CIVIL 369: “Structural Stability”

School of Architecture, Civil & Environmental Engineering
Civil Engineering Institute
Resilient Steel Structures Laboratory (RESSLab)

Plate Buckling – Part 1

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Objectives of the lecture

- Motivation to study plate stability
- Introduce the theoretical background to estimate plate:
  - Linear elastic buckling loads
  - Post-buckling behavior
- Look into design applications:
  - Section classification
  - Stiffened plates
  - Class 4 cross-section resistance
  - Concentrated loading
  - Post-critical web resistance
  - Shear links in EBFs
  - …

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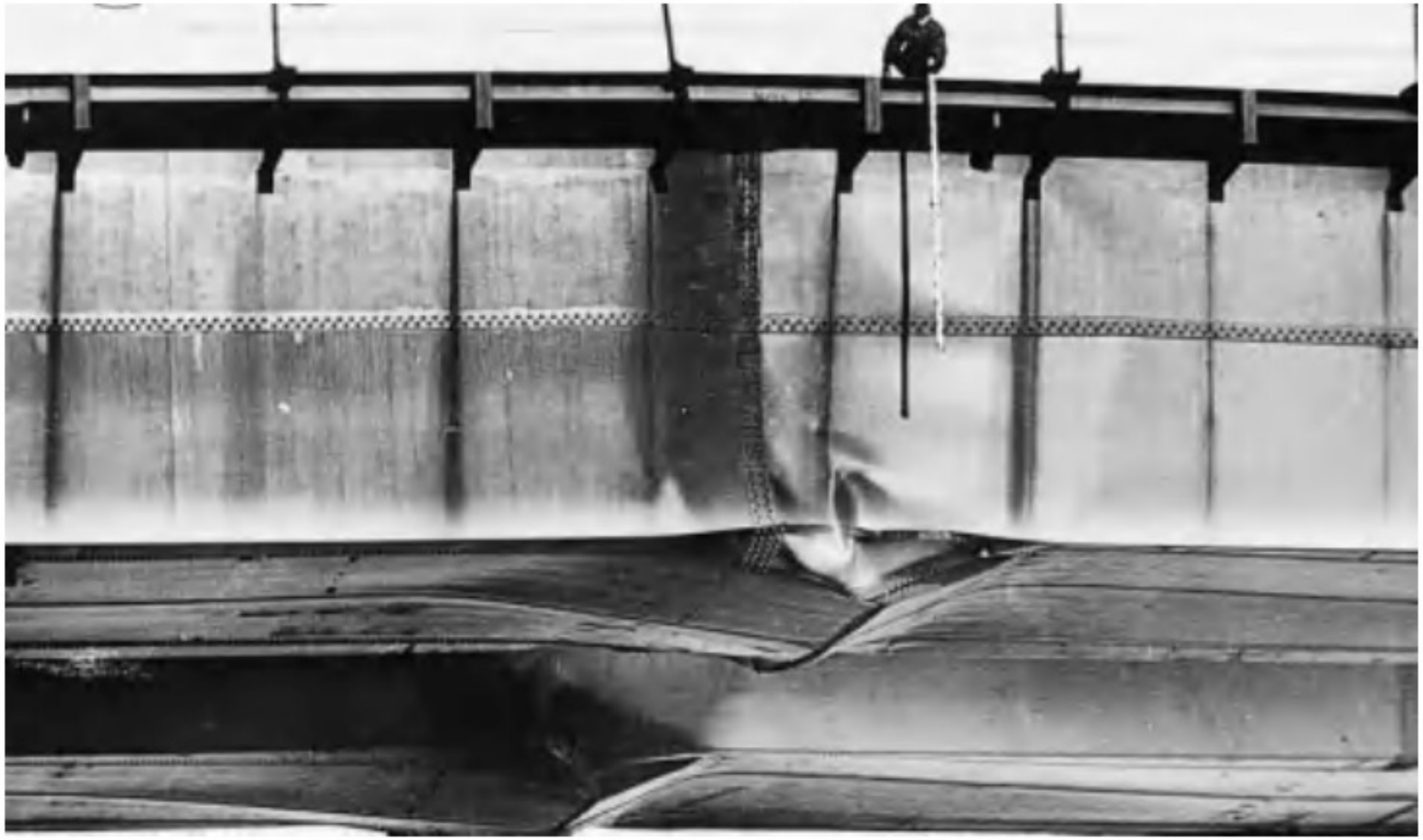
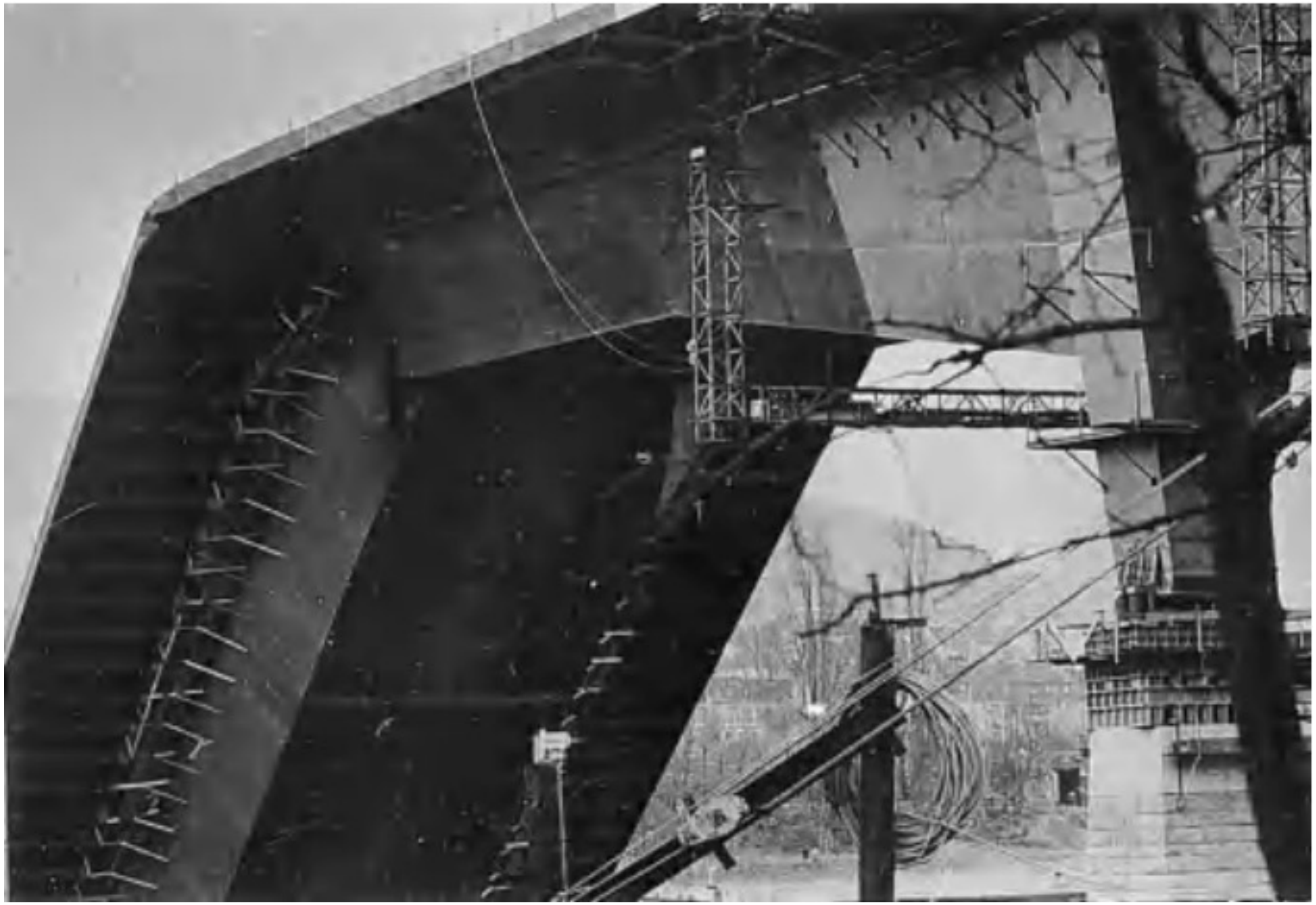
Motivation
Structural failures



4th Danube Bridge, Vienna, 1969. [Scheer, 2010]

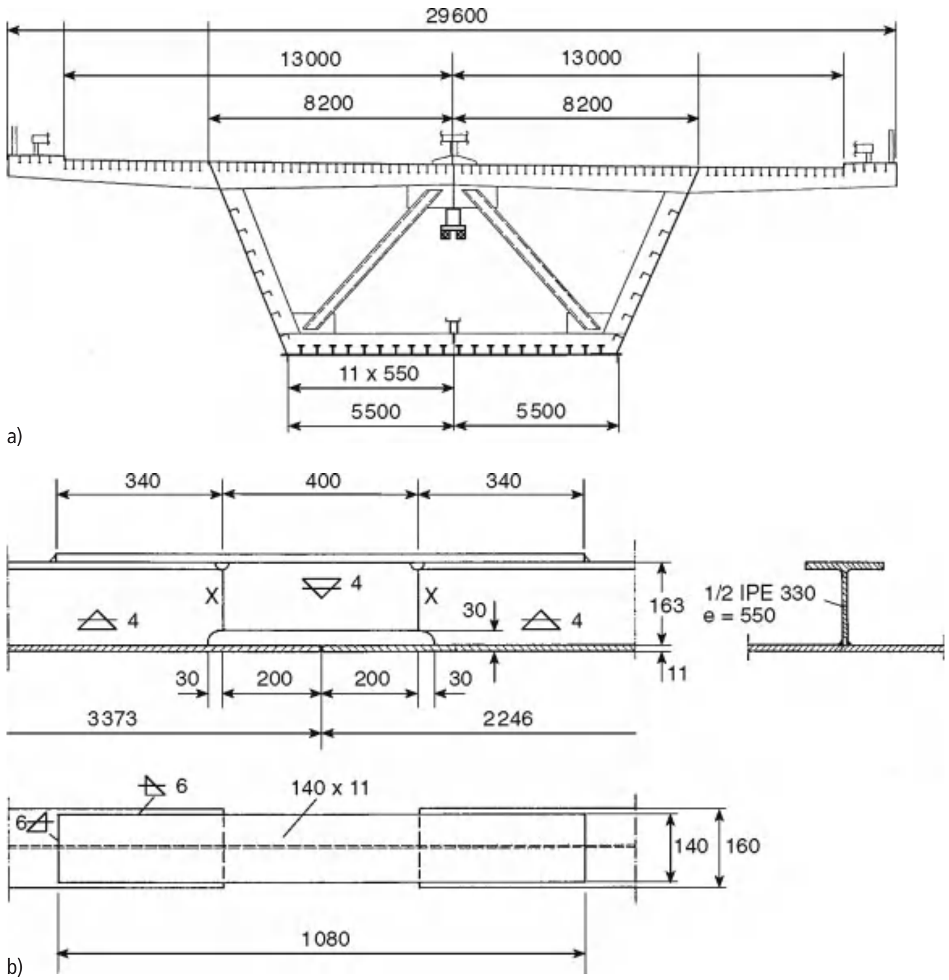
Motivation
Structural failures



Rhine Bridge, Koblenz, 1971. [Scheer, 2010]

Motivation

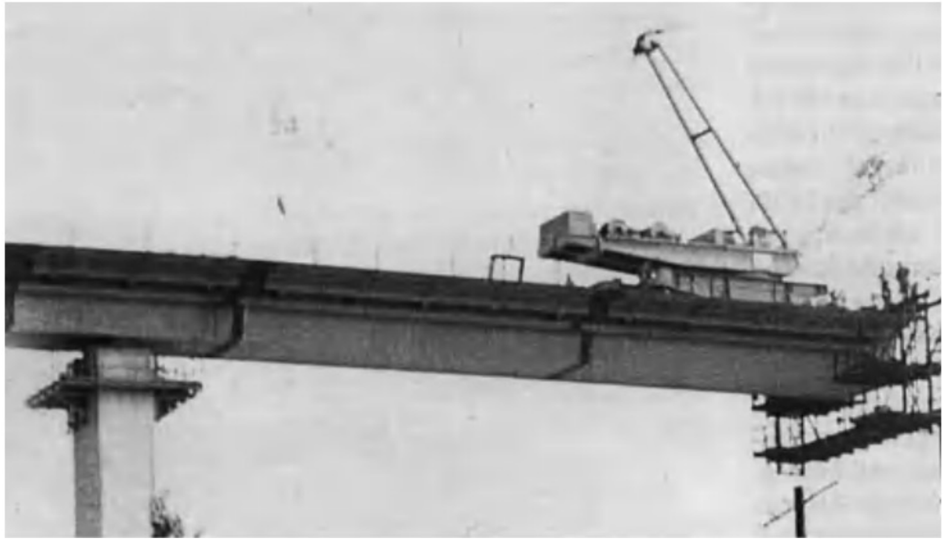
Structural failures



Rhine Bridge, Koblenz, 1971. [Scheer, 2010]

Rhine Bridge, Koblenz, 1971. Detailing experiment. [Scheer, 2010]

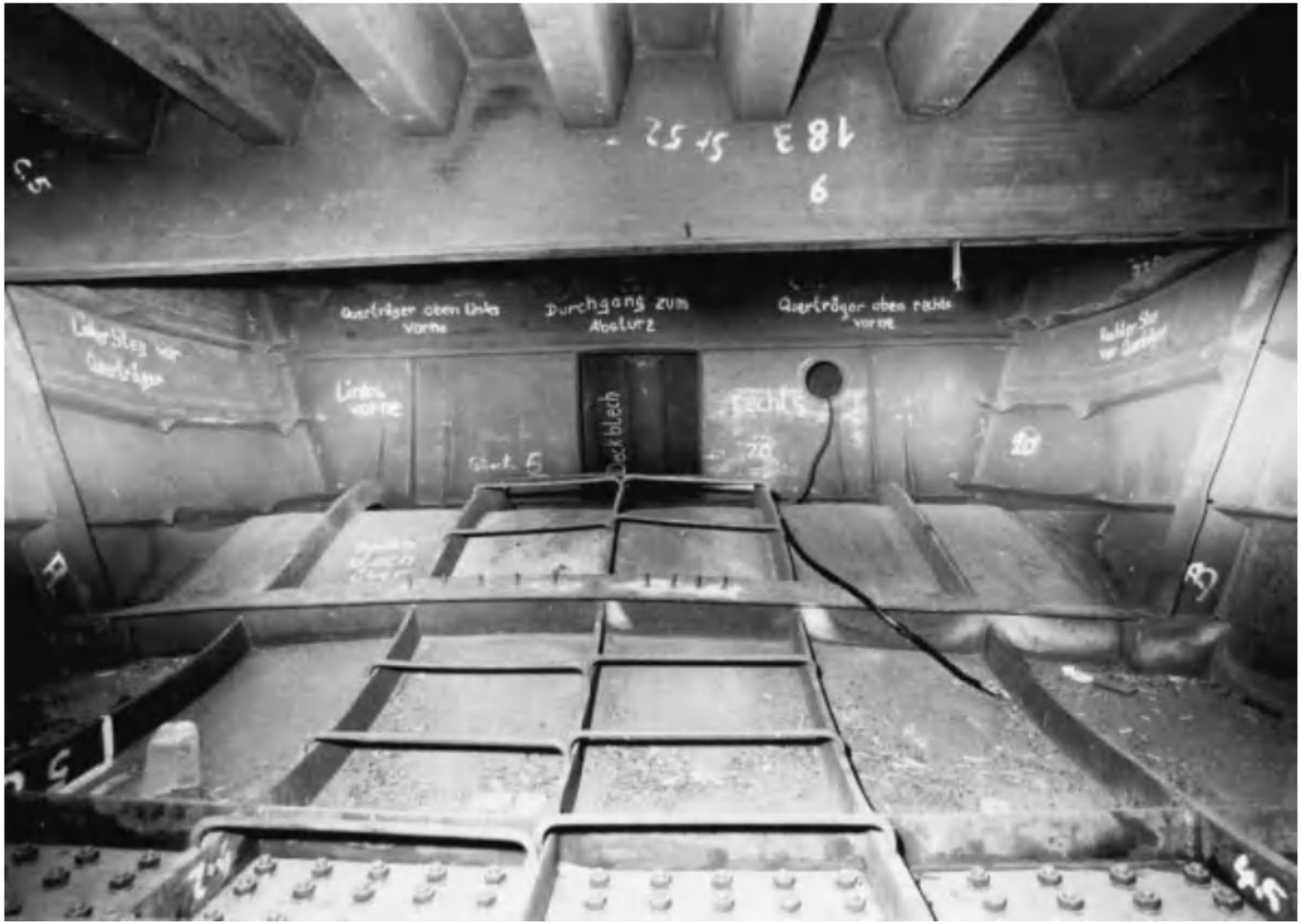
Motivation
Structural failures


Zeulenroda reservoir Bridge, 1973-- before failure. [Scheer, 2010]


Zeulenroda reservoir Bridge, 1973– after failure. [Scheer, 2010]

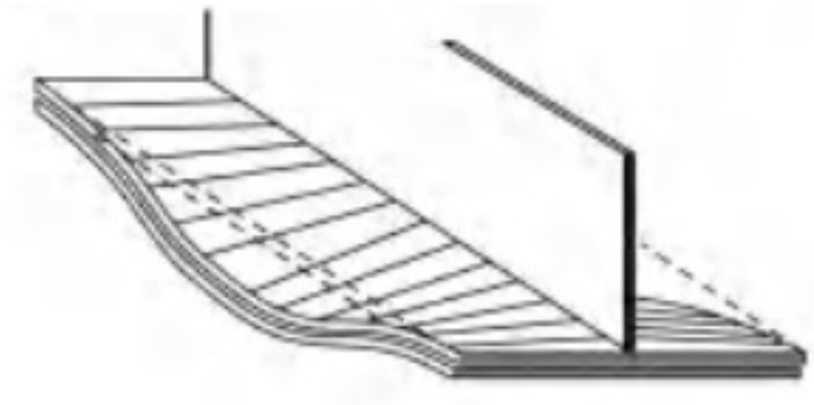
Motivation

Structural failures



Zeulenroda reservoir Bridge, 1973. [Scheer, 2010]

Motivation
Structural failures



[Scheer, 2010]

Werra Bridge,Hedemünden, 1991. [Scheer, 2010]

Plate buckling
Preliminary definitions

- **A thin surface laminar element** is characterized by:
  1. A surface defined by a curvilinear reference system in, say, directions $\{x, y\}$;
  2. A thickness t much smaller than the length scale of the surface and perpendicular to the surface in, say, direction $\{z\}$.

- Thin surface laminar elements can be divided in the following categories:
  1. **Plate** – element with **zero curvature in both surface directions--** $(1/\mathrm{R}_x = 0 \wedge 1/R_y = 0)$, with R the radius of curvature;
  2. Cylindrical Shell – with one direction of non-zero curvature-- $\left(1/\mathrm{R}_x = 0 \wedge 1/R_y \neq 0\right) \vee (1/\mathrm{R}_x \neq 0 \wedge 1/R_y = 0)$;
  3. Shell – with non-zero curvature in both directions $\left(1/\mathrm{R}_x \neq 0 \wedge 1/R_y \neq 0\right)$.

Plate buckling
Preliminary definitions

- Linear elastic buckling of plates:
    1. Objective to calculate stresses that lead to loss of stability of plate(**limit point**);
    2. Based on the first-order, small-displacement, deformed configuration of the surface;
    3. Assumes a linear elastic material;
    4. Typically does not take into account initial imperfections or residual stresses;

- Post-buckling of plates:
    1. Objective is to describe equilibrium path **after limit point** is reached;
    2. Based on the second-order(+), small- or large-displacement, deformed configuration;
    3. Slender plate behavior is considerably stable after limit point is reached;

- Inelastic buckling of plates
    1. Describes situations in which significant material nonlinearity takes place. This can happen **before or after** linear elastic limit point;
    2. Based on the second-order(+), small- or large-displacement, deformed configuration;
    3. Plate ultimate loads depend significantly on initial imperfections, residual stresses, and material nonlinear model.

