Freight transportation

Some logistics problems

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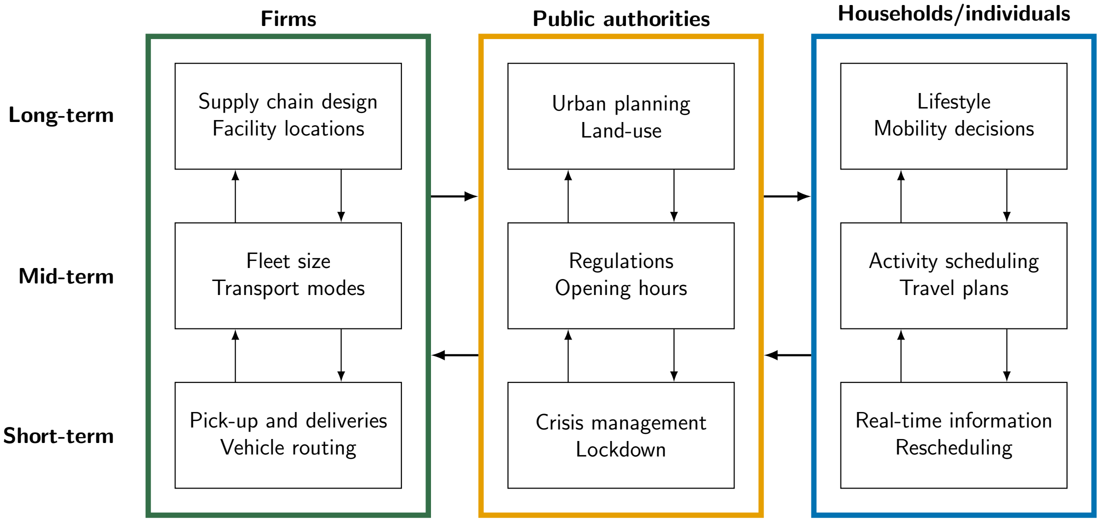
Introduction to transportation systems

Freight transportation

Differences between transporting people and goods?

- Goods have no behavior.
- Decisions are centralized or coordinated.
- Costs are the main driving indicators.
- Comfort, convenience, etc. have little role.

Choices and decisions



Logistics

Definition

Management of the flow of things between the point of origin and the point of consumption.

Source: Wikipedia

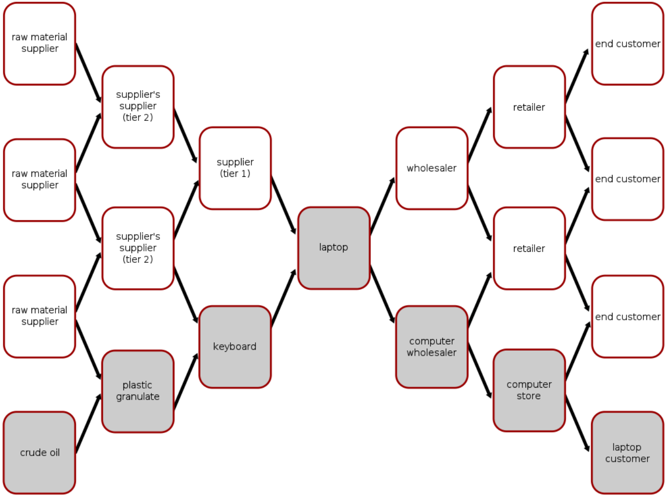
Philosophy

The right products in the right quantity to the right customers at the right time.

Logistics vs. supply chain

- Logistics: one actor.
- Supply chain: multiple actors.

Supply chain



Source: Wikipedia. Wieland and Wallenburg.

In this course

Analysis of three problems

- Long-term: facility location.
- Medium-term: inventory management.
- Short-term: vehicle routing problem.

Facility location

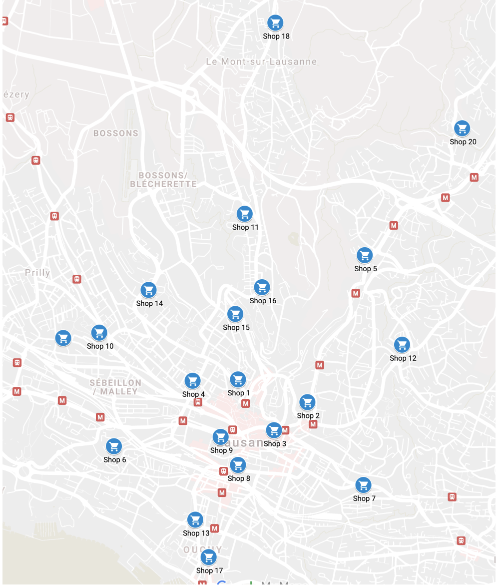
Problem data

- Set of customers: $\mathcal{C}$.
- Demand: d_j, $j \in \mathcal{C}$.
- Set of potential depots: $\mathcal{D}$.
- Setup cost: c_i, $i \in \mathcal{D}$.
- Capacity: ℓ_i, $i \in \mathcal{D}$.
- Trip duration: t_{ij}, $i \in \mathcal{D}$, $j \in \mathcal{C}$.

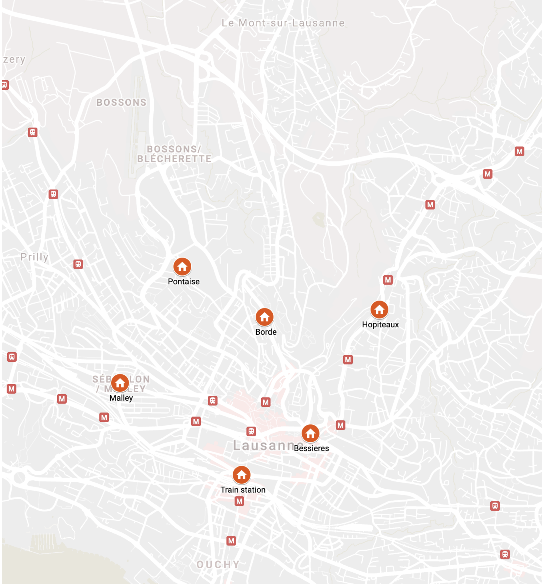
Objectives

- Decide which depots to open.
- Decide which depot serves which customer(s).

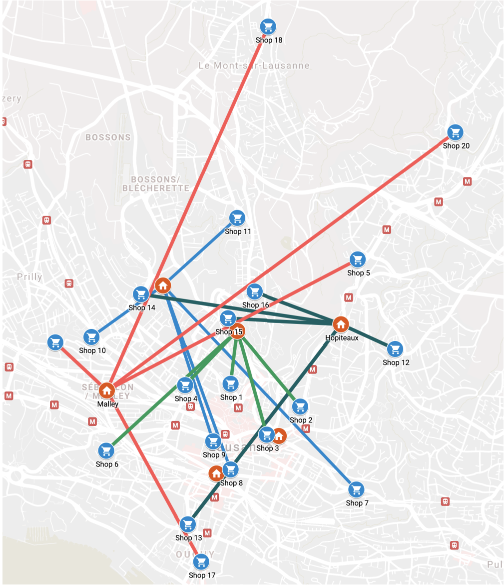
Facility location: 20 shops in Lausanne



Facility location: 5 potential locations for depots



Facility location: example of solution



Facility location: the model

Decision variables

- $x_i \in \{0, 1\}$, $i \in \mathcal{D}$: 1 if depot i is open, 0 otherwise.
- $y_{ij} \in \mathbb{R}$, $i \in \mathcal{D}$, $j \in \mathcal{C}$: proportion of the demand for j served by i.

Costs

- Fixed costs: $\sum_{i \in \mathcal{D}} c_i x_i$.
- Variable costs: $\sum_{i \in \mathcal{D}} \sum_{j \in \mathcal{C}} \gamma t_{ij} d_j y_{ij}$.
- γt_{ij} is the cost of transporting one unit with travel time t_{ij}.
- $d_j y_{ij}$ is the quantity transported from i to j.

