EPFL Introduction to transportation systems (2024-2025) TRANSP-OR

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Modeling — March 12, 2025

Question 1: Contingency table

We would like to analyze the morning peak hour by modeling the departure time of commuters in the morning. We build our model on the hypothesis that it depends on their professional occupation. We have interviewed a total of 1004 morning commuters and asked them two questions:

1. At what time did you depart from home yesterday for your morning commute? [Before 7:30 / Between 7:30 and 8:30 / After 8:30]
2. What is your professional occupation? [Full time job / Part time job]

The results of the data collection is summarized in the following contingency table:

	Full time	Part time
Before 7:30	122	62
7:30 – 8:30	238	356
After 8:30	45	181

We would like to develop a model.

1. What is the dependent variable of the model, and what type of variable is it?
2. What is the independent variable of the model, and what type of variable is it?
3. What are the unknown parameters of the model?
4. Write the log likelihood function.
5. Use the contingency table to obtain the estimates of the parameters.
6. It is expected that, in 5 years from now, 80% of the working population will be working full time, and 20% will be working part time. What will be the share of commuters departing in each period in the morning? Compare it with the shares today.

Question 2: Choice model

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We would like to develop a random utility model.

1. What is the choice set?
2. Define the utility function of each alternative in the choice set.
3. What is the probability provided by the logit model that a part-time worker departs before 7:30?

Question 3: Convolution

Consider two independent random variables X and Y, associated with the probability density functions f_X and f_Y, respectively, and with the cumulative distribution functions F_X and F_Y, respectively. They both can take any real value. Consider the random variable $Z = X + Y$. What are the probability density functions and cumulative distribution functions of Z?
Hint: for a given realization of X, the distribution of Z is closely related to the distribution of Y. Calculate first $\Pr(Z \leq z \mid X = x)$ using the CDF of Y. Then use it to deduce the CDF and the pdf of Z.