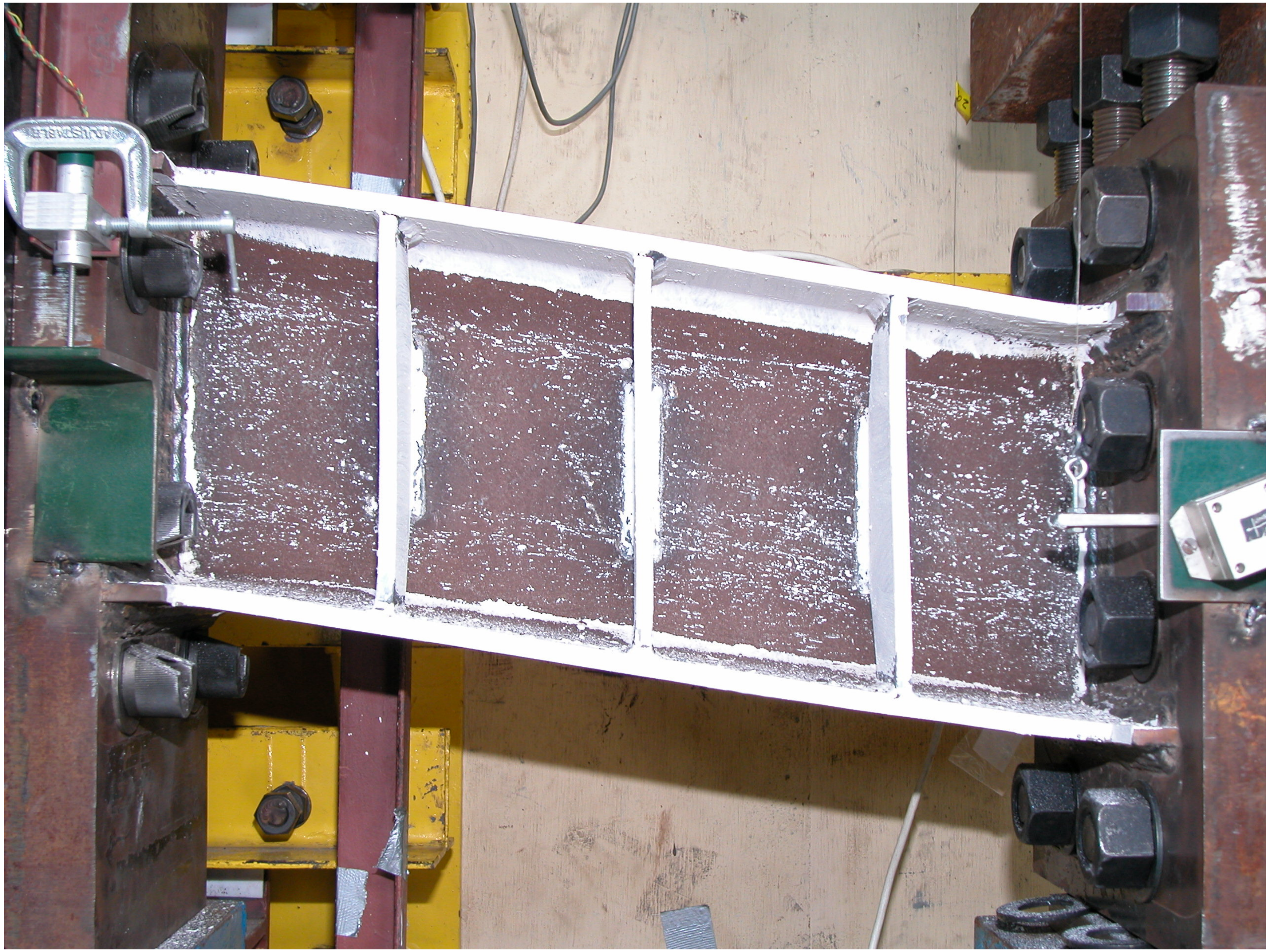
$e \leq 1.6\, M_{p,link} / V_{p,link}$



$$s \leq \begin{cases} 30\, t_w - \dfrac{d}{5} & for\ \gamma_p = 0.08\, rads \\ 52\, t_w - \dfrac{d}{5} & for\ \gamma_p = 0.02\, rads \\ \text{interpolate} & \text{for } 0.02 < \gamma_p < 0.08 rads \end{cases}$$