Facility location: the model

Objective function

$$\min_{x,y} \sum_{i\in\mathcal{D}} c_i x_i + \gamma \sum_{i\in\mathcal{D}} \sum_{j\in\mathcal{C}} t_{ij} d_j y_{ij}.$$

Constraints

- Each customer must be served completely:

$$\sum_{i\in\mathcal{D}} y_{ij} = 1,\ \forall j \in \mathcal{C}.$$

- Depots cannot serve more than capacity

$$\sum_{j\in\mathcal{C}} d_j y_{ij} \leq \ell_i x_i,\ \forall i \in \mathcal{D}.$$

Facility location: the model

$$\min_{x,y} \sum_{i \in \mathcal{D}} c_i x_i + \gamma \sum_{i \in \mathcal{D}} \sum_{j \in \mathcal{C}} t_{ij} d_j y_{ij}.$$

subject to

$$\begin{aligned}
\sum_{i \in \mathcal{D}} y_{ij} &= 1, && \forall j \in \mathcal{C}, \\
\sum_{j \in \mathcal{C}} d_j y_{ij} &\leq \ell_i x_i, && \forall i \in \mathcal{D}, \\
x_i &\in \{0, 1\}, && \forall i \in \mathcal{D}, \\
y_{ij} &\geq 0, && \forall i \in \mathcal{D}, \forall j \in \mathcal{C}.
\end{aligned}$$

Shops in Lausanne: travel time in seconds

	Bessieres	Borde	Hopiteaux	Malley	Pontaise	Train station
Shop 1	141.7	125.8	262.3	177.6	121.4	220.1
Shop 2	92.5	197.2	115.2	319.9	320.4	248.7
Shop 3	139.2	177.7	257.9	207.8	300.9	192.4
Shop 4	247.9	227.8	385.2	149.9	181.8	192.4
Shop 5	237.4	342.1	88.0	464.8	366.3	393.6
Shop 6	257.6	321.2	394.9	146.0	275.2	157.1
Shop 7	98.3	242.5	249.8	285.7	350.2	147.3
Shop 8	151.8	289.5	297.0	198.0	273.8	17.3
Shop 9	113.3	243.1	250.6	153.8	229.6	116.7
Shop 10	357.7	361.2	495.0	170.9	239.4	321.7
Shop 11	326.1	184.7	416.1	366.7	215.3	409.2
Shop 12	265.3	370.0	277.8	492.7	493.2	421.5
Shop 13	234.2	369.4	379.4	247.6	323.4	102.6
Shop 14	280.7	216.3	401.3	172.3	79.1	282.2
Shop 15	214.9	186.6	335.5	250.8	134.2	293.3
Shop 16	280.4	139.0	370.4	321.0	203.6	363.5
Shop 17	224.9	362.6	370.1	302.4	378.2	138.5
Shop 18	462.1	320.7	438.2	502.7	333.4	545.2
Shop 19	383.7	412.4	521.0	140.2	325.9	324.4
Shop 20	452.2	556.9	302.8	662.2	477.3	608.4

Source: OpenStreetMap.org

Facility location: scenario 1

Setup costs

- Train station: 0,
- Pontaise: 0,
- Hopiteaux: 0,
- Malley: 0,
- Bessieres: 0,
- Borde: 0.

Capacity

- Train station: 50,
- Pontaise: 50,
- Hopiteaux: 50,
- Malley: 50,
- Bessieres: 50,
- Borde: 50.

Conversion parameter

$\gamma = 0.01$

Demand

- Shop 1: 1,
- Shop 2: 1,
- Shop 3: 1,
- Shop 4: 1,
- Shop 5: 1,
- Shop 6: 1,
- Shop 7: 1,
- Shop 8: 1,
- Shop 9: 1,
- Shop 10: 1,
- Shop 11: 1,
- Shop 12: 1,
- Shop 13: 1,
- Shop 14: 1,
- Shop 15: 1,
- Shop 16: 1,
- Shop 17: 1,
- Shop 18: 1,
- Shop 19: 1,
- Shop 20 : 1.

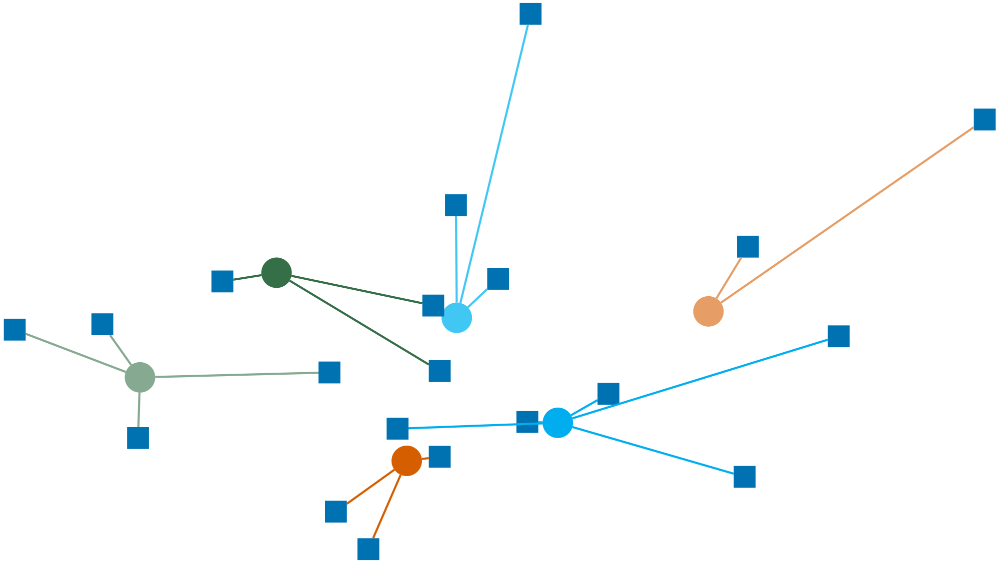
Scenario 1

Discussion: What can we expect?

Comments

- No setup cost.
- Only transport costs are optimized.
- All depots are open.
- Customers are assigned to the closest depot.

Scenario 1: optimal cost: 29.44



Facility location: scenario 2

Setup costs

- Train station: 5,
- Pontaise: 5,
- Hopiteaux: 5,
- Malley: 5,
- Bessieres: 5,
- Borde: 5.

Capacity

- Train station: 50,
- Pontaise: 50,
- Hopiteaux: 50,
- Malley: 50,
- Bessieres: 50,
- Borde: 50.

Conversion parameter

$\gamma = 0.01$

Demand

- Shop 1: 1,
- Shop 2: 1,
- Shop 3: 1,
- Shop 4: 1,
- Shop 5: 1,
- Shop 6: 1,
- Shop 7: 1,
- Shop 8: 1,
- Shop 9: 1,
- Shop 10: 1,
- Shop 11: 1,
- Shop 12: 1,
- Shop 13: 1,
- Shop 14: 1,
- Shop 15: 1,
- Shop 16: 1,
- Shop 17: 1,
- Shop 18: 1,
- Shop 19: 1,
- Shop 20 : 1.

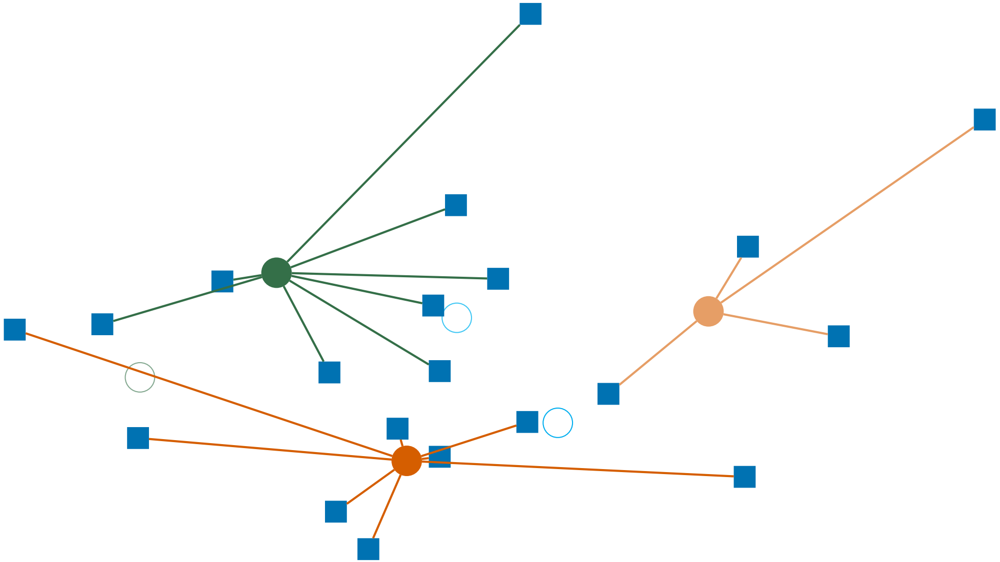
Scenario 2

Discussion: What can we expect?

Comments

- Difficult to say a priori.
- There is now a price to open a depot.
- It depends on the trade-off between transportation costs and setup costs.
- We expect that some depots will not be open.
- If so, each depot has to serve more customers.

Scenario 2: optimal cost: 49.88



Facility location: scenario 3

Setup costs

- Train station: 5,
- Pontaise: 5,
- Hopiteaux: 5,
- Malley: 5,
- Bessieres: 5,
- Borde: 5.

Capacity

- Train station: 5,
- Pontaise: 5,
- Hopiteaux: 5,
- Malley: 5,
- Bessieres: 5,
- Borde: 5.

Conversion parameter

$\gamma = 0.01$

Demand

- Shop 1: 1,
- Shop 2: 1,
- Shop 3: 1,
- Shop 4: 1,
- Shop 5: 1,
- Shop 6: 1,
- Shop 7: 1,
- Shop 8: 1,
- Shop 9: 1,
- Shop 10: 1,
- Shop 11: 1,
- Shop 12: 1,
- Shop 13: 1,
- Shop 14: 1,
- Shop 15: 1,
- Shop 16: 1,
- Shop 17: 1,
- Shop 18: 1,
- Shop 19: 1,
- Shop 20 : 1.

