Discrete choice

Utility and value of time

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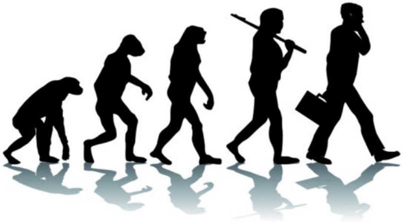
Introduction to transportation systems

Behavioral assumptions



Concept

Homo economicus

Decision maker

- is consistently rational,
- is narrowly self-interested,
- optimizes her outcome.

Nature of the decisions

- continuous: last lecture.
- discrete: this lecture.

Discrete choice

Choice situation

- Option 1: travel with public transportation.
- Option 2: travel by car.

Utility theory

Attributes

Alternatives	Attributes	
	Travel time (t)	Travel cost (c)
PT (1)	t_1	c_1
not PT (2)	t_2	c_2

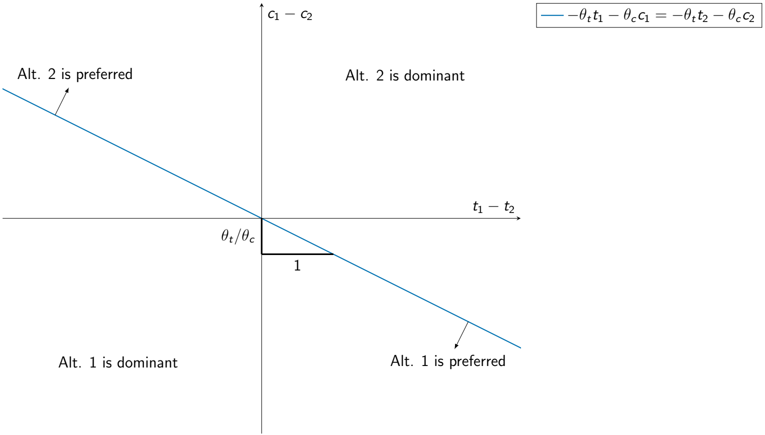
Utility functions

$$u_1 = -\theta_t t_1 - \theta_c c_1$$

$$u_2 = -\theta_t t_2 - \theta_c c_2$$

where $\theta_t > 0$ and $\theta_c > 0$ are parameters.

Utility theory



Utility model

Variables

- Travel time: t_{in}.
- Travel cost: c_{in}.
- Others (headway, etc.): x_{in}.

Function

$$u_{in} = u(t_{in}, c_{in}, x_{in})$$

Example:

$$u_{in} = -\theta_t t_{in} - \theta_c c_{in} - \theta_x x_{in},$$

where $\theta_t, \theta_c, \theta_x \geq 0$.

Utility model

$$u_{in} = -\theta_t t_{in} - \theta_c c_{in} - \theta_x x_{in}.$$

where $\theta_t, \theta_c, \theta_x \geq 0$.

Units

- Utility has no unit.
- If t_{in}: minutes, then θ_t: 1/minutes.
- If t_{in}: hours, then θ_t: 1/hours.
- If c_{in}: CHF, then θ_c: 1/CHF.
- If x_{in}: unit, then θ_x: 1/unit.

Utility model

$$u_{in} = -\theta_t t_{in} - \theta_c c_{in} - \theta_x x_{in}.$$

Changing the units

- Multiplying/dividing by a positive constant does not change the ranking.

$$u^x_{in} = \frac{u_{in}}{\theta_x} = -\frac{\theta_t}{\theta_x} t_{in} - \frac{\theta_c}{\theta_x} c_{in} - x_{in},$$

- $u_{in} \geq u_{jn} \iff u^x_{in} \geq u^x_{jn}$.
- Units of u^x_{in}: same as x_{in}

Utility model

$$u_{in} = -\theta_t t_{in} - \theta_c c_{in} - \theta_x x_{in}.$$

Monetary units

- Any variable can be used as a reference.
- Cost, for example:

$$u^c_{in} = \frac{u_{in}}{\theta_c} = -\frac{\theta_t}{\theta_c} t_{in} - c_{in} - \frac{\theta_x}{\theta_c} x_{in},$$

- $u_{in} \geq u_{jn} \iff u^c_{in} \geq u^c_{jn}$.
- Units of u^c_{in}: CHF.
- Its opposite is sometimes called generalized cost.

