

# Travel demand

## Four step approach

Michel Bierlaire

Introduction to transportation systems

**EPFL**

# Trip-based model: the 4-step approach

## 4-step approach

- ✓ Trip generation
- ▶ Trip distribution
- ▶ Modal split
- ▶ Assignment

### Production and attraction

- ▶  $O_r, D_r$  for each zone/centroid  $r$ .
- ▶ Random variables: result of linear regression.

### Transportation networks

- ▶ One for each mode  $i$ .
- ▶ Network performance.
- ▶ Generalized cost:  $c_{rs}^i$ , for each mode  $i$  and each OD pair  $(r, s)$ .
- ▶ Assumed deterministic.
- ▶ Define  $c_{rs} = \min_i c_{rs}^i$  for each  $(r, s)$ .

# Trip distribution

## Objective

- ▶ Origin-destination table.
- ▶  $f_{rs}$  for each OD pair  $(r, s)$ ,
- ▶ such that  $E[\sum_s f_{rs}] = E[O_r]$
- ▶ such that  $E[\sum_r f_{rs}] = E[D_s]$

Issue: incompatibility  
when  $\sum_r E[O_r] \neq \sum_s E[D_s]$

## Solutions

- ▶ Use random variables
- ▶ Do not impose equality:  
 $E[\sum_s f_{rs}] \approx E[O_r]$

Issue: under-determination  
infinite number of solutions.

## Solutions

- ▶ More data.
- ▶ More assumptions.

# Dealing with under-determination: more data

## Surveys

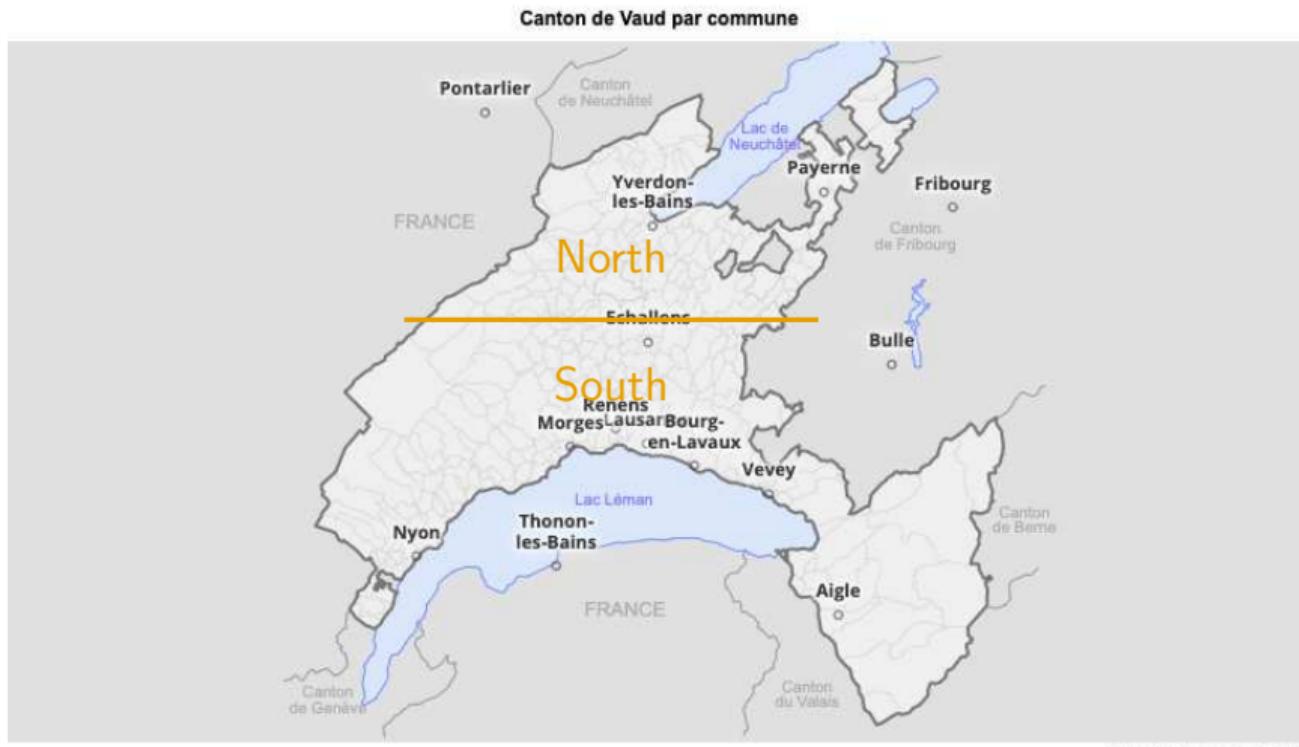


- ▶ Roadside interviews.
- ▶ License plate mail-out surveys.
- ▶ GPS data.
- ▶ etc.