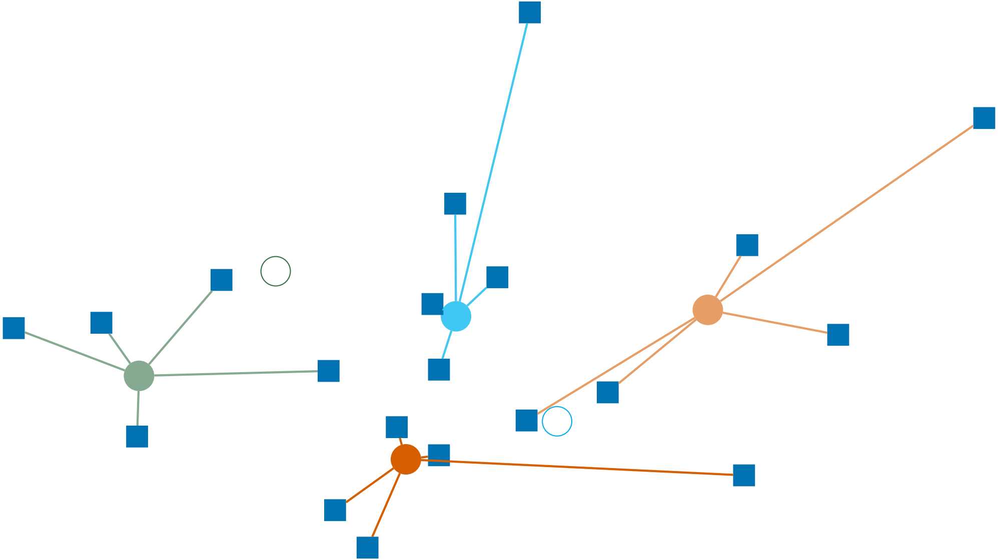
Scenario 3

Discussion: What can we expect?

Comments

- Each depot can serve maximum 5 customers.
- We need more depots than for the previous scenario.
- Again, difficult to say a priori how many.

Scenario 3: optimal cost: 53



Facility location: scenario 4

Setup costs

- Train station: 5,
- Pontaise: 5,
- Hopiteaux: 5,
- Malley: 5,
- Bessieres: 5,
- Borde: 5.

Capacity

- Train station: 5,
- Pontaise: 5,
- Hopiteaux: 5,
- Malley: 5,
- Bessieres: 5,
- Borde: 5.

Conversion parameter

$\gamma = 0.01$

Demand

- Shop 1: 6,
- Shop 2: 1,
- Shop 3: 1,
- Shop 4: 1,
- Shop 5: 1,
- Shop 6: 1,
- Shop 7: 1,
- Shop 8: 1,
- Shop 9: 1,
- Shop 10: 1,
- Shop 11: 1,
- Shop 12: 1,
- Shop 13: 1,
- Shop 14: 1,
- Shop 15: 1,
- Shop 16: 1,
- Shop 17: 1,
- Shop 18: 1,
- Shop 19: 1,
- Shop 20 : 1.

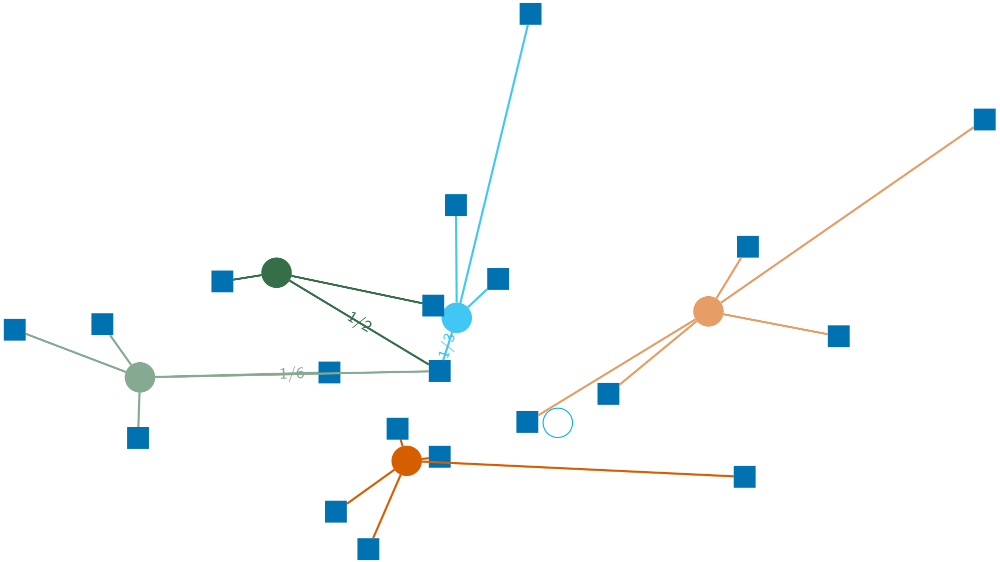
Scenario 4

Discussion: What can we expect?

Comments

- The demand of one customer exceeds the capacity of the depots.
- This customer is served by multiple depots.

Scenario 4: optimal cost: 63.22



Facility location problem

Summary

- Decide which depots to open, and which depots serve each customer.
- Complex optimization problem.
- Many variants of the problem exist, depending on the context.

Inventory management

Motivation

- Items are not shipped as soon as they are produced.
- Items arrive at destination before they are consumed.
- They must be stored on both ends.
- This generates extra costs:
  - Storage costs: space and facilities, staff, insurance, etc.
  - Waiting costs: opportunity costs of capital, obsolescence, etc.
- How to manage the inventory?

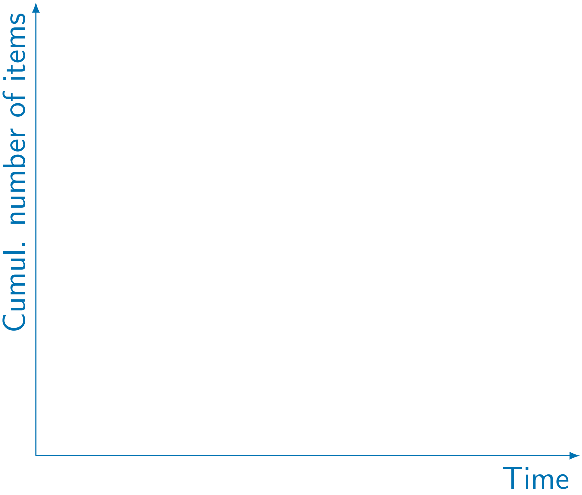
Inventory management

Context of the analysis

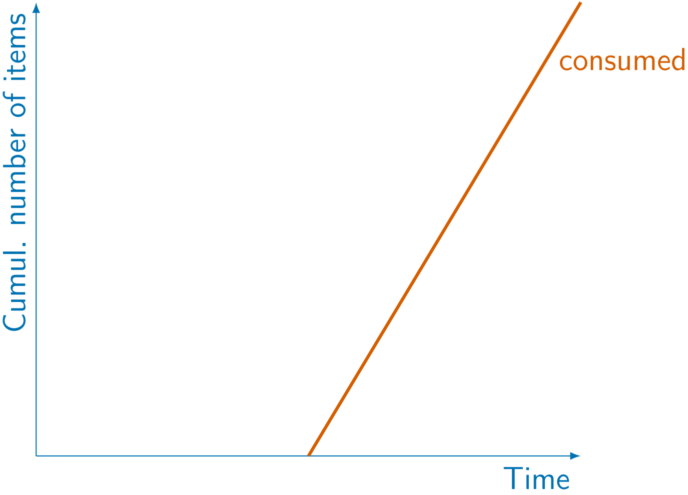
- Items are produced and shipped at one location.
- They are received and consumed at another location.
- Constant demand: f.
- Time horizon: t_H.
- Question: should we ship the items one by one, or two by two, etc.?
- Shipment size: quantity s of items shipped.
- Headway: time between two shipments: $s = hf$.

