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Introduction — February 19, 2025

Question 1: Road maintenance

Consider the road maintenance problem, where the quality of the road is measured by an index ranging from g_0 (the lowest acceptable road quality) to $g_{\max}$ (the road is in perfect, brand-new condition). The road deteriorates at a rate of τ quality units per day. The cost of repairing the road depends on its condition and is expressed as:

$$c(t) = c_f + c_v \tau t,$$

where c_f is the fixed cost (in KCHF), and c_v is the variable cost per quality unit (in KCHF per quality unit improved).

The engineer responsible for road maintenance must decide the optimal frequency of repairs over a time horizon of t_H days. If δ_t represents the number of days between two consecutive maintenance works, the cumulative quality of the road over this period is given by:

$$g(\delta_t) = t_H \left(g_{\max} - \frac{\tau \delta_t}{2} \right),$$

and the total cost of maintenance is:

$$c(\delta_t) = t_H \left(\frac{c_f}{\delta_t} + c_v \tau \right).$$

The engineer aims to select δ_t such that the marginal increase in cost is equal to the marginal increase in quality. Indeed, if the marginal cost is higher than the marginal quality improvement, resources are being wasted on overly frequent repairs. Conversely, if the marginal quality increase is higher than the marginal cost, the road may deteriorate too much before repairs, leading to potentially higher future costs or a decrease in service quality.

Derive the formula for δ_t that achieves this, and calculate its value for the following parameters:

$$\tau = 1, \quad g_{\max} = 100, \quad t_H = 365, \quad c_f = 100, \quad c_v = 5.$$

Question 2: Externalities

An externality is a side effect or consequence that affects other parties without this being reflected in the costs. Mention some negative externalities of private car transportation in cities, and suggest some possible “internalizations” of these externalities, that is, some ideas to reflect them in the costs.

Question 3: Shuttle service

At the end of a football game, there is a flow of f passengers per minute that need to be transported by shuttle to the train station. The goal is to optimize the shuttle operations over a time horizon t_H minutes. The model introduced in the lecture provides the following parameters and relationships.

As the rate of travelers arriving to take the shuttle is f passengers per minute, the total number of travelers over the time horizon t_H is ft_H passengers. The decision variable in this problem is the headway between shuttle departures, denoted by δ_t minutes.

The total number of trips made by the shuttles during the time horizon is t_H/δ_t, and each trip transports $\delta_t f$ passengers. The cost per trip is given by c CHF, so the total cost of the operation is $t_H c/\delta_t$ CHF.

The waiting time for passengers is also a consideration. The waiting time per trip is $\delta_t^2 f/2$ passenger-minutes, and thus the total waiting time over the entire horizon is $t_H \delta_t f/2$ passenger-minutes.

1. Assume that travelers are willing to pay W CHF to reduce their waiting time by one minute. The operator considers the generalized cost, which combines the monetary cost of operating the shuttle with the perceived cost of travelers' waiting time. Provide the expression for the generalized cost as a function of the headway and determine the headway value that minimizes the generalized cost.

2. Calculate the optimal headway with the following example: $t_H = 60$ min., $f = 60$ passengers/min., $c = 200$ CHF and $W = 1/3$ CHF/min.

3. Suppose each vehicle can accommodate a maximum of q passengers, making it potentially necessary to use multiple vehicles to transport all passengers. Each vehicle incurs an additional cost of p CHF. In this scenario, what is the total generalized cost? What is the value of the headway that minimizes it? Perform the calculation first allowing a fractional number of vehicles, and then considering only an integral number of vehicles.

4. Let us illustrate two scenarios: one that does not account for capacity, using $t_H = 60$ minutes, $f = 60$ passengers per minute, $c = 200$ CHF, and $W = 1/3$ CHF per minute, and another that incorporates capacity with $q = 60$ and $p = 200$. Figure 1 illustrates the total generalized cost as a function of headway without considering capacity, while Figure 2 on the next page includes the additional cost per bus.

   Compare these two graphs by identifying the terms in the cost expression responsible for the observed differences. Additionally, discuss how the shape of Figure 2 on the following page would be affected if only an integer number of vehicles were allowed.

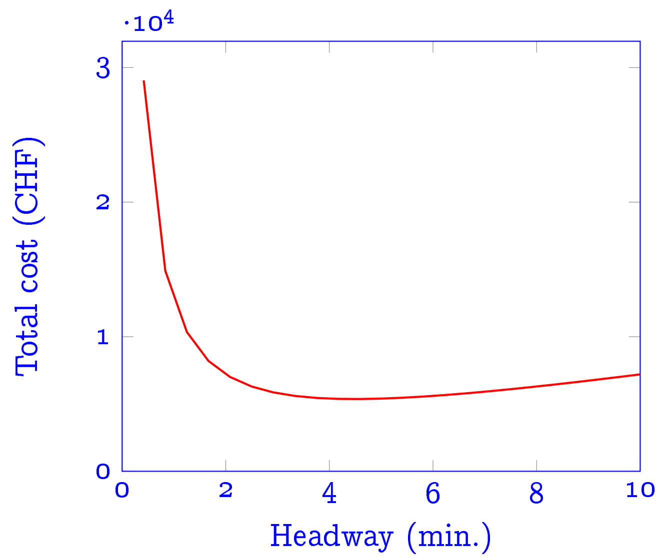

Figure 1: Generalized cost for the numerical example

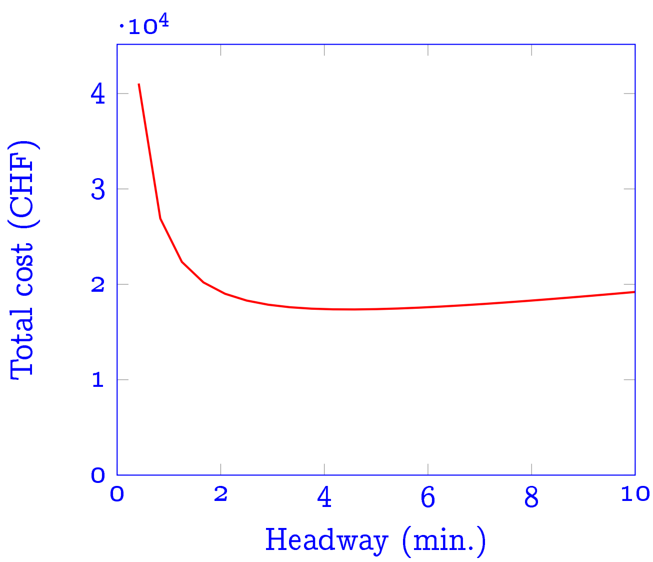

Figure 2: Generalized cost including the extra cost per bus