Linear elastic buckling of plates

Introduction

On the Stability of a Plane Plate under Thrusts in its own Plane, with Applications to the "Buckling" of the Sides of a Ship.

By G. H. BRYAN.

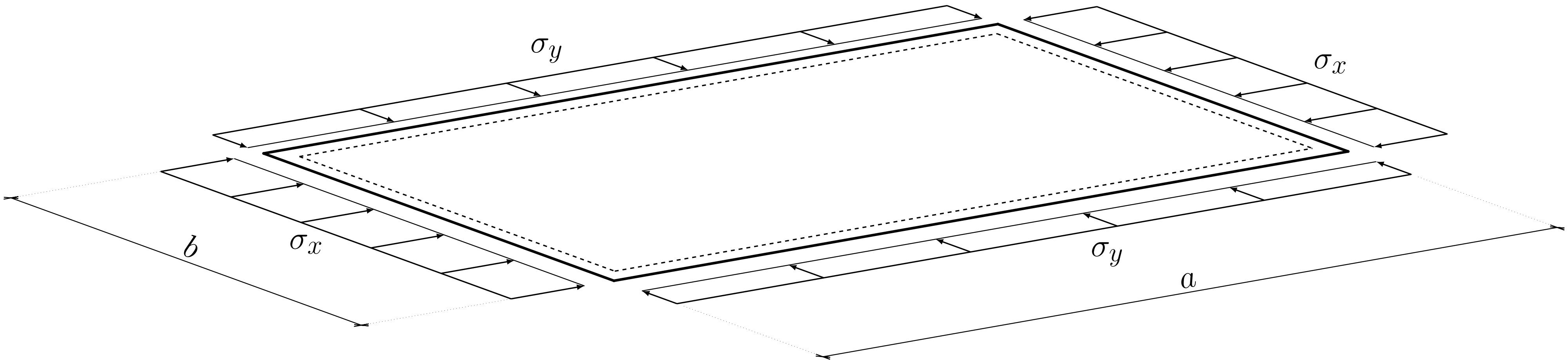
[*Read Dec.* 11*th*, 1890.]

Introduction.

1. The problems discussed in this paper are the analogues for a plane rectangular or circular plate of the well-known investigations of the stability of a thin wire or shaft, due in the first place to Euler, and since developed by Greenhill. I have employed the energy criterion of stability, the use of which I have already illustrated in this connexion in two papers published in the *Proceedings of the Cambridge Philosophical Society.**

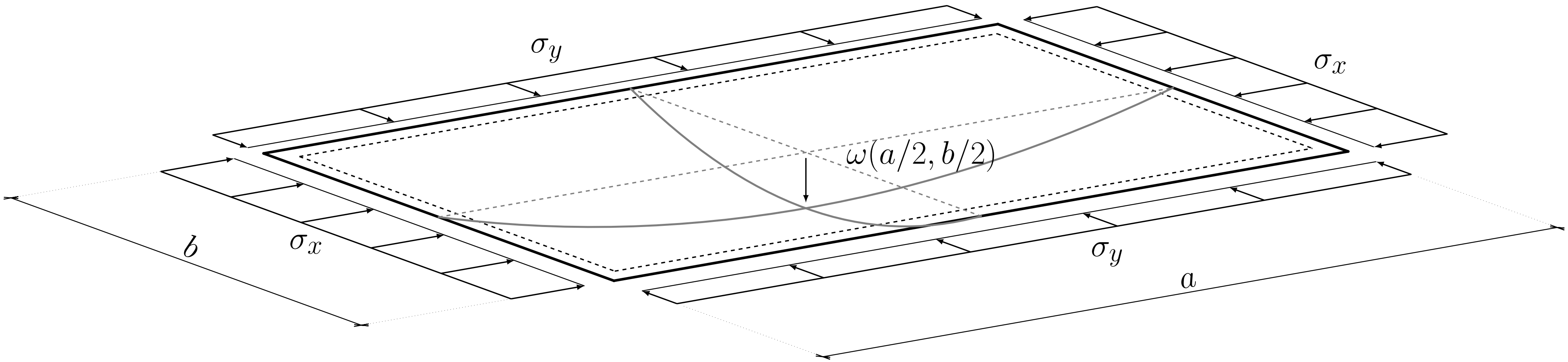
[Bryan, 1890]

Linear elastic buckling of plates
Introduction



Linear elastic buckling of plates

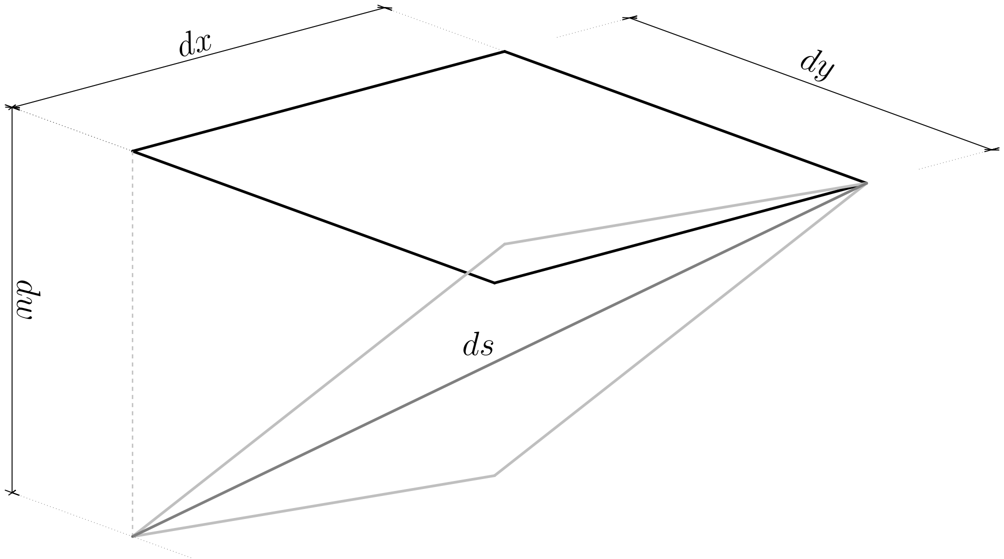
Introduction



Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

Consider an infinitesimal plate element,



The length between two points in the deformed configuration can expressed by,

$$ds^2 = dx^2 + dy^2 + dw^2$$

The displacement is a function of the position on the plate and so,

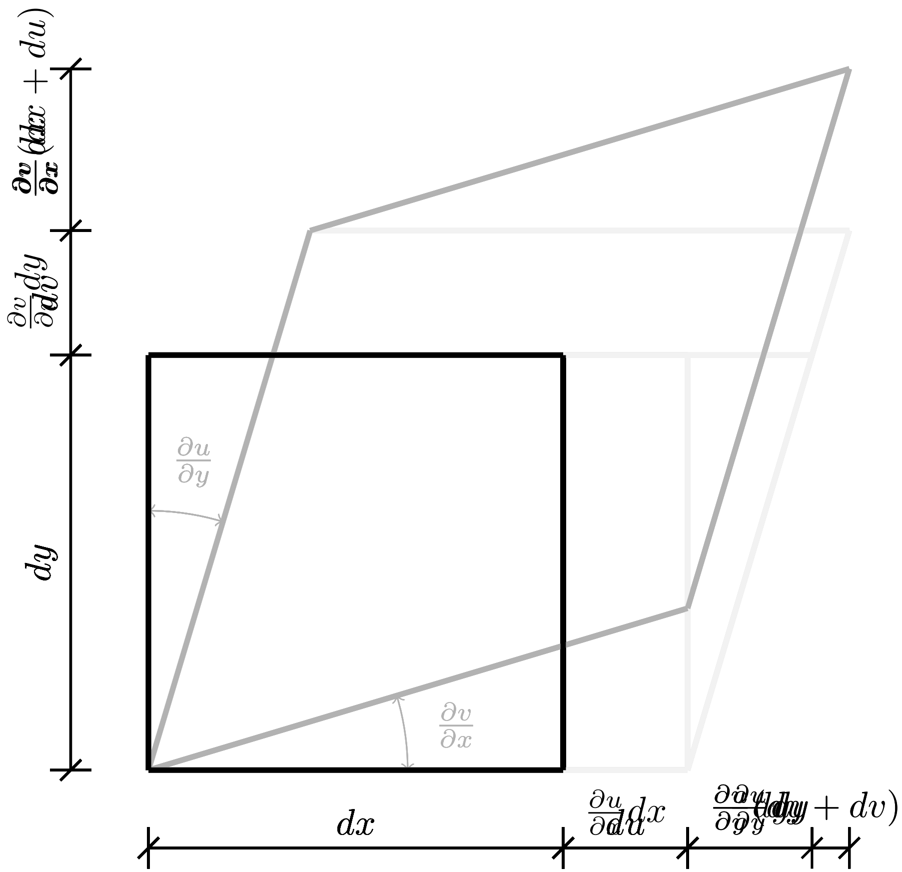
$$dw = \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy$$

Substituting,

$$ds^2 = \left[1 + \left(\frac{\partial w}{\partial x}\right)^2\right] dx^2 + 2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} dx\, dy + \left[1 + \left(\frac{\partial w}{\partial y}\right)^2\right] dy^2 \quad (1)$$

Linear elastic buckling of plates

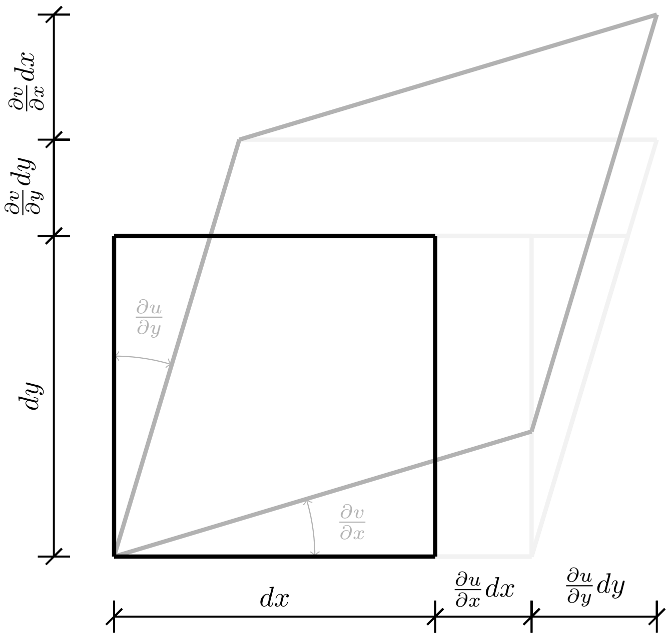
An energy approach to the buckling of rectangular plates



Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

The length between two points in the deformed configuration can also be expressed by the stretch and distortion of the plate,



$$ds^2 = (dx + du)^2 + (dy + dv)^2 =$$

$$= \left(dx + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy\right)^2 + \left(dy + \frac{\partial v}{\partial v}dv + \frac{\partial v}{\partial x}dx\right)^2 =$$

$$= \left(\left[1 + \frac{\partial u}{\partial x}\right]dx + \frac{\partial u}{\partial y}dy\right)^2 + \left(\left[1 + \frac{\partial v}{\partial y}\right]dy + \frac{\partial v}{\partial x}dx\right)^2 =$$

$$= \left[1 + \frac{\partial u}{\partial x}\right]^2 dx^2 + 2\left(1 + \frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial y}dxdy + \left(\frac{\partial u}{\partial y}\right)^2 dy^2 +$$

$$+ \left[1 + \frac{\partial v}{\partial y}\right]^2 dy^2 + 2\left(1 + \frac{\partial v}{\partial y}\right)\frac{\partial v}{\partial x}dxdy + \left(\frac{\partial v}{\partial x}\right)^2 dx^2 =$$

Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

The length between two points in the deformed configuration can also be expressed by the stretch and distortion of the plate,

$$ds^2 = (dx + du)^2 + (dy + dv)^2 =$$

$$= \left[1 + 2\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2\right] dx^2 + 2\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right) dxdy + \left(\frac{\partial u}{\partial y}\right)^2 dy^2 +$$

$$+ \left[1 + 2\frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y}\right)^2\right] dy^2 + 2\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial v}{\partial x}\right) dxdy + \left(\frac{\partial v}{\partial x}\right)^2 dx^2 =$$

Linear elastic buckling of plates
An energy approach to the buckling of rectangular plates

The length between two points in the deformed configuration can also be expressed by the stretch and distortion of the plate,

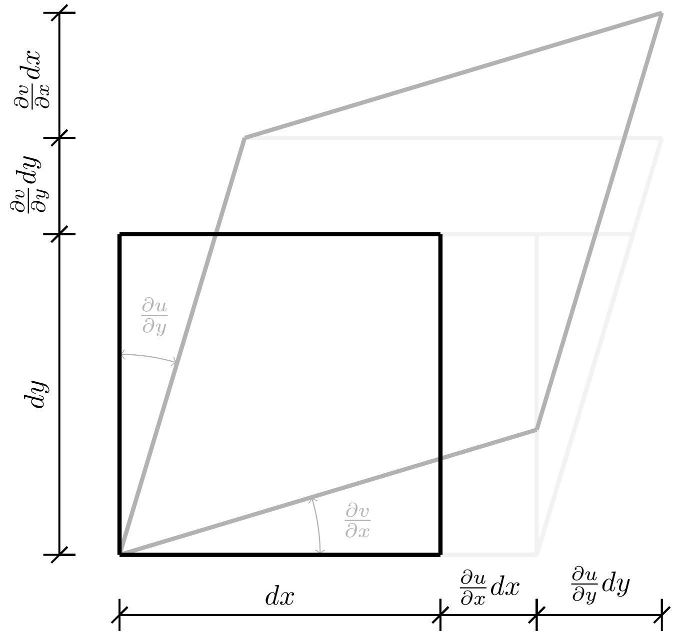
$$ds^2 = (dx + du)^2 + (dy + dv)^2 =$$

$$= \left[1 + 2\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2\right] dx^2 + 2\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right) dxdy + \left(\frac{\partial u}{\partial y}\right)^2 dy^2 +$$

$$+ \left[1 + 2\frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y}\right)^2\right] dy^2 + 2\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial v}{\partial x}\right) dxdy + \left(\frac{\partial v}{\partial x}\right)^2 dx^2 =$$

In this linearized analysis, the **second-order** terms of the strain are **neglected**

An energy approach to the buckling of rectangular plates



So finally,

$$ds^2 = \left[1 + 2\frac{du}{dx}\right]dx^2 + \\ + 2\varpi dx\, dy + \left[1 + 2\frac{dv}{dy}\right]dy^2 \quad (2)$$

In which ϖ is the shear angle.

$$\varpi = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

For small strains, the squared term of the strains is neglected, and so comparing Eq. (1) with (2) yields,

$$\frac{du}{dx} = \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \qquad \varpi = \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \qquad \frac{dv}{dy} = \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$$

Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

The work performed by the external forces acting on the plate can thus be expressed as,

$$V = -t \int_0^a \int_0^b \sigma_x \frac{du}{dx} + \tau_{xy} \varpi + \sigma_y \frac{dv}{dy} \, dx \, dy =$$

$$= -\frac{1}{2} t \int_0^a \int_0^b \sigma_x \left(\frac{\partial w}{\partial x}\right)^2 + 2\tau_{xy} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} + \sigma_y \left(\frac{\partial w}{\partial y}\right)^2 dx \, dy \quad (3)$$

Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

The internal work that the plate conducts in **bending** due to the displacement field $w(x, y)$, can be expressed as

$$U = \frac{1}{2} D \int_0^a \int_0^b \left[\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1 - \nu) \left(\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right) \right] dx\, dy \quad (4)$$

with D, the plate bending stiffness, equal to

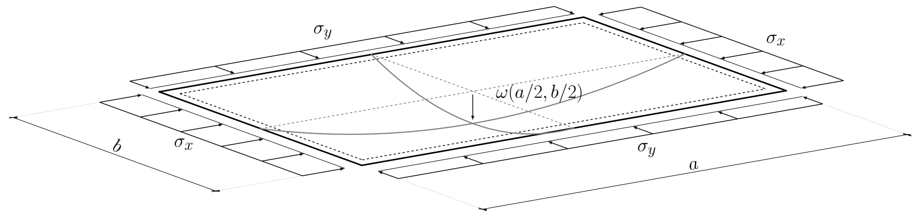
$$D = \frac{E t^3}{12(1 - \upsilon^2)}$$

Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

For a rectangular plate supported at its edges, the vertical displacement can be expressed by,

$$w(x,y)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A_{mn}\sin\frac{m\pi}{a}x\,\sin\frac{n\pi}{b}y \quad (5)$$



Assume for the moment that the **shear stress are zero** and the **normal stresses are constant**. Then, performing the corresponding derivatives of Eq. 5 and substituting in the expressions for internal and external work, it can be shown that

$$V=-\frac{1}{8}ab\pi^2 t\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left(\frac{\sigma_x m^2}{a^2}+\frac{\sigma_y n^2}{b^2}\right)A_{mn}^2 \quad (6)$$

$$U=\frac{1}{8}ab\pi^4 D\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 A_{mn}^2 \quad (7)$$

Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

From the previous lecture we saw that the total potential energy at an equilibrium point can be defined as,

$$\Pi = U + V \qquad \text{with stability defined as,} \qquad \frac{\partial \Pi}{\partial w} > 0$$

Substituting in the stability definition our expression for the internal and external work yields the following condition, with respect to the amplitude of each vertical displacement,

$$\frac{\partial}{\partial A_{mn}}\left(\frac{1}{8}ab\pi^4 D \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 A_{mn}^2 - \frac{1}{8}ab\pi^2 t \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left(\frac{\sigma_x m^2}{a^2}+\frac{\sigma_y n^2}{b^2}\right) A_{mn}^2\right) > 0 \ (8)$$

Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

From Eq.8 it can be seen that each buckling is independent of each other and, as such, the stability can be expressed for each mode as,

$$\frac{1}{4}ab\pi^4 D\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 A_{mn} > \frac{1}{8}ab\pi^2 t\left(\frac{\sigma_x m^2}{a^2}+\frac{\sigma_y n^2}{b^2}\right)A_{mn} \quad (9)$$

And so,

$$\pi^2 D > \frac{t\frac{\sigma_x m^2}{a^2}+t\frac{\sigma_y n^2}{b^2}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad (10)$$

With mode shape,

$$w(x,y) = A_{mn}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y$$

Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

Consider now the case where the plate loaded **uniaxially** ($\sigma_y = 0$),

$$\pi^2 D = \frac{t\,\dfrac{\sigma_{cr,x} m^2}{a^2}}{\left(\dfrac{m^2}{a^2} + \dfrac{n^2}{b^2}\right)^2} \Leftrightarrow \sigma_{cr,x} = \frac{\pi^2 D a^2}{t\, m^2}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2$$

The critical stress will be **minimal** when $n = 1$, that is, when the buckling **mode only has one half-wave perpendicular to the load**.

$$\sigma_{cr,x} = \frac{\pi^2 D a^2}{t\, m^2}\left(\frac{m^2}{a^2} + \frac{1}{b^2}\right)^2 \quad (11)$$

Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates

When we re-work Eq. 11, we are left with **one of the most important expressions in this lecture (Eq. 12)**,

$$\sigma_{cr,x} = \frac{\pi^2 D a^2}{t\, m^2}\left(\frac{m^2}{a^2}+\frac{1}{b^2}\right)^2 = \frac{\pi^2 D}{t\, b^2}\left(\frac{mb}{a}+\frac{a}{mb}\right)^2 \Rightarrow$$

$$\sigma_{cr,x} = k \cdot \frac{\pi^2 E}{12(1-\nu^2)\left(\frac{b}{t}\right)^2} = k \cdot \sigma_E \quad (12)$$

