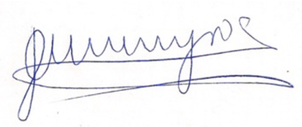
FACULTE ENVIRONNEMENT NATUREL, ARCHITECTURAL ET CONSTRUIT ENAC
INSTITUT D'INGÉNIERIE CIVILE IIC
Laboratoire des Structures Métalliques Résilientes - RESSLab

Téléphone : +41 21 693 24 27
Fax : +41 21 693 28 68
E-mail : dimitrios.lignos@epfl.ch
Site web : http://resslab.epfl.ch
Address: EPFL ENAC IIC RESSLAB
GC B3 485, Station 18,
CH-1015, Lausanne

Final Examination
Date: Monday, June 21st 2021
Time: 8h15 – 12h15
Room No: "Take home examination"

Instructor: Prof. Dr. Dimitrios G. Lignos, RESSLab
Co-Instructor: Dr. Albano de Castro e Sousa, RESSLab



CIVIL 369: STRUCTURAL STABILITY

INSTRUCTIONS

- This is an **OPEN BOOK** Take home examination.
- You are not allowed to work or talk with others during the take home examination.
- The total exam time is 4 hours (240minutes). The total time you need to answer the exam **shall not be more than 3 hours** (180minutes). The additional hour is given to you to prepare your exam sheets and submit them online.
- Your solutions shall be written with your own handwriting. The exam sheets will be subject to similarity checking after the end of the exam with specialized software.
- You **MUST SCAN** a copy of the examination sheets you used **WITH YOUR NAME** clearly written in all pages and upload those through Moodle by 12h15. **According to the EPFL guide, you must click the submit button and accept the submission statement in Moodle.** Submitted examinations **WILL NOT** be acceptable after 12h15 of June 21st 2021.
- If you are facing a technical difficulty during the exam (not related to the exam questions) you may call at 021 693 2427 (Office Line) or email at dimitrios.lignos@epfl.ch .
- The examination consists of four (4) questions. You must answer to all the questions. (total points 100/100).
- The total number of pages of the exam is seven (7) including the cover page.
- The exam questions are clear and have been proof-read by two different individuals. Therefore, during the examination period we are not going to answer to any question related to the exam.

Question 1 – Easy Statics / Plastic Analysis (25 points)

Part A (10 points / 2.5 points each)

The four frames shown in Figure 1 are made of steel ($E = 200GPa$). Compute the lateral stiffness for all four cases. If the steel beams are not rigid, then assume, $I_b = 4.16x10^8 mm^4$. For all the steel columns assume, $I_c = 4.16x10^8 mm^4$.

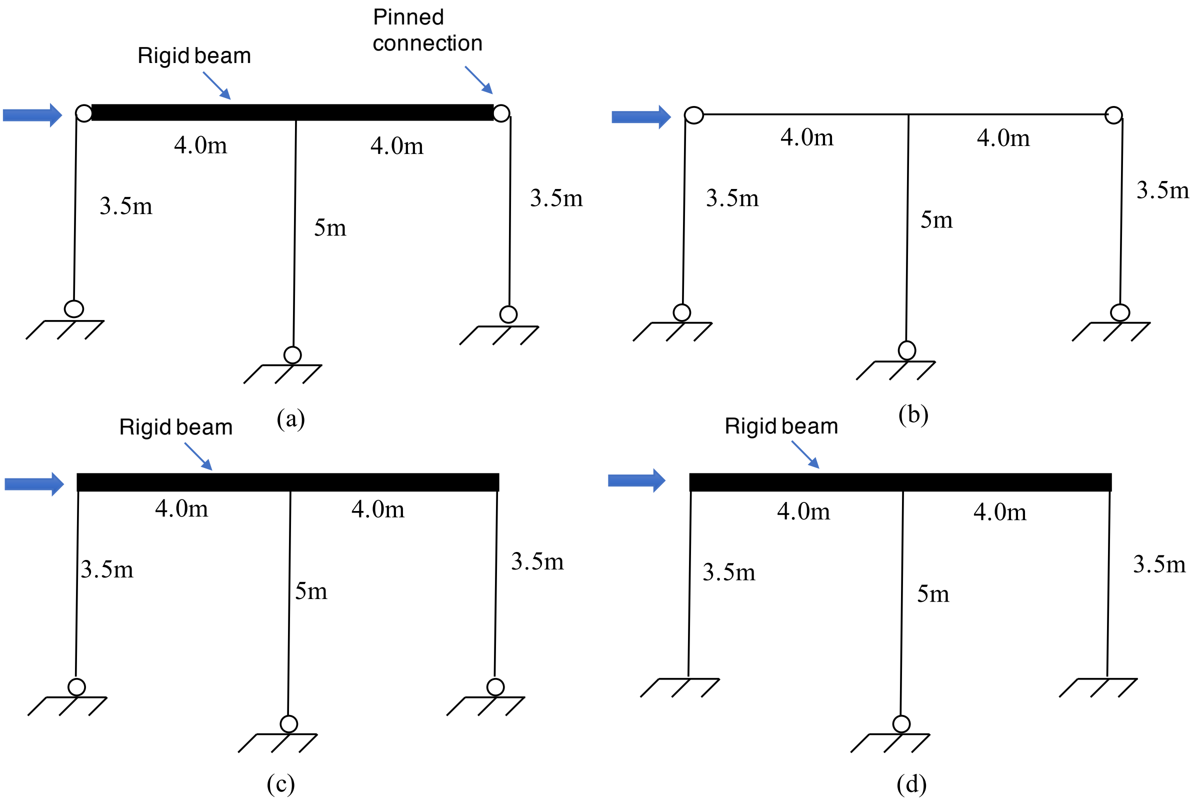


Figure 1 – Steel frames under lateral loading

Part B (15 points)

The single storey frame structure shown in Figure 2 is subjected to gravity load $N = 2000kN$ at each column as well as lateral load F. The final geometry and cross-section profiles are shown in the figure. The steel material is S355J2 ($E = 200GPa, f_y = 355MPa$). For all the steel columns assume $I_c = 4.16x10^8 mm^4$ and $W_{pl,y} = 2000x10^3 mm^3$.