Inventory management

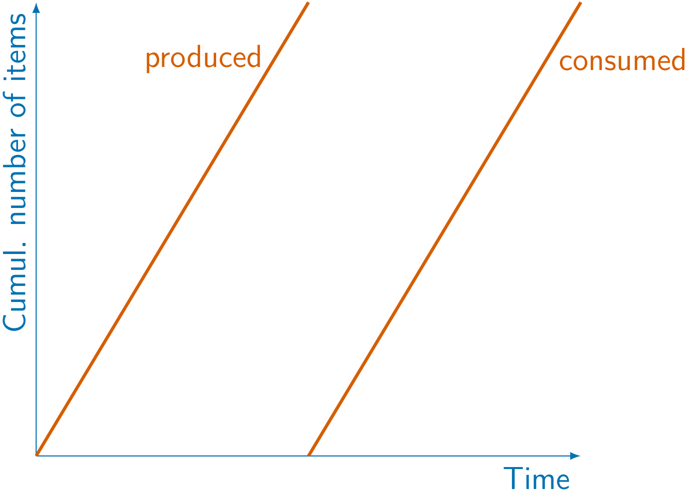
Inventory management



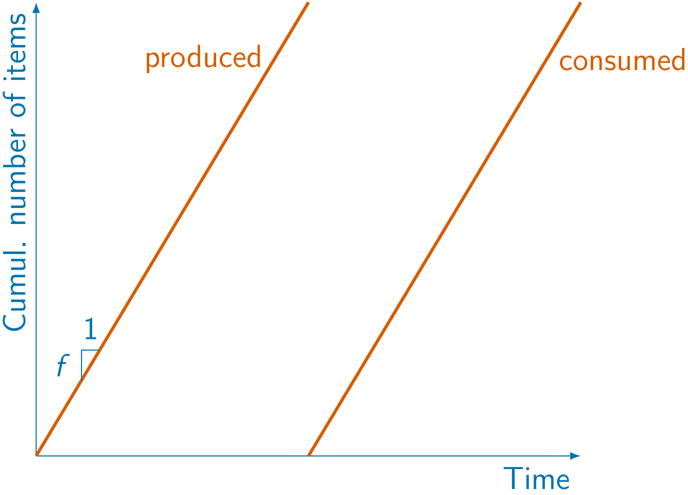
Inventory management



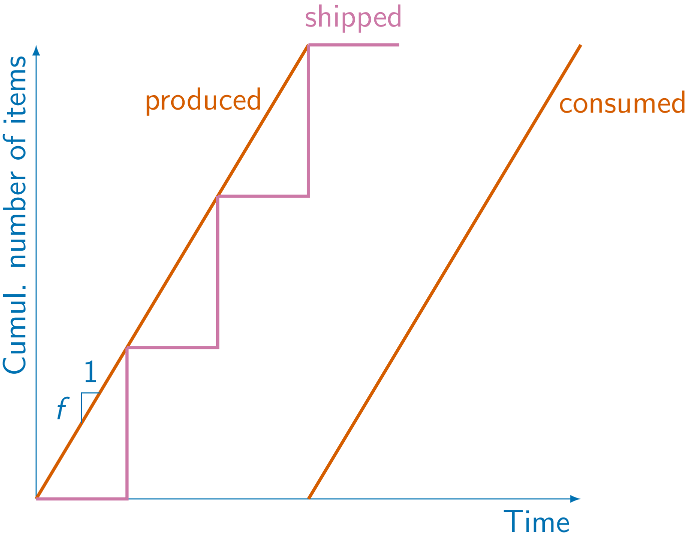
Inventory management



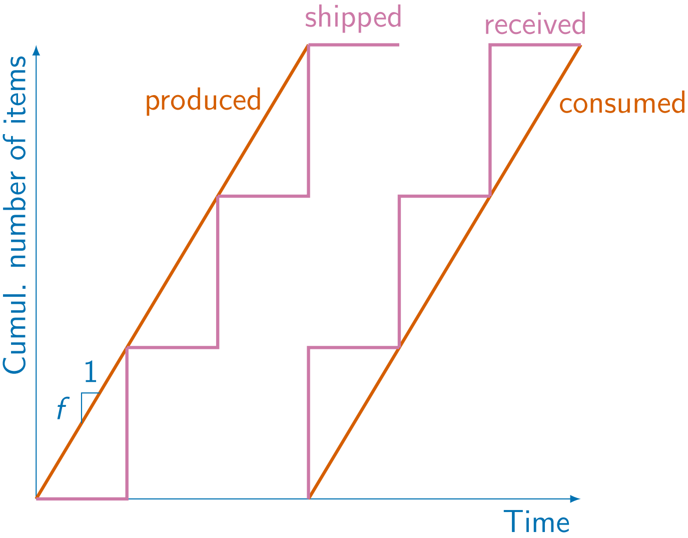
Inventory management



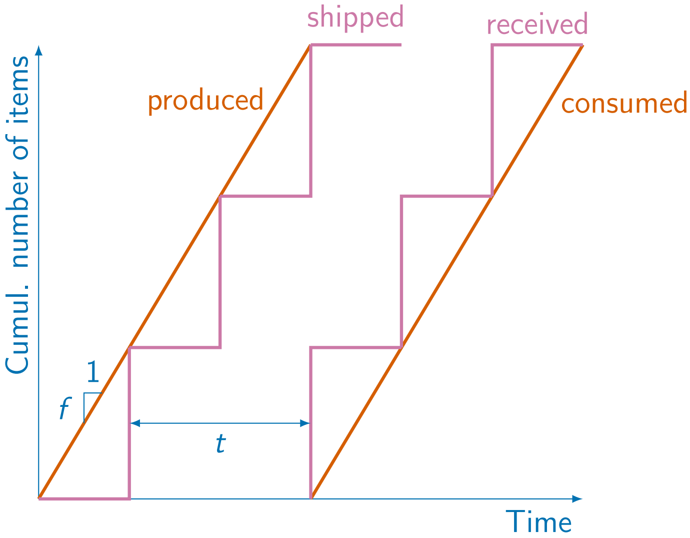
Inventory management



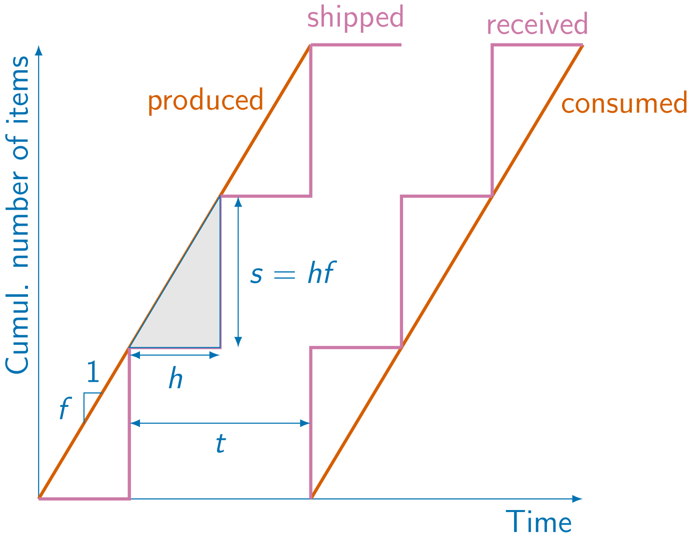
Inventory management



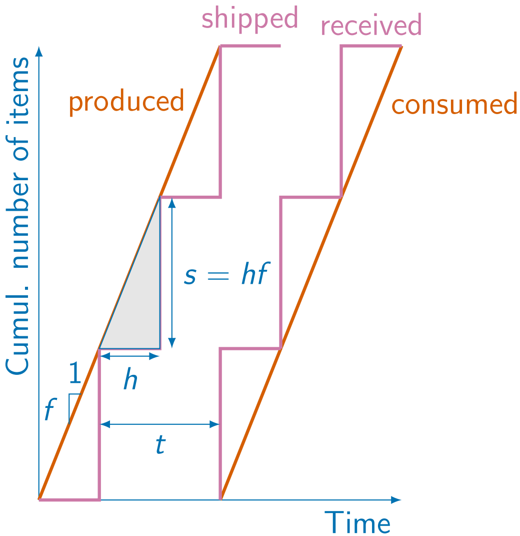
Inventory management



Inventory management



Inventory management



- Shipment size: s
- Number of shipments: $ft_H/s = t_H/h$
- Storage costs: $c_r s = c_r hf$
- Waiting time: $h + t$
- Waiting costs: $c_w f(h + t)$
- One shipment costs: $c_f + c_v s$
- All shipments costs: $t_H c_f/h + t_H c_v s/h = t_H c_f/h + t_H c_v f$
- Total costs: $c_r hf + c_w f(h + t) + t_H c_f/h + t_H c_v f$

Inventory management

Units

- s: items.
- h: time unit (minutes, days, years).
- f: items/day.
- c_r: CHF/item.
- c_w: CHF/item.
- c_f: CHF.
- c_v: CHF/item.

Optimal headway

Total costs

$$c_r hf + c_w f(h+t) + t_H c_f / h + t_H c_v f$$

Derivative

$$c_r f + c_w f - \frac{t_H c_f}{h^2}$$

Optimal headway

$$\sqrt{\frac{c_f t_H}{f(c_r + c_w)}}$$

Optimal headway

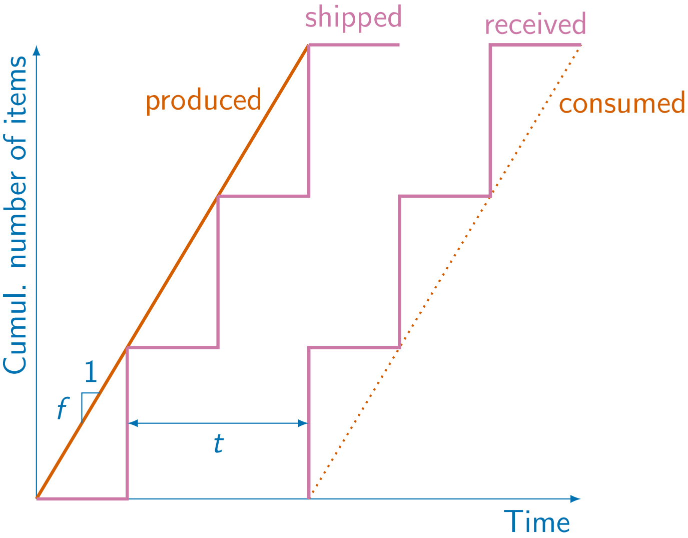
$$\sqrt{\frac{c_f t_H}{f(c_r + c_w)}}$$

Comments

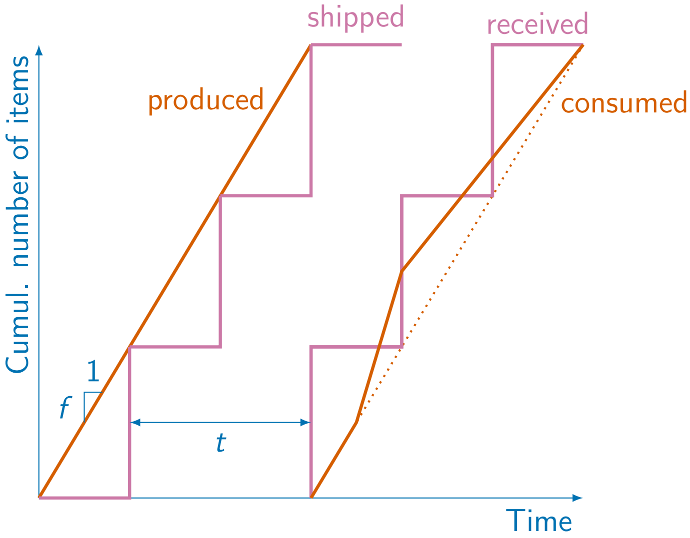
- The higher the fixed transportation costs, the higher the headway.
- Example: container ships, with 24K TEU (between 12K and 24K containers).
- The higher the holding costs, the lower the headway.
- Example: car manufacturing, “just-in-time”.

