Spring 2025

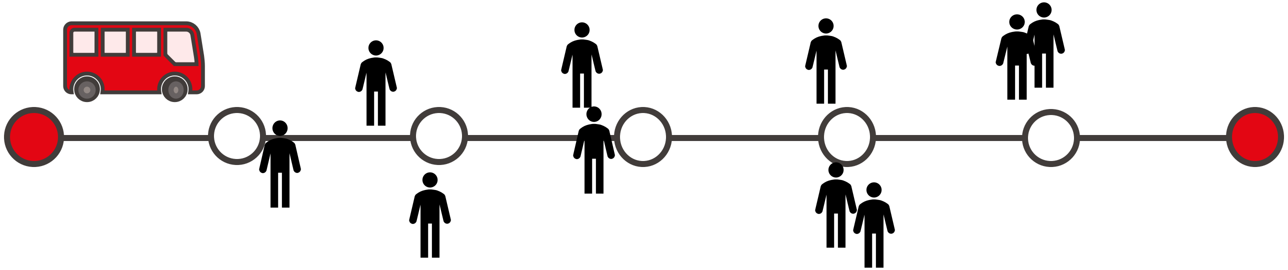
08 Flexible-Route Transit

CIVIL-324 Urban public transport systems

École polytechnique fédérale de Lausanne

Collective vs individual transport

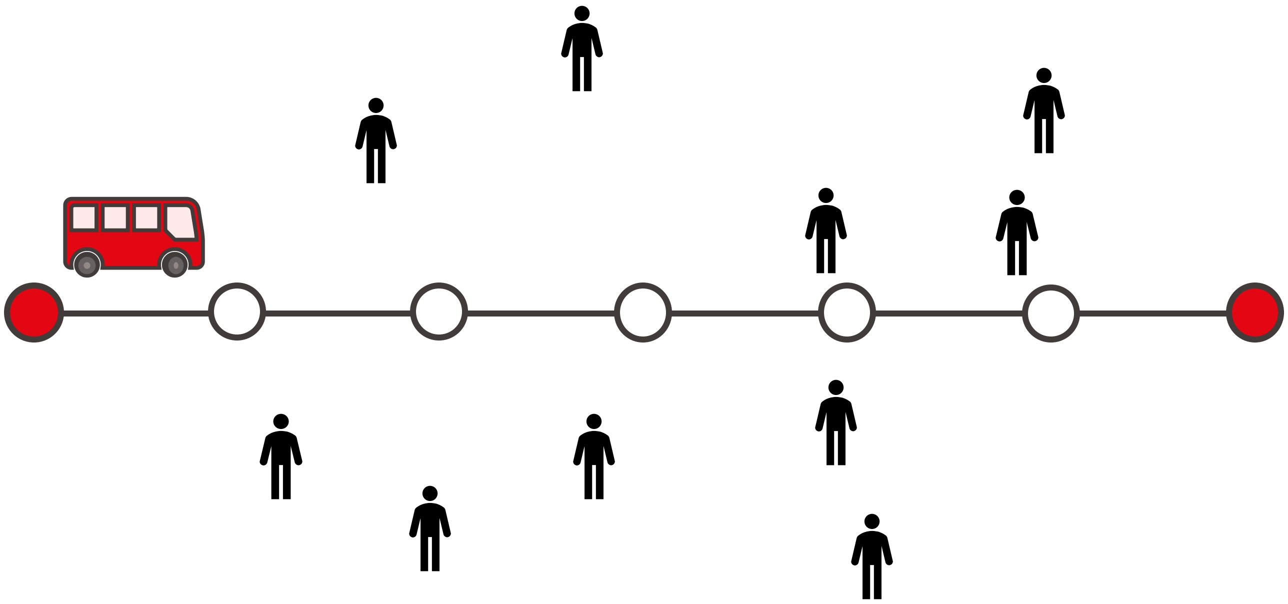
- Mass transit is efficient when travel demand shows spatial aggregation
  - e.g.,
    - Shuttle: between two points
    - Corridor: along a route
    - Network: with high density
      - e.g., 11.5k per/km^2 for subway; 7k per/ /km^2 for light rail transit[1]



[1] Pembina Institute. Building transit where we need it. 2011.