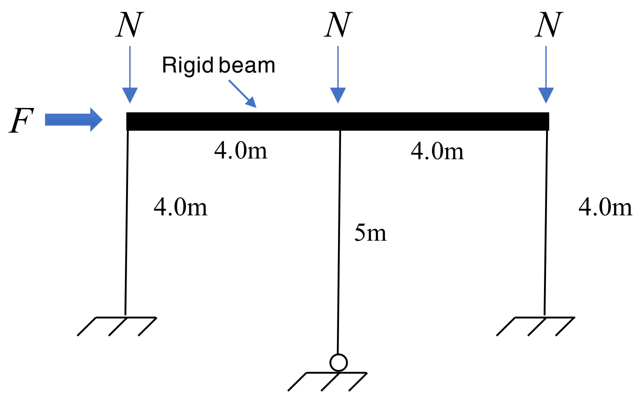


Figure 2 – Collapse analysis of single storey steel MRF

Q1 (8 points). Calculate the collapse load of the structure till it becomes a complete collapse mechanism by neglecting P-Delta effects (i.e., neglect N in this case).

Q2 (7 points). Calculate the collapse load of the structure by considering P-Delta effects in your step-by-step calculations. Calculate the deflection at which the structure loses its lateral load resistance. Justify your answer.

Question 2 – Lateral Torsional Buckling of Bridge Girders (25 Points)

Two simply supported steel girders spaced 2400mm apart with a span of 45720mm must support a factored moment of $M_u = 3900kNm$ in each girder during the construction stage. The cross-section in each and other pertinent geometric details are given in Figure 3. There are five intermediate cross frames spaced at 7620mm. Assuming the deck will be poured in one continuous operation, you are asked to verify the safety of this structure during construction. Assume that the steel is S355J2 ($E = 200GPa$, $f_y = 355MPa$).

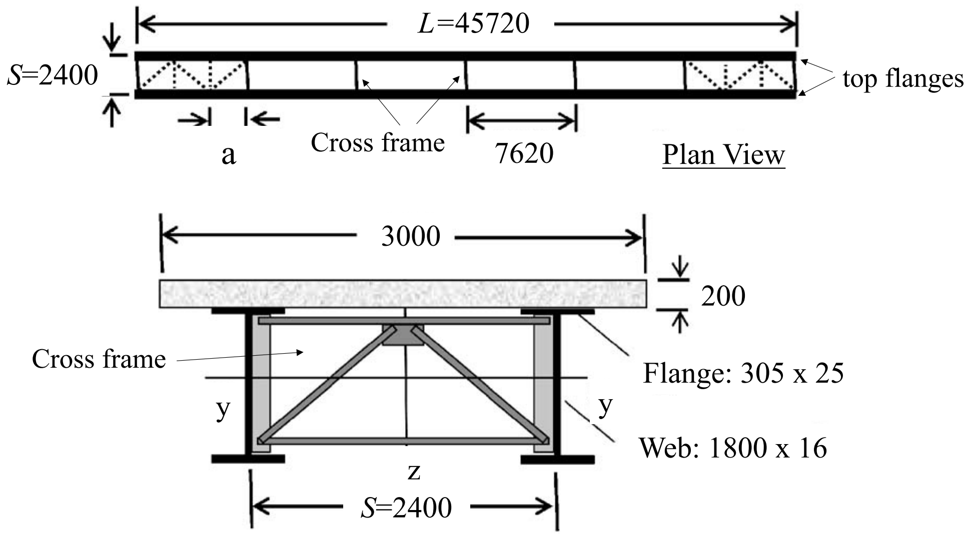


Figure 3 – Bridge girder (dimensions in millimeters)

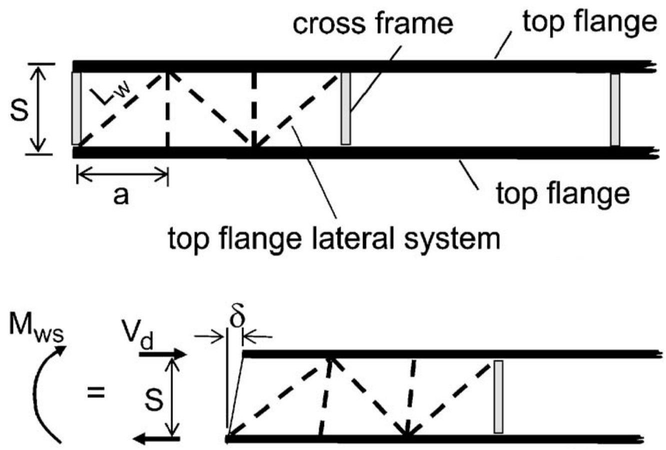


Figure 4 – End warping restraint

Q1 (3 points): Calculate the moment of inertia I_{yy}, I_{zz}, warping and torsional constants of the main girders.

Q2 (2 points): What is the section classification of the main girders?

Q3 (10 points): Check if the bending resistance of the girders against lateral torsional buckling is adequate for the factored moment during the construction stage. Assume that the load is applied at the top flange of the girder.