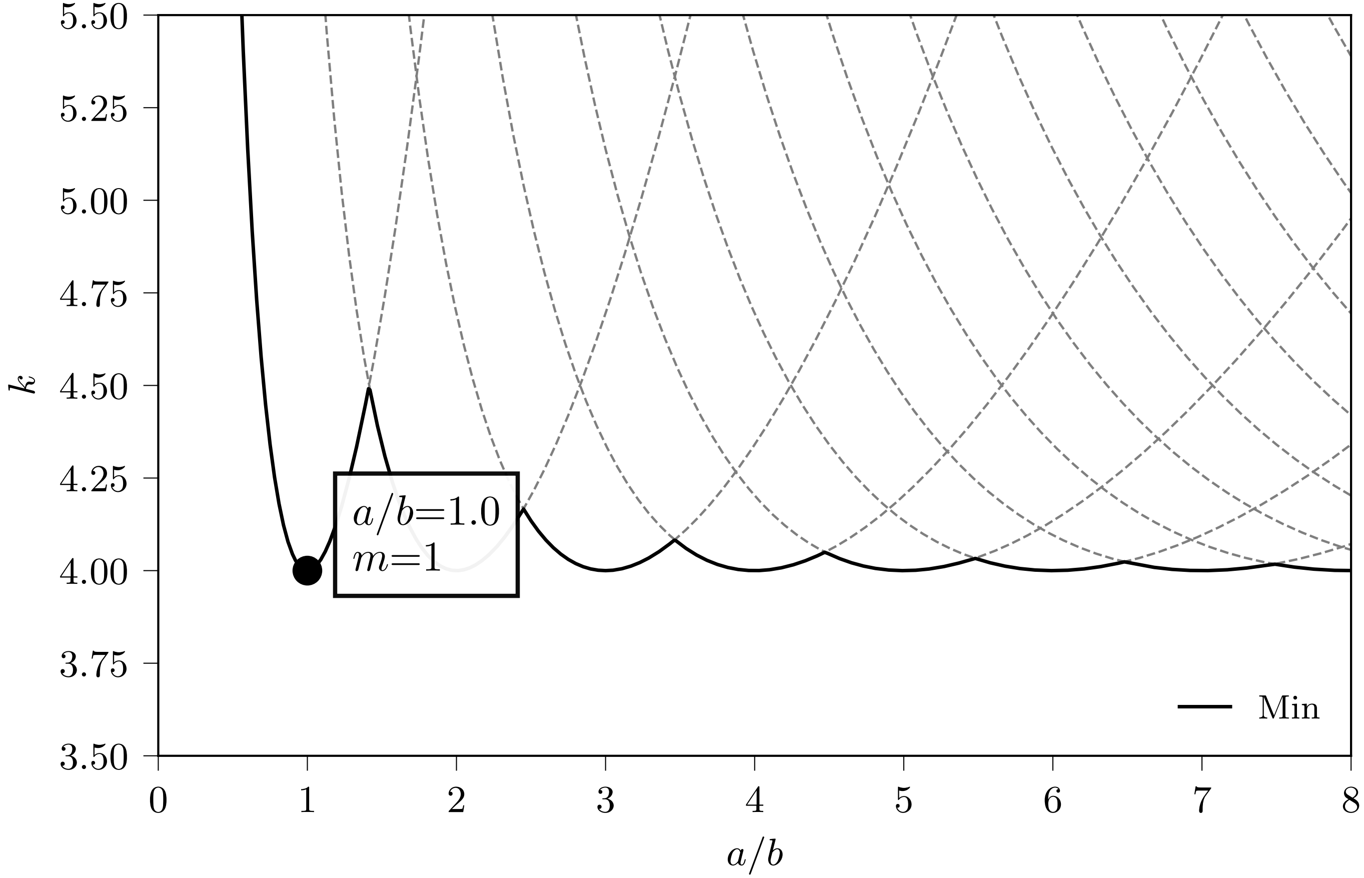
with k, **the plate factor or plate buckling coefficient**, and σ_E,

$$k = \left(\frac{mb}{a}+\frac{a}{mb}\right)^2 \quad (13)$$

$$\sigma_E = \frac{\pi^2 E}{12(1-\nu^2)\left(\frac{b}{t}\right)^2} \approx 0.9E\left(\frac{t}{b}\right)^2 \quad (14)$$

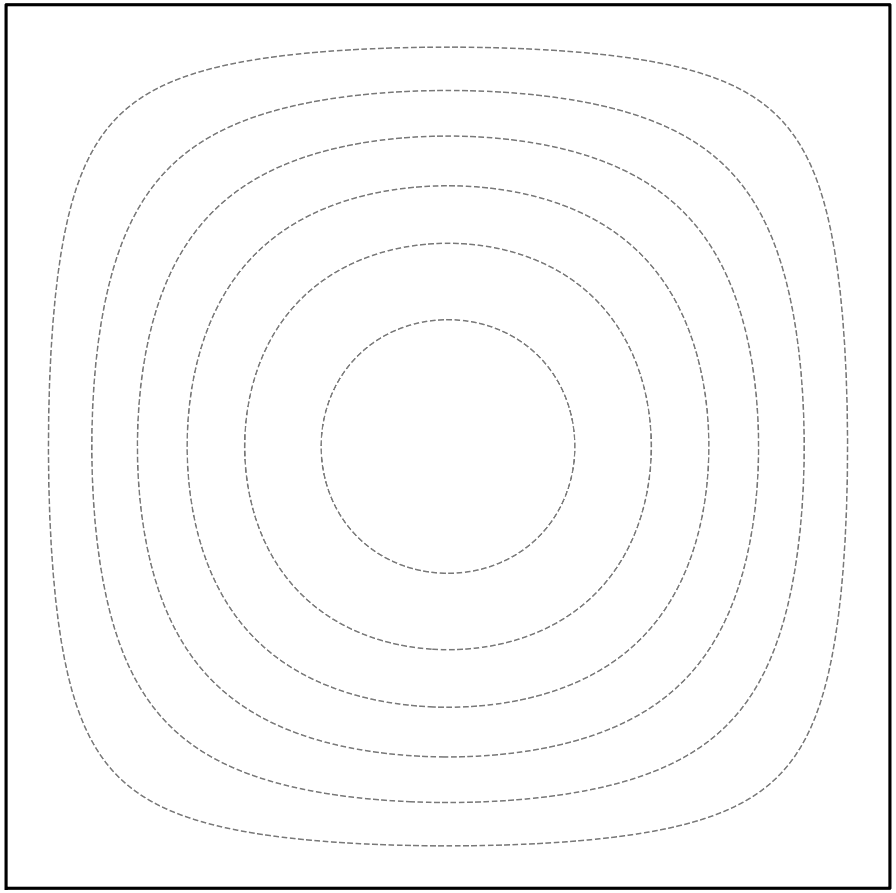
Linear elastic buckling of plates

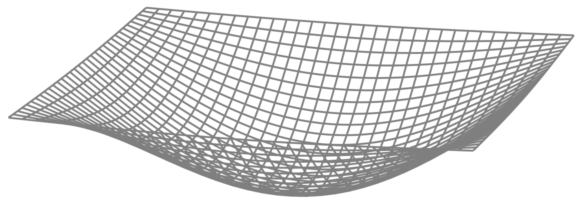
An energy approach to the buckling of rectangular plates



Linear elastic buckling of plates

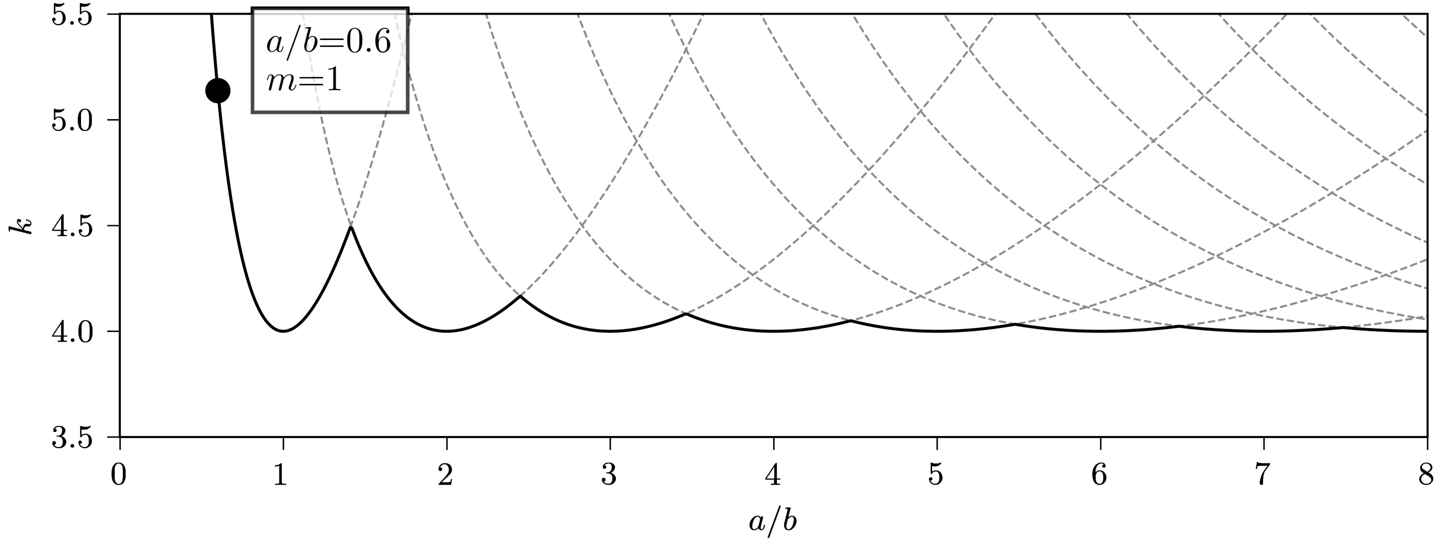
An energy approach to the buckling of rectangular plates

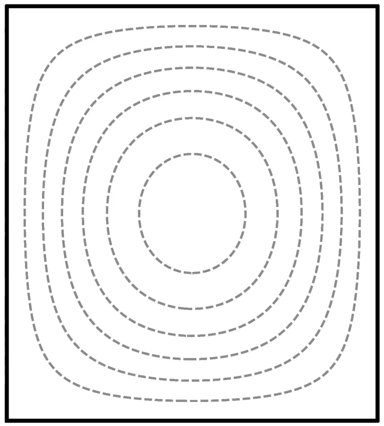


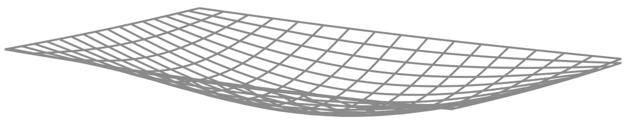


Linear elastic buckling of plates

An energy approach to the buckling of rectangular plates







Linear elastic buckling of plates

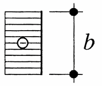
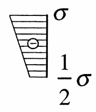
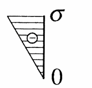
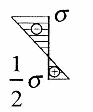
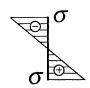
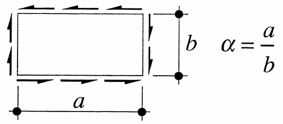
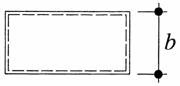
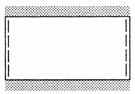
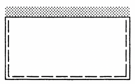
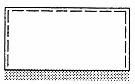
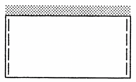
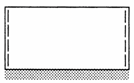
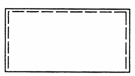
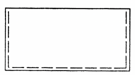
Influence of boundary and loading conditions

a, b	ψ	Boundary conditions							
$-\sigma$ $-\sigma$	1	4,00	6,97	5,41	5,41	1,28	1,28	0,426	0,426
$-\sigma$	0	7,81	13,54	11,73	9,54	5,91	1,608	1,702	0,567
$-\sigma$ $+\sigma$	–1	23,90	39,52	39,52	23,94		2,134		0,851
$-\sigma$ $-\psi\cdot\sigma$	$k_{min} \approx \dfrac{16}{\sqrt{(1+\psi)^2 + 0{,}112\,(1-\psi)^2} + (1+\psi)}$ valable pour (boundary condition symbol) et $\psi \geq -1{,}2$								

SIA263, §4.5.3 - [SIA, 2013]

Linear elastic buckling of plates

Influence of boundary and loading conditions

Boundary conditions	Normal stresses					Shear stresses (approximate formulas)
						
	4.00	5.32	7.81	13.40	23.9	$\alpha \geq 1 : k = 5.34 + (4.00/\alpha^2)$ $\alpha \leq 1 : k = 4.00 + (5.34/\alpha^2)$
	6.97	9.27	13.54	24.5	39.52	$\alpha \geq 1 : k = 9.00 + (3.30/\alpha^2)$ $\alpha \leq 1 : k = 7.00 + (5.30/\alpha^2)$
	5.41		11.73		39.52	$\alpha \geq 1 : k = 7.50 + (4.00/\alpha^2)$ $\alpha \leq 1 : k = 6.50 + (5.00/\alpha^2)$
	5.41		9.54		23.94	
	1.28		5.91			
	1.28		1.608		2.134	
	0.426		1.702			
	0.426		0.567		0.851	

TGC10, §12 - [Hirt et al., 2011]

Linear elastic buckling of plates
Influence of boundary and loading conditions

TGC10 §12 also discusses ways to compute equivalent critical stresses for mixed loading (uniaxial compression and shear)

$$\sigma_{gcr} = \frac{\sqrt{\sigma_x^2 + 3\tau^2}}{\frac{1+\psi}{4}\frac{\sigma_x}{\sigma_{cr,x}} + \sqrt{\left(\frac{3-\psi}{4}\frac{\sigma_x}{\sigma_{cr,x}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)}} \quad (15)$$

where,

ψ – is the ratio between the smallest and largest uniaxial stress (signs included)

σ_x – is the largest uniaxial stress

$\sigma_{cr,x}$ – is the limit stress in taking into account **only** the uniaxial stresses

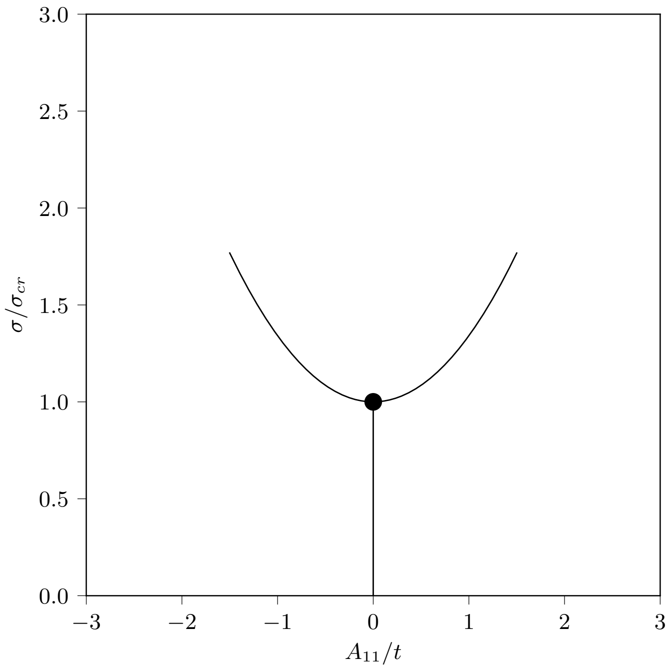
τ_{cr} – is the limit stress in taking into account **only** the shear stresses

Post buckling of plates

Equilibrium paths

Post-buckling analysis leads us to analyzed the previous problem taking into account higher order terms in the linearization of the problem. Analytically this can be performed by:

1. An energy approach [Koiter, 1945]
2. Equilibrium in the deformed configuration [von Karman et. al, 1932]



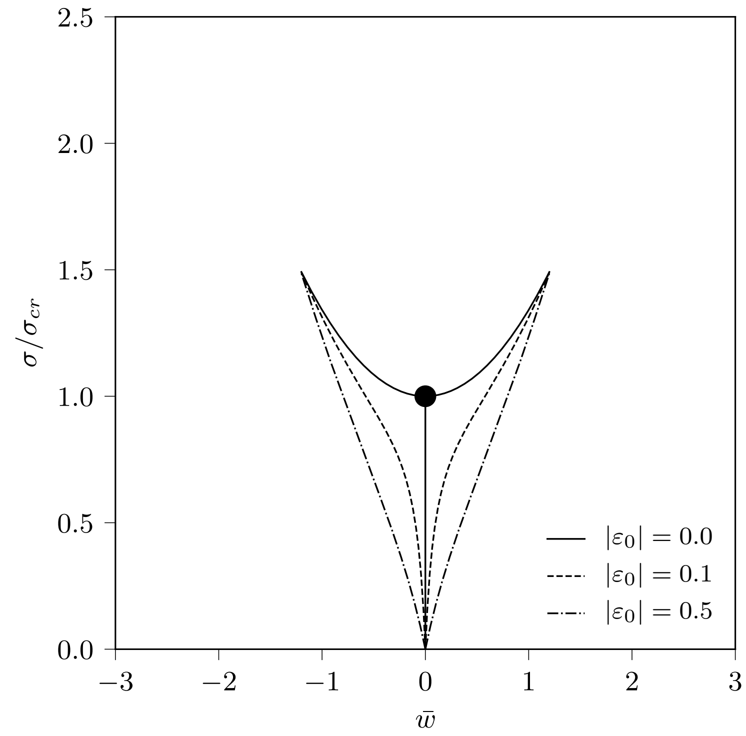
Both approaches can be shown to yield the same result in the neighborhood of the linear elastic limit point for a simply supported square plate:

$$\frac{\sigma}{\sigma_{cr}} = 1 + \frac{3}{8}(1 - \nu^{2})\left(\frac{A_{11}}{t}\right)^{2} \quad (15)$$

Note that for deflections on the order of the thickness of the plate, the post-buckling stress is around 35% larger than σ_{cr}

Post buckling of plates

Equilibrium path



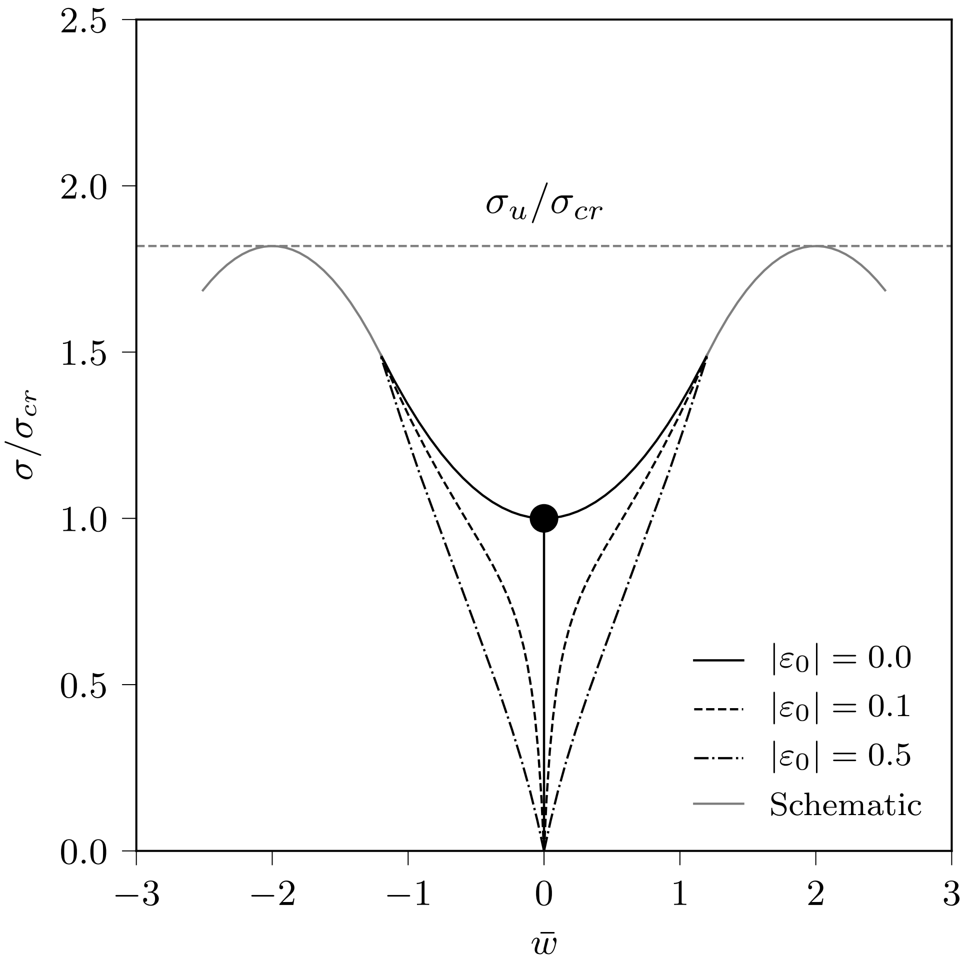
For a case with initial imperfections $\varepsilon_0 = \frac{w_0}{t}$, Volmir [1967] reports,

$$\frac{\sigma}{\sigma_{cr}} = 1 + \frac{3}{8}(1 - \nu^2)[\bar{w}^2 + 3\bar{w}\varepsilon_0]\frac{\bar{w}}{\bar{w} + \varepsilon_0} \quad (16)$$