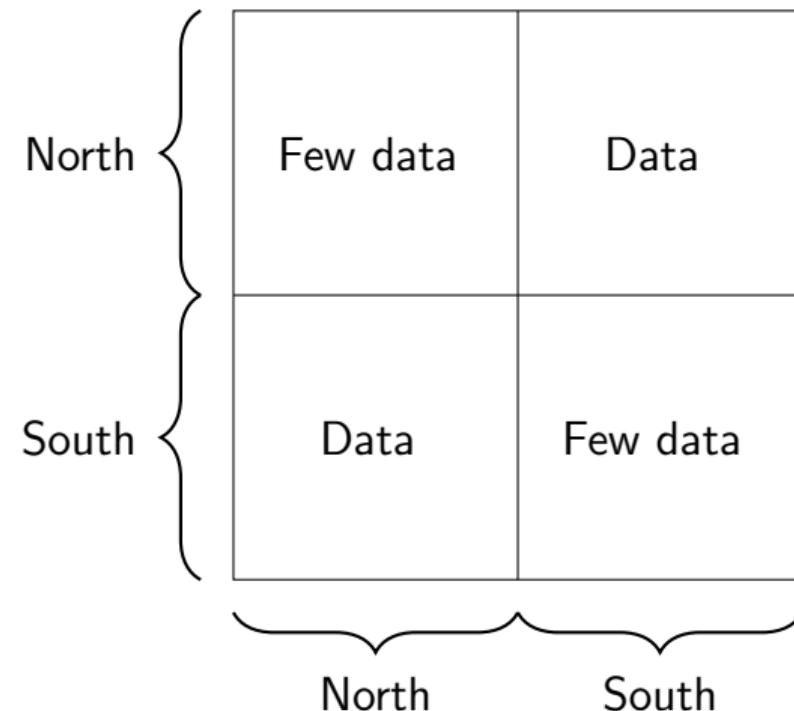
## Traffic counts

- ▶ Loop detectors.
- ▶ Pneumatic Road tube.
- ▶ Magnetic sensors.
- ▶ etc.

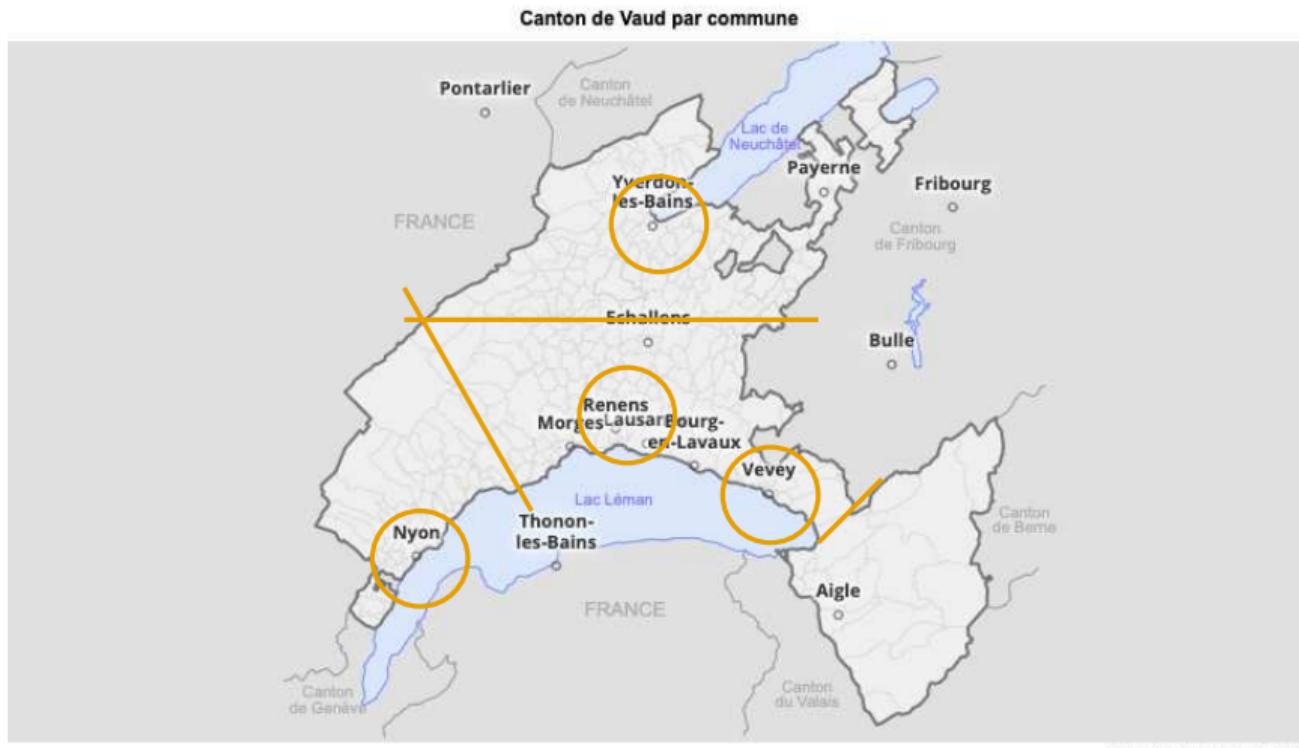
# Road side interviews: screening



## Road-side interviews



# Road side interviews: screening



## Road side interviews: issues

### Sampling rate

- ▶ Expensive data collection.
- ▶ Example: budget for 1000 interviews.
- ▶ One screen line: 1000 pieces of data per line.
- ▶ Seven screen lines: 143 pieces of data per line.

### Logistics

- ▶ Cars: interruption of traffic. May require police intervention.
- ▶ Public transportation: in-vehicle interviews.

### Biases

- ▶ In-vehicle interviews: travelers with long journeys.
- ▶ Some travelers may cross several lines.

## Elevator example

### True table

	0	1	2	3	4	
0		0.0	500.0	10.0	0.0	510.0
1	100.0		0.0	0.0	0.0	100.0
2	30.0	0.0		0.0	0.0	30.0
3	60.0	0.0	10.0		0.0	70.0
4	70.0	0.0	0.0	10.0		80.0
	260.0	0.0	510.0	20.0	0.0	

### Data

- ▶ Sum of rows: 515.5, 98.9, 16.4, 51.3, 96.2. Total: 778.3.
- ▶ Sum of columns: 248.8, 0, 506.4, 9.6, 0. Total: 764.8.