t_w = link web thickness d = link depth

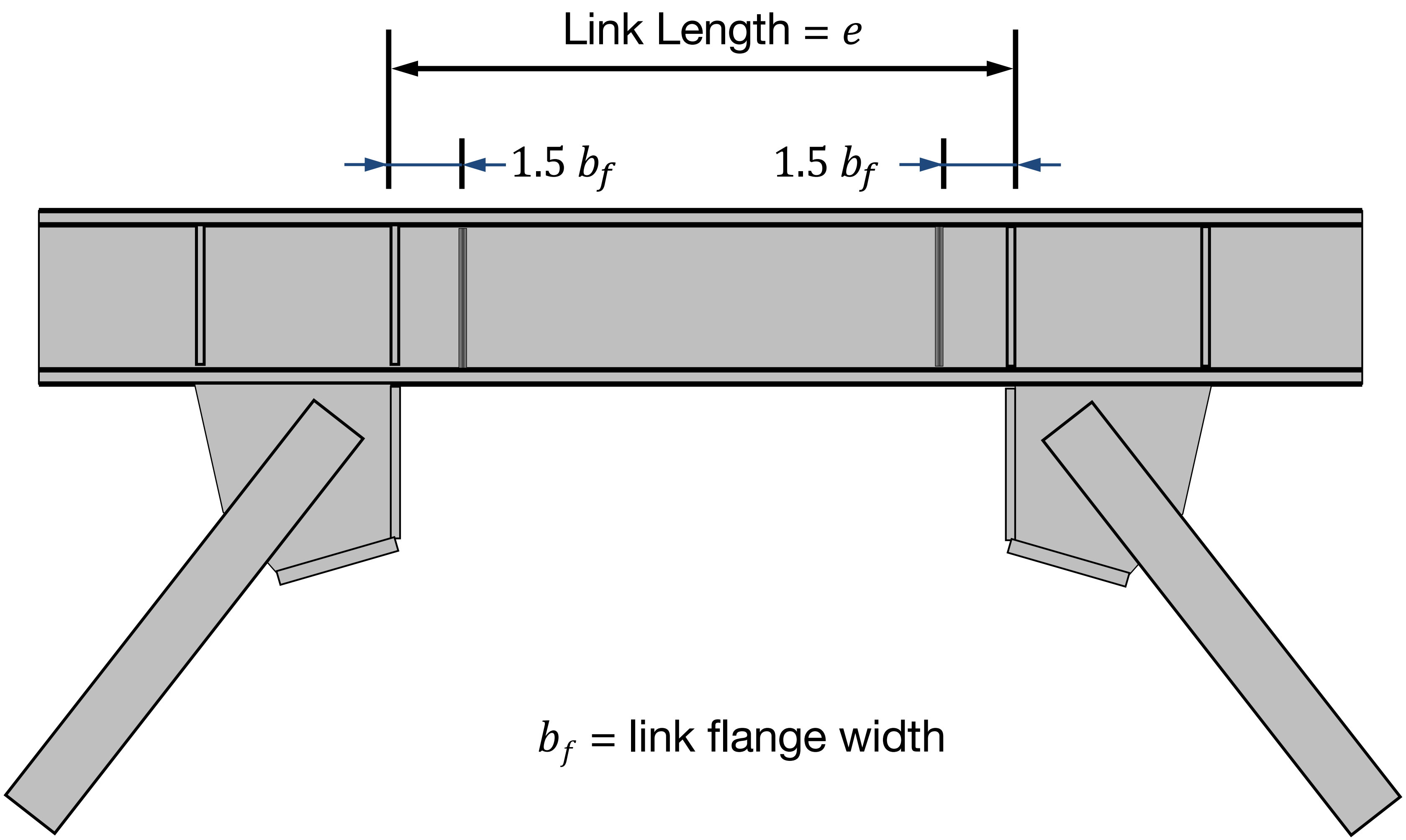
Short Link Stiffeners: Illustration



Figures courtesy of Prof. Tremblay & Christopoulos

Long Link Stiffeners (Flexural Yielding Links)

$$3\,M_{p,\,link}\,/\,V_{p,\,link} \;<\; e \;<\; 5\,M_{p,\,link}\,/\,V_{p,\,link}$$

