EPFL

Spring 2025

09 Taxi & Ride-hailing

CIVIL-324 Urban public transport systems

École
polytechnique
fédérale
de Lausanne

Collective vs individual transport

Individual

Private driving

On-demand mobility service

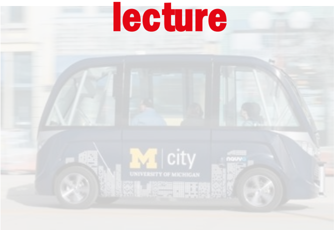
Out of the scope

Collective

Flexible transit

Mass transit

Topic of last lecture



First half of this course

Ride-hailing service

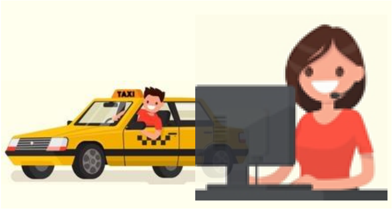
Passenger-vehicle matching

Physical

taxi stand

street-hailing

Virtual



radio-dispatch

e-hailing

Ride-hailing service

Passenger-vehicle matching

Physical

Virtual

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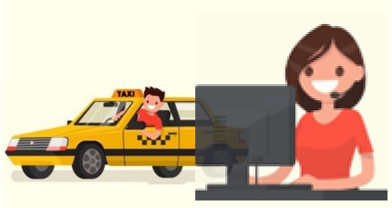


radio-dispatch

classic taxi services

e-hailing

ride-pooling

ride-sourcing services

Matching in ride-hailing

- Total travel time of each passenger trip

$$t = \tau_a + w^c + \tau_b + \delta$$

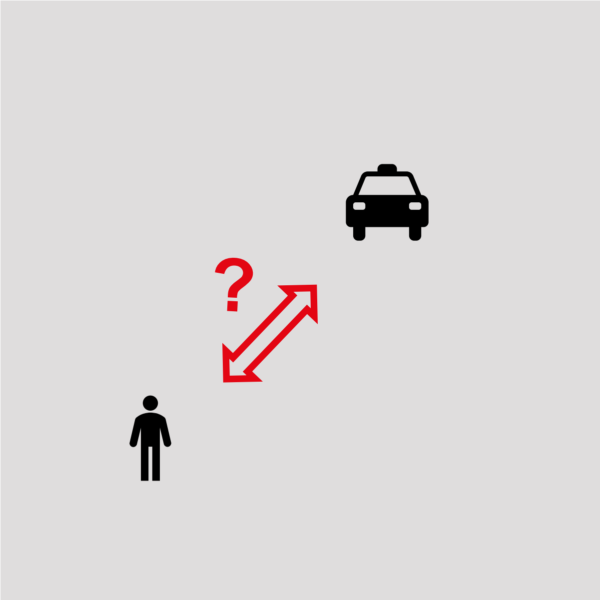
- Access time $\tau_a = 0$
  - ride-hailing services usually provide door-to-door trips
- Waiting time w^c
  - determined by the passenger-vehicle matching
- Riding time $\tau_b = \ell / v_b$
  - estimated by average trip distance ℓ and vehicle speed v_b
  - independent of the service type
- Stopping time $\delta \approx 0$
  - Pickup and dropoff times are almost zero
- ***Q: What are the main factors of waiting time?***

Matching in ride-hailing

- General service region of area A (km^2)

- Homogeneous travel demand
  - trip origins and destinations are uniformly distributed in the region
  - travelers arrive randomly at rate λ^c (pax/hr)

- Homogeneous vehicle supply
  - a fleet of N vehicles
  - serve door-to-door trips
  - average vehicle speed v_b (km/hr)
  - zero pickup and dropoff times

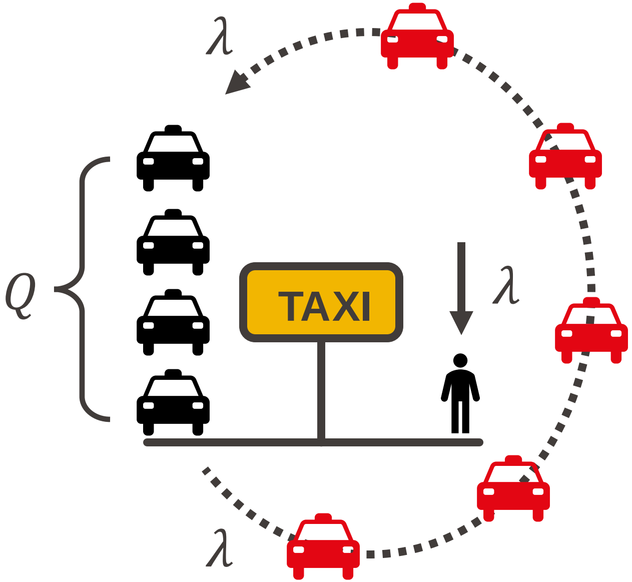


Matching in ride-hailing

- Vehicle time conservation

$$N = N^v + \lambda(t_p + \tau_b + t_d)$$

  - N: fleet size (total vehicle time)
  - N^v: vacant vehicles (vacant vehicle time)
  - λ: pickup rate
  - t_p, t_d : time before pickup and dropoff



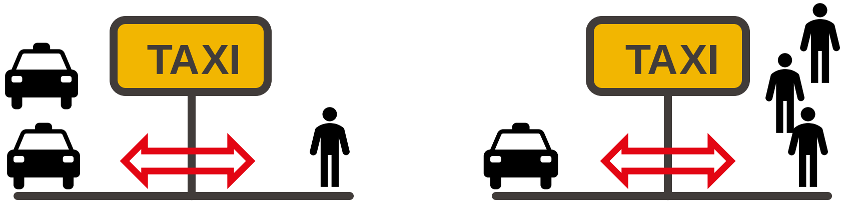
- Little's law in queuing theory

$$Q = \lambda w$$

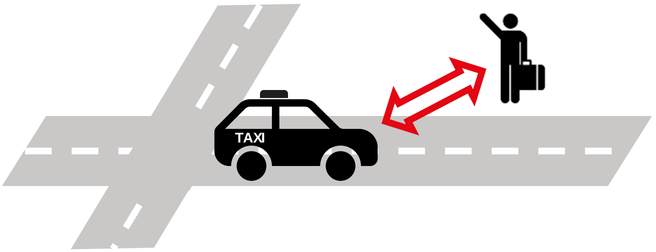
  - Q: long-term average queue length
  - λ: long-term effective arrival rate λ
  - w: average waiting time