# Interviews: ground floor

True table

	0	1	2	3	4	
0		0.0	500.0	10.0	0.0	510.0
1	100.0		0.0	0.0	0.0	100.0
2	30.0	0.0		0.0	0.0	30.0
3	60.0	0.0	10.0		0.0	70.0
4	70.0	0.0	0.0	10.0		80.0
	260.0	0.0	510.0	20.0	0.0	

Data

	0	1	2	3	4
0		0.0	501.9	9.6	0.0
1	100.7				
2	29.7				
3	59.5				
4	70.9				

## Least squares

$$\min_f \sum_r \left( O_r - \sum_s f_{rs} \right)^2 + \sum_s \left( D_s - \sum_r f_{rs} \right)^2 + \sum_{rs} \left( \hat{f}_{rs} - f_{rs} \right)^2$$

## Least squares

$$\min_f \sum_r \left( O_r - \sum_s f_{rs} \right)^2 + \sum_s \left( D_s - \sum_r f_{rs} \right)^2 + \sum_{rs} \left( \hat{f}_{rs} - f_{rs} \right)^2$$

## Results

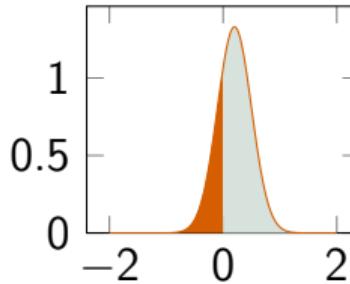
	0	1	2	3	4	
0		-18.0	483.9	-8.4	-18.0	439.4
1	55.8		-154.1	-76.2	197.0	22.5
2	-15.2	36.3		39.3	-120.4	-60.0
3	14.6	74.3	-141.5		27.5	-25.1
4	26.0	-16.1	-111.9	121.8		19.8
	81.3	76.4	76.4	76.4	86.0	396.5

# Linear regression

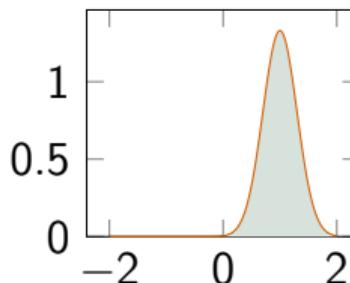
## Issues

- ▶ Maximum likelihood estimators are normally distributed.
- ▶ Normal distribution has infinite support.
- ▶ If the true value is close to zero, the estimate may be negative with high probability.

Close to zero



Far from zero



# Solutions

## Enforce non negativity

- ▶  $f_{rs} = \exp(\tau_{rs})$ ,  $\tau_{rs} \in \mathbb{R}$
- ▶ Estimator of  $\tau$  is normally distributed
- ▶ Estimator of  $f$  is log normally distributed
- ▶ Advantage: maximum likelihood.
- ▶ Inconvenient: nonlinear model.

## Nonlinear least squares

$$\min_{\tau} \sum_r \left( O_r - \sum_s e^{\tau_{rs}} \right)^2 + \sum_s \left( D_s - \sum_r e^{\tau_{rs}} \right)^2 + \sum_{rs} \left( \bar{f}_{rs} - e^{\tau_{rs}} \right)^2$$

## Results

	0	1	2	3	4	
0		57.2	252.1	65.0	65.0	439.2
1	44.1		0.0	0.0	0.0	44.1
2	0.0	0.0		0.0	0.0	0.0
3	0.0	0.0	0.0		0.0	0.0
4	26.2	6.4	0.0	0.0		32.6
	70.2	63.6	252.1	65.0	65.0	515.9

# Nonlinear least squares

True table

	0	1	2	3	4	
0		0.0	500.0	10.0	0.0	510.0
1	100.0		0.0	0.0	0.0	100.0
2	30.0	0.0		0.0	0.0	30.0
3	60.0	0.0	10.0		0.0	70.0
4	70.0	0.0	0.0	10.0		80.0
	260.0	0.0	510.0	20.0	0.0	

Estimated table

	0	1	2	3	4	
0		57.2	252.1	65.0	65.0	439.2
1	44.1		0.0	0.0	0.0	44.1
2	0.0	0.0		0.0	0.0	0.0
3	0.0	0.0	0.0		0.0	0.0
4	26.2	6.4	0.0	0.0		32.6
	70.2	63.6	252.1	65.0	65.0	515.9

We should put more emphasis on the survey data

## Weighted least squares

$$\min_{\tau} w_o^2 \sum_r \left( O_r - \sum_s e^{\tau_{rs}} \right)^2 + w_d^2 \sum_s \left( D_s - \sum_r e^{\tau_{rs}} \right)^2 + w_f^2 \sum_{rs} \left( \bar{f}_{rs} - e^{\tau_{rs}} \right)^2$$

Results with  $w_o = w_d = 1, w_f = 100$

	0	1	2	3	4	
0		0.0	500.8	9.6	0.0	510.4
1	100.2		0.0	0.0	4.1	104.3
2	29.2	0.0		0.0	0.0	29.2
3	58.9	0.0	0.0		0.2	59.1
4	70.3	11.8	0.0	2.3		84.4
	258.6	11.8	500.8	11.9	4.3	787.4

# Weighted least squares

True table

	0	1	2	3	4	
0		0.0	500.0	10.0	0.0	510.0
1	100.0		0.0	0.0	0.0	100.0
2	30.0	0.0		0.0	0.0	30.0
3	60.0	0.0	10.0		0.0	70.0
4	70.0	0.0	0.0	10.0		80.0
	260.0	0.0	510.0	20.0	0.0	

Estimated table

	0	1	2	3	4	
0		0.0	500.8	9.6	0.0	510.4
1	100.2		0.0	0.0	4.1	104.3
2	29.2	0.0		0.0	0.0	29.2
3	58.9	0.0	0.0		0.2	59.1
4	70.3	11.8	0.0	2.3		84.4
	258.6	11.8	500.8	11.9	4.3	787.4

# Modeling

## Sets

- ▶ Centroids:  $r = 1, \dots, N$ .
- ▶ Survey zones:  $p = 1, \dots, P$ .
- ▶ Set of centroids of zone  $p$ :  $\mathcal{S}_p$

## Data

- ▶ Production:  $O_r$ ,  $r = 1, \dots, N$ .
- ▶ Attraction:  $D_s$ ,  $s = 1, \dots, N$ .
- ▶ Survey data:  $\bar{f}_{rs}$  if  $r \in \mathcal{S}_p$  and  $s \notin \mathcal{S}_p$ .