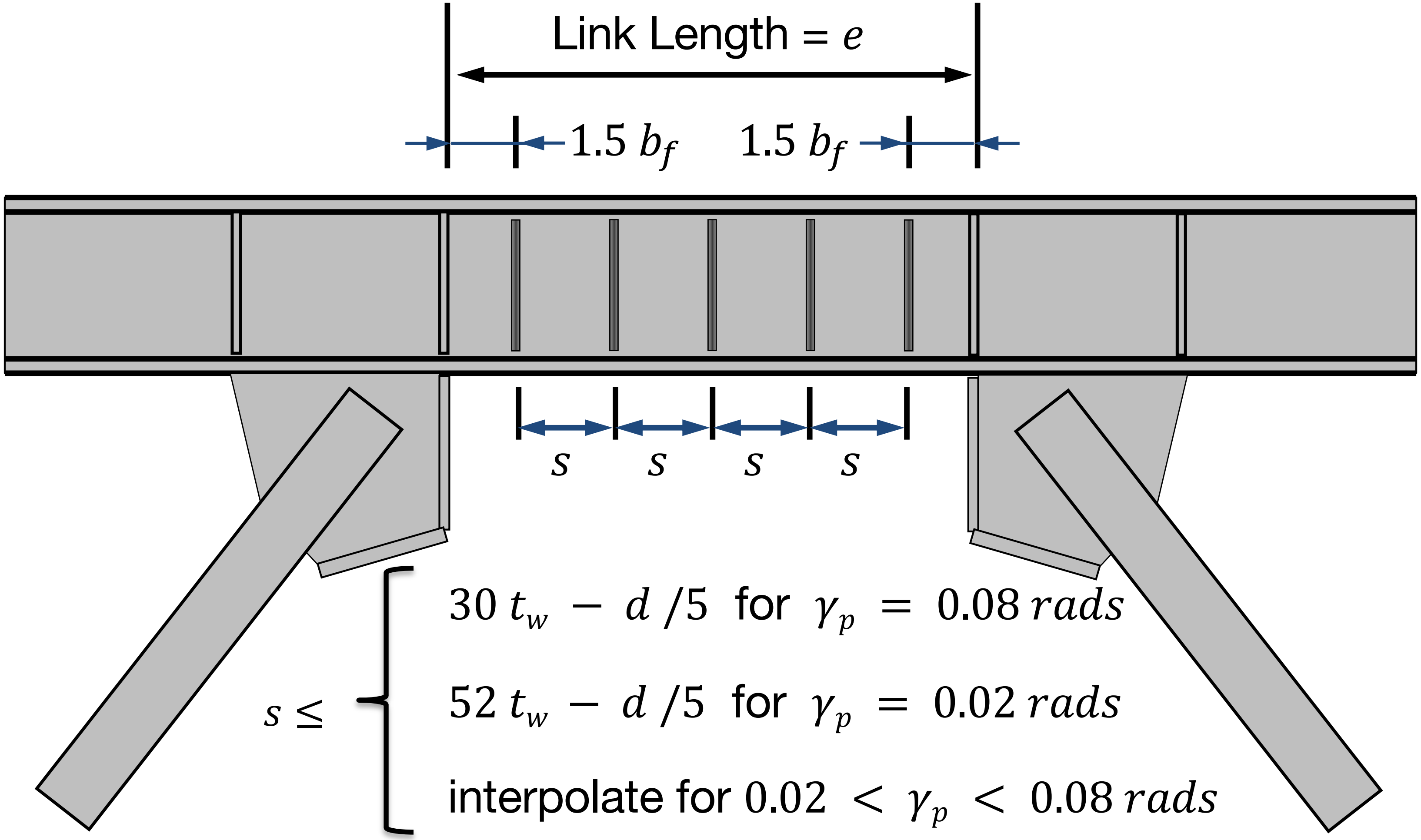


Long Link Stiffeners: Illustration

Figures courtesy of Prof. Tremblay & Christopoulos

Intermediate Link Stiffeners (Shear & Flexural Yielding Links)