Matching in ride-hailing

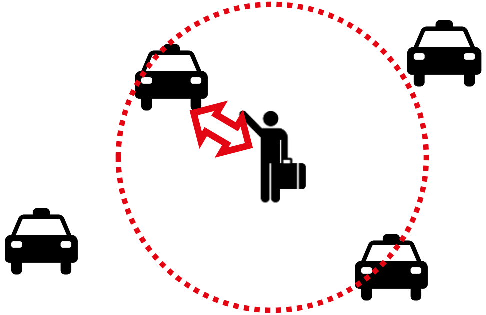
- Taxi stand
  - Matching a at point



- Street-hailing
  - Matching along a line

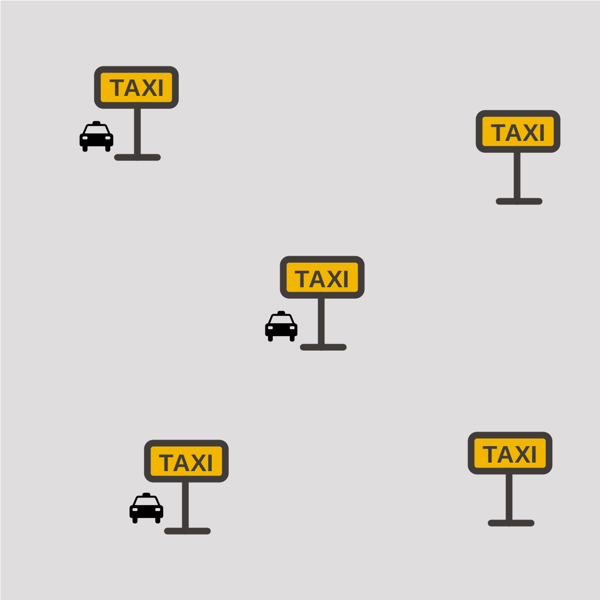


- Radio-dispatch/e-hailing
  - Matching over space



Matching at taxi stand

- General service region of area A (km^2)
  - K stands uniformly distributed with unlimited capacity
- Homogeneous travel demand
  - trip origins and destinations are uniformly distributed in the region
  - travelers arrive randomly at each stand at rate λ^c/K (pax/hr)
- Homogeneous vehicle supply
  - a fleet of N vehicles
  - serve door-to-door trips and return to the closest stand after each trip
  - average vehicle speed v_b (km/hr)
  - zero pickup and dropoff times



Matching at taxi stand

- Scenario I: Vehicles form a queue N_k^v at each stand $k = 1, \dots, K$
  - Vehicle waiting time

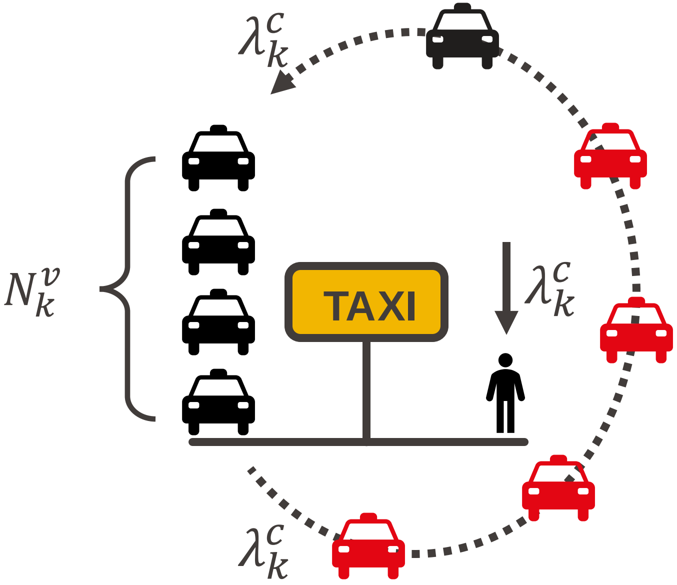
$$w_k^v = \frac{N_k^v}{\lambda_k^c}$$

    - λ_k^c: passenger arrival rate

  - Travel time from dropoff location to the closest stand[1]

$$t_d = \frac{\delta}{v_b}\sqrt{A/K}$$

    - A: area of service region
    - δ: parameter related to the region shape (e.g., $\delta = 0.5$ for circular region)



[1] Daganzo (1978) An approximate analytic model of many-to-many demand responsive transportation systems.

Matching at taxi stand

- Scenario I: Vehicles form a queue N_k^v at each stand $k = 1, \dots, K$
  - Vehicle time conservation

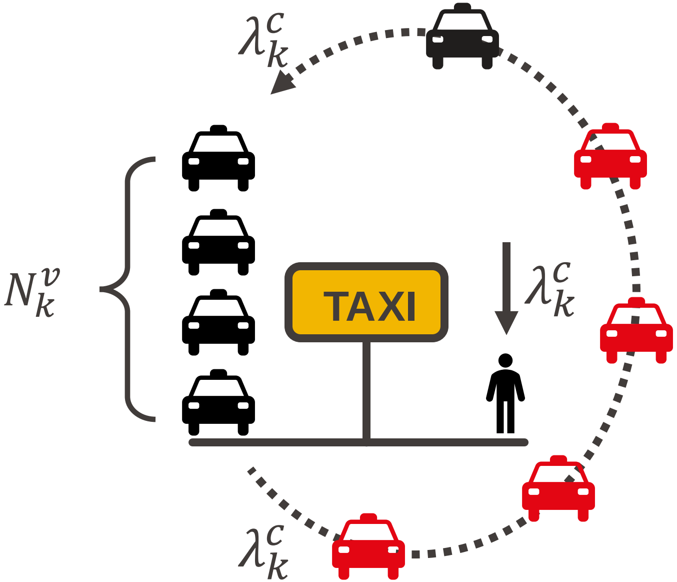
$$N = K[N_k^v + \lambda_k^c(\tau_b + t_d)]$$

  - Little's law

$$\lambda_k^c = \frac{\lambda^c}{K} = \frac{N_k^v}{w_k^v}$$

  - Stationary condition

$$\frac{\lambda^c}{K} = \frac{N_k^v}{w_k^v} = \frac{N/K - N_k^v}{\tau_b + t_d}$$



Matching at taxi stand

- Scenario I: Vehicles form a queue N_k^v at each stand $k = 1, \dots, K$
  - Queue length and waiting time at stationary state
    - passenger

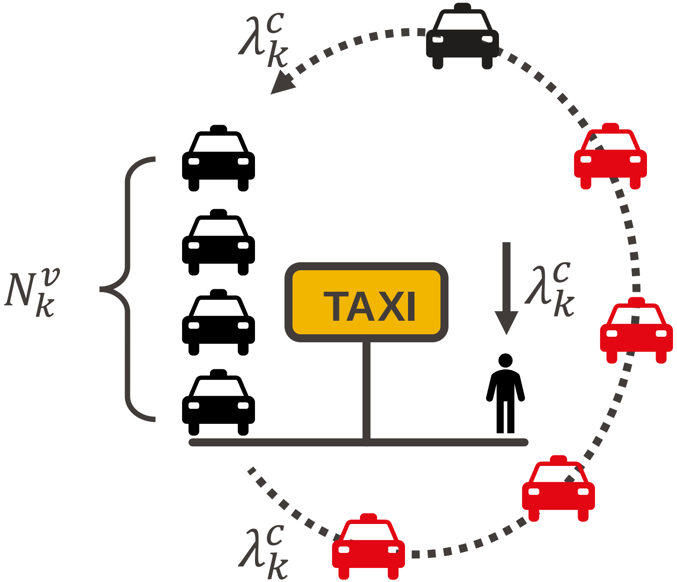
$$N_k^c = 0$$

$$w_k^c = 0$$

    - vehicle

$$N_k^v = \frac{N - \lambda^c\left(\tau_b + \frac{\delta}{v_b}\sqrt{A/K}\right)}{K}$$

$$w_k^v = \frac{N}{\lambda^c} - \left(\tau_b + \frac{\delta}{v_b}\sqrt{A/K}\right)$$



Matching at taxi stand

- Scenario II: Passengers form a queue N_k^c at each stand $k = 1, \dots, K$
  - Passenger waiting time

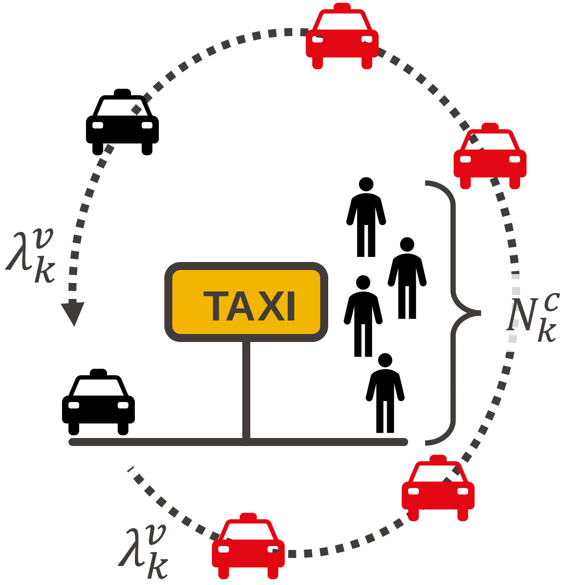
$$w_k^c = \frac{N_k^c}{\lambda_k^v}$$

    - λ_k^v: vehicle arrival rate

  - Travel time from dropoff location to the closest stand[1]

$$t_d = \frac{\delta}{v_b}\sqrt{A/K}$$

    - A: area of service region
    - δ: parameter related to the region shape (e.g., $\delta = 0.5$ for circular region)



[1] Daganzo (1978) An approximate analytic model of many-to-many demand responsive transportation systems.