Q4 (5 points): Check if the bending resistance of the girders against global buckling is adequate for the factored moment during the construction stage. For this verification, you can assume that the resistance against global buckling can be estimated as follows:

$$M_{gl} = C_1 \frac{\pi^2 SE}{L^2} \sqrt{I_{zz} I_{yy}}$$

Q5 (5 points): Referring to Figure 4, the end warping restraint (units kNm/rad) of a double girder system may be approximated as follows,

$$M_{ws} = \frac{3\left(M_u - M_{gl}\right)L}{h_o}$$

This may be transformed to a pair of forces V_d that cause relative deflection, δ between the two girders as shown in Figure 4. In order to reduce this deformation, we often install a top flange lateral system as shown in Figure 4. Prove that the required area A_d of the diagonal elements shown in Figure 4 can be calculated as follows:

$$A_d = \frac{M_{ws}}{S^2} \cdot \frac{L_w^3 + S^3}{m \cdot \mathrm{a}^2 \cdot E}$$

In which,

- S is the distance between the two girders ($S = 2400mm$ in this case as shown in Figure 3)
- L is the full length of the bridge girder as shown in Figure 3
- h_o is the centerline distance between the two flanges of a girder ($h_o = 1825mm$)
- m is the number of braced panels ($m = 3$ as shown in Figure 4)
- $C_1 = 1.12$ for uniform loading

Question 3 – (25 points)

Consider the cable-stayed column under compression load P in Figure 5. The axial direction is defined as "z" and the out of plane direction "y". Out-of-plane deflections are defined as "v" and its magnitude at the tip of the column is defined as δ (cf. deformed shape in grey). Consider that the cables only have axial stiffness and do not work in compression, and that the column only works in bending. The cables have area A, the column has bending inerta I, and members have Young's modulus E.

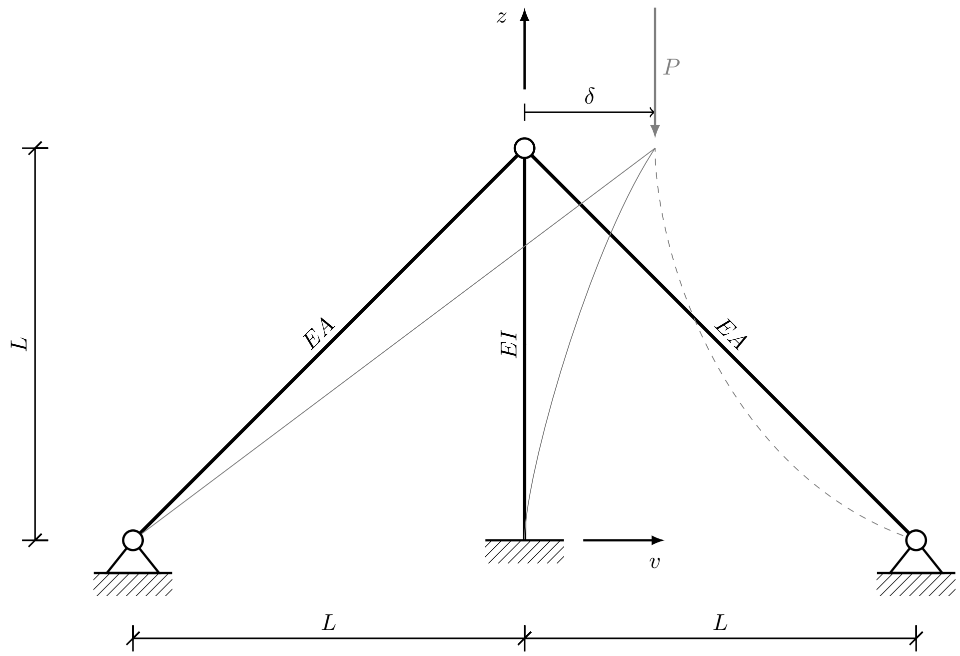


Figure 5 Cable stayed column with axial load P.

If the out-of-plane deflections of beam follow $v(z) = \frac{\delta}{L^2} z^2$, answer the following questions:

Q.1 (5 points) Compute the internal energy U of the column and cable structure;
Q.2 (5 points) Computed the external work V of the uniform load P;
Q.3 (5 points) Compute the critical load P_{crit} based on the two previous questions.

Now, assume that the out-of-plane deflections of beam follow $v(z) = \frac{\delta}{2L^3} z^2(3L - z)$, and answer the following question:

Q.4 (10 points) Re-compute the internal energy U of the column, the external work, and the critical load. Which approximation of the out-of-plane deflections is more accurate? Provide a numerical justification.

When solving this question consider the small angle approximations:

$$v'(z) = \tan\theta \approx \theta \,; \sin\theta \approx \theta \,; \cos\theta \approx 1 - \frac{\theta^2}{2}$$

Question 4 – Plate Buckling (25 points)

Consider the bridge girder open cross-section depicted in Figure 6. Assume that the bridge girder is made from an S355J2 material. Assume also that the girder has transverse stiffeners spaced out by 2200mm and that end stiffeners are sufficient to provide anchorage of the ties.