With, $\bar{w} = \frac{w}{t}$

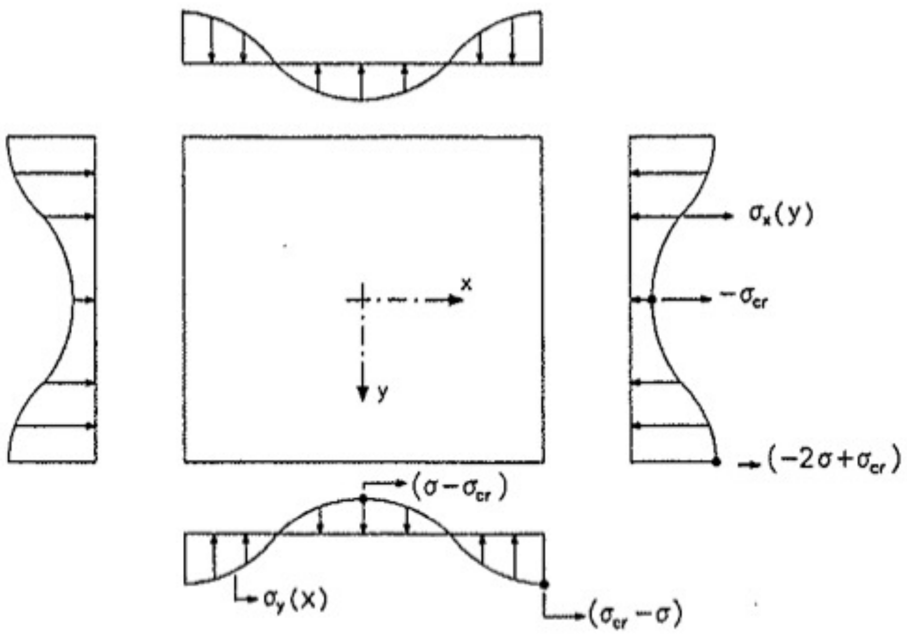
The **key idea** here is that there is a significant amount of reserve capacity, even in the presence of initial imperfections

Post buckling of plates
Ultimate plate load



Post buckling of plates

Stress distribution at edges



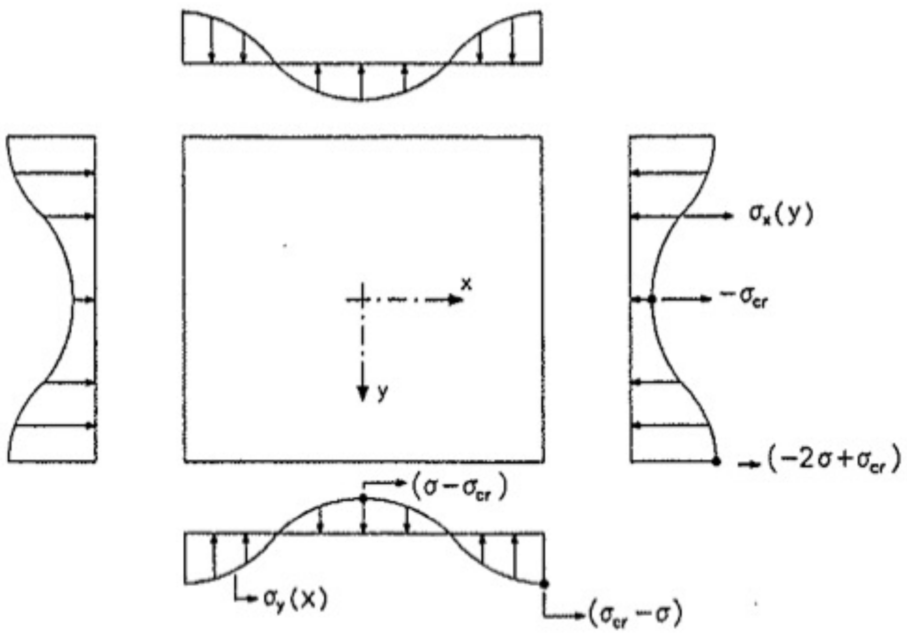
[Reis and Camotim, 2001]

For a rectangular plate (a=b), the stress distributions along its edges, in the post buckling phase, assumes the following form,

$$\sigma_x(y) = -\sigma + (\sigma - \sigma_{cr})\cos\frac{2\pi y}{b}$$

$$\sigma_y(x) = (\sigma - \sigma_{cr})\cos\frac{2\pi x}{b}$$

Post buckling of plates
Effective width



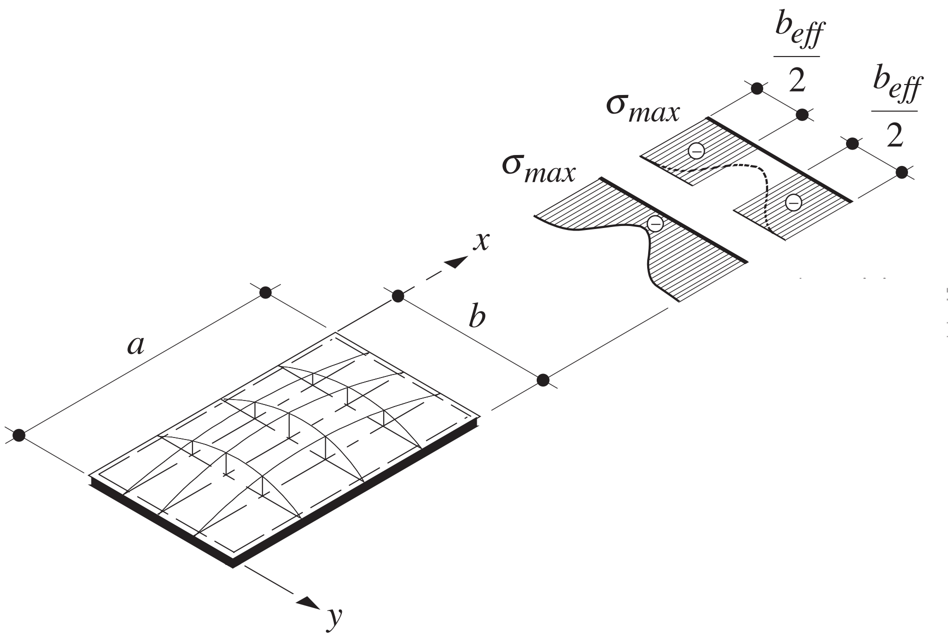
[Reis and Camotim, 2001]

Take the integral along the compressed edges,

$$\int_{-b/2}^{b/2} \sigma_x(y)dy = \bar{\sigma}b = \sigma_{max}b_{eff}$$

with, $\sigma_{max} = 2\bar{\sigma} - \sigma_{cr}$

Post buckling of plates
Effective width



[Hirt et al., 2011]

Take the integral along the compressed edges,

$$\int_{-b/2}^{b/2} \sigma_x(y)dy = \bar{\sigma}b = \sigma_{max}b_{eff}$$

with, $\sigma_{max} = 2\bar{\sigma} - \sigma_{cr}$

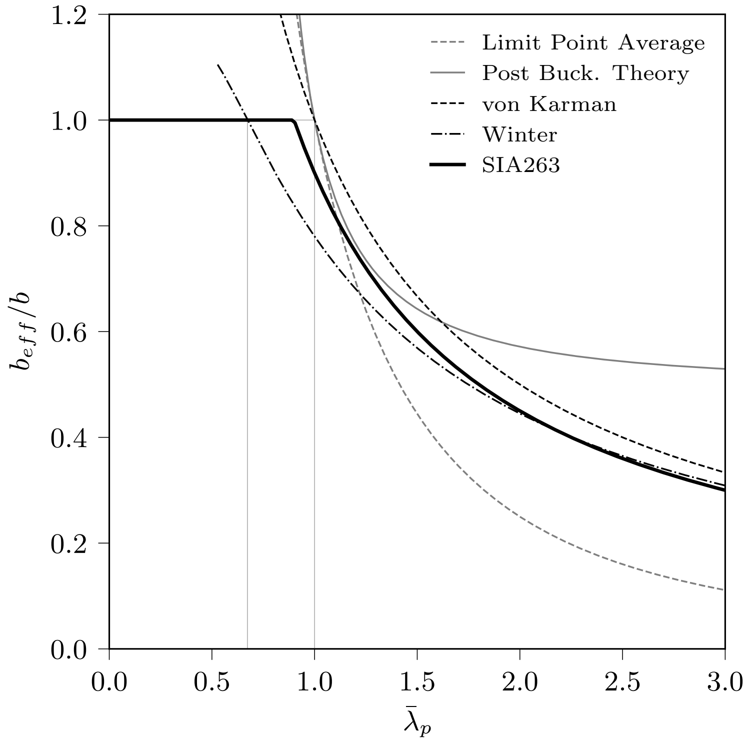
$$\frac{b_{eff}}{b} = \frac{\bar{\sigma}}{2\bar{\sigma} - \sigma_{cr}} = \frac{1}{2 - \frac{\sigma_{cr}}{\bar{\sigma}}} \quad (17)$$

Post buckling of plates

Effective width – in practice

Let's consider a reduced plate slenderness defined as,

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \quad (18)$$



Limit point average criterion,

$$\int_{-b/2}^{b/2} \sigma_x(y)dy = \sigma_{cr} b = f_y b_{eff}$$

$$\frac{b_{eff}}{b} = \frac{1}{\bar{\lambda}_p^2} \quad (19)$$

Post buckling theory suggests, from (17),

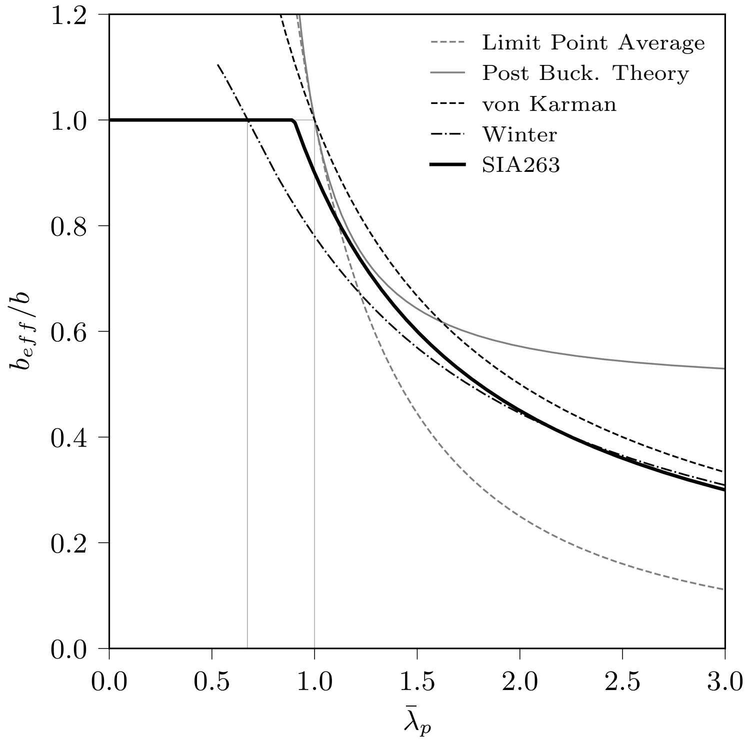
$$\frac{b_{eff}}{b} = \frac{1}{2 - \frac{1}{\bar{\lambda}_p^2}} \quad (20)$$

Post buckling of plates

Effective width – in practice

Let's consider a reduced plate slenderness defined as,

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \quad (18)$$



Empirically, a number of authors propose expressions for effective widths:

- von Karman et al. [1932] :

$$\frac{b_{eff}}{b} = \frac{1}{\bar{\lambda}_p} \quad (21)$$

- Winter [1947]:

$$\frac{b_{eff}}{b} = \frac{1}{\bar{\lambda}_p}\left(1 - \frac{0.22}{\bar{\lambda}_p}\right) \quad (22)$$

- SIA263-[SIA,2013] :

$$\frac{b_{eff}}{b} = \frac{0.9}{\bar{\lambda}_p} \quad (23)$$

Post buckling of plates
Effective width – in practice

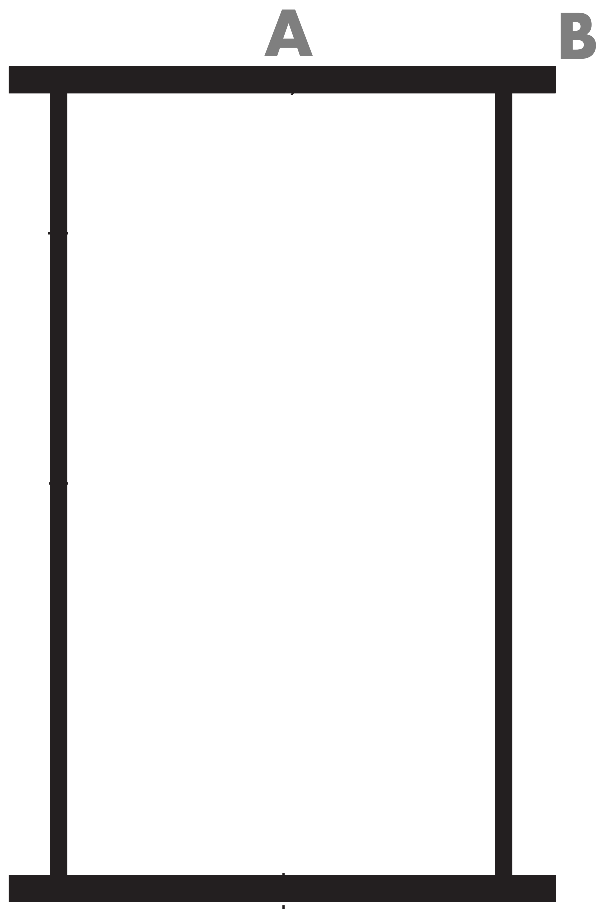
- SIA263-[SIA,2013] – the box section exception (§5.6.4.3) :

For panels simply supported at both ends (**A**),

$$\frac{b_{eff}}{b} = \frac{\bar{\lambda}_p - 0.055(3+\psi)}{\bar{\lambda}_p^2}$$

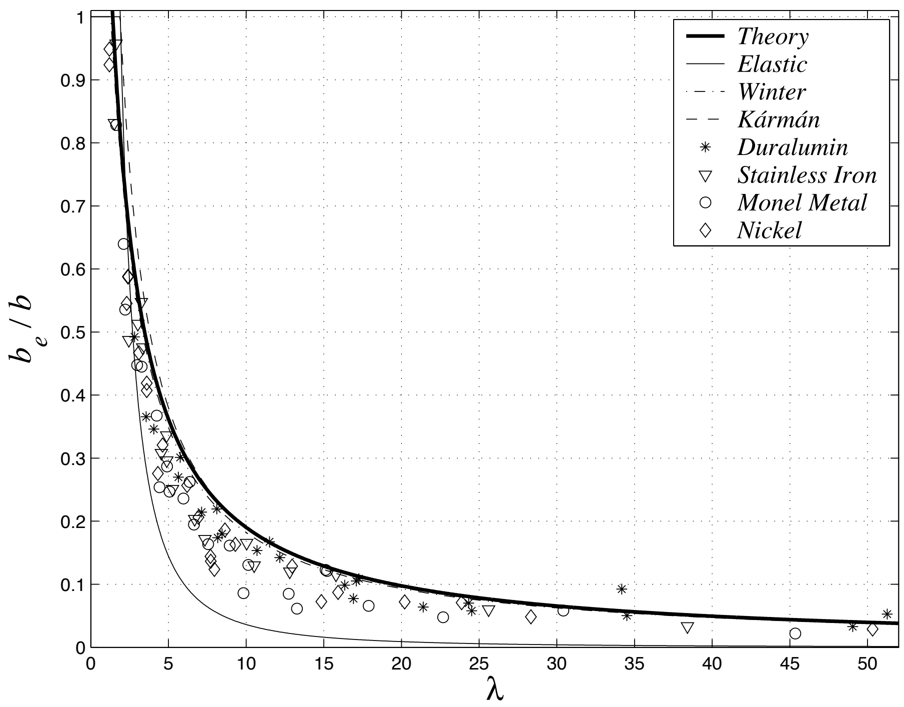
For panels simply supported at one ends (**B**),

$$\frac{b_{eff}}{b} = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2}$$

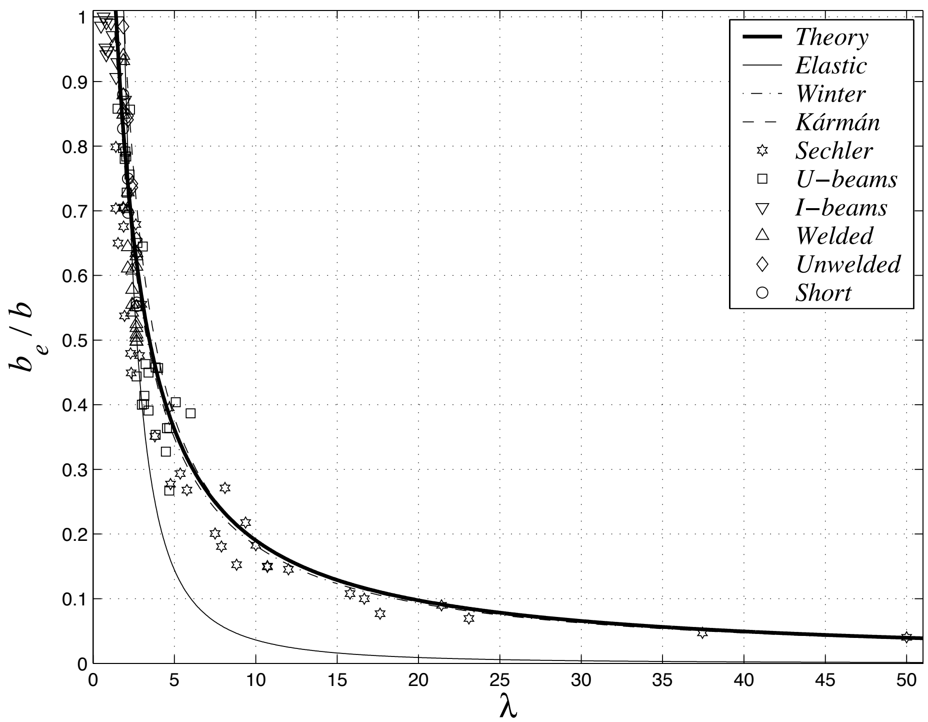


Post buckling of plates

Effective width – in practice



[Hansen., 2006]