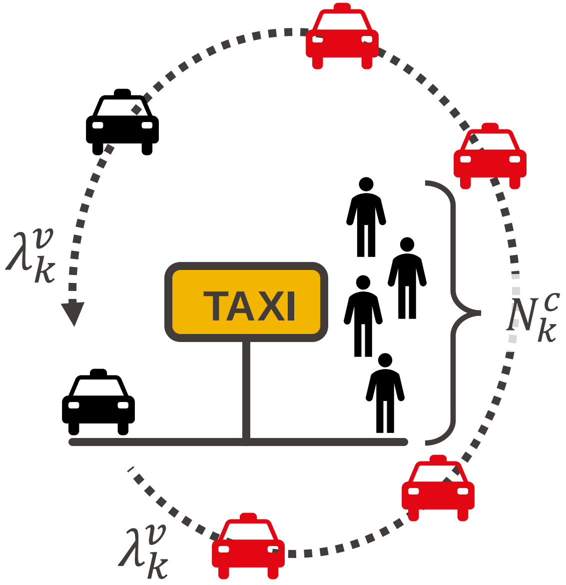
Matching at taxi stand

- Scenario II: Passengers form a queue N_k^c at each stand $k = 1, \ldots, K$
  - Vehicle time conservation

$$N = K\lambda_k^v(\tau_b + t_d)$$

  - Little's law

$$\lambda_k^v = \frac{N_k^c}{w_k^c}$$



- ***Q: How to construct the stationary condition?***

Matching at taxi stand

- Scenario II: Passengers form a queue N_k^c at each stand $k = 1, \dots, K$
  - M/M/1 queue:
    - both passenger and vehicle arrivals follow Poisson process

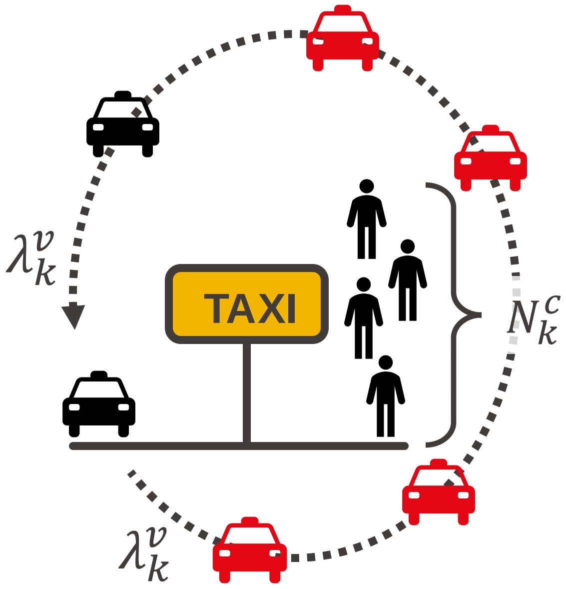
$$w_k^c = \frac{1}{\lambda_k^v - \lambda_k^c} - \frac{1}{\lambda_k^v}$$

  - Vehicle time conservation

$$N = K\lambda_k^v(\tau_b + t_d)$$

  - Little's law

$$\lambda_k^v = \frac{N_k^c}{w_k^c}$$



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Matching at taxi stand

- Scenario II: Passengers form a queue N_k^c at each stand $k = 1, \ldots, K$
  - Queue length and waiting time at stationary state
    - passenger

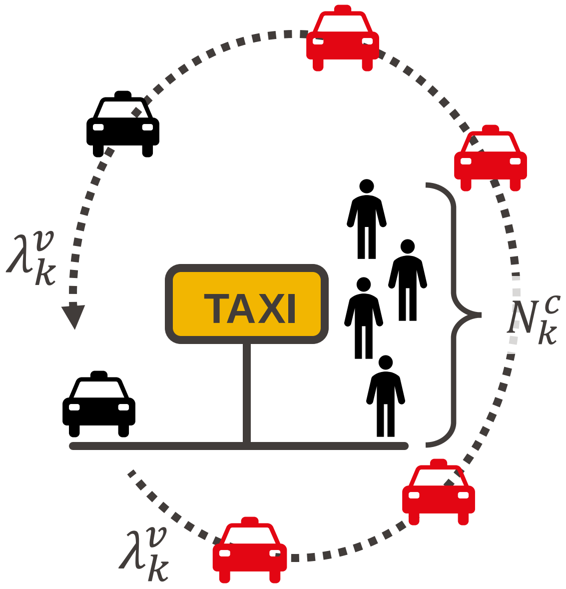
$$N_k^c = \frac{\lambda^c\left(\tau_b + \frac{\delta}{v_b}\sqrt{A/K}\right)}{N - \lambda^c\left(\tau_b + \frac{\delta}{v_b}\sqrt{A/K}\right)}$$

$$w_k^c = \frac{K\lambda^c\left(\tau_b + \frac{\delta}{v_b}\sqrt{A/K}\right)^2}{N\left[N - \lambda^c\left(\tau_b + \frac{\delta}{v_b}\sqrt{A/K}\right)\right]}$$

    - vehicle

$$N_k^v = 0$$

$$w_k^v = 0$$

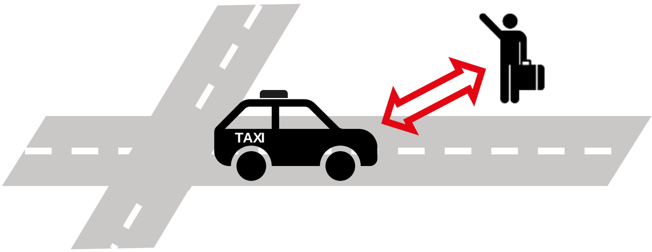


Questions?