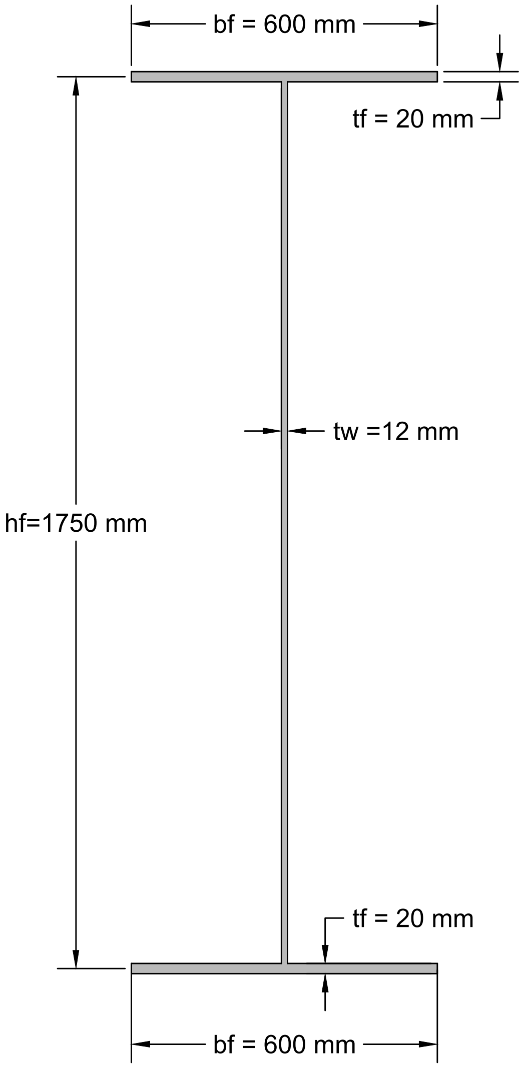


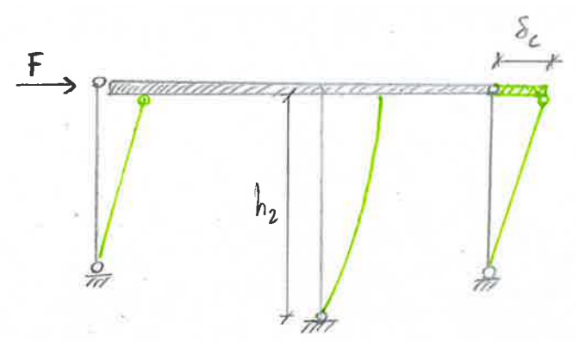
Figure 6 - Bridge cross-section

Q.1 (10 points) Compute, by using the Basler model, the shear resistance of the bridge girder;
Q.2 (15 points) Compute the sectional flexural resistance.

Question 1 – Solution

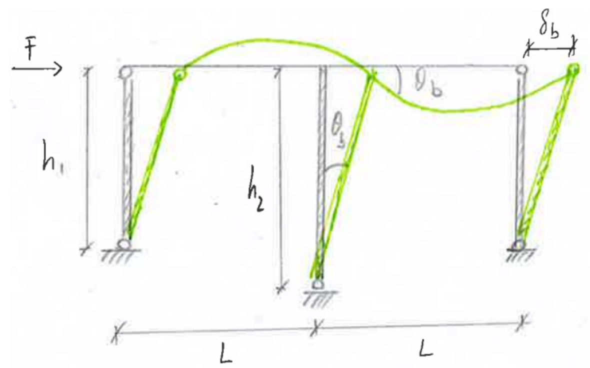
Solution – Part A

a) Stiffness only provided from central column: $K_{tot} = \frac{3EI_c}{h_2^3} = 2.00 \left[\frac{kN}{mm}\right]$



$$\delta_c = \frac{F}{K_c} = \frac{F}{\frac{3EI_c}{h_2^3}}$$

b) The total lateral displacement δ_{tot} should be due to bending in the beam and column:



$$\delta_b = \theta_b h_2 = \frac{M_b}{K_b} h_2 = \frac{M_b}{\frac{3EI_b}{L}} h_2 = \frac{F}{2} h_2 \frac{L}{3EI_b} h_2 = \frac{FLh_2^2}{6EI_b}$$

$$\delta_{tot} = \delta_c + \delta_b = \frac{Fh_2^3}{3EI_c} + \frac{FLh_2^2}{6EI_b}$$

$$K_{tot} = \frac{F}{\delta_{tot}} = \frac{1}{\frac{h_2^3}{3EI_c} + \frac{h_c^2 L}{6EI_b}} = 1.43 \left[\frac{kN}{mm}\right]$$

c) The total stiffness is the sum of the 3 columns: $K_{tot} = 2\frac{3EI_c}{h_1^3} + \frac{3EI_c}{h_2^3} = 13.60 \left[\frac{kN}{mm}\right]$

d) The total stiffness is the sum of the 3 columns but the 2 external columns are fixed at their both ends:

$$K_{tot} = 2\frac{12EI_c}{h_1^3} + \frac{3EI_c}{h_2^3} = 48.60\ \left[\frac{kN}{mm}\right]$$

Solution – Part B

Q1: The moment diagram prior to the onset of the first plastic hinge is as follows,

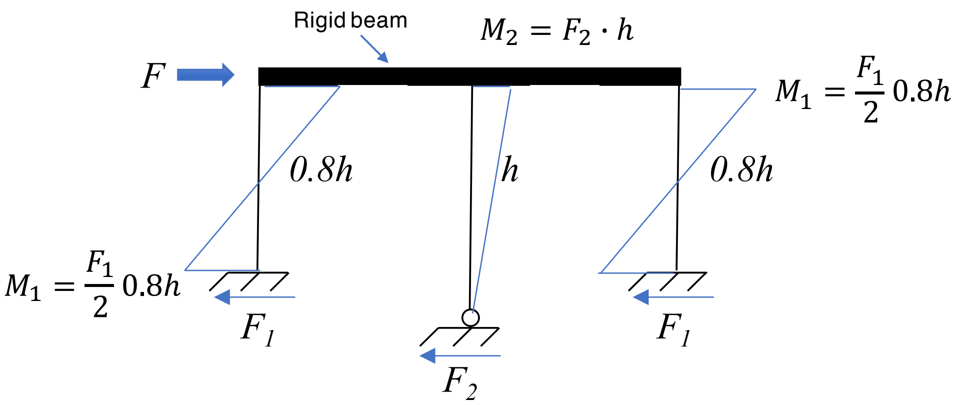


Figure 1. Moment diagram

$$K_1 = \frac{12EI}{(0.8h)^3} = \frac{12EI}{0.512h^3}$$

$$K_2 = \frac{3EI}{h^3}$$

$$F_1 = \frac{K_1}{2K_1 + K_2}F = \frac{\frac{12EI}{0.512h^3}}{2 \cdot \frac{12EI}{0.512h^3} + \frac{3EI}{h^3}} = 0.47F$$