[Hansen., 2006]

Applications
Section classification

Let's take our previous definitions,

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} \quad (18)$$

$$\sigma_{cr} = k \cdot \frac{\pi^2 E}{12(1-\nu^2)\left(\frac{b}{t}\right)^2} \quad (12)$$

And let's put them as a function of b/t:

$$\frac{b}{t} = \sqrt{\frac{\bar{\lambda}_p^2}{f_y} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}}$$

Section classification are typically made with respect to a reference material (S235), and so the above expression can be re-written as,

$$\frac{b}{t} = \sqrt{\frac{\bar{\lambda}_p^2}{235} \frac{235}{f_y} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} =$$

$$= \sqrt{\frac{235}{f_y}} \sqrt{\frac{\bar{\lambda}_p^2}{235} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} =$$

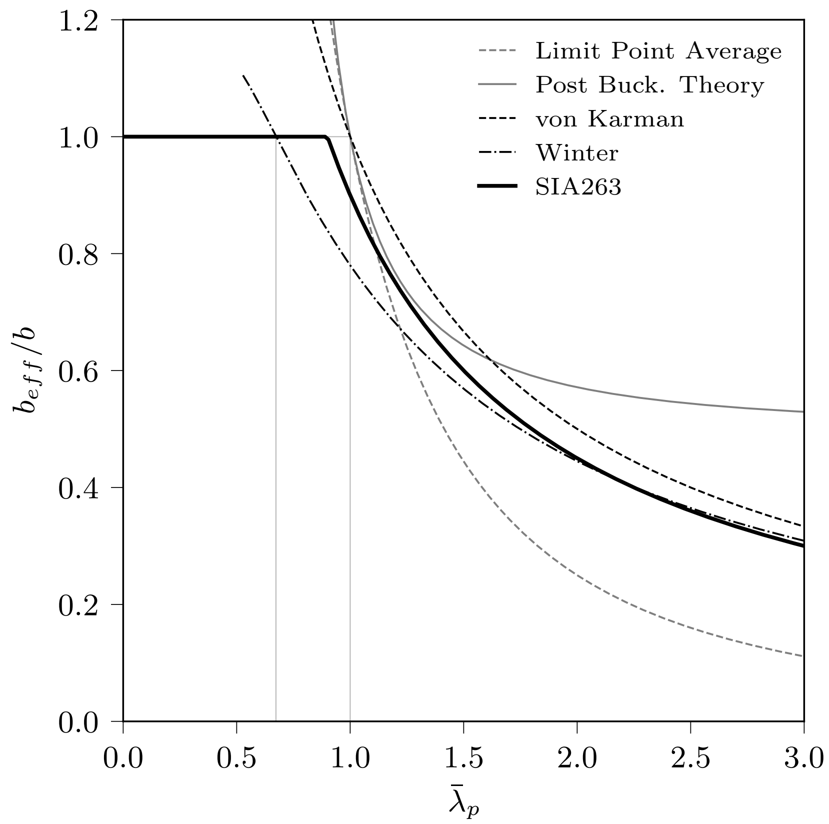
$$= \varepsilon \sqrt{\frac{\bar{\lambda}_p^2}{235} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \quad (24)$$

with, $\varepsilon = \sqrt{\frac{235}{f_y}}$

Applications
Section classification

Going from class 4 to class 3 cross-sections represents a threshold where you can consider the total plate area as being effective in carrying axial load.

$$\frac{b}{t} = \varepsilon \sqrt{\frac{\bar{\lambda}_p^2}{235} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \quad (24)$$



Applications
Section classification

a, b	ψ	Conditions de bord							
$-\sigma$ … $-\sigma$	1	4,00	6,97	5,41	5,41	1,28	1,28	0,426	0,426
$-\sigma$	0	7,81	13,54	11,73	9,54	5,91	1,608	1,702	0,567
$-\sigma$ … $+\sigma$	–1	23,90	39,52	39,52	23,94		2,134		0,851
$-\sigma$ … $-\psi\cdot\sigma$	$k_{min} \approx \dfrac{16}{\sqrt{(1+\psi)^2 + 0{,}112\,(1-\psi)^2} + (1+\psi)}$ valable pour … et $\psi \geq -1{,}2$								

Géométrie	Mode de sollicitation	Elancement limite b/t maximal: Classe de section 1	Classe de section 2	Classe de section 3
b, t, 1,5t, Panneau	Compression	$33\,\varepsilon$	$38\,\varepsilon$	$42\,\varepsilon$
	Flexion simple	$72\,\varepsilon$	$83\,\varepsilon$	$124\,\varepsilon$
	Compression avec flexion $\psi > -1$ (Comp. +)	$\dfrac{396\,\varepsilon}{13\alpha - 1}$, $\alpha \geq 0{,}5$	$\dfrac{456\,\varepsilon}{13\alpha - 1}$, $\alpha \geq 0{,}5$	$\dfrac{42\,\varepsilon}{0{,}67 + 0{,}33\,\psi}$
	Traction avec flexion $\psi \leq -1$ (Comp. +)	$\dfrac{36\varepsilon}{\alpha}$, $\alpha \leq 0{,}5$	$\dfrac{41{,}5\varepsilon}{\alpha}$, $\alpha \leq 0{,}5$	$62\,\varepsilon(1-\psi)\sqrt{-\psi}$
Cas particulier Tubes (t, D)	Flexion et/ou compression	$\dfrac{D}{t} \leq 50\,\varepsilon^2$	$\dfrac{D}{t} \leq 70\,\varepsilon^2$	$\dfrac{D}{t} \leq 90\,\varepsilon^2$

SIA263 - [SIA, 2013]

From Eq. 24,

$$f_y = 235\ MPa$$
$$E = 210GPa$$
$$\nu = 0.3$$
$$\bar{\lambda}_p = 0.75$$
$$k = 4.0$$

$$\frac{b}{t} = 42.63$$

Applications
Section classification

a, b	ψ	Conditions de bord							
$-\sigma$ … $-\sigma$	1	4,00	6,97	5,41	5,41	1,28	1,28	0,426	0,426
$-\sigma$	0	7,81	13,54	11,73	9,54	5,91	1,608	1,702	0,567
$-\sigma$ … $+\sigma$	–1	23,90	39,52	39,52	23,94		2,134		0,851
$-\sigma$ … $-\psi\cdot\sigma$	$k_{min} \approx \dfrac{16}{\sqrt{(1+\psi)^2 + 0{,}112\,(1-\psi)^2} + (1+\psi)}$ valable pour … et $\psi \geq -1{,}2$								

Géométrie	Sollicitation	bord tenu / bord libre	Elancement limite b/t maximal: Classe de section 1	Classe de section 2	Classe de section 3
b, t	Compression	f_y D f_y, b	9ε	10ε	14ε
	Compression avec flexion, bord libre comprimé	D f_y, f_y Z $\alpha \cdot b$; $\psi \cdot f_y$ Z D f_y	$\dfrac{9\varepsilon}{\alpha}$	$\dfrac{10\varepsilon}{\alpha}$	$21\varepsilon\sqrt{k_1}$
Panneau: b	Compression avec flexion, bord libre tendu	f_y D, $\alpha \cdot b$ Z f_y; f_y D Z $\psi \cdot f_y$	$\dfrac{9\varepsilon}{\alpha^{1,5}}$	$\dfrac{10\varepsilon}{\alpha^{1,5}}$	$21\varepsilon\sqrt{k_2}$
Coefficients de voilement k_1 et k_2: (ψ: rapport des contraintes)	$k_1 = 0{,}57 - 0{,}21\psi + 0{,}07\psi^2$ pour $1 \geq \psi \geq -3$; $k_2 = 0{,}578/(0{,}34+\psi)$ pour $1 \geq \psi \geq 0$; $k_2 = 1{,}7 - 5\psi + 17{,}1\psi^2$ pour $0 \geq \psi \geq -1$ (compression positive)				
Facteur de réduction pour les aciers à plus haute limite d'élasticité: $\varepsilon = \sqrt{\dfrac{235}{f_y}}$	S 235: ε = 1,0; S 275: ε = 0,924; S 355: ε = 0,814; S 460: ε = 0,715				

SIA263 - [SIA, 2013]

From Eq. 24,

$$f_y = 235\ MPa$$
$$E = 210GPa$$
$$\nu = 0.3$$
$$\bar{\lambda}_p = 0.75$$
$$k = 0.426$$

$$\frac{b}{t} = 13.9$$

Applications
Section classification

a, b	ψ	Conditions de bord							
$-\sigma$ … $-\sigma$	1	4,00	6,97	5,41	5,41	1,28	1,28	0,426	0,426
$-\sigma$	0	7,81	13,54	11,73	9,54	5,91	1,608	1,702	0,567
$-\sigma$ … $+\sigma$	–1	23,90	39,52	39,52	23,94		2,134		0,851
$-\sigma$ … $-\psi\cdot\sigma$	$k_{min} \approx \dfrac{16}{\sqrt{(1+\psi)^2 + 0{,}112\,(1-\psi)^2} + (1+\psi)}$ valable pour et $\psi \geq -1{,}2$								

Géométrie	Sollicitation	bord tenu / bord libre	Elancement limite b/t maximal: Classe de section 1	Classe de section 2	Classe de section 3
b, t	Compression	f_y, D, f_y, b	9ε	10ε	14ε
	Compression avec flexion, bord libre comprimé	D, f_y, Z, $\alpha\cdot b$, $\psi\cdot f_y$	$\dfrac{9\varepsilon}{\alpha}$	$\dfrac{10\varepsilon}{\alpha}$	$21\varepsilon\sqrt{k_1}$
Panneau: b	Compression avec flexion, bord libre tendu	f_y, D, $\alpha\cdot b$, Z, $\psi\cdot f_y$	$\dfrac{9\varepsilon}{\alpha^{1,5}}$	$\dfrac{10\varepsilon}{\alpha^{1,5}}$	$21\varepsilon\sqrt{k_2}$
Coefficients de voilement k_1 et k_2: (ψ: rapport des contraintes)	$k_1 = 0{,}57 - 0{,}21\psi + 0{,}07\psi^2$ pour $1 \geq \psi \geq -3$; $k_2 = 0{,}578/(0{,}34+\psi)$ pour $1 \geq \psi \geq 0$; $k_2 = 1{,}7 - 5\psi + 17{,}1\psi^2$ pour $0 \geq \psi \geq -1$	(compression positive)			
Facteur de réduction pour les aciers à plus haute limite d'élasticité: $\varepsilon = \sqrt{\dfrac{235}{f_y}}$	S 235: $\varepsilon = 1{,}0$; S 275: $\varepsilon = 0{,}924$; S 355: $\varepsilon = 0{,}814$; S 460: $\varepsilon = 0{,}715$				

SIA263 - [SIA, 2013]

From Eq. 24,

$$f_y = 235\ MPa$$
$$E = 210GPa$$
$$\nu = 0.3$$
$$\bar{\lambda}_p = 0.75$$
$$k = 0.851$$

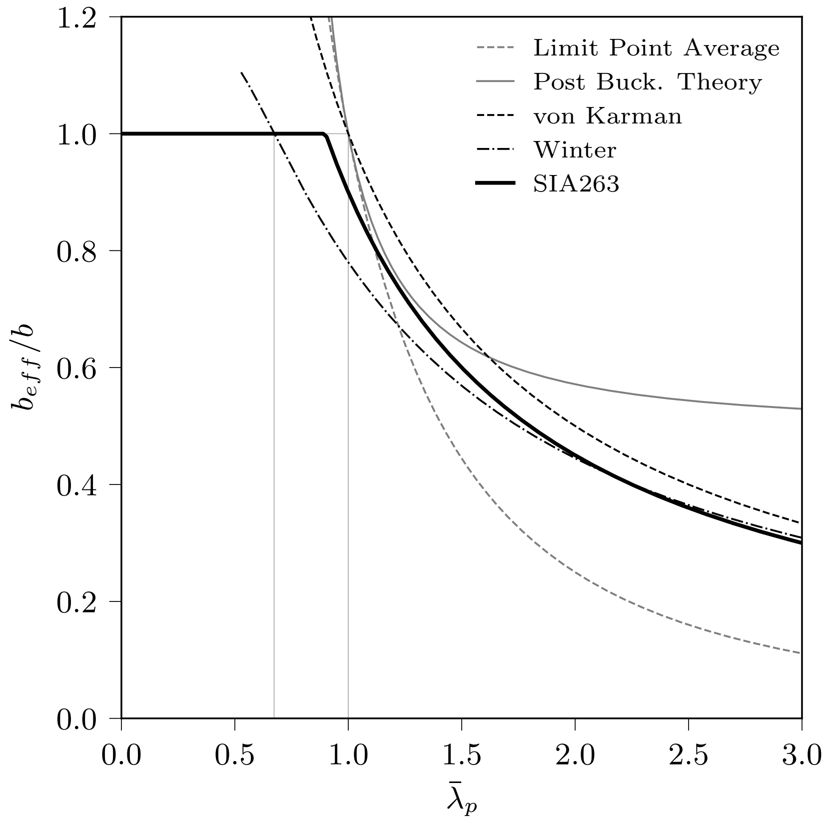
$$\frac{b}{t} = 19.66$$

$$21 * (0.57 - 0.21 * -1 + 0.07)^{0.5} = 19.36$$

Applications
Section classification

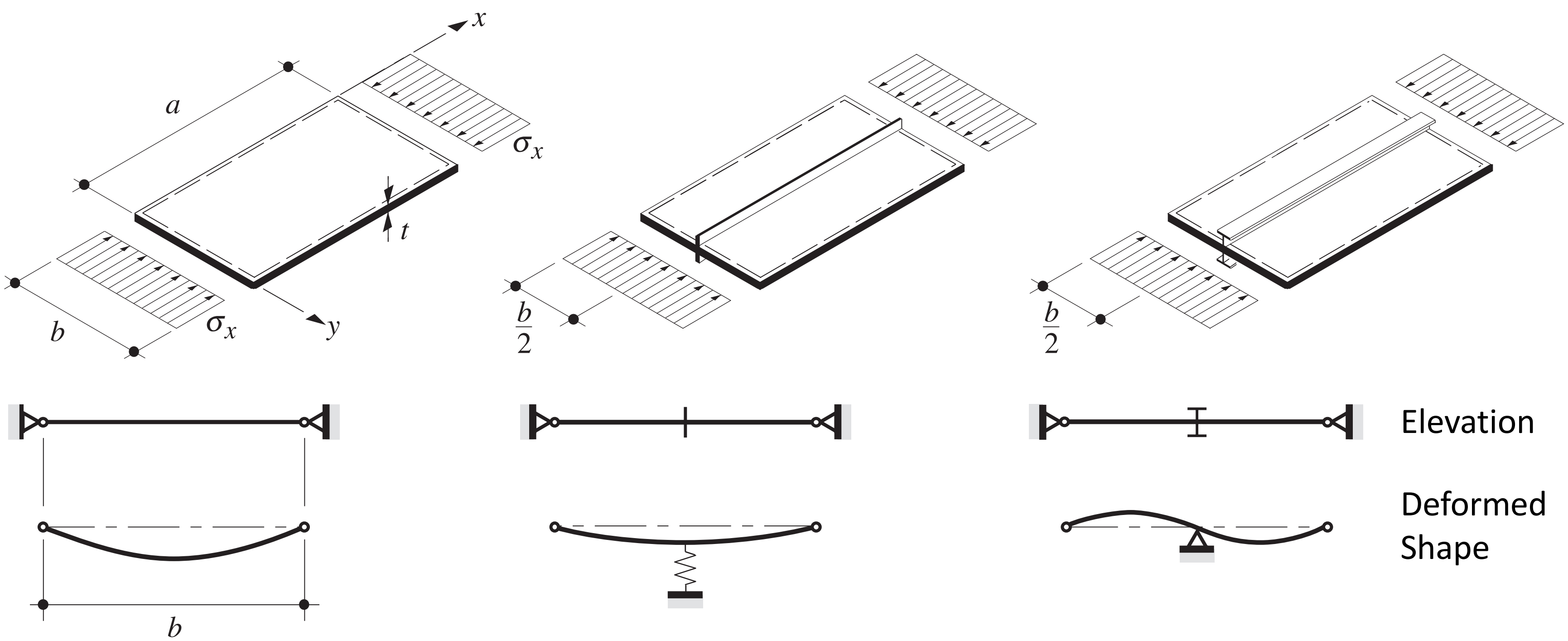
Going from **class 4 to class 3** cross-sections represents a threshold where you can consider the total plate area as being effective in carrying axial load.

$$\frac{b}{t} = \varepsilon \sqrt{\frac{\bar{\lambda}_p^2}{235} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \quad (24)$$



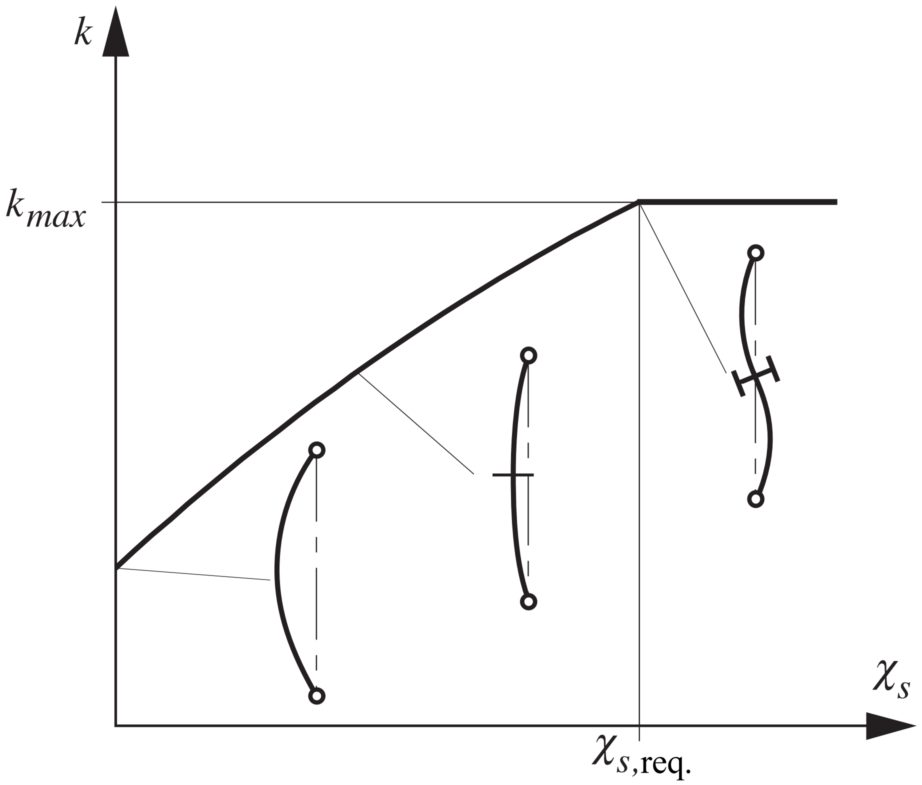
However, plate buckling is **not the only** criterion influencing section classification. Rotation capacity of members also play a role and so the $\bar{\lambda}_p = 0.75$ is to be taken more as an indicative value rather than a hard limit for class 3 sections.