Matching in ride-hailing

- Taxi stand
  - Matching a at point

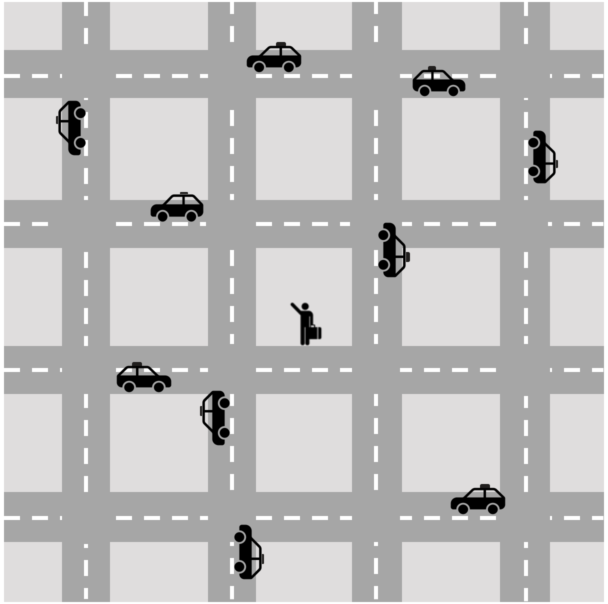
- Street-hailing
  - Matching along a line



- Radio-dispatch/e-hailing
  - Matching over space

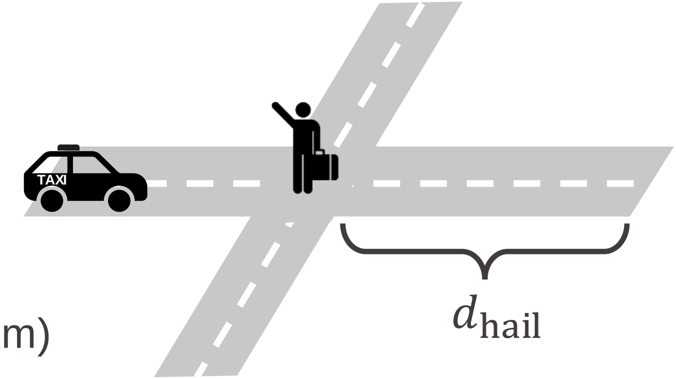
Matching in street-hailing

- General service region of area A (km^2)
  - road density ρ (km/km^2)
- Homogeneous travel demand
  - trip origins and destinations are uniformly distributed in the region
  - travelers arrive randomly on streets at rate $\lambda^c/(\rho A)$ (pax/hr/km)
- Homogeneous vehicle supply
  - a fleet of N vehicles
    - randomly cruise on streets
  - serve door-to-door trips
  - average vehicle speed v_b (km/hr)
  - zero pickup ad dropoff times



Matching in street-hailing

- Waiting passenger and cruising vehicle coexist
  - passenger queue and waiting time $N^c, w^c > 0$
  - vehicle queue and waiting time $N^v, w^v > 0$



- Assume passengers wait at intersections
  - maximum hail distance d_{hail} (km) along each direction, which yields a hail region of $4d_{\text{hail}}$ (km)

Matching in street-hailing

- Cruising vehicles distribute on streets at density

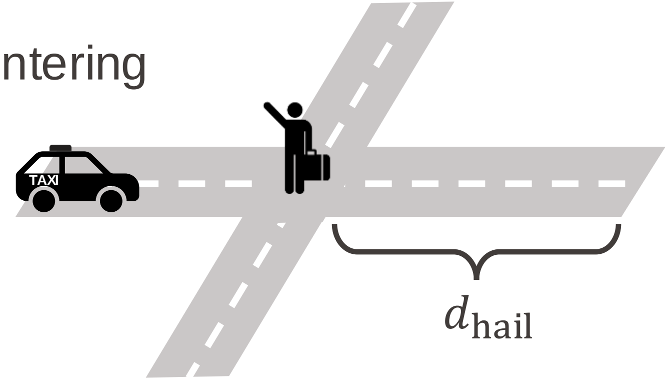
$$\Lambda^v = \frac{N^v}{\rho A}$$

- Let $V(t)$ be the number of cruising vehicles entering the hail region up to time t
  - $V(t)$ follows a Poisson process with rate

$$\lambda^v = (4d_{\text{hail}})\Lambda^v$$

  - probability that n vehicles enter the hail region

$$\mathbb{P}[V(t) = n] = \frac{(\lambda^v t)^n}{n!}\exp(-\lambda^v t)$$



Matching in street-hailing

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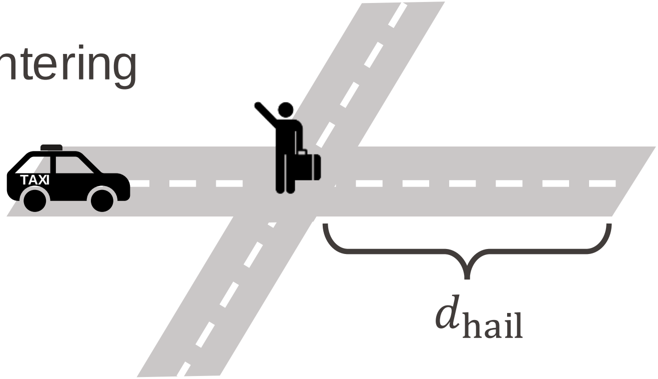
- Let $V(t)$ be the number of cruising vehicles entering the hail region up to time t
  - $V(t)$ follows a Poisson process with rate

$$\lambda^v = (4d_{\text{hail}})\Lambda^v$$

  - probability that no vehicle enters the hail region

$$\mathbb{P}[V(t) = 0] = \exp(-\lambda^v t)$$



- ***Q: What is the probability of waiting time $w^c = t$?***

Matching in street-hailing

- Cruising vehicles distribute on streets at density

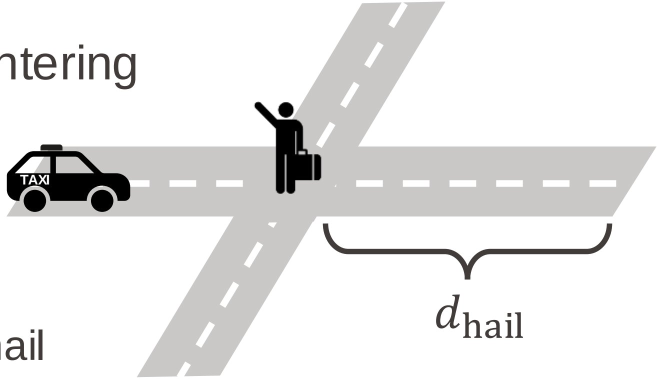
$$\Lambda^v = \frac{N^v}{\rho A}$$

- Let $V(t)$ be the number of cruising vehicles entering the hail region up to time t
  - $V(t)$ follows a Poisson process with rate

$$\lambda^v = (4d_{\text{hail}})\Lambda^v$$

  - probability that at least one vehicle enters the hail region

$$1 - \mathbb{P}[V(t) = 0] = 1 - \exp(-\lambda^v t)$$
$$= \mathbb{P}[w^c = t]$$



- ***Q: What is the expected waiting time $\mathbb{E}[w^c]$?***

Matching in street-hailing

- Cruising vehicles distribute on streets at density

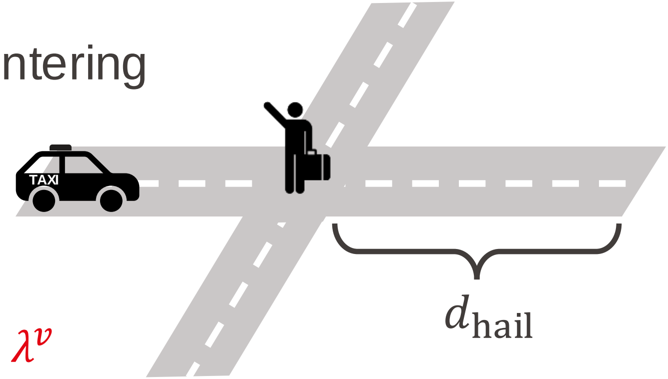
$$\Lambda^v = \frac{N^v}{\rho A}$$

- Let $V(t)$ be the number of cruising vehicles entering the hail region up to time t
  - $V(t)$ follows a Poisson process with rate

$$\lambda^v = (4d_{\text{hail}})\Lambda^v$$

  - w^c follows an exponential distribution with rate λ^v

$$\mathbb{E}[w^c] = \frac{1}{\lambda^v} = \frac{1}{4d_{\text{hail}}\Lambda^v} = \frac{\rho A}{4d_{\text{hail}}N^v}$$