Therefore,

$$F_2 = F - 2F_1 = F - 2 \cdot 0.47F = 0.06F$$

$$M_1 = \frac{F_1}{2}0.8h = 0.4F_1h = 0.4 \cdot 0.47 \cdot F \cdot h = 0.188 \cdot F \cdot h$$

$$M_2 = F_2 \cdot h = 0.06 \cdot F \cdot h$$

$$M_{pl,y} = W_{pl} \cdot f_y = 2000x10^3 \cdot 0.355 = 7.1x10^5 kNmm$$

The first plastic hinge will form at the edge columns based on moment distribution diagram of Figure above. Therefore,

$$W_{pl,y}f_y = 0.188 \cdot F \cdot h \rightarrow F = \frac{W_{pl,y}f_y}{0.188 \cdot h} = 755kN$$

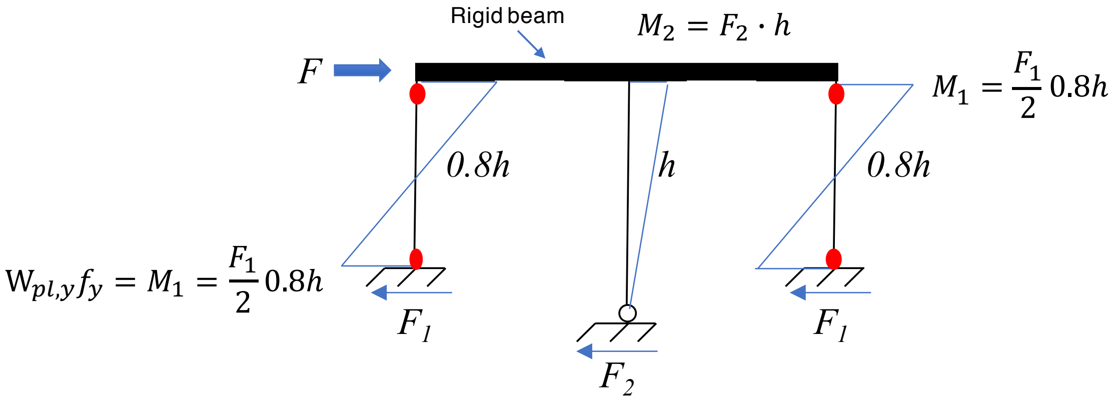


Figure 2. Assumed plastic hinge positions

The corresponding deflection at this point is as follows,

Therefore,

$$\delta_1 = \frac{F_1}{2K_1 + K_2} = \frac{755}{2 \cdot 15.6 + 1.9968} = 22.8mm$$

The moment diagram in this case is as follows,

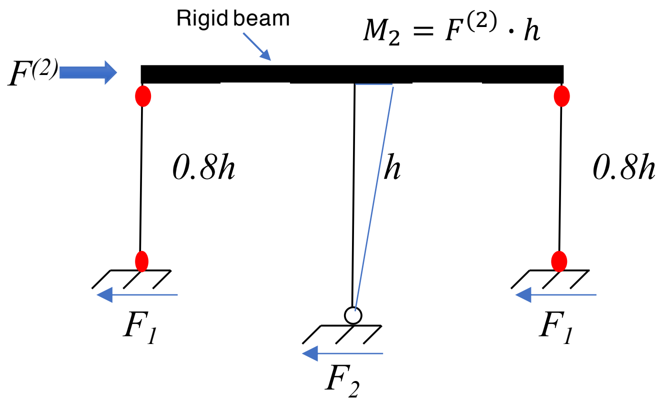


Figure 3. Moment diagram after the formation of the first plastic hinge

Therefore,

$$0.06 \cdot F \cdot h + F^{(2)} \cdot h = W_{pl,y} f_y \rightarrow F^{(2)} = \frac{W_{pl,y} f_y - 0.06 \cdot F \cdot h}{h} = 96.7kN$$

The corresponding deflection at this point is as follows,

$$\delta_2 = \frac{F^{(2)} \cdot h^3}{3 \cdot E \cdot I} = \frac{96.7 \cdot 5000^3}{3 \cdot 200 \cdot 4.16x10^8} = 48.4mm$$

Therefore,

$\delta = \delta_1 + \delta_2 = 71.2mm$

Q2: When P-Delta effects are considered in the analysis, the load deformation equilibrium path rotates by $N \cdot \frac{\delta}{h}$; in this case, the load, N is the total vertical load that is supported by the floor. Therefore, $N = 6000kN$.

At the corresponding deflections, δ_1, and $\delta_{tot} = \delta_1 + \delta_2$, the P-Delta forces should be:

$$V_{P-\Delta}^{(1)} = 4000 \cdot \frac{22.8}{4000} + 2000 \cdot \frac{22.8}{5000} = 31.9kN$$

$$V_{P-\Delta}^{(tot)} = 4000 \cdot \frac{71.2}{4000} + 2000 \cdot \frac{71.2}{5000} = 99.6kN$$

Therefore, the corresponding load in this case should be:

$$F_c = F^{(1)} + F^{(2)} - V_{P-\Delta}^{tot} = 852 - 99.6 = 752kN$$

The two solutions from the previous two questions are compared in an equilibrium path. Moreover, notice that once the collapse load is attained, the equilibrium path has a negative stiffness; therefore, it is unstable. Once this path hits the x-axis this will be the corresponding deflection that the structure loses its lateral load resistance. Therefore, the lateral displacement is 609mm.