## Regressions

- ▶  $O_r = \sum_{s=1}^N e^{\tau_{rs}} + \sigma_o \varepsilon_r^o$
- ▶  $D_s = \sum_{r=1}^N e^{\tau_{rs}} + \sigma_d \varepsilon_s^d$
- ▶  $\ln \bar{f}_{rs} = \tau_{rs} + \sigma_{rs} \varepsilon_{rs}$

## Warning

- ▶  $\sigma$  parameter not the same for all observations.
- ▶ Assumption of ordinary regression violated.
- ▶ We must use **weights**

# Weighted least squares

## Weights

- ▶ Each observation is associated with a weight.
- ▶ The more precise the observation, the higher the weight.
- ▶ Must be defined before hand.

## Roadside interviews

- ▶ A different weight for each set of data.
- ▶ Production:  $w_o$
- ▶ Attraction:  $w_d$
- ▶ Surveys:  $w_f$
- ▶ Weights must reflect the quality of the data.
- ▶ For instance:  $w_f \geq w_o \approx w_d$

## Weighted least squares

$$\min_{\tau} w_o^2 \sum_r \left( O_r - \sum_s e^{\tau_{rs}} \right)^2 + w_d^2 \sum_s \left( D_s - \sum_r e^{\tau_{rs}} \right)^2 + w_f^2 \sum_{rs} \left( \bar{f}_{rs} - e^{\tau_{rs}} \right)^2$$

## Regression equations

$$O_r = \sum_{s=1}^N e^{\tau_{rs}} + \sigma_o \varepsilon_r^o = \sum_{s=1}^N e^{\tau_{rs}} + \frac{\sigma}{w_o} \varepsilon_r^o$$

$$D_s = \sum_{r=1}^N e^{\tau_{rs}} + \sigma_d \varepsilon_s^d = \sum_{r=1}^N e^{\tau_{rs}} + \frac{\sigma}{w_d} \varepsilon_s^d$$

$$\bar{f}_{rs} = e^{\tau_{rs} + \sigma_{rs} \varepsilon_{rs}} = e^{\tau_{rs} + \frac{\sigma}{w_f} \varepsilon_{rs}}$$

Large weight = small variance

# Traffic count data



## Data

- ▶ Flow  $\bar{x}_\ell$  on link  $\ell$ , for some links.
- ▶ Assignment matrix.

## Assignment matrix

- ▶ Transforms OD flows into link flows.
- ▶ Number of rows = number of links.
- ▶ Number of columns = number of OD pairs.
- ▶ Available only after the assignment phase.

$$x = Qf, \quad x_\ell = \sum_q Q_{\ell q} f_q, \quad \text{where } q = (r, s).$$

## Assignment matrix

### Network topology: path-link

- ▶ Dimensions: number of links  $\times$  number of paths.
- ▶  $P_{\ell p} = 1$  if link  $\ell$  belongs to path  $p$ , 0 otherwise.

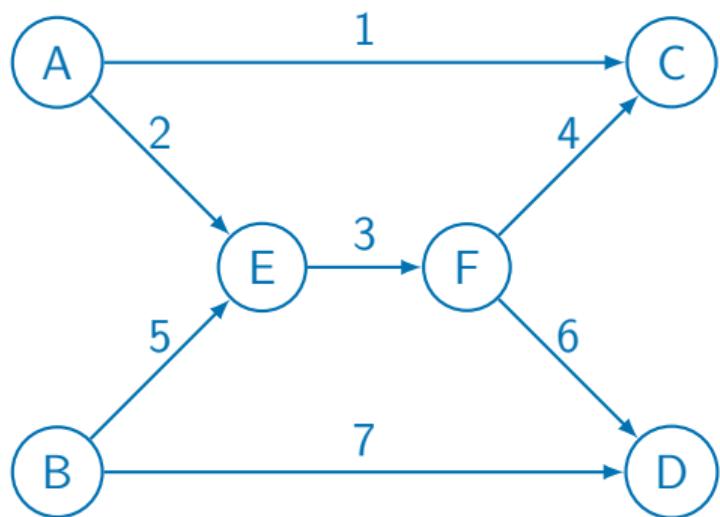
### Route choice: OD-path

- ▶ Dimensions: number of paths  $\times$  number of OD pairs.
- ▶  $R_{pq}$  is the proportion of OD flow  $q$  using path  $p$ .