Source: Wikimedia Commons

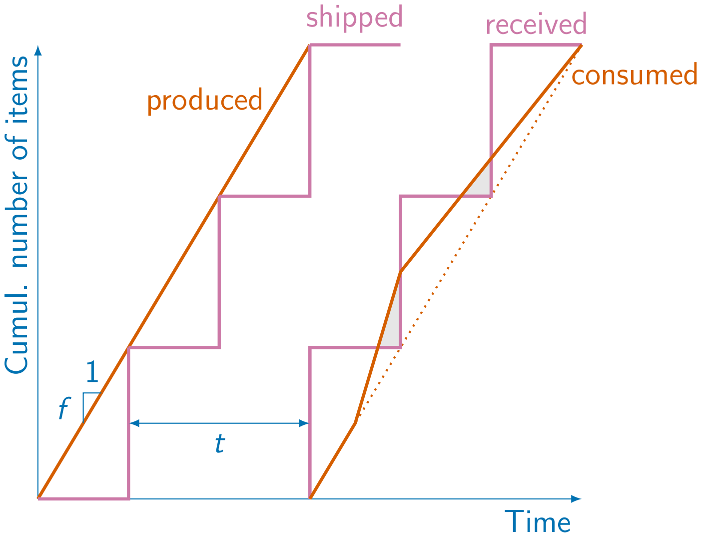
Variable consumption



Variable consumption



Variable consumption



Variable consumption

Comments

- Time varying decisions.
- Unsatisfied demand should be avoided.

Approach

- Multi-period model.
- Within a period everything is constant.

Multi-period inventory management

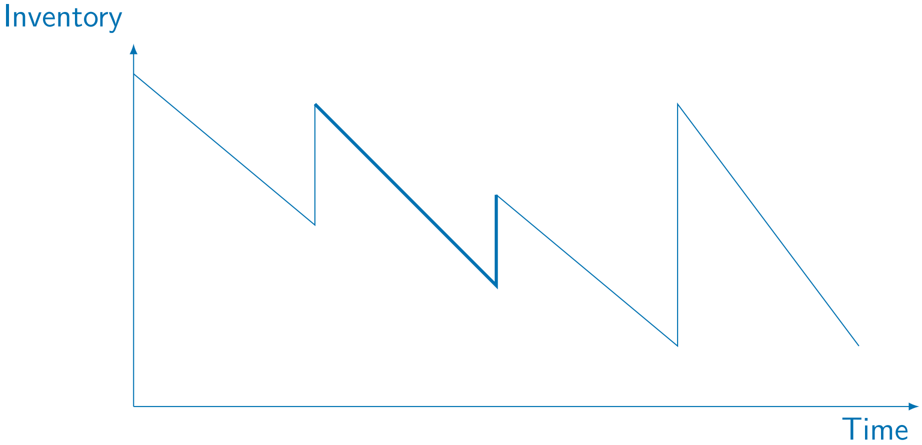
Data

- Time horizon: $[0, t_H]$.
- Divided into K periods indexed $t = 1, 2, \ldots K$.
- δ_t: duration of period t.
- Demand: $f_1, f_2, \ldots, f_K$
- Initial inventory: i_0.
- Costs: c_w^t, c_f^t, c_v^t. Fixed storage costs.

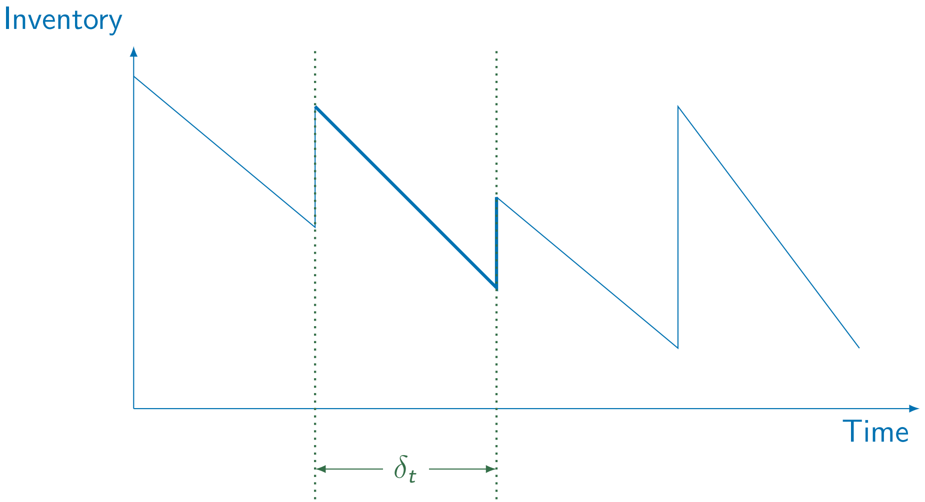
Variables

- Quantity to order: s_t.
- Inventory at the end of the period: $i_t = i_{t-1} - f_t\delta_t + s_t$.