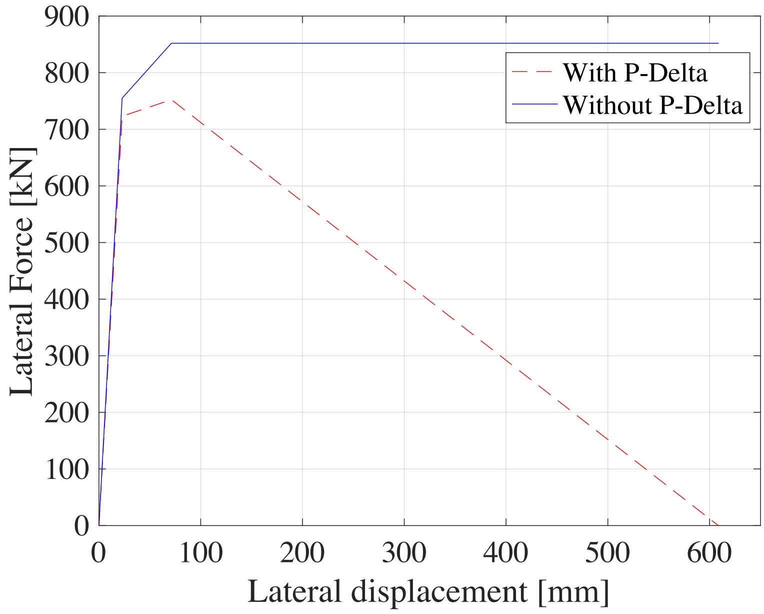


Figure 8. Equilibrium path of the structure by excluding and including P-Delta effects

Question 2 - Solution

2.1 – section properties

$$I_{yy} = 25^3 \cdot \frac{305}{12} + (25 \cdot 305) \cdot (900 + 12.5)^2 \cdot 2 + 16 \cdot \frac{1800^3}{12} = 2.05 \cdot 10^{10} mm^4$$

$$I_{zz} = 2 \cdot 25 \cdot \frac{305^3}{12} + 1800 \cdot \frac{16^3}{12} = 1.19 \cdot 10^8 mm^4$$

$$K = \frac{1}{3}(2 \cdot 305 \cdot 25^3 + 1800 \cdot 16^3) = 5.635 \cdot 10^6 mm^4$$

For doubly symmetric I-sections the following formula holds true:

$$I_w = \frac{1}{4} I_{zz} h^2 = \frac{1}{4} \cdot (1.19 \cdot 10^8)(1800 + 25)^2 = 9.892 \cdot 10^{13} mm^6$$

Q2: - Section classification

Assume pure bending:

$$\frac{c}{t_w} = \frac{1800 - 2 \cdot 7\sqrt{2}}{16} = 111.3 > 124\varepsilon = 100.9$$

(web class 4)

No matter what is the flange the web is class 4; therefore, the cross-section is class 4.

Q3: - Computation of lateral torsional buckling resistance

The resistance against lateral torsional buckling should be calculated as follows:

$$M_b = \frac{C_1 \pi^2 E I_{zz}}{k_v k\varphi L_D^2} \left(\sqrt{(C_2 \zeta_a + C_3 \beta)\text{^}2 + \frac{I_w}{I_{zz}} (\frac{G K k_\varphi^2 L_D^2}{\pi^2 E I_w} + 1)} + C_2 \zeta_a + C_3 \beta \right)$$

$C_1 = 1.13, C_2 = 0.46, C_3 = 0.53$

$k_\varphi = 1.0, k_v = 1.0$

$\beta = 0$

$\zeta_a = -925mm$

E=200GPa, G=81GPa

Therefore, $M_b = 2888.9KNm < 3900kNm$

Q4:

$$M_{gl} = 1.12 \cdot \pi^2 \cdot 2400 \cdot \frac{200000}{45720^2} \cdot \sqrt{1.19 \cdot 10^8 \cdot 2.05 \cdot 10^{10}} = 3964.6kNm > 3900kNm$$

Therefore, global buckling is prevented.

Q5

The lateral stiffness of one panel by considering that only the diagonal is primarily contributing to the lateral stiffness is equivalent to the work we presented during Week #1 in class. Particularly,

$$K = \frac{EA_d L_w^2 \cos^2\theta}{(L_w^2 \cos^2\theta + S^2)^{\frac{3}{2}}}$$

The stability of a panel is guaranteed as follows:

$$A_d \geq \frac{M_w}{S^2} \cdot \frac{(a^2 + S^2)^{\frac{3}{2}}}{Ea^2} = \frac{M_w}{s^2} \cdot \frac{(L_w^2 - S^2 + S^2)^{\frac{3}{2}}}{Ea^2} = \frac{M_w}{S^2} \cdot \frac{L_w^3}{Ea^2}$$

Assuming that we have m panels:

$$A_d \geq \frac{M_w}{S^2} \cdot \frac{L_w^3}{Ea^2 m}$$

Assuming that the vertical elements of *m* panels have the same area with that of the diagonals, then that contribution is as follows:

$$A_d \geq \frac{M_w}{S^2} \cdot \frac{S^3}{Ea^2 m}$$

Therefore, the superposition of deflections of the two elements (elements in parallel) would give

$$A_d \geq \frac{M_w}{S^2} \cdot \frac{L_w^3 + S^3}{Ea^2 m}$$

Question 3 – Solution

Q1

The internal energy of the column is computed by:

$$U_{col} = \frac{1}{2}\int_0^L EI\left(v''(z)\right)^2 dz$$

Taking the deformed shape assumption,

$$v(z) = \frac{\delta}{L^2}z^2$$

$$v'(z) = 2\frac{\delta}{L^2}z$$

$$v''(z) = 2\frac{\delta}{L^2}$$

The internal energy becomes,

$$U_{col} = \frac{1}{2}\int_0^L EI\left(2\frac{\delta}{L^2}\right)^2 dz = \frac{2EI}{L^3}\delta^2$$