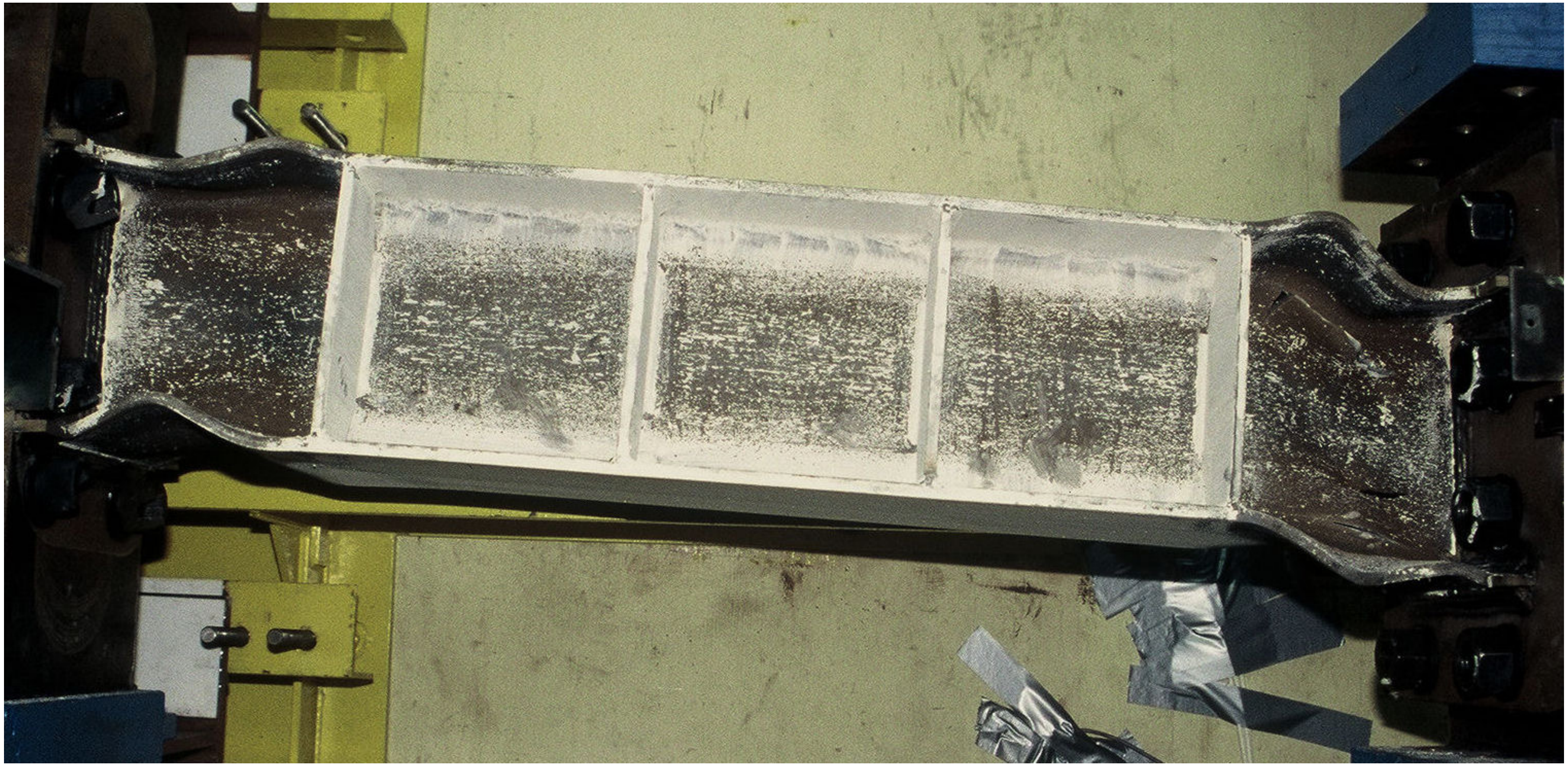
$$1.6\, M_{p,\,link} \,/\, V_{p,\,link} \;<\; e \;<\; 3\, M_{p,\,link} \,/\, V_{p,\,link}$$



$$s \leq \begin{cases} 30\, t_w - d/5 \text{ for } \gamma_p = 0.08\, rads \\ 52\, t_w - d/5 \text{ for } \gamma_p = 0.02\, rads \\ \text{interpolate for } 0.02 < \gamma_p < 0.08\, rads \end{cases}$$