Matching in street-hailing

- Stationary condition
  - Vehicle time conservation

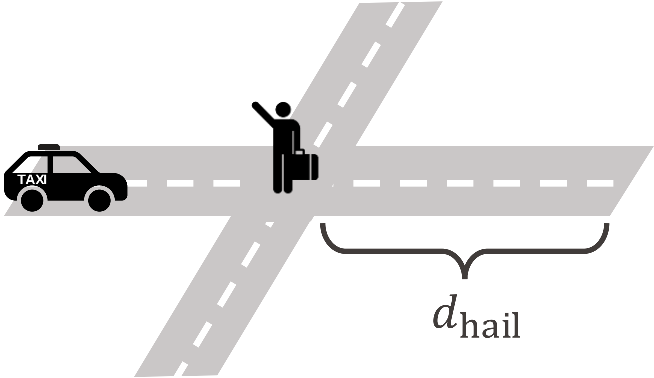
$$N = N^v + \lambda^c \tau_b$$

  - Little's law
    - pickup rate = demand rate

$$\lambda^c = \frac{N^v}{w^v} = \frac{N^c}{w^c}$$

  - Expected waiting time

$$w^c = \frac{\rho A}{4 d_{\text{hail}} N^v}$$

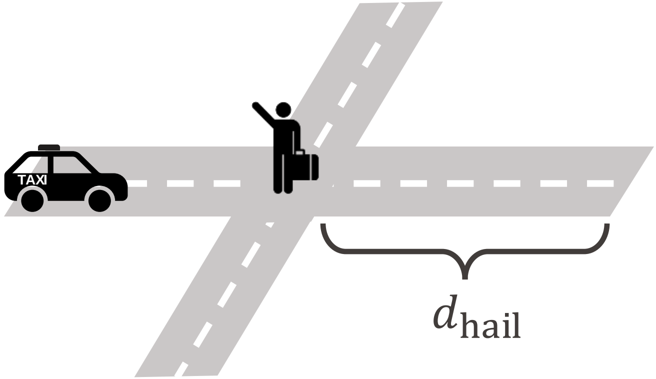


Matching in street-hailing

- Queue length and waiting time at stationary state
  - Passenger

$$N^c = \frac{\lambda^c \rho A}{4 d_{\text{hail}} (N - \lambda^c \tau_b)}$$

$$w^c = \frac{\rho A}{4 d_{\text{hail}} (N - \lambda^c \tau_b)}$$



  - Vehicle

$$N^v = N - \lambda^c \tau_b$$

$$w^v = \frac{N}{\lambda^c} - \tau_b$$

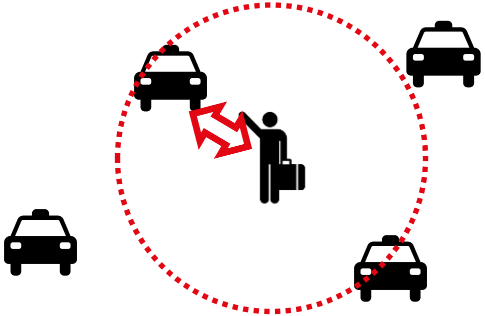
Questions?

Matching in ride-hailing

- Taxi stand
  - Matching a at point

- Street-hailing
  - Matching along a line

- Radio-dispatch/e-hailing
  - Matching over space
  - Low-demand (radio dispatch)

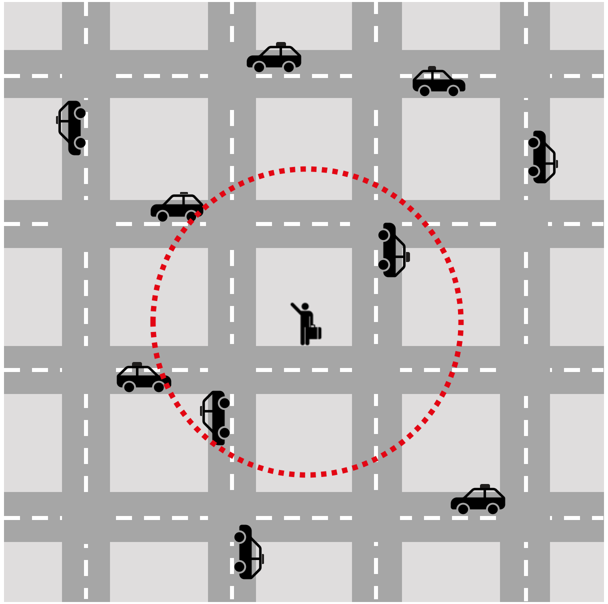


Matching in radio-dispatch

- General service region of area A (km^2)

- Homogeneous travel demand
  - trip origins and destinations are uniformly distributed in the region
  - travelers arrive randomly at λ^c (pax/hr)
  - picked up by the closest vehicle
  - zero trip dispatch time

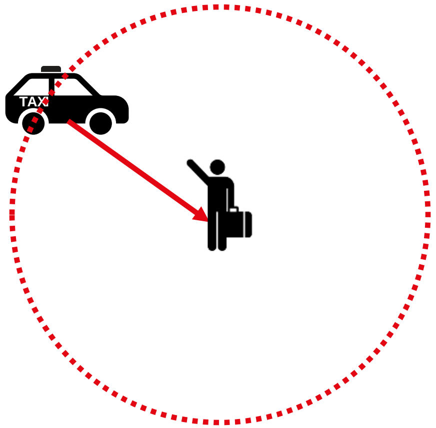
- Homogeneous vehicle supply
  - a fleet of N vehicles uniformly distribute when waiting for trip requests
  - serve door-to-door trips
  - average vehicle speed v_b (km/hr)
  - zero pickup and dropoff times



Matching in radio-dispatch

- Waiting passenger and cruising vehicle coexist
  - Passenger queue and waiting time $N^c, w^c > 0$
  - Vehicle queue and waiting time $N^v, w^v > 0$

- Assume passengers do not have waiting time threshold
  - infinite hail distance $d_{\text{hail}} = \infty$ (km)



Matching in radio-dispatch

- Idle vehicles distribute on streets at density

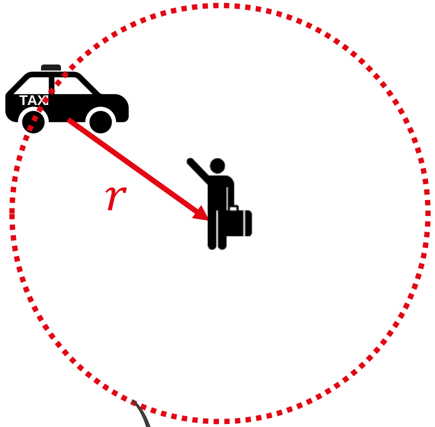
$$\Lambda^v = \frac{N^v}{A}$$

- Let $V(r)$ be the number of cruising vehicles within distance r
  - $V(r)$ follows a Poisson process at rate

$$\lambda^v(r) = 2\pi r \Lambda^v$$

  - probability that n vehicles are within distance r

$$\mathbb{P}[V(r) = n] = \frac{\left(\int_0^r \lambda^v(u)\mathrm{d}u\right)^n}{n!} \exp\left(-\int_0^r \lambda^v(u)\mathrm{d}u\right)$$