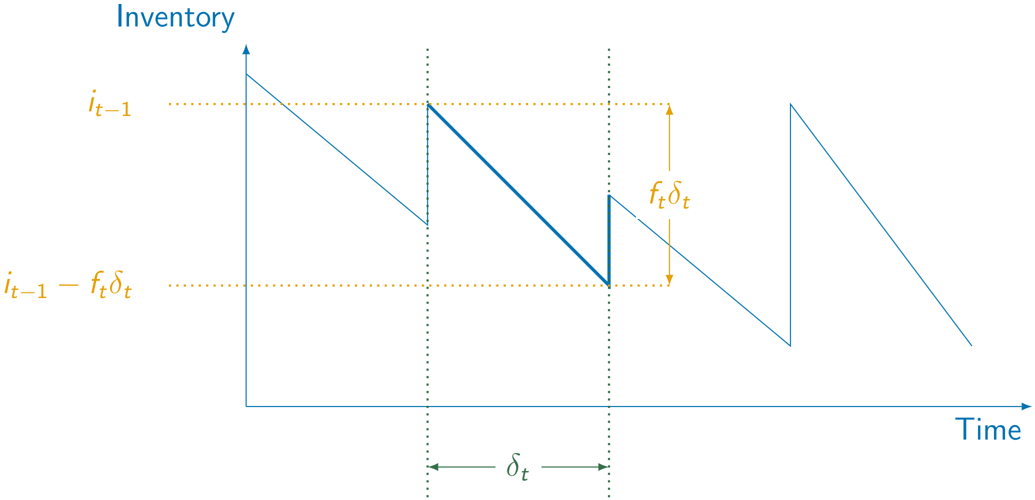
Multi-period inventory management



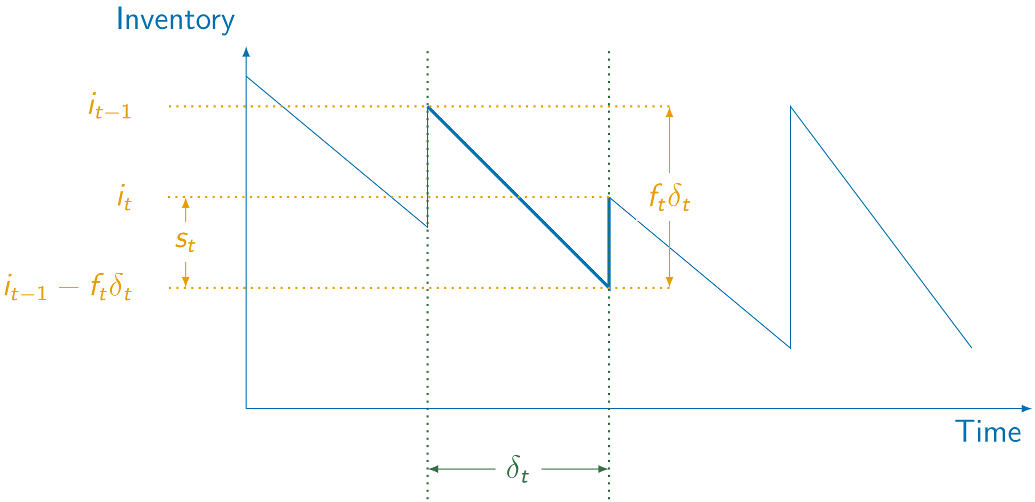
Multi-period inventory management



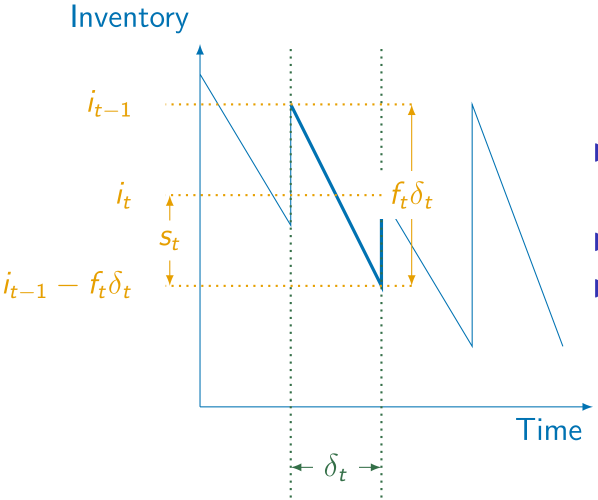
Multi-period inventory management



Multi-period inventory management



Multi-period inventory management



- Stored items:
  $\frac{1}{\delta_t}(i_{t-1}\delta_t - \frac{1}{2}f_t\delta_t^2) = i_{t-1} - \frac{1}{2}f_t\delta_t$.
- Waiting costs: $c_w^t(i_{t-1} - \frac{1}{2}f_t\delta_t)$
- Transportation costs: $c_f^t + c_V^t s_t$.

Multi-period inventory management

Objective function

$$\min_{s,i} \sum_{t=1}^{K} c_w^t i_{t-1} - \cancel{\frac{1}{2} c_w^t f_t \delta_t} + \cancel{c_f^t} + c_V^t s_t$$

Inventory dynamics constraints

$$i_t = i_{t-1} - f_t \delta_t + s_t, \ t = 1, \ldots, K.$$

Demand satisfaction constraints

$$i_{t-1} - f_t \delta_t \geq 0, \ t = 1, \ldots, K.$$

Multi-period inventory management

Comments

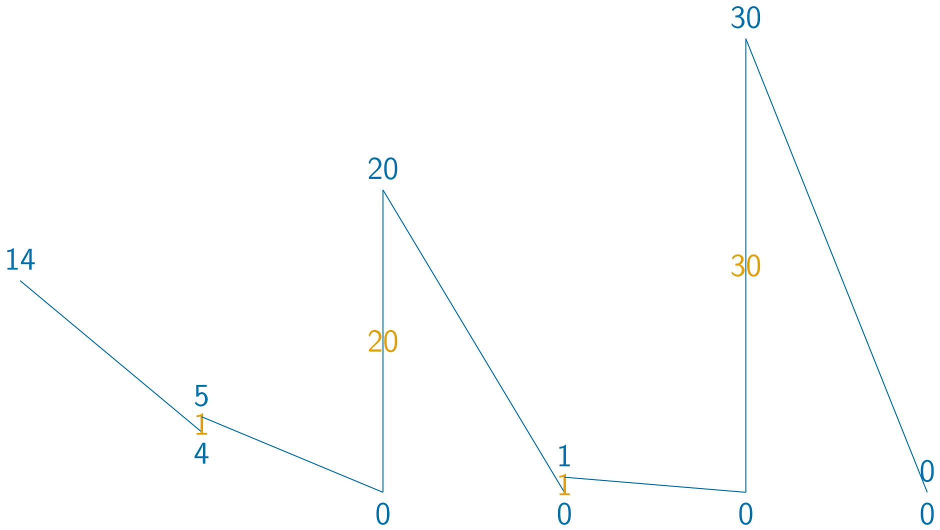
- Linear optimization problem.
- As we order at each period, the fixed transportation costs do not depend on the decision variables.

Scenario 1

Data

- $K = 5$ intervals of duration 1
- Initial inventory: 14
- Demand: [10, 5, 20, 1, 30]
- Waiting costs: [1, 1, 1, 1, 1]
- Transportation costs: [1, 1, 1, 1, 1]

Scenario 1: optimal solution



Scenario 1

Comments

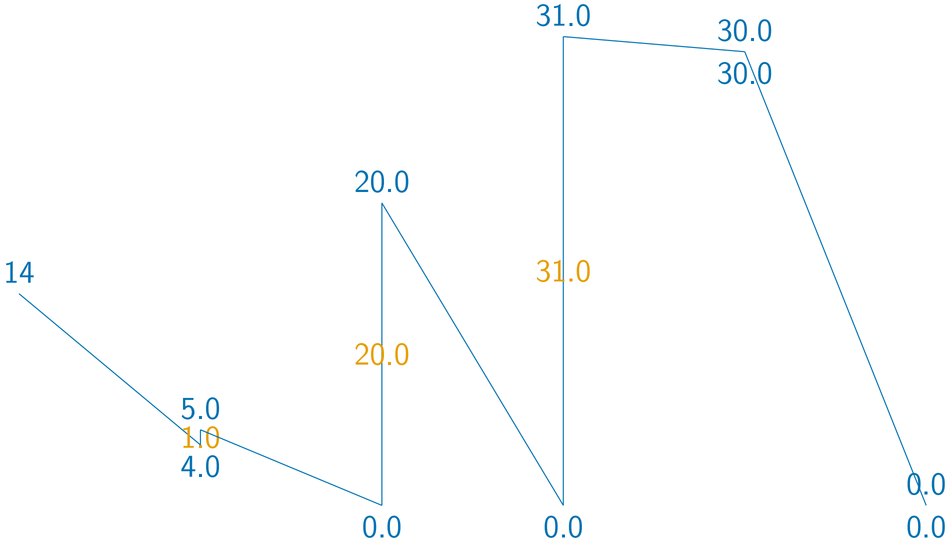
- Transportation costs are low.
- Beginning of each period: inventory = demand.
- "just-in-time" strategy.
- Perfect anticipation.

Scenario 2

Data

- $K = 5$ intervals of duration 1
- Initial inventory: 14
- Demand: [10, 5, 20, 1, 30]
- Waiting costs: [1, 1, 1, 1, 1]
- Transportation costs: [1, 1, 1, 100, 1]

Scenario 2: optimal solution



Scenario 2

Comments

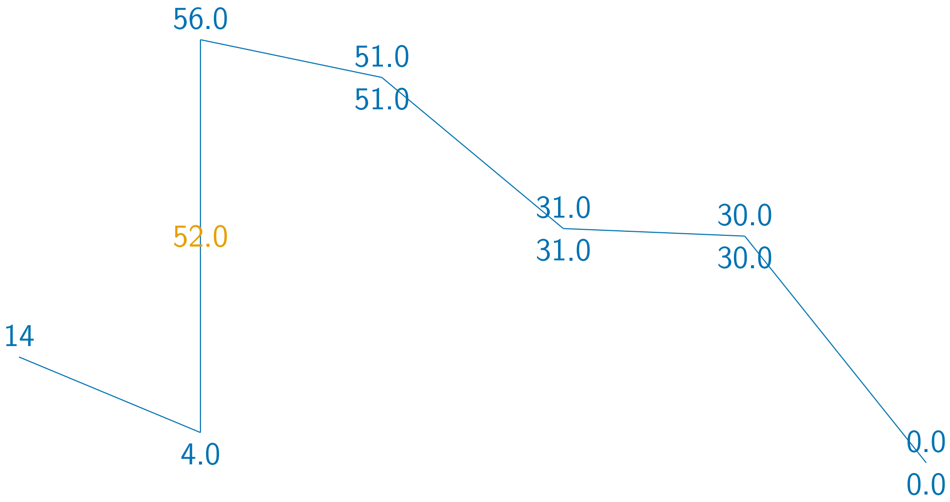
- Transportation costs are high during one period.
- The demand for two periods is ordered when costs are cheap.
- Again, perfect anticipation.

Scenario 3

Data

- $K = 5$ intervals of duration 1
- Initial inventory: 14
- Demand: [10, 5, 20, 1, 30]
- Waiting costs: [1, 1, 1, 1, 1]
- Transportation costs: [1, 100, 100, 100, 100]

Scenario 3: optimal solution



Scenario 3

Comments

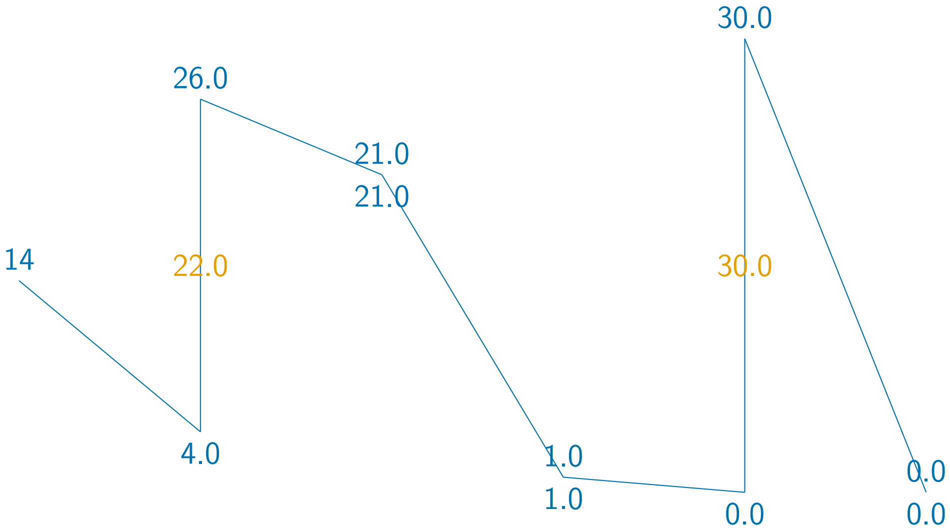
- Transportation costs are always high, except during the first period.
- The demand for all periods is ordered when costs are cheap.
- Again, perfect anticipation.