### Assignment matrix: OD-link

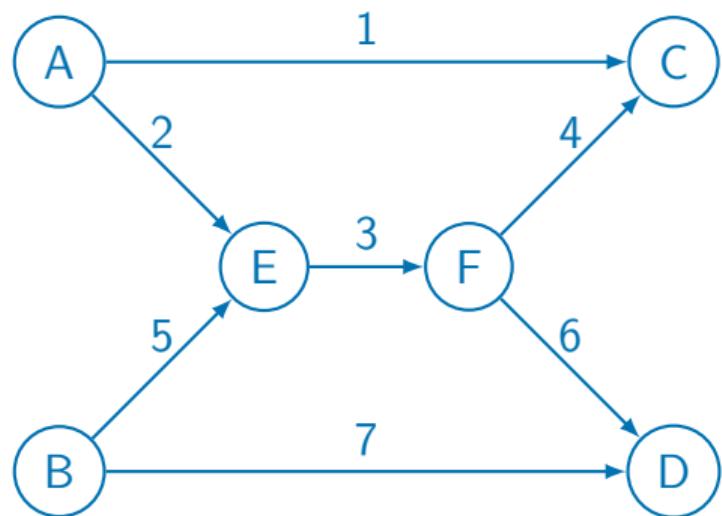
- ▶ Dimensions: number of links  $\times$  number of OD pairs.
- ▶  $Q = PR$ .
- ▶  $Q_{\ell q}$  is the proportion of OD flow  $q$  using link  $\ell$ .

## Network topology



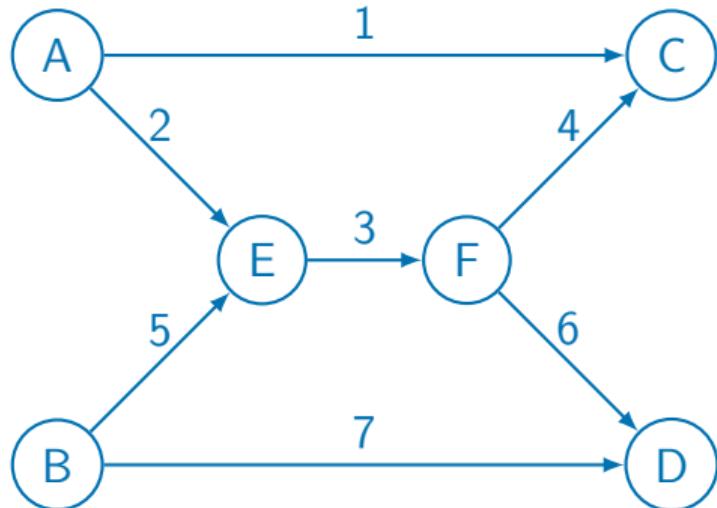
	$A-C$	$A-E-F-C$	$A-E-F-D$	$B-E-F-C$	$B-E-F-D$	$B-D$
1	1	0	0	0	0	0
2	0	1	1	0	0	0
3	0	1	1	1	1	0
4	0	1	0	1	0	0
5	0	0	0	1	1	0
6	0	0	1	0	1	0
7	0	0	0	0	0	1

## Route choice



	A-C	A-D	B-C	B-D
A-C	0.5	0	0	0
A-E-F-C	0.5	0	0	0
A-E-F-D	0	1	0	0
B-E-F-C	0	0	1	0
B-E-F-D	0	0	0	0.5
B-D	0	0	0	0.5

# Assignment matrix



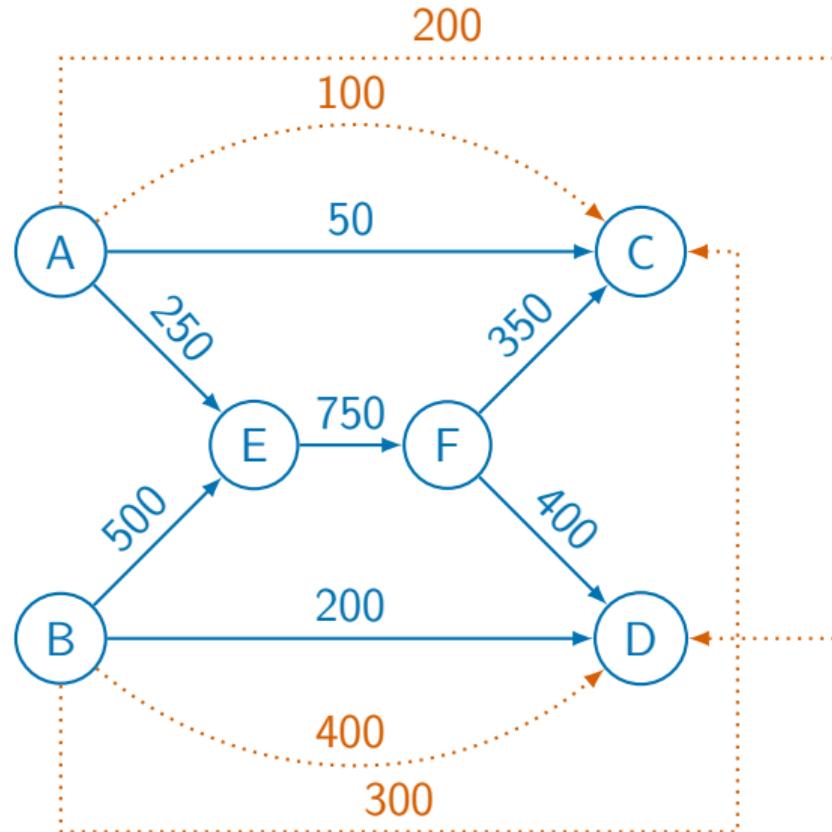
	A-C	A-D	B-C	B-D
1	0.5	0	0	0
2	0.5	1	0	0
3	0.5	1	1	0.5
4	0.5	0	1	0
5	0	0	1	0.5
6	0	1	0	0.5
7	0	0	0	0.5

Assignment matrix  $\times$  OD flows = Link flows

# Assignment

$$\begin{array}{l} \text{Assignment} \\ \text{matrix} \\ \\ \left( \begin{array}{cccc} 0.5 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0.5 & 1 & 1 & 0.5 \\ 0.5 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 \end{array} \right) \end{array} \quad \begin{array}{l} \text{OD} \\ \text{flows} \\ \\ \left( \begin{array}{c} 100 \\ 200 \\ 300 \\ 400 \end{array} \right) \end{array} = \quad \begin{array}{l} \text{Link} \\ \text{flows} \\ \\ \left( \begin{array}{c} 50 \\ 250 \\ 750 \\ 350 \\ 500 \\ 400 \\ 200 \end{array} \right) \end{array}$$

