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Question 1: Equilibrium

Consider the small network represented in Figure 1, where the label of each node and each arc is reported. There are 4 units of flow that travel from node A to node C. The performance functions are

$$t_1 = 2 + x_1^2,$$
$$t_2 = 1 + 3x_2,$$
$$t_3 = 3 + x_3.$$

Find the user equilibrium flow and the equilibrium travel time on each link.

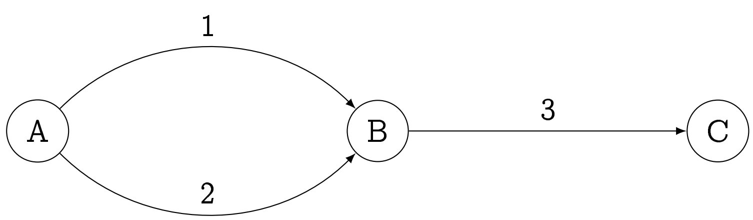

Figure 1: Small network, with nodes and arcs labels.

Question 2: Algorithm

Consider the small network represented in Figure 2, where the label of each node and each arc is reported. There are 4 units of flow that travel from node A to node B. The performance functions are

$$t_1 = 2 + x_1^2,$$
$$t_2 = 1 + 3x_2.$$

Apply the solution algorithm in order to find the link flows at equilibrium. Remember that the solution algorithm is defined as follows.

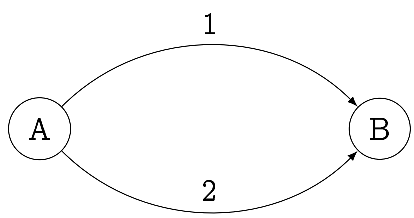

Figure 2: Small network, with nodes and arcs labels.

Initialization Empty network.

- Link costs: $t_\ell(0)$.
- Link flows from all-or-nothing assignment: x^0.
- $k = 0$.

Step 1 Calculate link costs: $t_\ell^k = t_\ell(x^k)$.

Step 2 Link flows from all-or-nothing assignment: $\tilde{x}^k$.

Step 3 Line search.

$$x^{k+1} = x^k + \alpha(\tilde{x}^k - x^k),\ 0 \leq \alpha \leq 1,$$

where α solves

$$\min_\alpha \sum_\ell \int_0^{x_\ell^{k+1}} t_\ell(z) dz$$

Step 4 Check convergence. If not, go to step 1.

Question 3: System optimum

Consider the small network represented in Figure 3, that represents an origin-destination pair connected with two links: one representing a highway, and one representing urban streets. There are 1.5 units of flow that travel from node A to node B. The performance functions are

$$t_{\text{highway}} = 3 + \frac{1}{2}x_1,$$
$$t_{\text{urban}} = 1 + x_2.$$

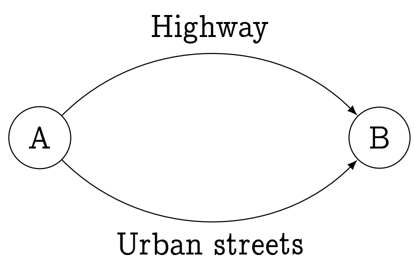

Figure 3: Small network

1. Calculate the flows and the total travel time at the user equilibrium.
2. Calculate the flows and the total travel time at the system optimum.
3. What is the price of anarchy?
4. Woud everybody be better off if the network was at system optimum compared to user equilibrium?