Utility model

$$u_{in}^{c} = -\frac{\theta_t}{\theta_c} t_{in} - c_{in} - \frac{\theta_x}{\theta_c} x_{in},$$

Units

- $\frac{\theta_t}{\theta_c}$: (1/minute)/(1/CHF) = CHF/minute
- $\frac{\theta_x}{\theta_c}$: (1/unit)/(1/CHF)=CHF/unit

The quantity $\frac{\theta_t}{\theta_c}$ is called the value of time.

Note: slope of the line on slide 5.

Value of time

Context

- The utility function contains a cost or price variable.
- What is the willingness of the traveler to pay for a modification of another variable of the model?
- Typical example in transportation: value of time.

Value of time

Price that a traveler is willing to pay to decrease the travel time.

Value of time

Definition

- Let c_{in} be the cost of alternative i for individual n.
- Let t_{in} be the value of travel time.
- Let $u(c_{in}, t_{in})$ be the value of the utility function.
- Consider a scenario where the travel time is reduced:

$$t'_{in} = t_{in} - \delta^t_{in}.$$

- We denote by δ^c_{in} the additional cost that would achieve the same utility, that is

$$u(c_{in} + \delta^c_{in}, t_{in} - \delta^t_{in}) = u(c_{in}, t_{in}).$$

- The value of time is the additional cost per unit of saved time, that is

$$\delta^c_{in}/\delta^t_{in}$$

Value of time

Calculation

Invoke Taylor's theorem:

$$\begin{aligned} u(c_{in}, t_{in}) &= u(c_{in} + \delta_{in}^c, t_{in} - \delta_{in}^t) \\ &\approx u(c_{in}, t_{in}) + \delta_{in}^c \frac{\partial u}{\partial c_{in}}(c_{in}, t_{in}) - \delta_{in}^t \frac{\partial u}{\partial t_{in}}(c_{in}, t_{in}) \end{aligned}$$

Therefore...

$$\frac{\delta_{in}^c}{\delta_{in}^t} = \frac{(\partial u / \partial t_{in})(c_{in}, t_{in})}{(\partial u / \partial c_{in})(c_{in}, t_{in})}$$

Value of time

For example, if

$$u_{in} = -\theta_t t_{in} - \theta_c c_{in} - \theta_x x_{in}.$$

then

$$\frac{\delta_{in}^c}{\delta_{in}^t} = \frac{(\partial u/\partial t_{in})(c_{in}, t_{in})}{(\partial u/\partial c_{in})(c_{in}, t_{in})} = \frac{\theta_t}{\theta_c}$$

Value of time in Switzerland

Per trip purpose

	Business	Work	Leasure	Shopping
Public transportation (CHF/h)	49.57	27.81	21.84	17.73
Car (CHF/h)	50.23	30.64	29.20	24.32

[Axhausen et al., 2008]

Willingness to pay

Other variables

- The same analysis can be done for other variables.
- Willingness to pay to have less transfers.
- Willingness to pay to have less waiting time.
- Willingness to pay to have WiFi in the bus.

Warning

Taylor's theorem can only be invoked for continuous variables.

Willingness to pay: example

$$u = -\theta_c c_{in} + \theta_w w_{in}$$

where $w_{in} = 1$ if WiFi is available in alternative i for traveler n, and 0 otherwise.

Utility without WiFi: $w_{in} = 0$

$$-\theta_c c_{in}$$

Utility with WiFi: $w_{in} = 1$

$$-\theta_c (c_{in} + \delta^c_{in}) + \theta_w$$

Willingness to pay for WiFi

$$-\theta_c c_{in} = -\theta_c (c_{in} + \delta^c_{in}) + \theta_w$$

$$0 = -\theta_c \delta^c_{in} + \theta_w$$

$$\delta^c_{in} = \frac{\theta_w}{\theta_c}.$$

Summary

Choice models

- Utility theory applies to discrete choices as well.
- Assumption: alternative associated with maximum utility is chosen.

Units of utility function

- Utility as such has no unit.
- It is possible to transform the function into the unit of any variable.
- Opposite of generalized cost, in CHF.

Willingness to pay

- Price that the traveler is willing to pay to improve another variable.
- Typical case: value of time = willingness to pay for travel time savings.

Bibliography

Axhausen, K., Hess, S., Koenig, A., Abay, G., Bates, J., and Bierlaire, M. (2008).
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