# Assignment



## Traffic count data

### Weighted least squares

$$\min_{\tau} w_o^2 \sum_r \left( O_r - \sum_s e^{\tau_{rs}} \right)^2 + w_d^2 \sum_s \left( D_s - \sum_r e^{\tau_{rs}} \right)^2 + w_\ell^2 \sum_\ell \left( \bar{x}_\ell - \sum_q Q_{\ell q} e^{\tau_q} \right)^2$$

# Dealing with under-determination: more assumptions



## Gravity model

- ▶  $f_{rs}$  proportional to  $O_r$ .
- ▶  $f_{rs}$  proportional to  $D_s$ .
- ▶ Decreases when generalized cost  $c_{rs}$  increases.

$$f_{rs} \approx \frac{\alpha_r O_r \beta_s D_s}{c_{rs}^2}$$

$$f_{rs} \approx \alpha_r O_r \beta_s D_s e^{-\gamma c_{rs}}$$

$$f_{rs} \approx \alpha_r O_r \beta_s D_s h(c_{rs}), \quad h'(c_{rs}) < 0$$

# The gravity model

$$\begin{aligned} & \min_{\tau, \alpha, \beta, \gamma} w_o^2 \sum_r \left( O_r - \sum_s e^{\tau_{rs}} \right)^2 \\ & + w_d^2 \sum_s \left( D_s - \sum_r e^{\tau_{rs}} \right)^2 \\ & + w_g^2 \sum_{rs} \left( \alpha_r O_r \beta_s D_s e^{-\gamma c_{rs}} - e^{\tau_{rs}} \right)^2 \end{aligned}$$

## Notes

- ▶ Here,  $w_g \leq w_o \approx w_d$ .
- ▶ Not appropriate for the elevator example. Why?

## The gravity model, survey data and traffic counts

$$\begin{aligned} & \min_{\tau, \alpha, \beta, \gamma} w_o^2 \sum_r \left( O_r - \sum_s e^{\tau_{rs}} \right)^2 \\ & + w_d^2 \sum_s \left( D_s - \sum_r e^{\tau_{rs}} \right)^2 \\ & + w_g^2 \sum_{rs} \left( \alpha_r O_r \beta_s D_s e^{-\gamma c_{rs}} - e^{\tau_{rs}} \right)^2 \\ & + w_f^2 \sum_{rs} \left( \bar{f}_{rs} - e^{\tau_{rs}} \right)^2 \\ & + w_\ell^2 \sum_\ell \left( \bar{x}_\ell - \sum_q Q_{\ell q} e^{\tau_q} \right)^2. \end{aligned}$$

# Trip-based model: the 4-step approach

## 4-step approach

- ✓ Trip generation
- ✓ Trip distribution
- ▶ Modal split
- ▶ Assignment

### Origin-destination table

- ▶  $f_{rs}$  for each pair of zones/centroids  $(r, s)$ .

### Transportation networks

- ▶ One for each mode  $i$ .
- ▶ Generalized cost:  $c_{rs}^i$ , for each mode  $i$  and each OD pair  $(r, s)$ .

# Modal split

## Mode choice model

- ▶ Consider an OD pair  $(r, s)$ .
- ▶ Set of modes for  $(r, s)$ :  $\mathcal{C}_{rs}$ .

## Logit model

Probability to choose mode  $i$  in  $\mathcal{C}_{rs}$ :

$$\pi_i^{rs} = \frac{e^{-\theta c_{rs}^i}}{\sum_{j \in \mathcal{C}_{rs}} e^{-\theta c_{rs}^j}}, \quad \theta \geq 0$$

# Elevator example

## Mode choice model

- ▶ Consider an OD pair  $(r, s)$ .
- ▶ Set of modes for  $(r, s)$ :  $\mathcal{C}_{rs} = \{\text{elevator, stairs}\}$ .
- ▶ Number of floors:  $d_{rs} = |r - s|$

## Logit model: utilities

$$u_{\text{elevator}} = 0$$

$$u_{\text{stairs}} = -\theta d_{rs}, \theta = 1.1$$

## Logit model: probabilities

Proportion who choose the stairs:

$$\pi_{\text{stairs}}^{rs} = \frac{e^{-1.1d_{rs}}}{1+e^{-1.1d_{rs}}}$$

$$\pi_{\text{stairs}}^{01} = 0.250, \pi_{\text{stairs}}^{02} = 0.100, \\ \pi_{\text{stairs}}^{03} = 0.0356, \pi_{\text{stairs}}^{04} = 0.0121.$$

## Elevator example

	0	1	2	3	4	
0		0.0	500.8	9.6	0.0	510.4
1	100.2		0.0	0.0	4.1	104.3
2	29.2	0.0		0.0	0.0	29.2
3	58.9	0.0	0.0		0.2	59.1
4	70.3	11.8	0.0	2.3		84.4
	258.6	11.8	500.8	11.9	4.3	787.4