The internal energy coming from one cable in tension,

$$U_{cable} = \frac{1}{2}kx^2 = \frac{1}{2}\frac{EA}{L}\cos^2 45°\,\delta^2$$

The total internal energy of the structural system is,

$$U = U_{col} + U_{cable} = \frac{2EI}{L^3}\delta^2 + \frac{1}{2}\frac{EA}{L}\cos^2 45°\,\delta^2$$

Q2

The external work performed by the vertical column load needs to be computed from the vertical displacement at the column tip. The vertical displacement can be computed by the integral over the height of the infinitesimal vertical displacement. This vertical displacement for a unit length infinitesimal segment is equal to

$$\bar{\delta}_v = 1 \cdot (1 - \cos\theta) = \left(1 - 1 + \frac{\theta^2}{2}\right) = \frac{\theta^2}{2} = \frac{1}{2}(v')^2$$

As such, displacement at the tip is,

$$\delta_v = \frac{1}{2}\int_0^L (v')^2 dz = \frac{1}{2}\int_0^L \left(2\frac{\delta}{L^2}z\right)^2 dz = 2\frac{\delta^2}{L^4}\frac{L^3}{3} = \frac{2}{3}\frac{\delta^2}{L}$$

$$V = -P\delta_v = -\frac{2}{3}\frac{\delta^2}{L}P_{crit}$$

Q3

The critical load can be computed from total energy in the system,

$$\Pi = U + V = \frac{2EI}{L^3}\delta^2 + \frac{1}{2}\frac{EA}{L}\cos^2 45°\,\delta^2 - \frac{2}{3}\frac{\delta^2}{L}P$$

Equilibrium is reached when,

$$\frac{\partial \Pi}{\partial \delta} = 0 \Leftrightarrow \frac{4EI}{L^3}\delta + \frac{EA}{L}\cos^2 45°\,\delta - \frac{4}{3}\frac{\delta}{L}P = 0 \;\Leftrightarrow$$

$$\Leftrightarrow\; P_{crit} = \frac{3EI}{L^2} + \frac{3}{4}EA \cdot cos^2 45°$$

Q4

Taking the deformed shape assumption,

$$v(z) = \frac{\delta}{2L^3}z^2(3L - z)$$

$$v'(z) = \frac{3\delta}{2L^3}(2Lz - z^2)$$

$$v''(z) = \frac{6\delta}{2L^2}\left(1 - \frac{z}{L}\right)$$

Internal energy of the column,

$$U_{col} = \frac{1}{2}\int_0^L EI\left(\frac{6\delta}{2L^2}\left(1 - \frac{1}{L}z\right)\right)^2 dz = \frac{9EI}{2L^4}\delta^2 \cdot \left(\frac{L^3}{3L^2} - \frac{L^2}{L} + L\right) = \frac{3EI}{2L^3}\delta^2$$

External work,

$$\delta_v = \frac{1}{2}\int_0^L (v')^2 dz = \frac{1}{2}\int_0^L \left(\frac{3\delta}{2L^2}\left(2z - \frac{1}{L}z^2\right)\right)^2 dz = \frac{1}{2}\frac{9\delta^2}{4L^4}\left(\frac{L^5}{5L^2} - \frac{L^4}{L} + \frac{4L^3}{3}\right)$$

$$= \frac{9\delta^2}{8L^4}L^3\left(\frac{1}{5} - 1 + \frac{4}{3}\right) = \frac{3 - 15 + 20}{15}\cdot\frac{9\delta^2}{8L} = \frac{72}{120}\frac{\delta^2}{L}$$

$$V = -P\delta_v = -\frac{72}{120}\frac{\delta^2}{L}P$$

For the total energy,

$$\Pi = U + V = \frac{3EI}{2L^3}\delta^2 + \frac{1}{2}\frac{EA}{L}\cos^2 45°\,\delta^2 - \frac{72}{120}\frac{\delta^2}{L}P$$

At equilibrium,

$$\frac{\partial\Pi}{\partial\delta} = 0 \Leftrightarrow \frac{6EI}{2L^3}\delta + \frac{EA}{L}\cos^2 45°\,\delta - \frac{144}{120}\frac{\delta}{L}P_{crit} = 0 \;\Leftrightarrow$$

$$\Leftrightarrow\; P_{crit} = \frac{120\cdot 6EI}{144\cdot 2\,L^2} + \frac{120}{144}EA\cdot cos^2 45° = 2.5\frac{EI}{L^2} + \frac{120}{144}EA\cdot cos^2 45°$$

Comparing the two assumptions, one can see that the second choice is a better one since the coefficient 2.5 is closer to the theoretical value of $\frac{\pi^2}{2^2}$, where 2 is the k-factor of the column.