Matching in radio-dispatch

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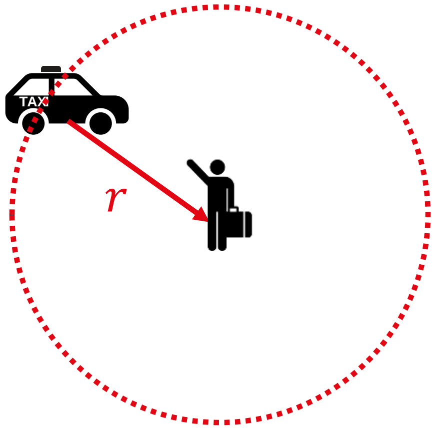
- Let $V(r)$ be the number of cruising vehicles within distance r
  - $V(r)$ follows a Poisson process at rate

$$\lambda^v(r) = 2\pi r \Lambda^v$$

  - probability that no vehicle is within distance r

$$\mathbb{P}[V(r) = 0] = \exp\left(-\int_0^r \lambda^v(u)\mathrm{d}u\right)$$
$$= \exp(-\pi\Lambda^v r^2)$$



Matching in radio-dispatch

- Idle vehicles distribute on streets at density

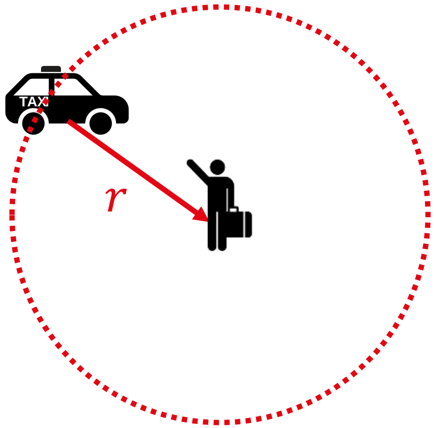
$$\Lambda^v = \frac{N^v}{A}$$

- Let $V(r)$ be the number of cruising vehicles within distance r
  - $V(r)$ follows a Poisson process at rate

$$\lambda^v(r) = 2\pi r \Lambda^v$$

  - probability that at least one vehicle is within distance r

$$1 - \mathbb{P}[V(r) = 0] = 1 - \exp(-\pi \Lambda^v r^2)$$
$$= \mathbb{P}[w^c v_b = r]$$



Matching in radio-dispatch

- Idle vehicles distribute on streets at density

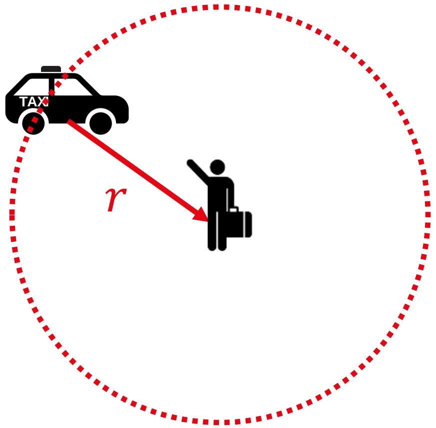
$$\Lambda^v = \frac{N^v}{A}$$

- Let $V(r)$ be the number of cruising vehicles within distance r
  - $V(r)$ follows a Poisson process at rate

$$\lambda^v(r) = 2\pi r \Lambda^v$$

  - Pickup distance $w^c v_b$ follows a Weibull with shape parameter $k = 2$ and rate $\lambda^v(r)$

$$\mathbb{E}[w^c] = \frac{1}{2v_b\sqrt{\Lambda^v}}$$



Matching in radio-dispatch

- Stationary condition
  - Vehicle time conservation

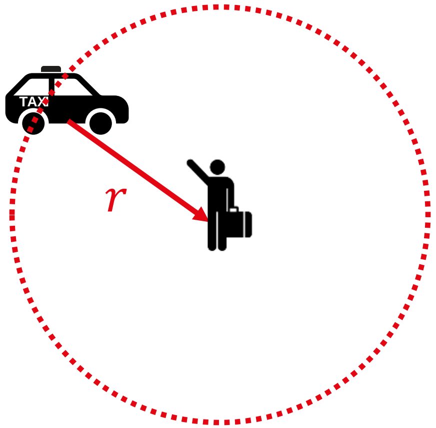
$$N = N^v + \lambda^c(\tau_b + w^c)$$

  - Little's law
    - pickup rate = demand rate

$$\lambda^c = \frac{N^v}{w^v} = \frac{N^c}{w^c}$$

  - Expected waiting time

$$w^c = \frac{1}{2v_b}\sqrt{A/N^v}$$



Matching in radio-dispatch

- Queue length and waiting time at stationary state
  - Passenger

$$N^c = \frac{\lambda^c}{2v_b}\sqrt{\frac{A}{N-\lambda^c(\tau_b+w^c)}} \approx \frac{\lambda^c}{2v_b}\sqrt{\frac{A}{N-\lambda^c\tau_b}}$$

* assume $N^v \approx N - \lambda^c \tau_b$

$$w^c = \frac{1}{2v_b}\sqrt{\frac{A}{N-\lambda^c(\tau_b+w^c)}} \approx \frac{1}{2v_b}\sqrt{\frac{A}{N-\lambda^c\tau_b}}$$

  - Vehicle

$$N^v = N - \lambda^c(\tau_b + w^c) \approx N - \lambda^c\left(\tau_b + \frac{1}{2v_b}\sqrt{\frac{A}{N-\lambda^c\tau_b}}\right)$$

$$w^v = \frac{N}{\lambda^c} - (\tau_b + w^c) \approx \frac{N}{\lambda^c} - \left(\tau_b + \frac{1}{2v_b}\sqrt{\frac{A}{N-\lambda^c\tau_b}}\right)$$

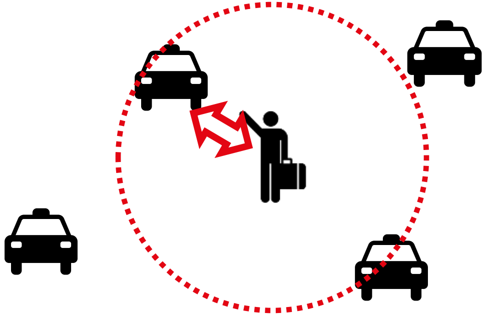
Questions?

Matching in ride-hailing

- Taxi stand
  - Matching a at point

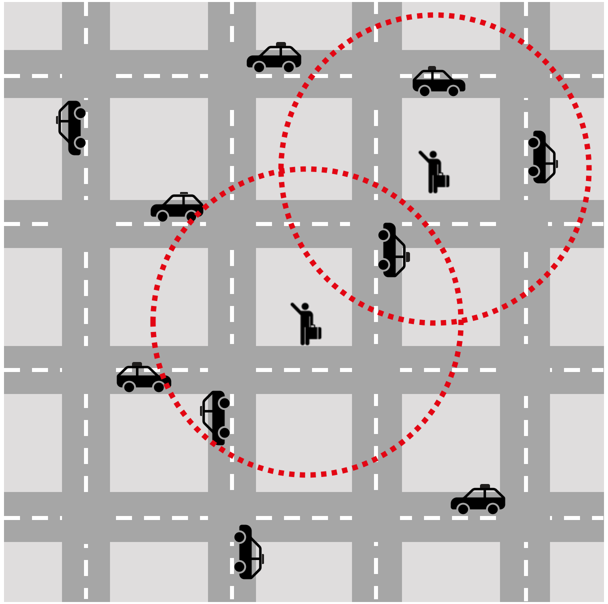
- Street-hailing
  - Matching along a line

- Radio-dispatch/e-hailing
  - Matching over space
  - High-demand (e-hailing)