EPFL

Intermediate Link Stiffeners: Illustration



Figures courtesy of Prof. Tremblay & Christopoulos

References

- [Scheer, 2010] – Scheer, Joachim (2010), “Failed bridges”, Ernst & Sohn, Berlin, Germany
- [Bryan, 1890] – Bryan, George H. (1890), “On the stability of a Plane Plate under Thrusts in its own Plane with Applications to the ‘Buckling’ of the Sides of a Ship”, London Mathematical Society, Volume s1-22, Issue 1, p. 54-67
- [Hirt et al., 2011] – Hirt, Manfred, Bez, Rolf and Nussbaumer, Alain (2011), “Traité Génie Civil, Volume 10 – Construction Métallique”, PPUR, Lausanne, Switzerland
- [SIA, 2013] – Societé suisse d’ingénieurs et architectes (2013), “SIA263:2013 – Construction en acier”, SIA, Zürich, Switzerland
- [Koiter, 1945] – Koiter, Warner T., “Over de stabiliteit van het Elastiche Evenwicht”, PhD Thesis, Delft Netherlands (translated to English by Edward Riks - https://apps.dtic.mil/dtic/tr/fulltext/u2/704124.pdf)
- [von Karman et al., 1932] – von Karman, T., Sechler, E. E., and Donnell, L.H (1932) “ The strength of thin plates in compression”, Transaction of the American Society of Mechanical Engineers (ASME), Vol 54, p. 53
- [Reis and Camotim, 2001] – Reis, António; Camotim, Dinar (2001) “Estabilidade Estrutural”, McGrawHill, Amadora, Portugal.
- [Winter, 1947] – Winter, G. (1947)., “Strength of Thin Steel Compression Flanges”, Transactions of the American Society of Mechanical Engineers(ASME), Vol. 112, p. 527
- [Hansen, 2006]– Hansen, Thomas(2006), Theory of Plasticity for steel structures – solutions for fillet welds. Plate girders, and thin plates, PhD Thesis, R-146, BYG-DTU, Copenhagen, Denmark (https://www.byg.dtu.dk/-/media/Institutter/Byg/publikationer/PhD/byg_r146.ashx?la=da&hash=33E70EFD516A84EC0A2F7F5C22D44FE09C1C0CCB)
- [Klöppel and Scheer] – Klöppel, K. and Scheer, J. (1960) “Beulwerte ausgesteifter Rechteckplatten”, Verlag von Wilhem Enst & Sohn, Berlin, Germany.
- [Lebet and Hirt, 2009] – Lebet, J.P. and Hirt, M. A. (2009) “Traité Génie Civil, Volume 12 – Ponts en Acier”, PPUR, Lausanne, Switzerland

References

- [Rockey and Skaloud, 1972] – Rockey K.C. and Skaloud, M. (1972), “The ultimate load behavior of plate girders loaded in shear”, The Structural Engineer, Vol. 50, n°1, p. 29-47. https://www.istructe.org/journal/volumes/volume-50-(published-in-1972)/issue-1/the-ultimate-load-behaviour-of-plate-girders-loade/;
- [Rockey et al., 1978] – Rockey K.C., Evans, H.R. and Porter, D.M. (1978), “A design method for predicting the collapse behavior of plate girders”, Proceedings of the Institution of Civil Engineers, Vol. 65, Issue 1 part-2, p. 85-112. https://www.icevirtuallibrary.com/doi/abs/10.1680/iicep.1978.2930?journalCode=jpric
- [Ostapenko and Chern, 1969] – Ostapenko, A. and Chern. C. (1969), “The ultimate strength of plate girders under shear”, Fritz Laboratory Reports, Paper 264, Bethlehem, USA. https://preserve.lehigh.edu/cgi/viewcontent.cgi?article=1263&context=engr-civil-environmental-fritz-lab-reports
- [Basler, 1969] – Basler, K. (1969) “Strength of plate girders in shear”, Proceedings American Society of Civil Engineers, 87, (St67), Reprint 186(61-13), Fritz Laboratory reports. Paper 70. https://preserve.lehigh.edu/cgi/viewcontent.cgi?article=1069&context=engr-civil-environmental-fritz-lab-reports
- [Höglund, 1993] – Höglund, T. (1993) “Design of thin plate I-girders in shear and bending with special reference to web buckling”, Division of Buildings Statics and Structural Engineering, Royal Institute of Technology, Bulletin n° 94, Stockholm, Sweden.