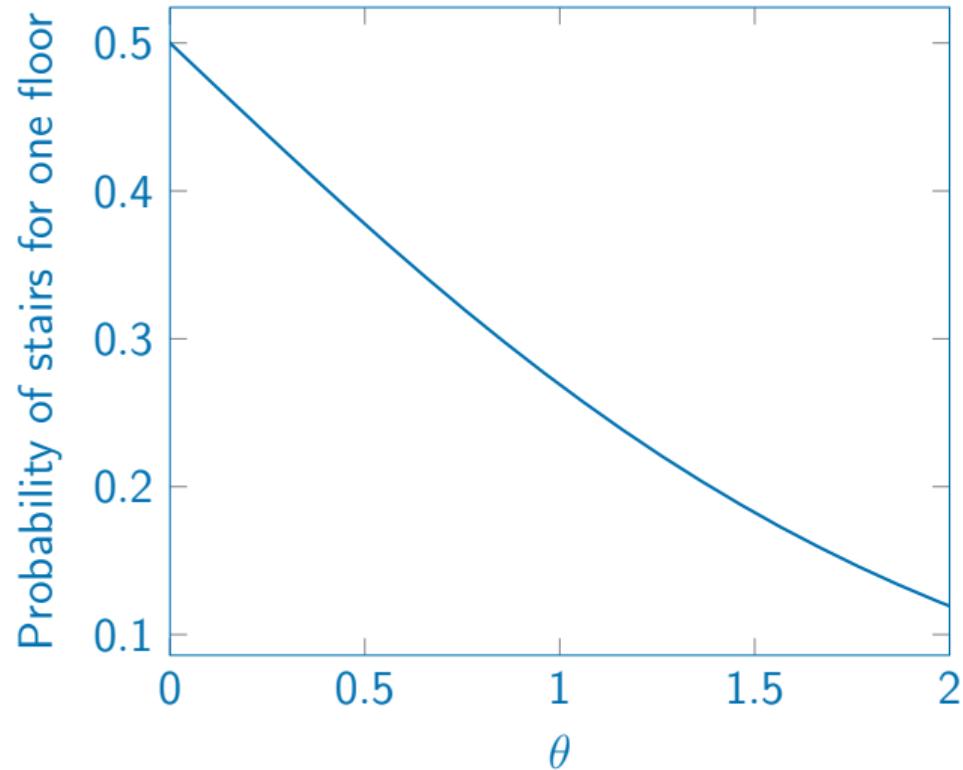
Elevator

	0	1	2	3	4	
0		0.0	450.9	9.2	0.0	460.1
1	75.2		0.0	0.0	4.0	79.2
2	26.3	0.0		0.0	0.0	26.3
3	56.8	0.0	0.0		0.1	56.9
4	69.5	11.3	0.0	1.7		82.5
	227.7	11.4	450.9	11.0	4.1	705.0

Stairs

	0	1	2	3	4	
0		0.0	50.0	0.3	0.0	50.3
1	25.0		0.0	0.0	0.1	25.2
2	2.9	0.0		0.0	0.0	2.9
3	2.1	0.0	0.0		0.0	2.1
4	0.9	0.4	0.0	0.6		1.9
	30.9	0.4	50.0	0.9	0.2	82.4

## Importance of the parameters



# Choice data

## Revealed preferences (RP)

- ▶ Observe actual choices made by travelers.
- ▶ Critical to reproduce the modal shares.
- ▶ Collect data for explanatory variables.

## Inconvenients of RP data

- ▶ Limited to existing modes, attributes, and attribute levels.
- ▶ Lack of variability of some attributes.
- ▶ High level of correlation.
- ▶ Expensive data collection.
- ▶ Lack of information on unchosen alternatives.

# Choice data

Pick Your Preferred Flight

Three flight options are described for your trip from Chicago to San Diego. These are options that might be available on this route or might be new options actively being considered for this route as well as replacing some options that are offered now. The options differ from each other in one or more of the features described on the left.

Please evaluate these options, assuming that everything about the options is the same except these particular features. Indicate your choices at the bottom of the appropriate column and press the Continue button.

FEATURES	Non-Stop (Option 1)	1 Stop (Option 2)	1 Stop (Option 3)
Departure time (local)	6:00 PM	4:30 PM	6:00 PM
Arrival time (local)	8:14 PM	9:44 PM	9:44 PM
Total time in air	4 hr 14 min	4 hr 44 min	4 hr 44 min
Total trip time	4 hr 14 min	6 hr 14 min	5 hr 44 min
Legroom	typical legroom	2-in more of legroom	4-in more of legroom
Airline [Airplane]	Depart Chicago Continental Airlines [B737] to San Diego	Depart Chicago Southwest Airlines [A320], connecting with Southwest Airlines [MD80] to San Diego	Depart Chicago Northeast Airlines [MD80], connecting with American Airlines [DC9] to San Diego
Fare	\$565	\$485	\$620

1. Which is MOST attractive?  Option 1  Option 2  Option 3

2. Which is LEAST attractive?  Option 1  Option 2  Option 3

3. If these were the ONLY three options available, I would NOT make this trip by air.  Yes  No

## Stated preferences (SP)

- ▶ Surveys, interviews.
- ▶ Hypothetical situations.
- ▶ Choice context defined by the analyst.
- ▶ “What would you choose if...?”

## Advantages of SP data

- ▶ Exploring new contexts.
- ▶ Control of the attributes variability.
- ▶ Control on all alternatives.
- ▶ Control on the level of correlation.
- ▶ One individual can answer several questions.

# Choice data

## Inconvenients of SP data

- ▶ Hypothetical situations.
- ▶ Cannot be used for market shares.
- ▶ Decision-makers do not have to assume their choice.
- ▶ Real constraints not involved.
- ▶ Credibility.
- ▶ Valid within the range of the experimental design.
- ▶ Policy bias (example: “every body else should take the bus” ).
- ▶ Justification bias (or inertia).
- ▶ Framing: phrasing of the question matters.
- ▶ Anchoring: one variable explains it all.
- ▶ Fatigue effect.

# Choice data

## Estimation of the parameters

- ▶ RP data is necessary.
- ▶ SP data is highly valuable.
- ▶ They are complementary.
- ▶ Maximum likelihood estimation.