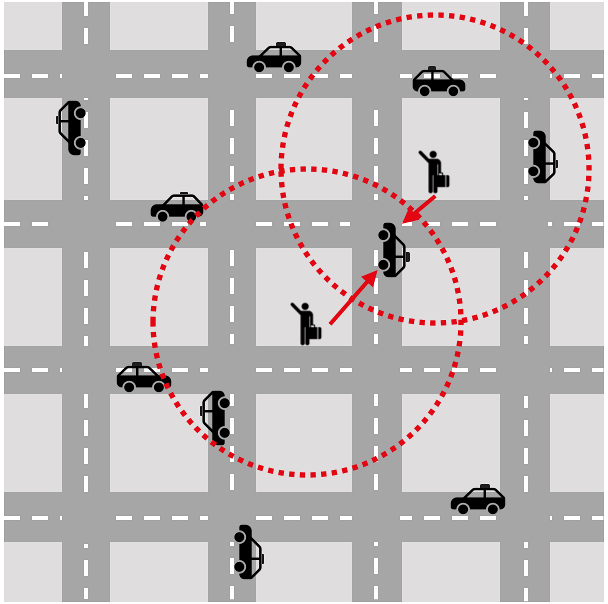
Matching in e-hailing

- General service region of area A (km^2)
- Homogeneous travel demand
  - trip origins and destinations are uniformly distributed in the region
  - travelers arrive randomly at λ^c [pax/hr]
  - picked up by the closest "matchable" vehicle
  - zero dispatch time
- Homogeneous vehicle supply
  - a fleet of N vehicles uniformly distribute when waiting for trip requests
  - serve door-to-door trips
  - average vehicle speed v_b (km/hr)
  - zero pickup and dropoff times



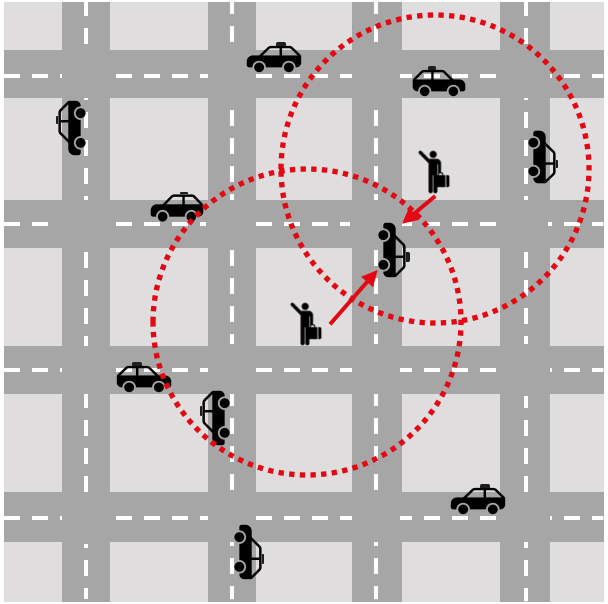
Matching in e-hailing

- E-hailing vs radio-dispatch
  - e-hailing is an advanced version of radio-dispatch
  - matching outcomes remain the same when demand rate is low
  - passenger competition emerges when demand rate is high



Matching in e-hailing

- E-hailing vs radio-dispatch
  - e-hailing is an advanced version of radio-dispatch
  - matching outcomes remain the same when demand rate is low
  - passenger competition emerges when demand rate is high
- Influence of passenger competition
  - Density of "matchable" vehicles $\Lambda^m(N^v, N^c)$
    - N^v: number of vacant vehicles
    - N^c: number of waiting passengers
- ***Q: How should Λ^m change with N^v, N^c?***



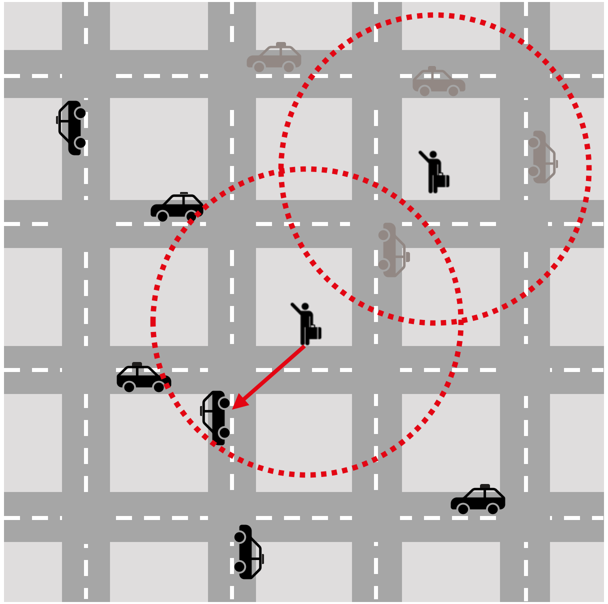
Matching in e-hailing

- Simple estimate of "matchable" vehicles
  - Assume vacant vehicles are uniformly distribute among waiting passengers

$$\Lambda^m(N^v, N^c) = \frac{N^v}{AN^c}$$

  - Expected waiting time

$$\mathbb{E}[w^c] = \frac{1}{2v_b}\sqrt{\frac{AN^c}{N^v}}$$



Matching in e-hailing

- Queue length and waiting time at stationary state
  - Passenger

$$N^c = \frac{\lambda^c}{2v_b}\sqrt{\frac{AN^c}{N-\lambda^c(\tau_b+w^c)}} \approx \frac{\lambda^c}{2v_b}\sqrt{\frac{AN^c}{N-\lambda^c\tau_b}}$$

* assume $N^v \approx N - \lambda^c\tau_b$

$$w^c = \frac{1}{2v_b}\sqrt{\frac{AN^c}{N-\lambda^c(\tau_b+w^c)}} \approx \frac{1}{2v_b}\sqrt{\frac{AN^c}{N-\lambda^c\tau_b}}$$

  - Vehicle

$$N^v = N - \lambda^c(\tau_b + w^c)$$

$$w^v = \frac{N}{\lambda^c} - (\tau_b + w^c)$$

Matching in e-hailing

- Queue length and waiting time at stationary state
  - Passenger

$$N^c = \frac{\lambda^c}{2v_b}\sqrt{\frac{AN^c}{N-\lambda^c(\tau_b+w^c)}} \approx \frac{\lambda^c}{2v_b}\sqrt{\frac{AN^c}{N-\lambda^c\tau_b}} \qquad N^c = \left(\frac{\lambda^c}{2v_b}\right)^2 \frac{A}{N-\lambda^c\tau_b}$$

$$w^c = \frac{1}{2v_b}\sqrt{\frac{AN^c}{N-\lambda^c(\tau_b+w^c)}} \approx \frac{1}{2v_b}\sqrt{\frac{AN^c}{N-\lambda^c\tau_b}} \qquad w^c = \frac{\lambda^c A}{4v_b^2(N-\lambda^c\tau_b)}$$

  - Vehicle

$$N^v = N - \lambda^c(\tau_b + w^c) \approx N - \lambda^c\left(\tau_b + \frac{\lambda^c A}{4v_b^2(N-\lambda^c\tau_b)}\right)$$

$$w^v = \frac{N}{\lambda^c} - (\tau_b + w^c) \approx \frac{N}{\lambda^c} - \left(\tau_b + \frac{\lambda^c A}{4v_b^2(N-\lambda^c\tau_b)}\right)$$

Questions?