Question 4 – Solution

4.1 – Shear resistance

$$\alpha = \frac{a}{h_f} = \frac{2200}{1750} = 1.257$$

§F.1.4 SIA263:2013

$$k = 5.34 + \frac{4.0}{\alpha^2} = 5.34 + \frac{4.0}{1.257^2} = 7.87$$

Eq. 104 – Annex F, SIA263:2013

$$\tau_{cr} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t_w}{h_f}\right)^2 = 7.87\frac{\pi^2 210e3}{12(1-0.3^2)}\left(\frac{12}{1750}\right)^2 = 70.23 \leq 0.8\tau_y = 0.8\frac{355}{\sqrt{3}} = 164\ N/mm2$$

Eq. 101 and Eq.102 – Annex F, SIA263:2013

$$V_{Rd} = \frac{1}{\gamma_{M1}}\left(\tau_{cr} + \frac{\sqrt{3}(\tau_y - \tau_{cr})}{2\sqrt{1+\alpha^2}}\right)A_w = \frac{1}{1.05}\left(70.23 + \frac{\sqrt{3}\left(\frac{355}{\sqrt{3}} - 70.23\right)}{2\sqrt{1+1.257^2}}\right)(1750\cdot 12) = 2.86e6\ N$$

4.2 – Flexural cross-section resistance

First let's check the slenderness limits for (1) web Euler buckling and (2) web breathing (cf. course slides 60 and 61)

$$\frac{h_f}{t_w} = \frac{1750}{12} = 145.8 \leq 240 - OK(1);\ \frac{h_c}{t_w} = \frac{\frac{1750}{2}\cdot}{12} = 72.9 \leq 100 - OK(2)$$

Let's now check section classification: ($\varepsilon = 0.81$ for S355)

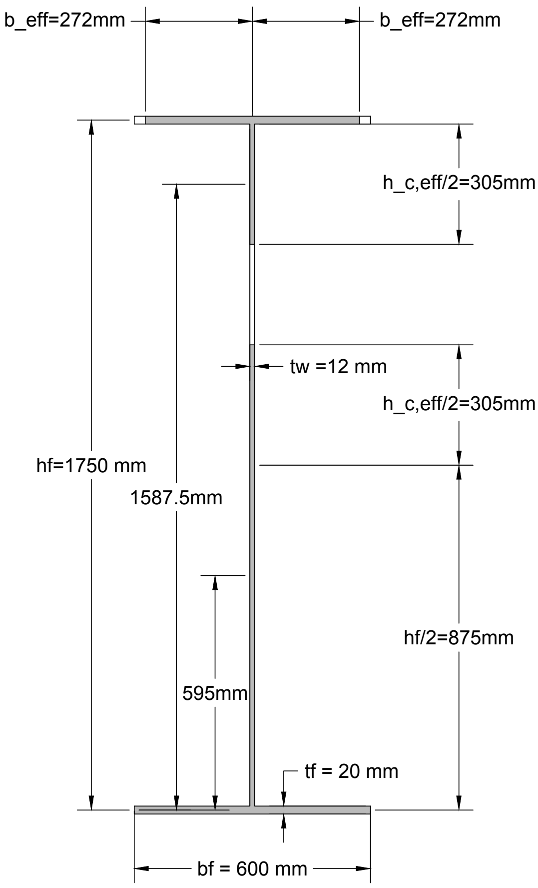
- Flange: $\frac{b}{t} = \frac{\frac{600}{2}}{20} = 15 \leq 14\varepsilon = 11.34$ is **KO**. This implies that the flange is class 4. The b_{eff} for class 4 is equal to (Eq. 32 from course with k=0.426 from Table in slide 12):

$$b_{eff} = 0.9\cdot\sqrt{\frac{1}{f_y}\cdot k\cdot\frac{\pi^2 E}{12(1-\nu^2)}}\cdot t = 0.9\cdot\sqrt{\frac{1}{355e3}\cdot 0.426\cdot\frac{\pi^2\cdot 210e6}{12(1-0.3^2)}}\cdot 20 \approx 272mm$$

- Web: $\frac{h_f}{t_w} = \frac{1750}{12} = 145 \leq 124\varepsilon = 100.44$ is **KO**. This implies that the web is class 4. The $h_{c,eff}$ for class 4 is equal to (Eq. 33 from course with k=23.9 from Table in slide 12):

$$h_{c,eff} = 0.9 \cdot \sqrt{\frac{1}{f_y} \cdot k \cdot \frac{\pi^2 E}{12(1-\nu^2)}} \cdot t \cdot \frac{h_c}{h_f} = 0.9 \cdot \sqrt{\frac{1}{355e3} \cdot 23.9 \cdot \frac{\pi^2 \cdot 210e6}{12(1-0.3^2)}} \cdot 12 \cdot \frac{875}{1750} \approx 610mm$$

These findings yield the following profile for the effective section of the girder,



Now let's calculate the effective properties,

Element	**Area (mm2)**	h **(mm)**	$A \cdot h$ **(mm3)**	$\bar{h} = h - h_G$ **(mm)**	$A \cdot \bar{h}^2$**(mm4)**	**I (mm4)**
Upper Flange	10880	1750	19040000	932.4	9.45e9	0.00e9
Upper Web	3660	1587.5	5810250	769.9	2.17e9	0.03e9
Lower Web	14160	595	8425200	-222.6	0.7e9	0.67e9
Lower Flange	12000	0	0	-817.6	8.02e9	0.00e9
Totals	40700		33275450		20.4	0.7e9

With,

$$h_G = \frac{33275450}{40700} = 817.6\, mm$$

As such, the effective cross-sectional inertia is given by :

$$I_{eff} = 20.4e9 + 0.7e9 = 21.1e9\ mm4$$

The cross-section modulus at the compression fiber can be given by,

$$W_{c,eff} = \frac{I_{eff}}{h_f - h_G} = \frac{21.1e9}{1750 - 817.6} = 22.63e6\ mm3$$

The cross-section flexural resistance can be computed by,

$$M_{Rd} = \frac{W_{c,eff} f_y}{\gamma_{M1}} = 22.63e6 \cdot \frac{355}{1.05} \cdot 1e - 6 = 7651\ kN.m$$