Applications
Stiffened plates – relative stiffness and optimal positions



TGC 10 - [Hirt et al., 2011]

Applications
Stiffened plates – relative stiffness and optimal positions



TGC 10 - [Hirt et al., 2011]

Let's define the ratio between the stiffener and the plate bending stiffnesses as χ_s,

$$\chi_s = \frac{EI_s}{Db} \quad (25)$$

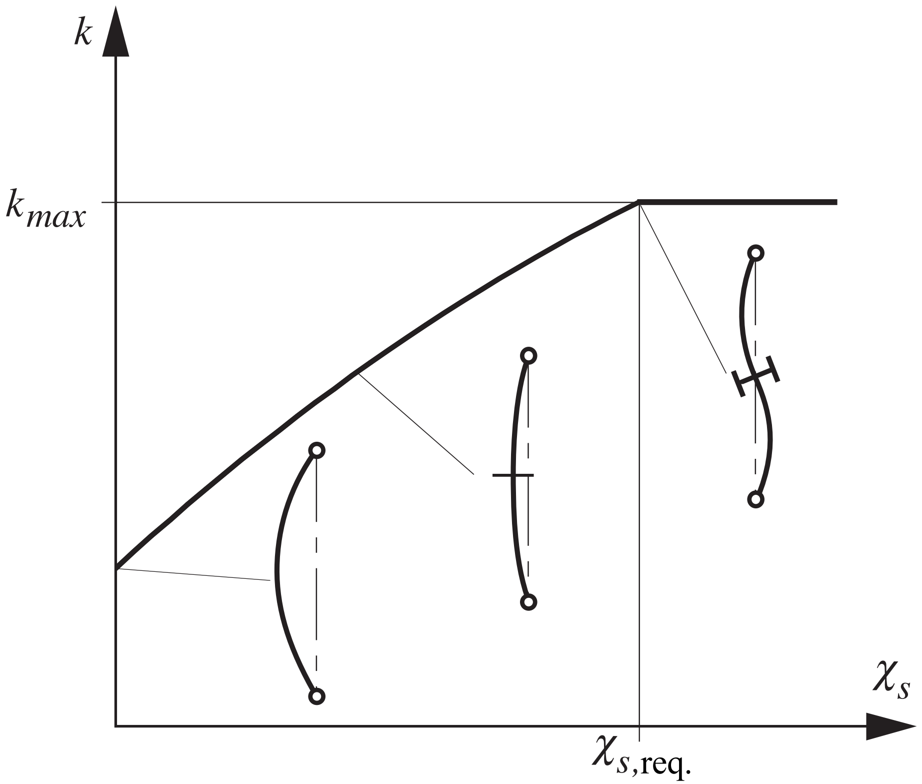
with,

I_s - the stiffener moment of inertia as **measured from the plate's middle plane**

D – the plate's bending stiffness as defined in slide 20

Applications

Stiffened plates – relative stiffness and optimal positions



TGC 10 - [Hirt et al., 2011]

Fixity is reached when χ_s is greater than a a required threshold we call $\chi_{s,req}$

$$\chi_s \geq \chi_{s,req} \; (26)$$

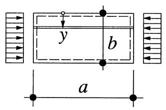
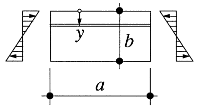
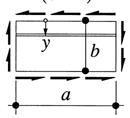
Determining the required stiffness ratio depends on a number of variables, namely:

1. The plate length and width ratio a/b
2. The relative area of the stiffener and the plate - δ_s

$$\delta_s = \frac{A_s}{bt} \; (27)$$

EPFL Applications

Stiffened plates – relative stiffness and optimal positions

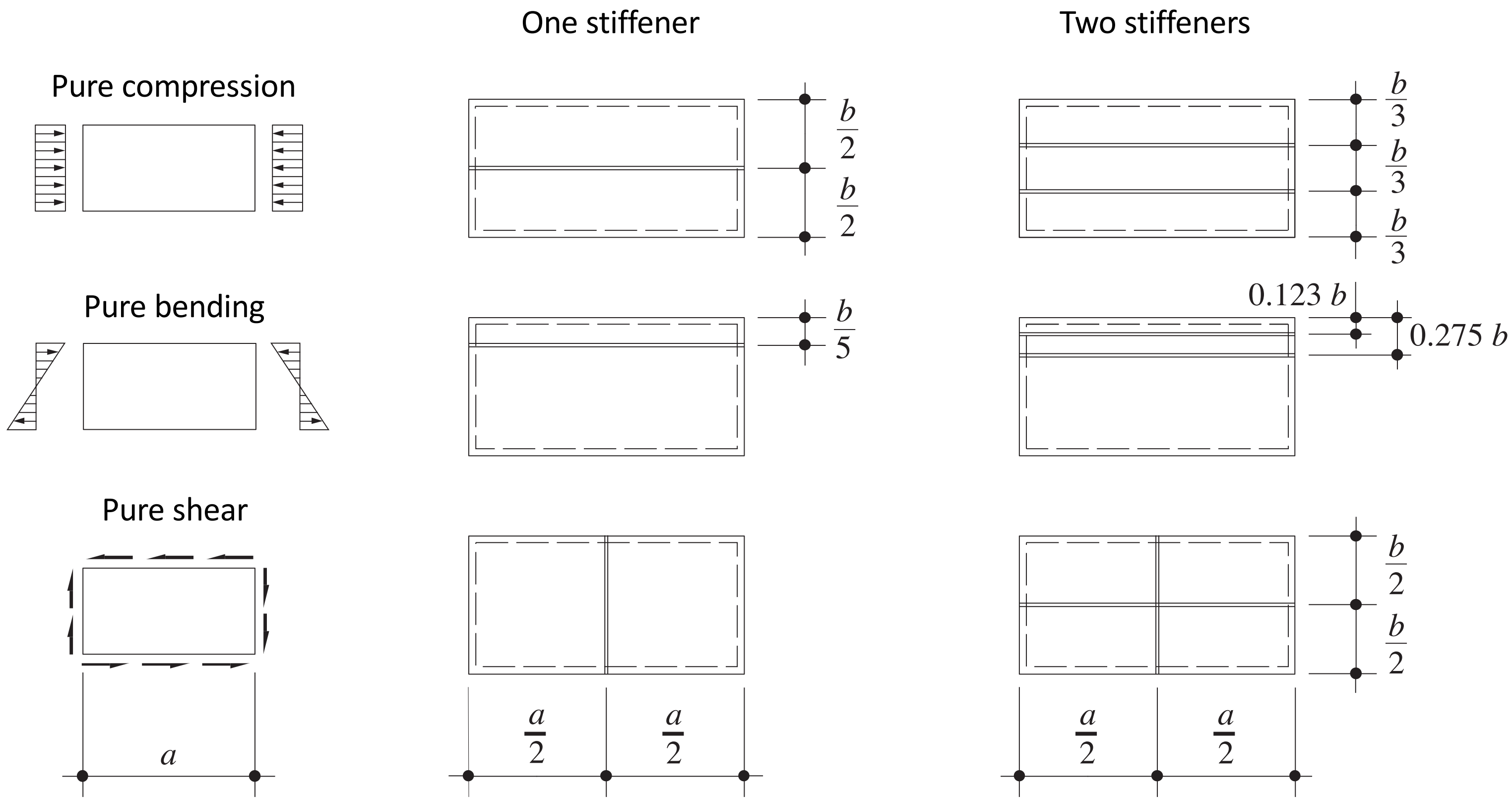
	y	$\alpha = \frac{a}{b}$	Rigidité relative nécessaire $\chi_{s,néc}$
Compression pure ($\tau = 0$) 	$\frac{b}{2}$	$\alpha \leq \sqrt{8\,(1+2\,\delta_s)-1}$	$-\frac{1}{2}\,\alpha^4 + [8\,(1+2\,\delta_s)-1]\,\alpha^2 + \delta_s + \frac{1}{2}$
		$\alpha \geq \sqrt{8\,(1+2\,\delta_s)-1}$	$\frac{1}{2}\,[8\,(1+2\,\delta_s)-1]^2 + \delta_s + \frac{1}{2}$
	$\frac{b}{3}$ et $\frac{2b}{3}$	$\alpha \leq \sqrt{18\,(1+3\,\delta_s)-1}$	$-\frac{1}{3}\,\alpha^4 + \frac{2}{3}\,[18\,(1+3\,\delta_s)-1]\,\alpha^2 + \delta_s + \frac{1}{3}$
		$\alpha \geq \sqrt{18\,(1+3\,\delta_s)-1}$	$\frac{1}{3}\,[18\,(1+3\,\delta_s)-1]^2 + \delta_s + \frac{1}{3}$
Flexion pure ($\tau = 0$) 	$\frac{b}{2}$	$\alpha \geq 0.5$	1.5
	$\frac{b}{3}$	$0.5 \leq \alpha \leq \sqrt{1.6 + 8\,\delta_s}$	$-1.1\,\alpha^4 + (3.5 + 17.6\,\delta_s)\,\alpha^2 + 0.7$
		$\alpha \geq \sqrt{1.6 + 8\,\delta_s}$	$3.4 + 27.7\,\delta_s + 70.4\,\delta_s^2$
	$\frac{b}{4}$	$0.5 \leq \alpha \leq \sqrt{3.06 + 17.4\,\delta_s}$	$-1.21\,\alpha^4 + (7.41 + 42\,\delta_s)\,\alpha^2 + 1$
		$\alpha \geq \sqrt{3.06 + 17.4\,\delta_s}$	$12.3 + 130\,\delta_s + 370\,\delta_s^2$
	$\frac{b}{5}$	$0.5 \leq \alpha \leq \sqrt{5.14 + 25.2\,\delta_s}$	$-1.54\,\alpha^4 + (15.82 + 77.6\,\delta_s)\,\alpha^2 + 3.55$
		$\alpha \geq \sqrt{5.14 + 25.2\,\delta_s}$	$43.4 + 381\,\delta_s + 1080\,\delta_s^2$
	$\frac{b}{3}$ et $\frac{2b}{3}$	$\alpha \geq 0.7$	$(3.33 + 16.67\,\delta_s)\,(\alpha - 0.1)$ mais $\chi_{s,néc} \leq 26\,\delta_s + 3$
	$\frac{b}{4}$ et $\frac{b}{2}$	$\alpha \geq 0.5$	$(7 + 50\,\delta_s + 27\,\delta_s^2)\,\alpha - (1.6 + 14.5\,\delta_s + 13.5\,\delta_s^2)$ mais $\chi_{s,néc} \leq 8 + 70\,\delta_s + 200\,\delta_s^2$
Cisaillement pur ($\sigma = 0$) 	$\frac{b}{2}$	$0.7 \leq \alpha \leq 1.1$	$210\,(\alpha - 0.4)^4 + 7,5$
		$1.1 \leq \alpha \leq 3.5$	$18\,(4.1 - \alpha)\,(\alpha - 2.1)^3 + 108\,(\alpha - 2.1) + 220$
	$\frac{b}{3}$	$0.7 \leq \alpha \leq 1.6$	$(10 - 2.38\,\alpha)\,\alpha^4$
		$1.6 \leq \alpha \leq 3.0$	$90.6 - [35\,/\,(\alpha - 0.9)]$
	$\frac{b}{4}$	$0.5 \leq \alpha \leq 1.5$	$4.16\,\alpha^4 + 5\,(\alpha - 1)\,(\alpha - 0.7)$
		$1.5 \leq \alpha \leq 4.0$	$40 - [5.1\,/\,(\alpha - 1.2)]$
	$\frac{b}{5}$	$0.5 \leq \alpha \leq 1.3$	$19.5\,(\alpha - 0.5)^3 + 0.5$
		$1.3 \leq \alpha \leq 2.5$	$35 - 17\,(2.5 - \alpha)^2$
	$\frac{b}{3}$ et $\frac{2b}{3}$	$0.5 \leq \alpha \leq 1.0$	$50\,\alpha^3 + 10\,\alpha$
		$1.0 \leq \alpha \leq 2.2$	$\frac{1}{3}\,(550\,\alpha - 370)$
		$2.2 \leq \alpha \leq 3.5$	$370 - 41\,(3.5 - \alpha)^3$
	$\frac{b}{4}$ et $\frac{b}{2}$	$0.7 \leq \alpha \leq 2.5$	$34.4\,\alpha - 19$
		$2.5 \leq \alpha \leq 4.0$	$66\,\alpha - 98$

TGC 10 - [Hirt et al., 2011], From Klöppel and Scheer[1960] tables

Applications

Stiffened plates – relative stiffness and optimal positions

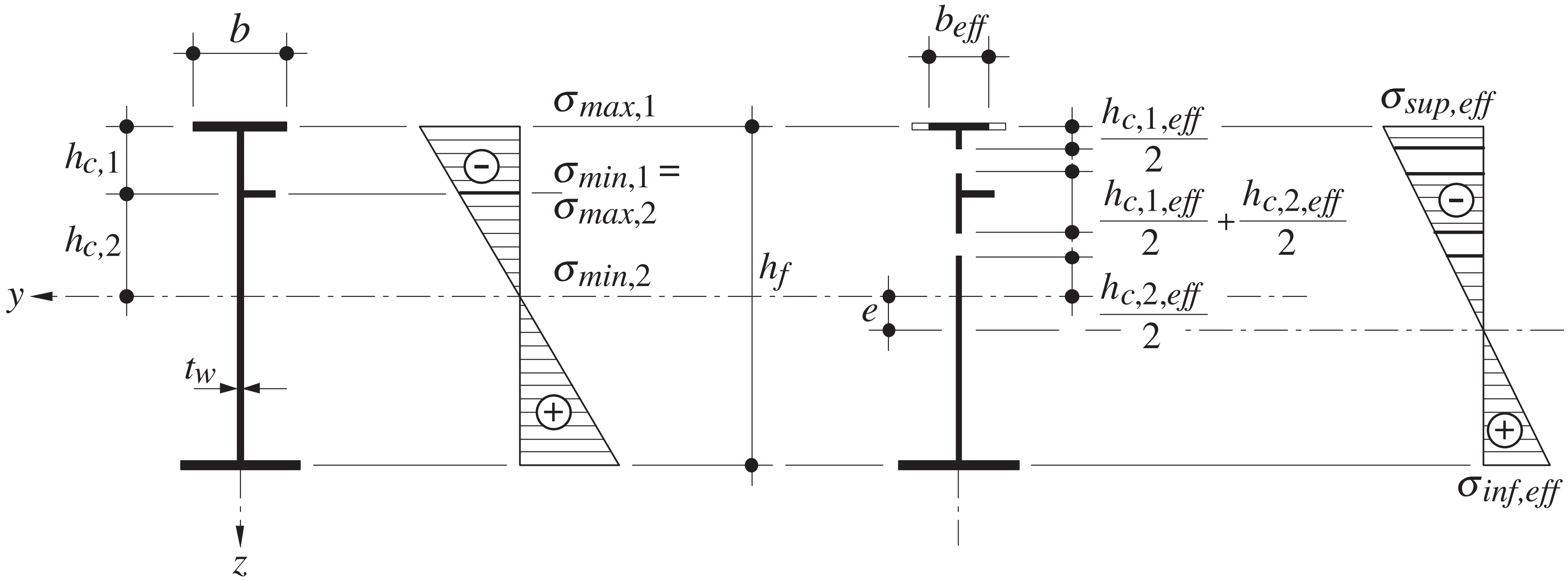
Optimal stiffener positions



TGC 10 - [Hirt et al., 2011]

Applications

Class 4 cross-sections



Gross section

Effective section

TGC 12 - [Lebet and Hirt, 2009]

Applications

Class 4 cross-sections – moment resistance – the big picture

Steps in calculating moment resistance of longitudinally stiffened class 4 cross-sections:

1. Ensure that the stiffener is sufficient rigid so as to consider it a laterally restrained boundary;

2. Determine effective widths for each element in the cross section (slenderness checks);

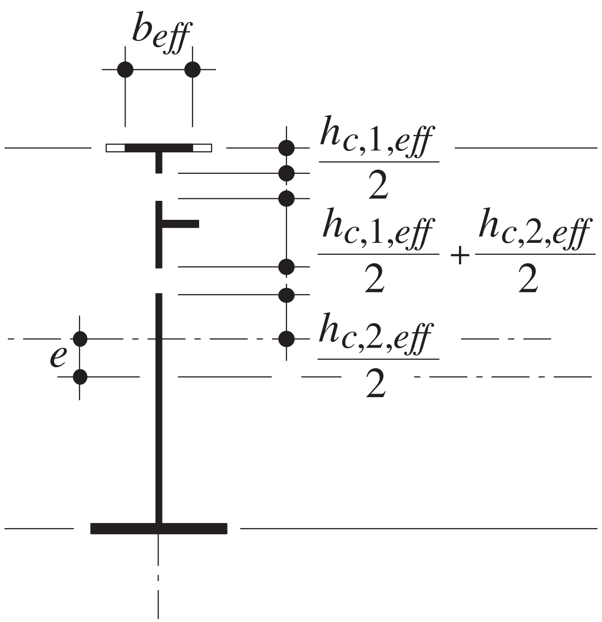
3. Calculate effective geometric properties for the cross section as a whole;

4. Check if you need to re-compute external loading with respect to new neutral axis (i.e. eccentricity + axial load induces an additional moment in the cross-section);

5. Perform code verifications;

Applications

Class 4 cross-sections – longitudinal stiffener design



TGC 12 - [Lebet and Hirt, 2009]

The TGC 12 discusses two criteria to design longitudinal stiffeners,

1. Imposing a minimum required relative stiffness - χ_s
2. Design the stiffener as an equivalent column whose resistance is greater than that required by the moment action - M_{Ed}

In approach 1. for bridge sections it is recommended that,

$$\chi_s \geq m\, \chi_{s,req} \quad (28)$$

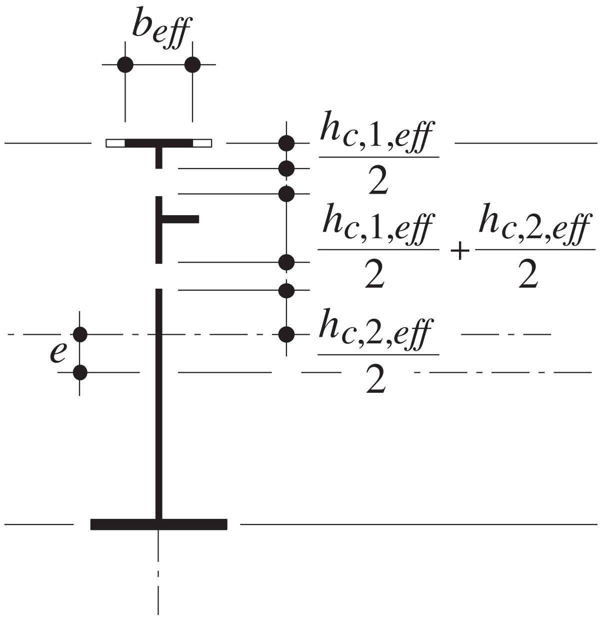
With,

m =5 for open-section stiffeners (no torsional stiffness)

m =3 for closed-section stiffeners (torsional stiffness)

Applications

Class 4 cross-sections – longitudinal stiffener design



TGC 12 - [Lebet and Hirt, 2009]

The TGC 12 discusses two criteria to design longitudinal stiffeners,

1. Imposing a minimum required relative stiffness - χ_s
2. Design the stiffener as an equivalent column whose resistance is greater than that required by the moment action - M_{Ed}

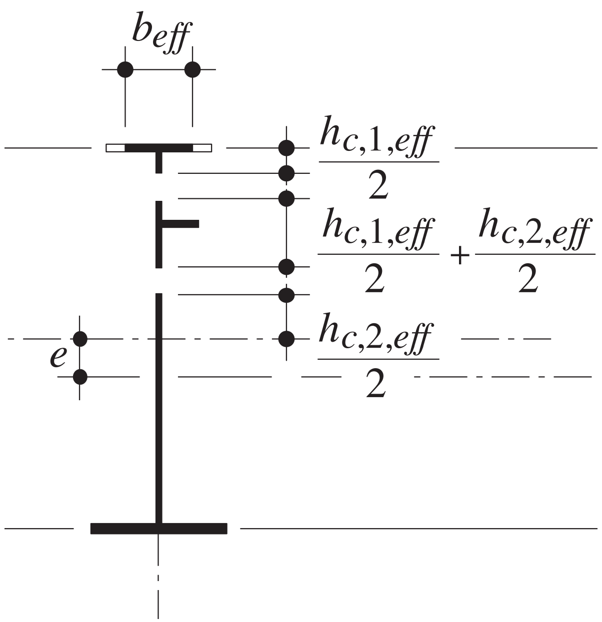
In approach 2.