# Mode choice models

## Disaggregate

- ▶ Actual choice models are disaggregate.
- ▶ They are different across segments of population.
- ▶ Typical characterization of segments:
  - ▶ trip purpose,
  - ▶ gender,
  - ▶ income,
  - ▶ age,
  - ▶ employment,
  - ▶ availability of mobility tools,
  - ▶ etc.

# Mode choice models

## Aggregation

- ▶ Suppose that the population is partitioned into  $N$  segments.
- ▶ The proportion of individuals in segment  $n$  for OD pair  $(r, s)$  is  $\pi_n^{rs}$ .
- ▶ The probability to choose mode  $i$  for OD pair  $(r, s)$  in segment  $n$  is  $\pi_{in}^{rs}$ .
- ▶ The proportion of individuals choosing mode  $i$  for OD pair  $(r, s)$  is therefore

$$\pi_i^{rs} = \sum_n \pi_{in}^{rs} \pi_n^{rs}.$$

# Elevator example

## Mode choice model

- ▶ Consider an OD pair  $(r, s)$ .
- ▶ Set of modes for  $(r, s)$ :  $\mathcal{C}_{rs} = \{\text{elevator, stairs}\}$ .
- ▶ Number of floors:  $d_{rs} = |r - s|$

### Young

$$u_{\text{elevator, young}} = 0$$

$$u_{\text{stairs, young}} = -\theta_{\text{young}} d_{rs}, \theta_{\text{young}} = 1.1$$

Proportion who choose the stairs:

$$\pi_{\text{stairs, young}}^{01} = 0.250,$$

$$\pi_{\text{stairs, young}}^{02} = 0.100,$$

$$\pi_{\text{stairs, young}}^{03} = 0.0356,$$

$$\pi_{\text{stairs, young}}^{04} = 0.0121.$$

### Old

$$u_{\text{elevator, old}} = 0$$

$$u_{\text{stairs, old}} = -\theta_{\text{old}} d_{rs}, \theta_{\text{old}} = 2.1$$

Proportion who choose the stairs:

$$\pi_{\text{stairs, old}}^{01} = 0.110,$$

$$\pi_{\text{stairs, old}}^{02} = 0.0148,$$

$$\pi_{\text{stairs, old}}^{03} = 0.00183,$$

$$\pi_{\text{stairs, old}}^{04} = 0.000225.$$

# Elevator example

## Young

Proportion who choose the stairs:

$$\pi_{\text{stairs, young}}^{01} = 0.250,$$

$$\pi_{\text{stairs, young}}^{02} = 0.100,$$

$$\pi_{\text{stairs, young}}^{03} = 0.0356,$$

$$\pi_{\text{stairs, young}}^{04} = 0.0121.$$

In the building:  $\pi_{\text{young}}^{rs} = 25\%, \forall(r, s)$

## Old

Proportion who choose the stairs:

$$\pi_{\text{stairs, old}}^{01} = 0.110,$$

$$\pi_{\text{stairs, old}}^{02} = 0.0148,$$

$$\pi_{\text{stairs, old}}^{03} = 0.00183,$$

$$\pi_{\text{stairs, old}}^{04} = 0.000225.$$

In the building:  $\pi_{\text{old}}^{rs} = 75\%, \forall(r, s)$

## Total population

Proportion who choose the stairs:

$$\pi_{\text{stairs}}^{01} = 0.25 \pi_{\text{stairs, young}}^{01} + 0.75 \pi_{\text{stairs, old}}^{01} = 0.144,$$

$$\pi_{\text{stairs}}^{02} = 0.25 \pi_{\text{stairs, young}}^{02} + 0.75 \pi_{\text{stairs, old}}^{02} = 0.0360,$$

$$\pi_{\text{stairs}}^{03} = 0.25 \pi_{\text{stairs, young}}^{03} + 0.75 \pi_{\text{stairs, old}}^{03} = 0.0103,$$

$$\pi_{\text{stairs}}^{04} = 0.25 \pi_{\text{stairs, young}}^{04} + 0.75 \pi_{\text{stairs, old}}^{04} = 0.003,$$

## Elevator example

	0	1	2	3	4	
0		0.0	500.8	9.6	0.0	510.4
1	100.2		0.0	0.0	4.1	104.3
2	29.2	0.0		0.0	0.0	29.2
3	58.9	0.0	0.0		0.2	59.1
4	70.3	11.8	0.0	2.3		84.4
	258.6	11.8	500.8	11.9	4.3	787.4

Elevator

	0	1	2	3	4	
0		0.0	482.8	9.5	0.0	492.3
1	85.7		0.0	0.0	4.1	89.8
2	28.1	0.0		0.0	0.0	28.1
3	58.3	0.0	0.0	0.1		58.4
4	70.1	11.6	0.0	2.0		83.7
	242.2	11.7	482.8	11.5	4.3	752.4

Stairs

	0	1	2	3	4	
0		0.0	18.0	0.1	0.0	18.1
1	14.5		0.0	0.0	0.0	14.5
2	1.1	0.0		0.0	0.0	1.1
3	0.6	0.0	0.0		0.0	0.6
4	0.2	0.1	0.0	0.3		0.7
	16.3	0.1	18.0	0.4	0.1	35.0

# Summary

## OD table estimation

- ▶ Zones production and attraction.
- ▶ More data (e.g. roadside interviews.)
- ▶ More assumptions: gravity model.
- ▶ Weighted least squares.
- ▶ Non negativity must be enforced.

## Modal split

- ▶ RP and SP data.
- ▶ Mode choice model.
- ▶ Need for aggregation.

## 4-step approach

- ✓ Trip generation
- ✓ Trip distribution
- ✓ Modal split
- ▶ Assignment