Scenario 4

Data

- $K = 5$ intervals of duration 1
- Initial inventory: 14
- Demand: [10, 5, 20, 1, 30]
- Waiting costs: [1, 1, 1, 100, 1]
- Transportation costs: [1, 100, 100, 100, 100]

Scenario 4: optimal solution



Scenario 4

Comments

- Waiting costs are high during period 4.
- Order just what is needed for the last period.
- All the rest is ordered when transportation is cheap.
- Inventory brought to zero when expensive.
- Again, perfect anticipation.

Inventory management

Summary

- Between production and consumption, items are stored and transported.
- Both generate costs.
- Minimizing storage costs implies short headways.
- Minimizing transportation costs implies long headways.
- Trade-off: optimization problem.
- Demand issue: variations around the mean.
- Multi-period model: linear optimization.
- Other demand issue: forecasting errors.
- Requires a stochastic version of the model.

Vehicle routing problem

Motivation

- Items must be delivered to customers.
- A fleet of vehicles is available at the depot.
- Which vehicle serves which customer?
- In what order must the customers be visited?

Vehicle routing problem

Problem data

- Set of customers: $\mathcal{C}$.
- Set of locations: $\mathcal{C}^+ = \mathcal{C} \cup \{\text{depot}\}$.
- Index of depot: 0.
- Demand: d_j, $j \in \mathcal{C}$.
- Number of vehicles: q.
- Capacity of each vehicle: ℓ.
- Trip duration between each location: t_{ij}, $i, j \in \mathcal{C}^+, i \neq j$.
- We assume symmetry: $t_{ij} = t_{ji}$, $i, j \in \mathcal{C}^+$.

Vehicle routing problem: the model

Decisions variables

$x_{ij} \in \{0,1\}$, $i,j \in \mathcal{C}^+$: 1 if j is visited just after i.

Objective function

$$\min \sum_{i,j \in \mathcal{C}^+} t_{ij} x_{ij}.$$

Vehicle routing problem: the model

Constraints: each customer has exactly one successor

$$\sum_{j \in \mathcal{C}^+} x_{ij} = 1, \ \forall i \in \mathcal{C}.$$

Constraints: each customer has exactly one predecessor

$$\sum_{i \in \mathcal{C}^+} x_{ij} = 1, \ \forall j \in \mathcal{C}.$$

Constraint: number of successors of the depot is the number of vehicles

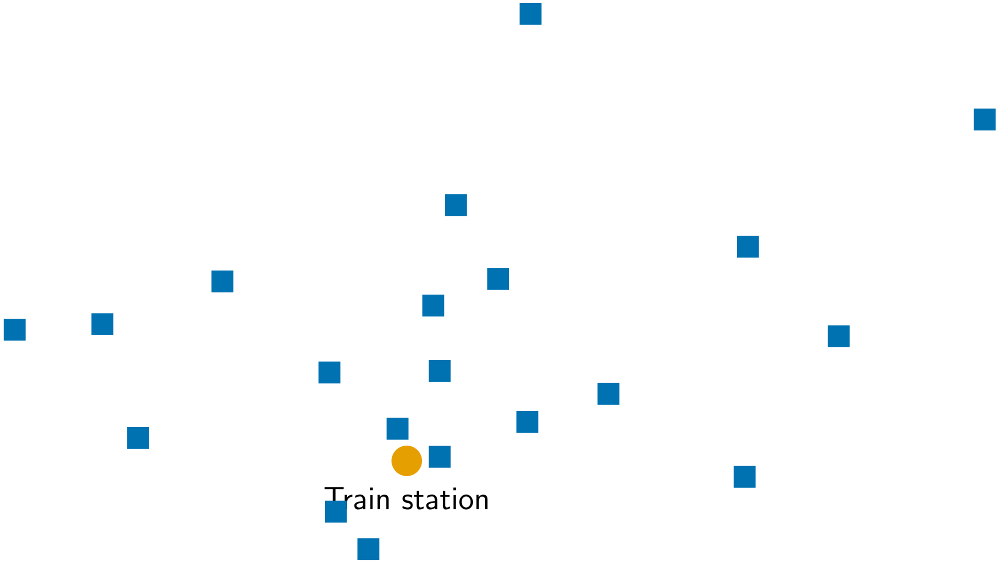
$$\sum_{j \in \mathcal{C}} x_{0j} = q.$$

Example

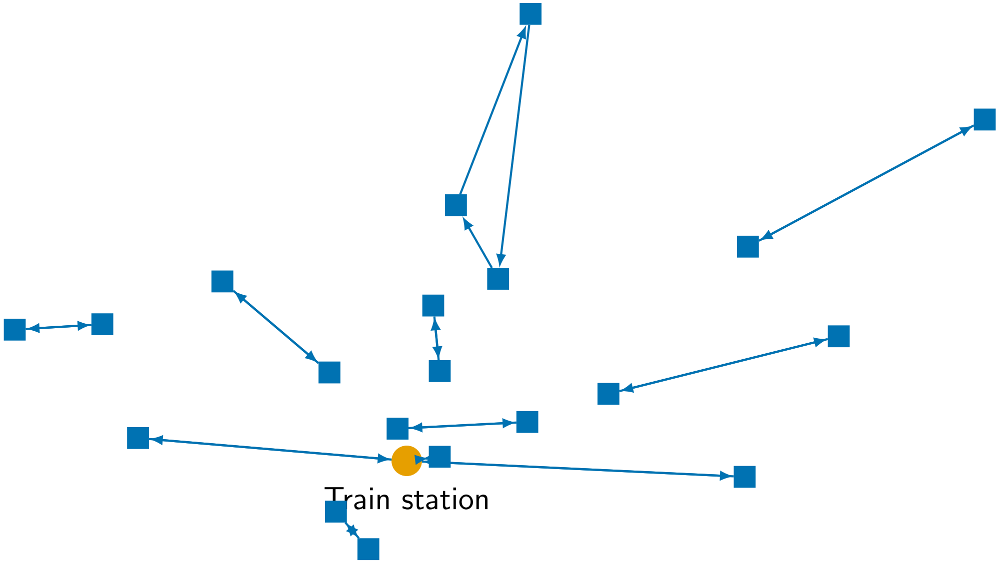
Problem data: scenario 1

- 20 customers.
- Depot: train station.
- Demand: $d_j = 1, \forall j \in \mathcal{C}$.
- Number of vehicles: 3.
- Capacity of each vehicle: 20.
- Trip duration between each location: OpenStreetMap.

Example: location of depot and customers



Scenario 1: solution



Invalid solution

Issues

- Presence of subtours.
- Capacity is ignored.

Notes

- Traveling salesman problem = vehicle routing with one vehicle.
- We use a similar solution for subtour elimination as for TSP.

Solution

- Introduce new variables.
- u_i: content of the vehicle when reaching customer i.
- Need for additional constraints.

Vehicle routing problem: the model

Constraints: capacity

$$u_j \leq \ell, \ \forall j \in \mathcal{C}.$$

Constraints: demand

$$u_j \geq d_j, \ \forall j \in \mathcal{C}.$$

Constraints: definition of u_i

$$u_i - u_j + \ell x_{ij} \leq \ell - d_j, \ \forall i, j \in \mathcal{C}.$$

Vehicle routing problem: the model

$$u_i - u_j + \ell x_{ij} \leq \ell - d_j.$$

If $x_{ij} = 1$

$$u_i - u_j + \cancel{\ell} \leq \cancel{\ell} - d_j.$$

$$u_j \geq u_i + d_j > u_i$$

Strictly increasing: no subtour.