$$N_{Ed} = \sigma_{s,Ed} A_{s,eff} \leq N_{K,Rd} = \frac{\chi_K f_y A_{s,eff}}{\gamma_a} \quad (29)$$

$$A_{s,eff} = A_s + \frac{h_{c,1,eff}}{2} t_w + \frac{h_{c,2,eff}}{2} t_w$$

Applications

Class 4 cross-sections – longitudinal stiffener design



TGC 12 - [Lebet and Hirt, 2009]

$$N_{Ed} = \sigma_{s,Ed} A_{s,eff} \leq N_{K,Rd} = \frac{\chi_K f_y A_{s,eff}}{\gamma_a} \quad (29)$$

$$A_{s,eff} = A_s + \frac{h_{c,1,eff}}{2} t_w + \frac{h_{c,2,eff}}{2} t_w$$

With,

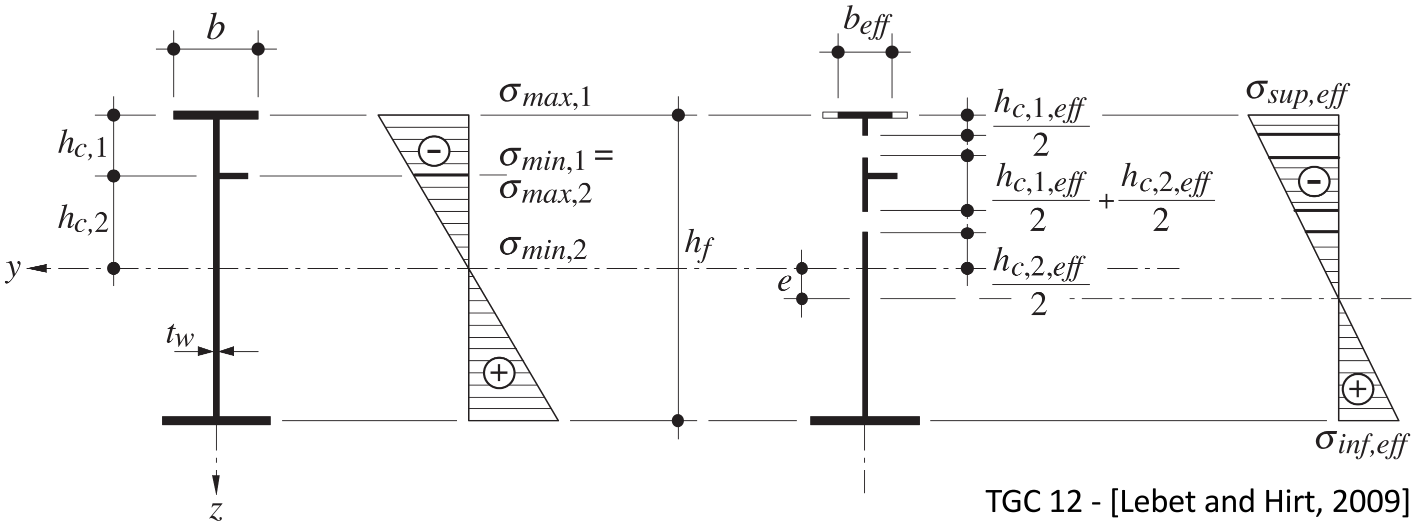
A_s - Stiffener area

$\sigma_{s,Ed}$ - Stiffener stress

$A_{s,eff}$ - Equivalent column area

χ_K - Equivalent column buckling reduction factor (seen in previous lectures) with imperfection factors $\alpha_K = 0.49$ for closed-section stiffeners and $\alpha_K = 0.64$ for open-section stiffeners. Column length l_K taken between **rigid** vertical stiffeners

Applications
Class 4 cross-sections – effective widths



TGC 12 - [Lebet and Hirt, 2009]

Before calculating the effective widths for longitudinally stiffened plates, it is important to mention when these situations arise. Such cases usually happen when the web thickness is small (webs are too slender). This is usually assessed by two criteria:

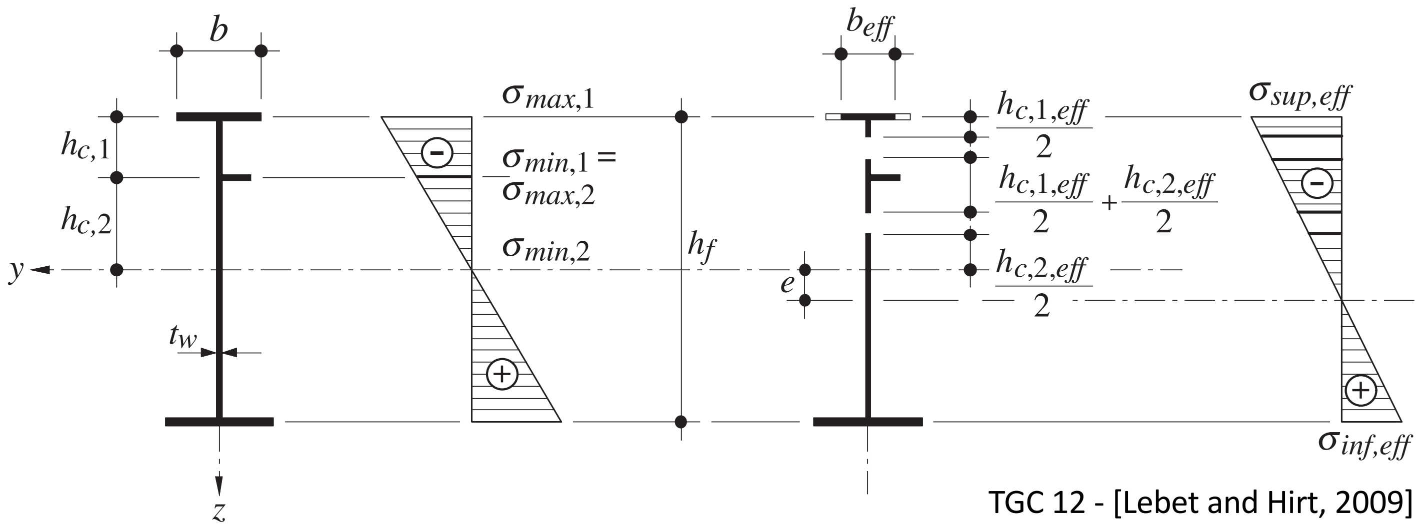
1 - Web Euler buckling:

$$\frac{h_f}{t_w} \leq 0.40\ \frac{E}{f_y} \quad (30)$$

If Eq. 30 is not satisfied then add a stiffener such that,

$$\frac{\frac{h_f}{2} + h_{c,2}}{t_w} \leq 0.40\ \frac{E}{f_y}$$

Applications
Class 4 cross-sections – effective widths



TGC 12 - [Lebet and Hirt, 2009]

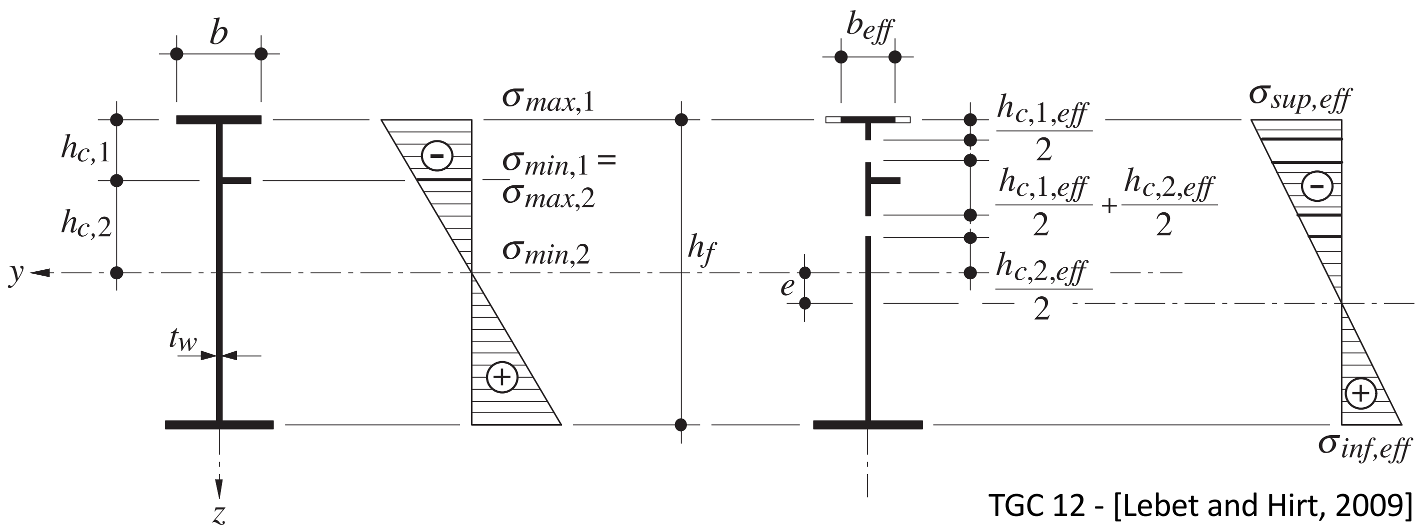
Before calculating the effective widths for longitudinally stiffened plates, it is important to mention when these situations arise. Such cases usually happen when the web thickness is small (webs are too slender). This is usually assessed by two criteria:

2 - Web breathing (fatigue): If Eq. 31 is not satisfied then add a stiffener such that,

$$\frac{h_{c,1} + h_{c,2}}{t_w} = \frac{h_c}{t_w} \leq 100 \ (31)$$

$$\frac{h_{c,1}}{t_w} \leq 100$$

Applications
Class 4 cross-sections – effective widths



TGC 12 - [Lebet and Hirt, 2009]

Let's use for the effective width the SIA263 criterion:

$$\frac{b_{eff}}{b} = \frac{0.9}{\bar{\lambda}_p} \ (23)$$

From Eq. (24),

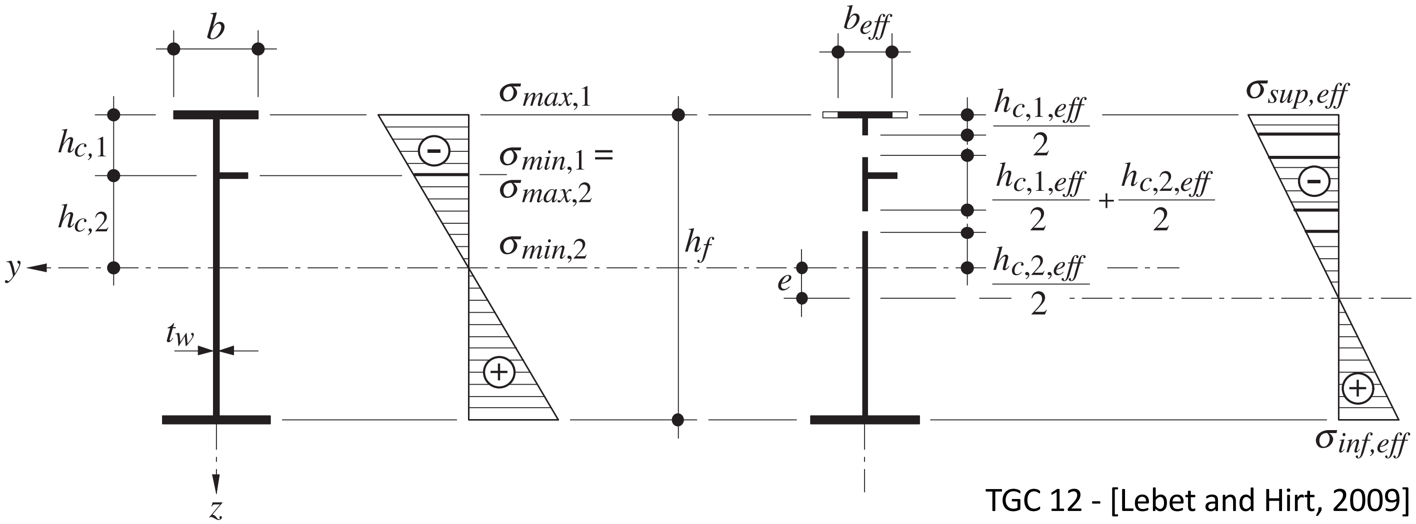
$$\frac{b}{t} = \sqrt{\frac{\bar{\lambda}_p^2}{f_y} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \ (24)$$

Combining both Equations,

$$b_{eff} = 0.9 \cdot \sqrt{\frac{1}{f_y} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \cdot t \ (32)$$

Applications

Class 4 cross-sections – effective widths



TGC 12 - [Lebet and Hirt, 2009]

Now let's try to apply Eq. 32 to our web panel. Let's at the top patch (1),

$$b_{eff} = 0.9 \cdot \sqrt{\frac{1}{f_y} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \cdot t \ (32) \Longrightarrow h_{c,1,eff} = 0.86\sqrt{k \cdot \frac{E}{f_y}} \cdot t_w$$

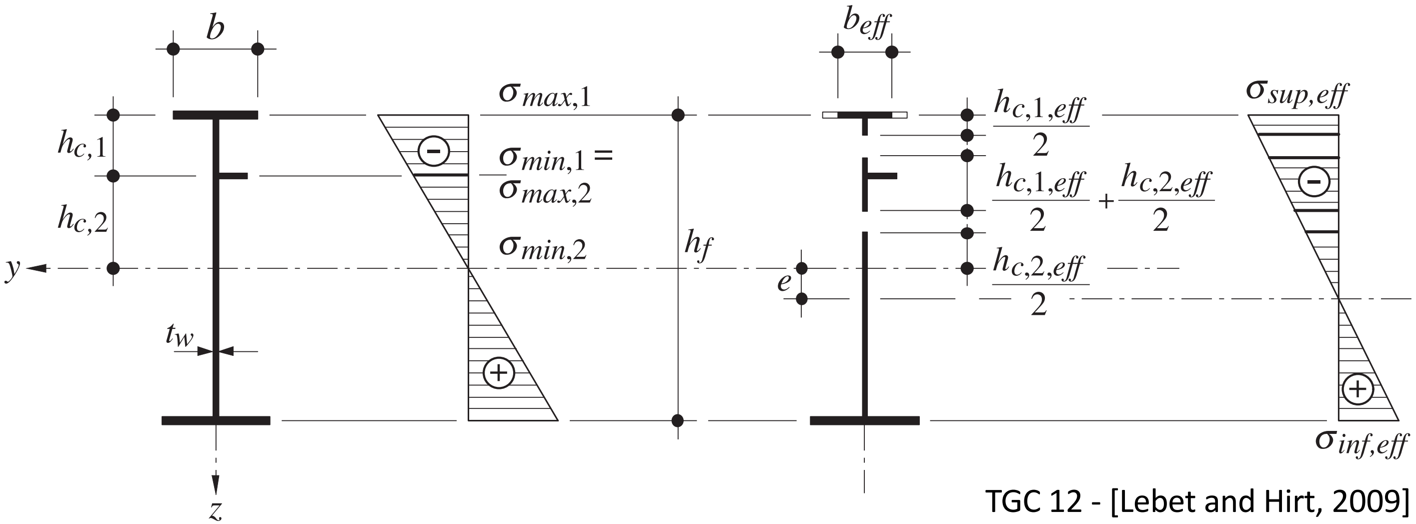
With k from SIA263's table in slide 29,

$$k = \frac{16}{\sqrt{(1+\psi)^2 + 0.112(1-\psi)^2} + 1 + \psi} \text{ and } \psi = \frac{\sigma_{min,1}}{\sigma_{max,1}}$$

EPFL

Applications

Class 4 cross-sections – effective widths



TGC 12 - [Lebet and Hirt, 2009]

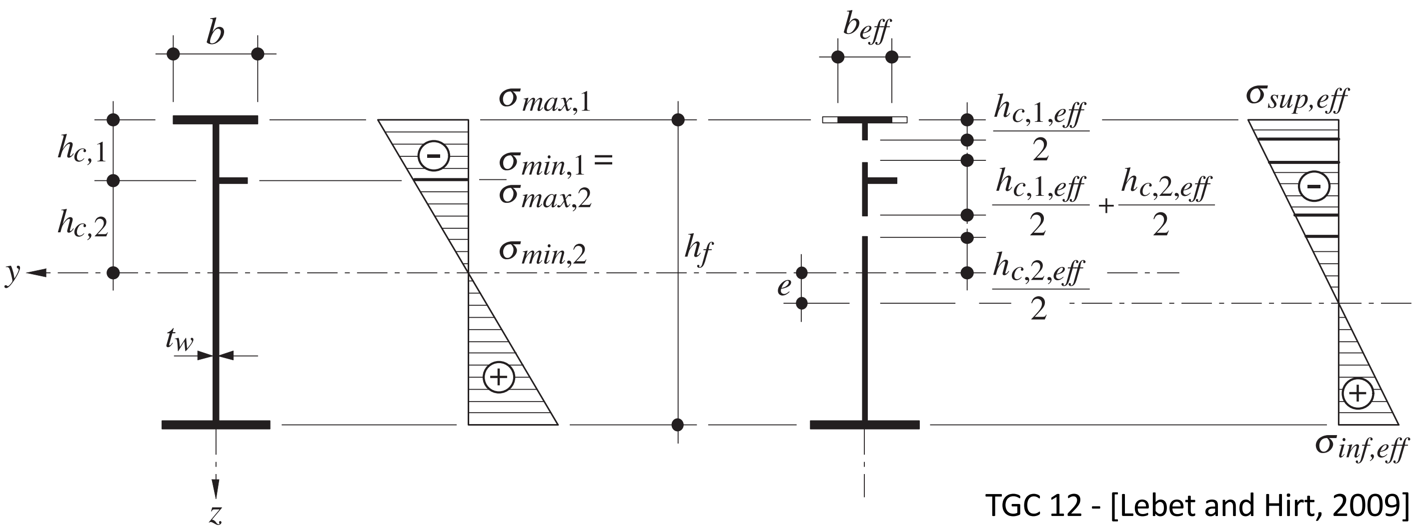
Now let's try to apply Eq. 32 to our web panel. Let's at the top patch (1),

$$b_{eff} = 0.9 \cdot \sqrt{\frac{1}{f_y} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \cdot t \ (32) \Longrightarrow h_{c,1,eff} = 0.86\sqrt{k \cdot \frac{E}{f_y}} \cdot t_w$$

The effective width for patch 1 is then assigned to **equally to each edge** of the patch, that is, $\frac{h_{c,1,eff}}{2}$ next to the top flange and $\frac{h_{c,1,eff}}{2}$ on the upper side of the stiffener.