Ride-hailing service design

- Scenario I: One-sided market
  - Design variables
    - Trip fare p (CHF/pax)
    - Fleet size N (veh)

$$\max_{p,N} \quad \Pi(p,N) = \lambda^c p - cN$$

$$s.t. \quad \lambda^c = g_\lambda(p, w^c) \quad \textbf{demand sensitivity}$$

$$w^c = g_w(\lambda^c, N) \quad \textbf{matching outcome}$$

- c:unit operation cost c (CHF/veh/hr)

Ride-hailing service design

$$w^c = g_w(\lambda^c, N)$$

	N^c	w^c	N^v	w^v
Taxi stand I	0	0	$N - \lambda^c(\tau_b + \frac{\delta}{v_b}\sqrt{A/K})$	$\frac{N}{\lambda^c} - (\tau_b + \frac{\delta}{v_b}\sqrt{A/K})$
Taxi stand II*	$\frac{\lambda^c K\left(\tau_b + \frac{\delta}{v_b}\sqrt{\frac{A}{K}}\right)}{N - \lambda^c\left(\tau_b + \frac{\delta}{v_b}\sqrt{\frac{A}{K}}\right)}$	$\frac{\lambda^c K\left(\tau_b + \frac{\delta}{v_b}\sqrt{\frac{A}{K}}\right)^2}{N\left[N - \lambda^c\left(\tau_b + \frac{\delta}{v_b}\sqrt{\frac{A}{K}}\right)\right]}$	0	0
Street-hailing	$\frac{\lambda^c \rho A}{4d_{\text{hail}}(N - \lambda^c \tau_b)}$	$\frac{\rho A}{4d_{\text{hail}}(N - \lambda^c \tau_b)}$	$N - \lambda^c \tau_b$	$\frac{N}{\lambda^c} - \tau_b$
Radio-dispatch#	$\frac{\lambda^c}{2v_b}\sqrt{\frac{A}{N - \lambda^c \tau_b}}$	$\frac{1}{2v_b}\sqrt{\frac{A}{N - \lambda^c \tau_b}}$	$N - \lambda^c\left(\tau_b + \frac{1}{2v_b}\sqrt{\frac{A}{N - \lambda^c \tau_b}}\right)$	$\frac{N}{\lambda^c} - \left(\tau_b + \frac{1}{2v_b}\sqrt{\frac{A}{N - \lambda^c \tau_b}}\right)$
E-hailing#	$\left(\frac{\lambda^c}{2v_b}\right)^2 \frac{A}{N - \lambda^c \tau_b}$	$\frac{\lambda^c A}{4v_b^2(N - \lambda^c \tau_b)}$	$N - \lambda^c\left[\tau_b + \left(\frac{\lambda^c}{2v_b}\right)^2 \frac{A}{N - \lambda^c \tau_b}\right]$	$\frac{N}{\lambda^c} - \left[\tau_b + \frac{\lambda^c A}{4v_b^2(N - \lambda^c \tau_b)}\right]$

* Assume M/M/1 queue   # Drop w^c in the right-hand-side of formula of w^c

Ride-hailing service design

- Scenario II: Two-sided market
  - Design variables
    - Trip fare p (CHF/pax)
    - Commission rate η (%)

$$\max_{p,\eta} \quad \Pi(p,\eta) = \eta\lambda^c p$$

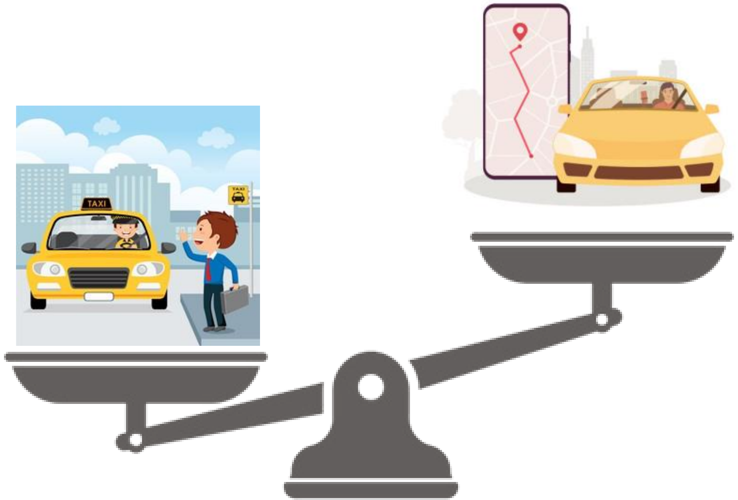
$$s.t. \quad \lambda^c = g_\lambda(p, w^c)$$ **demand sensitivity**

$$N = g_N\left(\frac{(1-\eta)\lambda^c p}{N}\right)$$ **supply sensitivity**

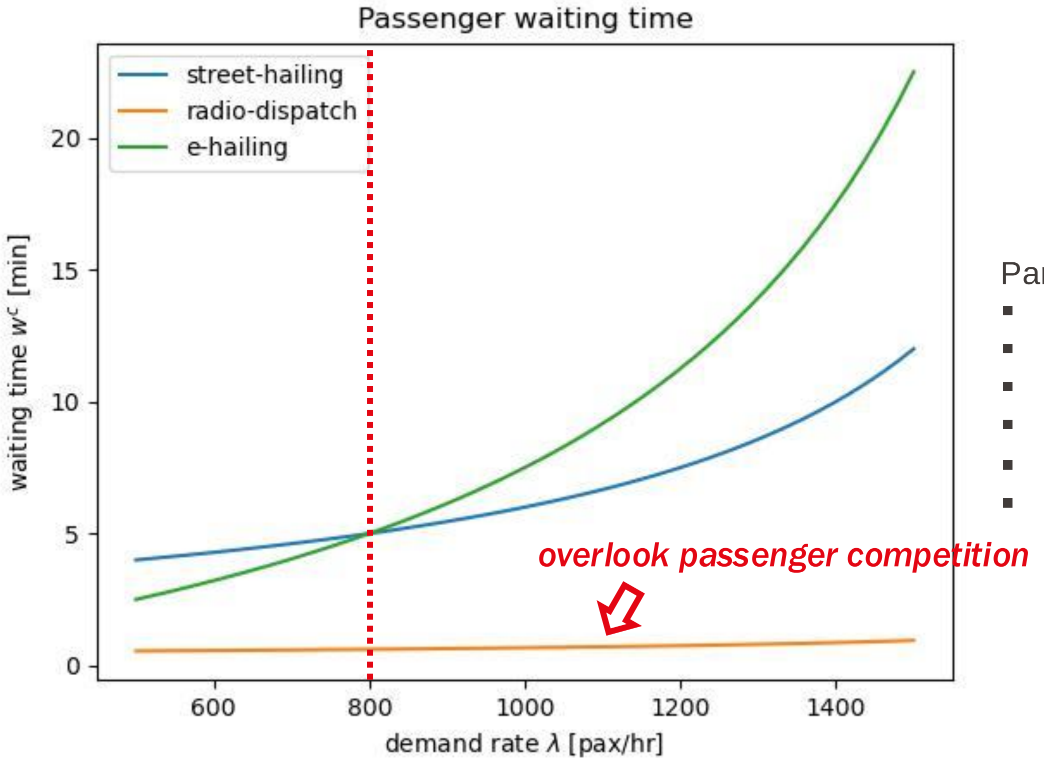
$$w^c = g_w(\lambda^c, N)$$ **matching outcome**

Case study

- Street-hailing vs e-hailing
  - Suppose a ride-hailing company wish to launch its service in a city
    - when does e-hailing provide better service?
    - does passenger competition cause a significant impact?



Case study



Parameters

- service area $A = 50$ [km^2]
- road density $\rho = 0.1$ [km/ km^2]
- fleet size $N = 500$ [veh]
- vehicle speed $v_b = 20$ [km/hr]
- average trip duration $\tau_b = 15$ [min]
- hail distance $d_{\text{hail}} = 50$ [m]