If $x_{ij} = 0$

$$u_i - u_j \leq \ell - d_j.$$

$$d_j - u_j \leq \ell - u_i.$$

Always true (cap. and demand constraints): negative number $\leq$ positive number

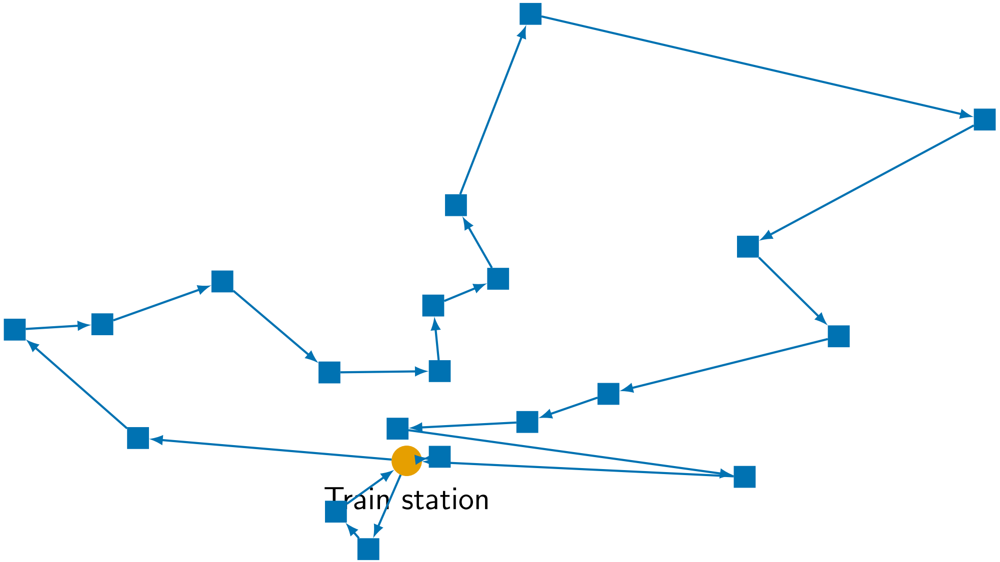
Vehicle routing problem: the model

$$\min_{x,y} \sum_{i,j \in \mathcal{C}^+} t_{ij} x_{ij}.$$

subject to

$$\begin{aligned}
\sum_{j \in \mathcal{C}^+} x_{ij} &= 1, && \forall i \in \mathcal{C}, \\
\sum_{i \in \mathcal{C}^+} x_{ij} &= 1, && \forall j \in \mathcal{C}, \\
\sum_{j \in \mathcal{C}} x_{0j} &= q, && \\
u_j &\leq \ell, && \forall j \in \mathcal{C}, \\
u_j &\geq d_j, && \forall j \in \mathcal{C}, \\
u_i - u_j + \ell x_{ij} &\leq \ell - d_j, && \forall i, j \in \mathcal{C}, \\
x_{ij} &\in \{0, 1\}, \forall i, j \in \mathcal{C}^+.
\end{aligned}$$

Scenario 1: solution



Scenario 1: solution

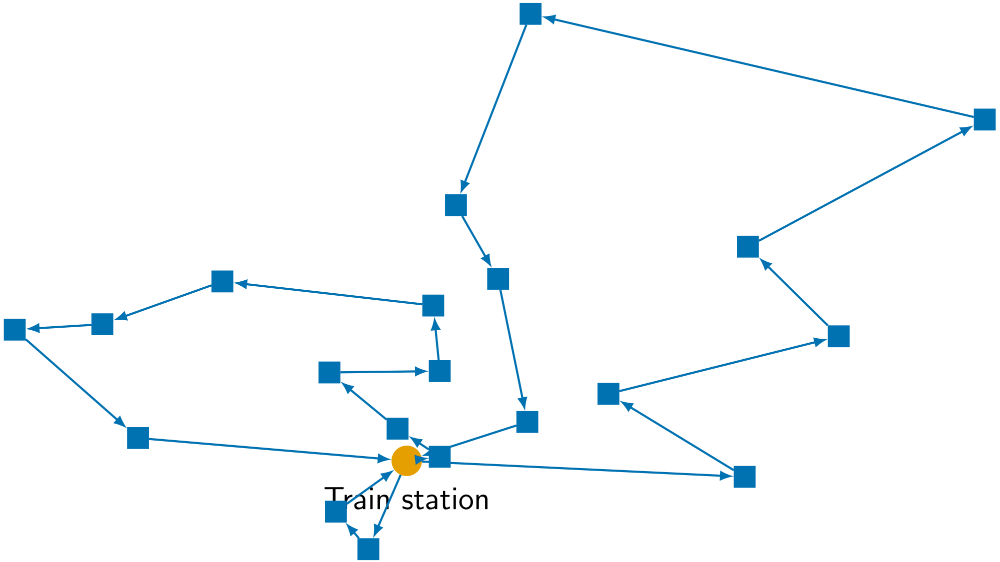
Comments

- Vehicle 1: 17 customers.
- Vehicle 2: 2 customers.
- Vehicle 3: 1 customer.

Scenario 2

Capacity of the vehicles: 9

Scenario 2: solution



Scenario 2: solution

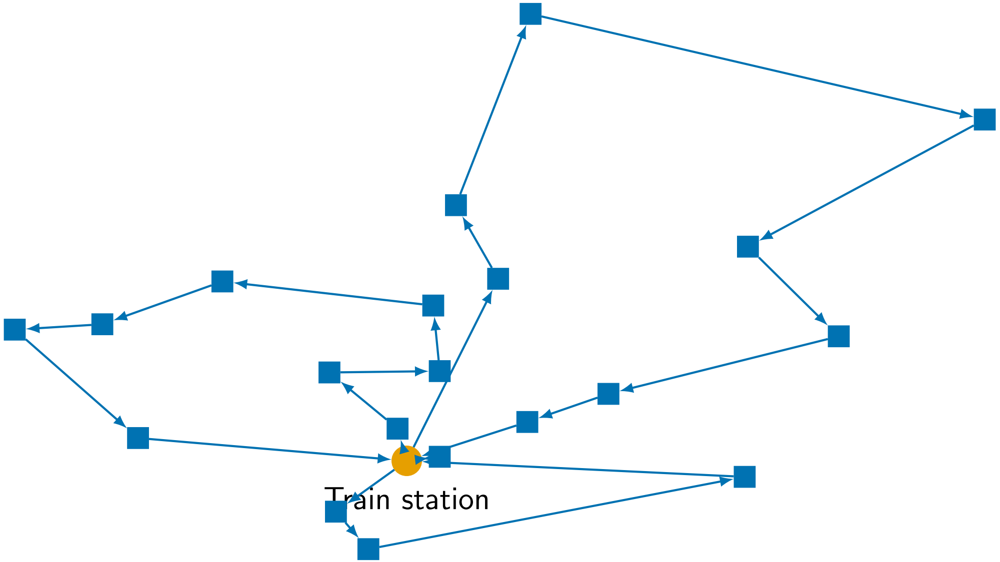
Comments

- Vehicle 1: 9 customers.
- Vehicle 2: 9 customers.
- Vehicle 3: 2 customer.

Scenario 3

- Number of vehicles: 4
- Capacity of the vehicles: 8

Scenario 3: solution



Scenario 3: solution

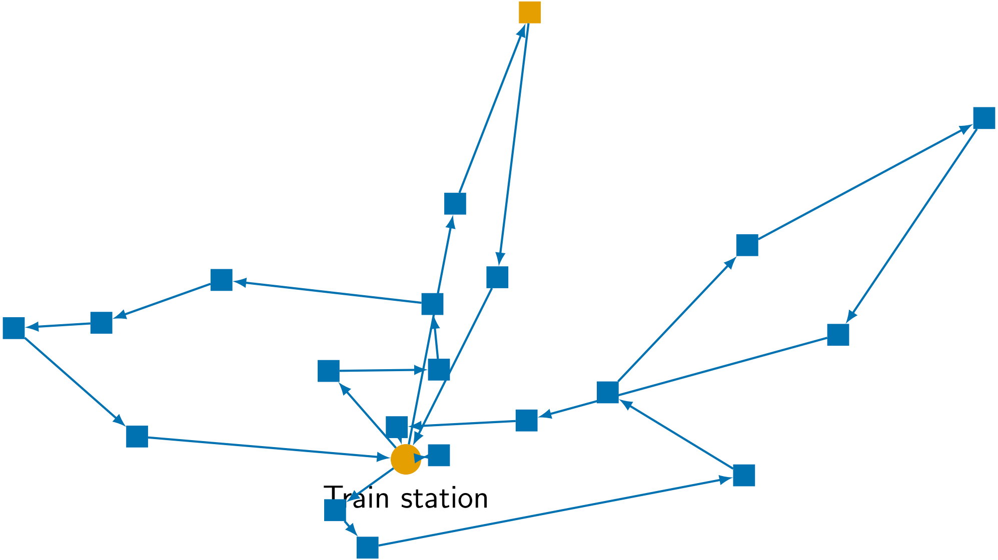
Comments

- Vehicle 1: 8 customers.
- Vehicle 2: 8 customers.
- Vehicle 3: 3 customers.
- Vehicle 4: 1 customer.

Scenario 4

- Number of vehicles: 4.
- Capacity of the vehicles: 10.
- Demand for client 18: 8.

Scenario 4: solution



Scenario 4

Solution

- Vehicle 1: 9 customers.
- Vehicle 2: 7 customers.
- Vehicle 3: 3 customers.
- Vehicle 4: 1 customer.

Complexity

More than 10 hours on EPFL Scitas High Performance Computing servers.

Vehicle routing problem

Many other variants

- Heterogeneous fleet.
- Pick up and delivery.
- Time windows.
- Split deliveries.
- Multiple depot.
- etc.

Vehicle routing problem

Complexity

- Complex optimization problem.
- Mathematical complexity due to subtour elimination and capacity constraints.
- There exist other formulations that are more efficient to solve.
- In practice, heuristics are often used.
- Indeed, the problem must be solved frequently with new data.

Freight transportation

Summary

- Less behavior, more optimization.
- Three examples of problems:
  - Long-term: facility location.
  - Medium-term: inventory management.
  - Short-term: vehicle routing.