Applications

Class 4 cross-sections – effective widths



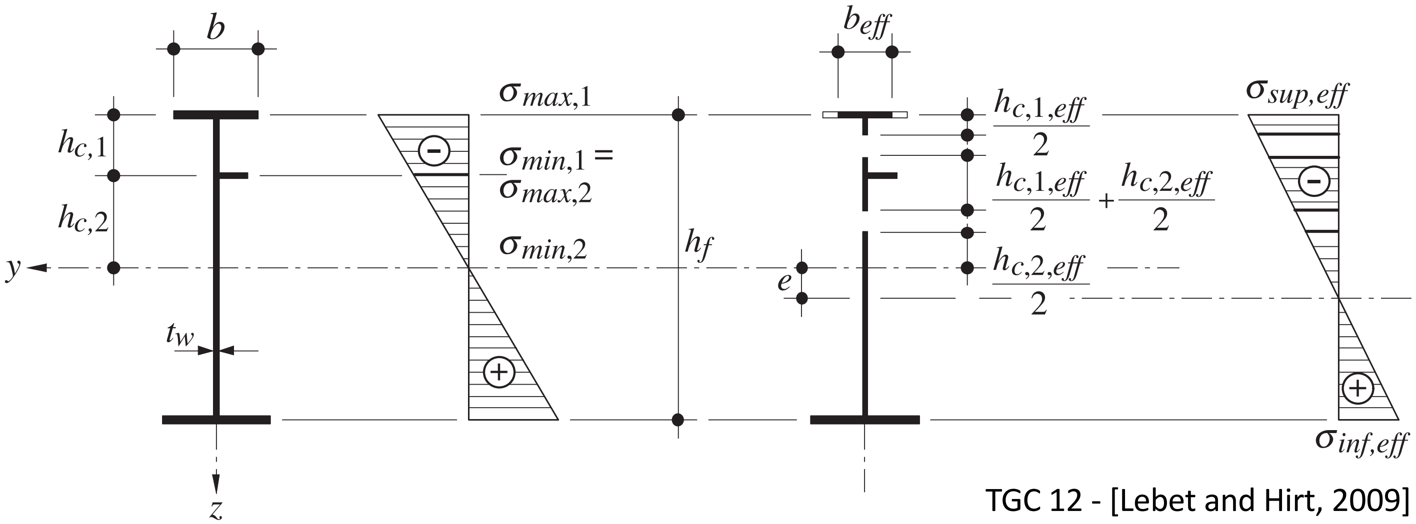
TGC 12 - [Lebet and Hirt, 2009]

Let's look now at the lower compressed patch (2). This patch has the particularity that it is not compressed throughout height of the panel it belongs to. In such cases, there is **an important modification** to Eq. (32), in which the effective width is calculated proportionally to the compressed portion of the total width (b_c), that is,

$$b_{eff} = \frac{b_c}{b} \cdot 0.9 \cdot \sqrt{\frac{1}{f_y} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \cdot t \ (33) \Rightarrow h_{c,2,eff} = \frac{h_{c,2}}{h_{c,2}+h_f/2} 0.86 \sqrt{k \cdot \frac{E}{f_y}} \cdot t_w$$

Applications

Class 4 cross-sections – effective widths



TGC 12 - [Lebet and Hirt, 2009]

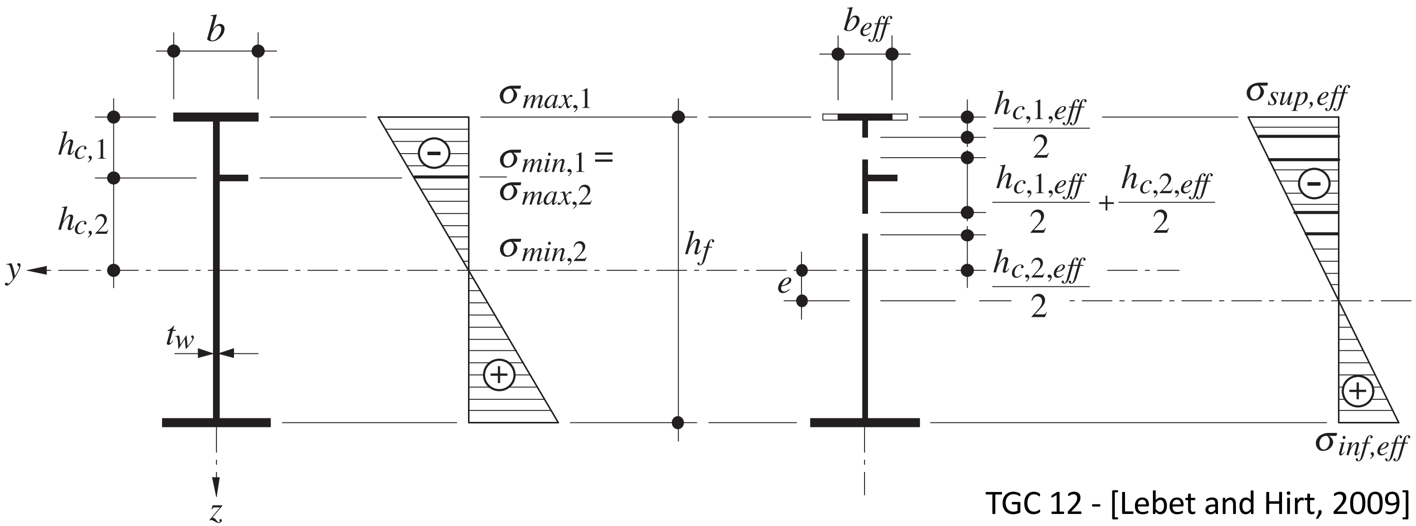
$$h_{c,2,eff} = \frac{h_{c,2}}{h_{c,2} + h_f/2} 0.86 \sqrt{k \cdot \frac{E}{f_y}} \cdot t_w$$

Again with k from SIA263's table in slide 29,

$$k = \frac{16}{\sqrt{(1+\psi)^2 + 0.112(1-\psi)^2} + 1 + \psi} \text{ and } \psi = \frac{\sigma_{inf}}{\sigma_{max,2}} \geq -1.2$$

Applications

Class 4 cross-sections – effective widths

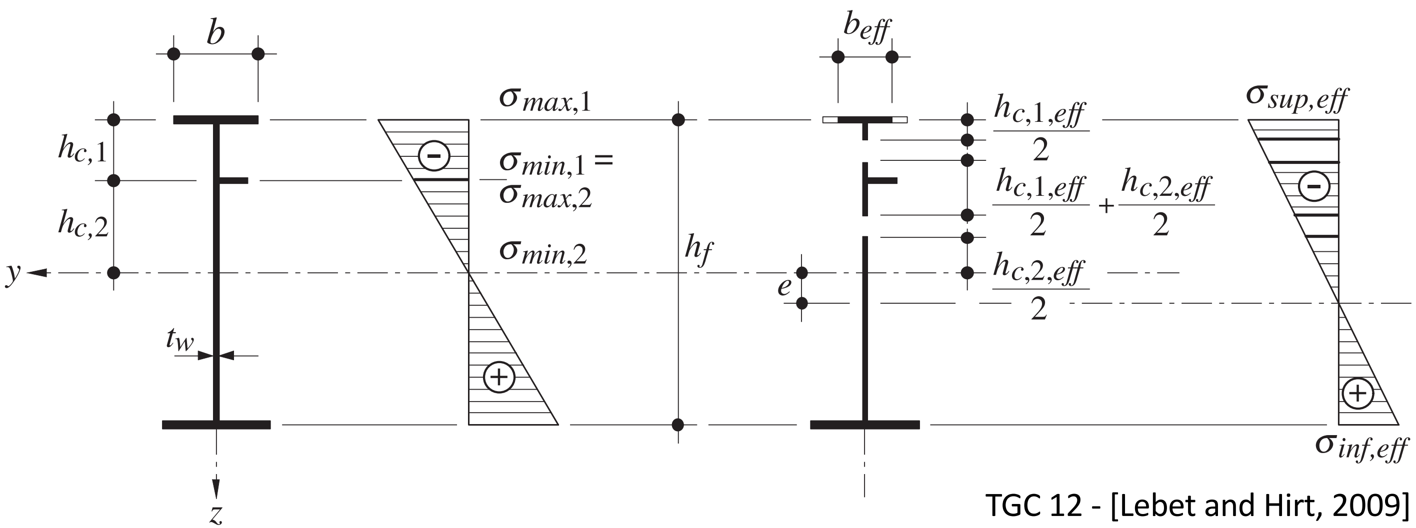


TGC 12 - [Lebet and Hirt, 2009]

$$h_{c,2,eff} = \frac{h_{c,2}}{h_{c,2} + h_f/2} 0.86 \sqrt{k \cdot \frac{E}{f_y}} \cdot t_w$$

The effective width for patch 2 is then assigned to **equally** between the compressed edge of the panel and the point of zero uniaxial stress, that is, $\frac{h_{c,2,eff}}{2}$ next to stiffener and $\frac{h_{c,2,eff}}{2}$ on the upper side of the gross-section's neutral axis.

Applications
Class 4 cross-sections – effective widths



TGC 12 - [Lebet and Hirt, 2009]

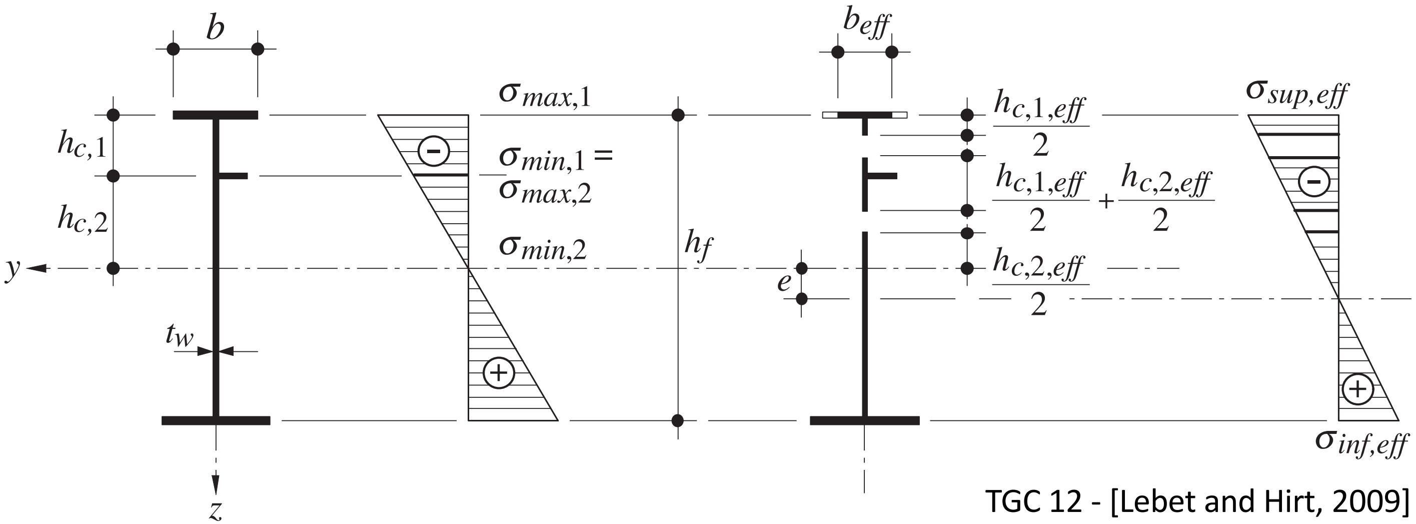
The next effective width to calculate is that of the compressed top flange. This can be done similarly to what but (i) with a cantilever model from the web ($b_{eff}/2$) and (ii) corresponding plate buckling factor k.

$$b_{eff} = 0.9 \cdot \sqrt{\frac{1}{f_y} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \cdot t \; (32) \Longrightarrow \frac{b_{eff}}{2} = 0.86\sqrt{k \cdot \frac{E}{f_y}} \cdot t_{f,top}$$

With k from SIA263's table in slide 29, $\quad k = 0.426$

Applications

Class 4 cross-sections – effective geometric properties



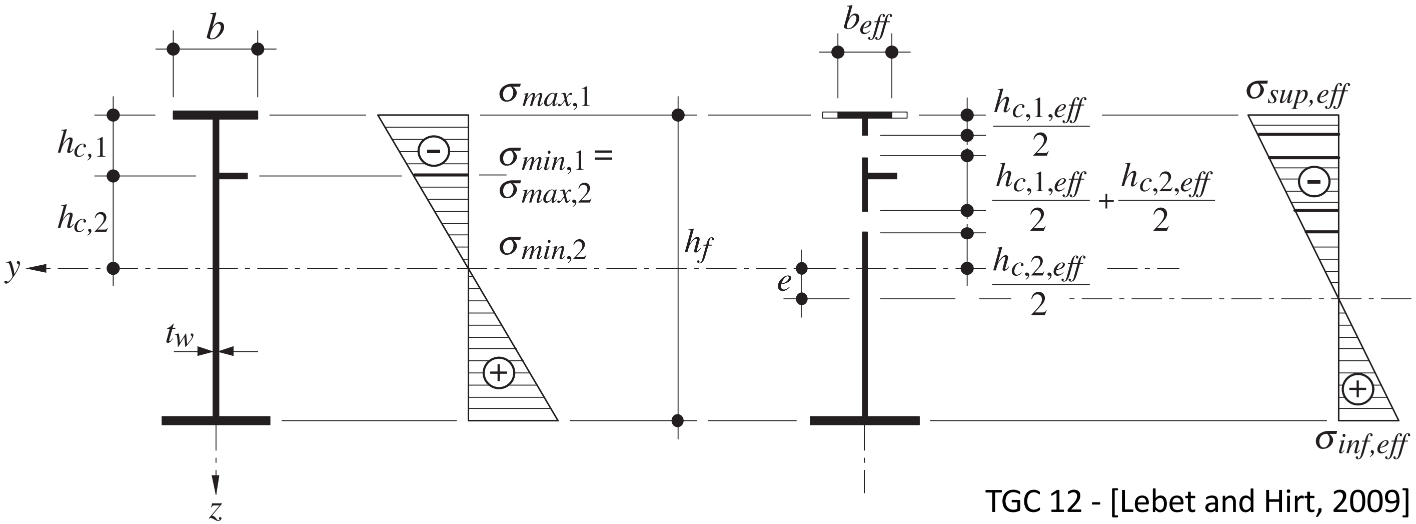
TGC 12 - [Lebet and Hirt, 2009]

Having calculated all the relevant effectives widths, the next step in cross-section verification involves in computing:

1. Effective area A_{eff} of the cross section (sum of all areas effective areas);
2. The position of effective neutral axis and obtaining eccentricity e (the difference in position between the gross and effective axes);
3. Effective moment of inertia I_{eff} (a systematic application of Steiner's theorem);
4. Section modulus to the farthest fiber, e.g. $W_{c,eff} = \frac{I_{eff}}{h_c + e}$

Applications

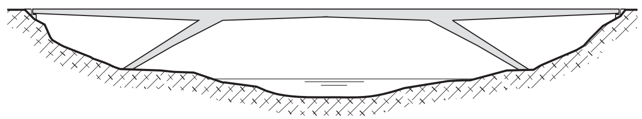
Class 4 cross-sections – eccentricity load update

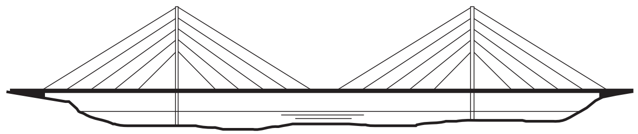


TGC 12 - [Lebet and Hirt, 2009]

If the load combination under analysis has axial load, then care must be taken to the fact the added eccentricity induces an extra moment in the cross,

$$\Delta M_y = \mathrm{N_{Ed}} \cdot e_z$$





TGC 12 –
[Lebet and Hirt, 2009]

Applications

Class 4 cross-sections – code checks

Cross section resistance in SIA 263 (§5.3.5) follows,

$$\sigma_{Ed} = \frac{\mathrm{N_{Ed}}}{A_{eff}} + \frac{\mathrm{M_{y,Ed}} + \mathrm{N_{Ed}} e_z}{W_{y,eff}} + \frac{\mathrm{M_{z,Ed}} + \mathrm{N_{Ed}} e_y}{W_{z,eff}} \leq \frac{f_y}{\gamma_{M1}} \quad (34)$$

And stability checks in SIA 263 (§5.3.9) follow,

$$\sigma_{Ed} = \frac{\mathrm{N_{Ed}}}{A_{eff}} + \frac{1}{1 - \frac{N_{Ed}}{N_{cr,y}}} \frac{\mathrm{M_{y,Ed}} + \mathrm{N_{Ed}}(e_z + w_{0,z})}{W_{y,eff}} + \frac{\mathrm{M_{z,Ed}} + \mathrm{N_{Ed}}(e_y + w_{0y})}{W_{z,eff}} \leq \frac{f_y}{\gamma_{M1}} \quad (35)$$

$$\sigma_{Ed} = \frac{\mathrm{N_{Ed}}}{A_{eff}} + \frac{\mathrm{M_{y,Ed}} + \mathrm{N_{Ed}}(e_z + w_{0,z})}{W_{y,eff}} + \frac{1}{1 - \frac{N_{Ed}}{N_{cr,z}}} \frac{\mathrm{M_{z,Ed}} + \mathrm{N_{Ed}}(e_y + w_{0y})}{W_{z,eff}} \leq \frac{f_y}{\gamma_{M1}} \quad (36)$$

Applications

Class 4 cross-sections – code checks

For open-sections, if the compressed flange is unrestrained, SIA 263 (§5.6.2) also asks to check its stability. This equivalent to checking for lateral torsional buckling with SIA's method.

$$\sigma_{Ed} = \frac{\mathrm{M_{Ed}}}{W_{c,eff}} \leq \frac{\sigma_D}{\gamma_{M1}} \quad (37)$$

The computation of critical stress σ_D was discussed in the lateral torsional buckling lecture and can also be found in Annex B of SIA263. For slender webs, the critical stress is more associated with the warping term and thus,

$$\sigma_D \approx \sigma_{Dw} = \frac{\pi^2 E}{\lambda_k^2}$$

with, $\lambda_K = L_k/i$ and i the radius of gyration with effective flange $t_f b_{eff}$, and effective web $\min\left(t_w \frac{h_{c,eff}}{2}; t_w \frac{h_c}{3}\right)$

EPFL References

- [Scheer, 2010] – Scheer, Joachim (2010), “Failed bridges”, Ernst & Sohn, Berlin, Germany
- [Bryan, 1890] – Bryan, George H. (1890), “On the stability of a Plane Plate under Thrusts in its own Plane with Applications to the ‘Buckling’ of the Sides of a Ship”, London Mathematical Society, Volume s1-22, Issue 1, p. 54-67
- [Hirt et al., 2011] – Hirt, Manfred, Bez, Rolf and Nussbaumer, Alain (2011), “Traité Génie Civil, Volume 10 – Construction Métallique”, PPUR, Lausanne, Switzerland
- [SIA, 2013] – Societé suisse d’ingénieurs et architectes (2013), “SIA263:2013 – Construction en acier”, SIA, Zürich, Switzerland
- [Koiter, 1945] – Koiter, Warner T., “Over de stabiliteit van het Elastiche Evenwicht”, PhD Thesis, Delft Netherlands (translated to English by Edward Riks - https://apps.dtic.mil/dtic/tr/fulltext/u2/704124.pdf)
- [von Karman et al., 1932] – von Karman, T., Sechler, E. E., and Donnell, L.H (1932) “ The strength of thin plates in compression”, Transaction of the American Society of Mechanical Engineers (ASME), Vol 54, p. 53
- [Reis and Camotim, 2001] – Reis, António; Camotim, Dinar (2001) “Estabilidade Estrutural”, McGrawHill, Amadora, Portugal.
- [Winter, 1947] – Winter, G. (1947)., “Strength of Thin Steel Compression Flanges”, Transactions of the American Society of Mechanical Engineers(ASME), Vol. 112, p. 527
- [Hansen, 2006]– Hansen, Thomas(2006), Theory of Plasticity for steel structures – solutions for fillet welds. Plate girders, and thin plates, PhD Thesis, R-146, BYG-DTU, Copenhagen, Denmark (https://www.byg.dtu.dk/-/media/Institutter/Byg/publikationer/PhD/byg_r146.ashx?la=da&hash=33E70EFD516A84EC0A2F7F5C22D44FE09C1C0CCB)
- [Klöppel and Scheer] – Klöppel, K. and Scheer, J. (1960) “Beulwerte ausgesteifter Rechteckplatten”, Verlag von Wilhem Enst & Sohn, Berlin, Germany.
- [Lebet and Hirt, 2009] – Lebet, J.P. and Hirt, M. A. (2009) “Traité Génie Civil, Volume 12 – Ponts en Acier”, PPUR, Lausanne, Switzerland