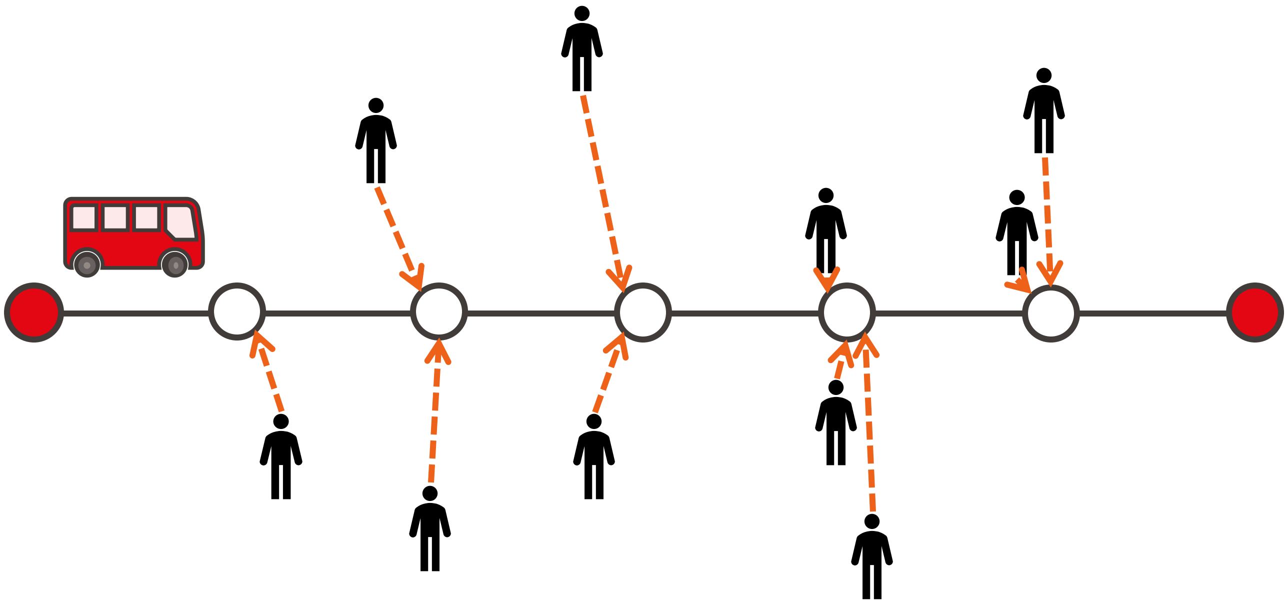
Collective vs individual transport

- However, this is not always the case in practice
  - Travel demand may distribute sparsely over the space



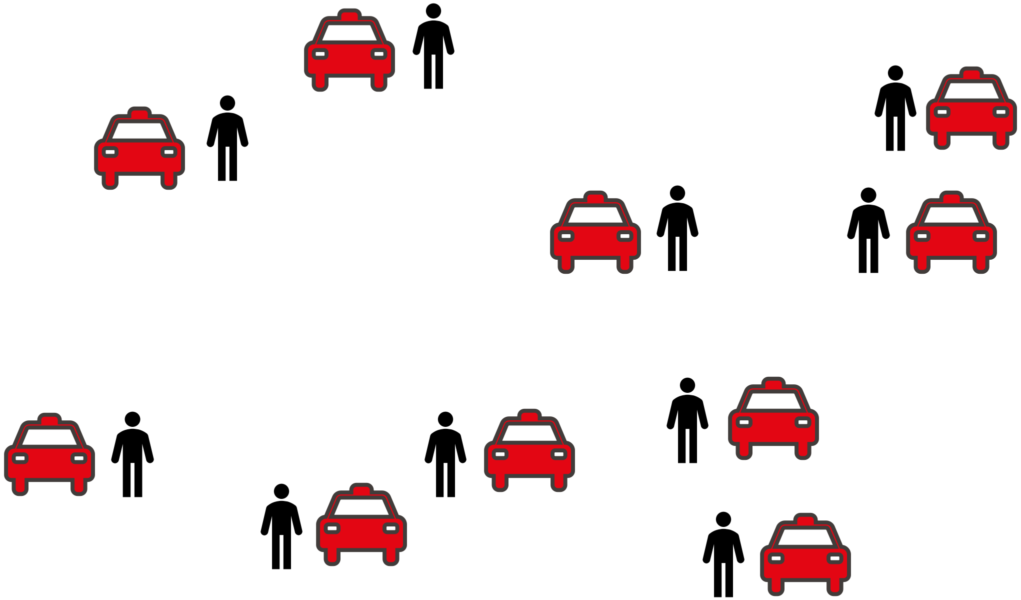
Collective vs individual transport

- However, this is not always the case in practice
  - Travel demand may distribute sparsely over the space
  - Classic transit services (e.g., bus) may lead to large user cost (e.g., long access time)



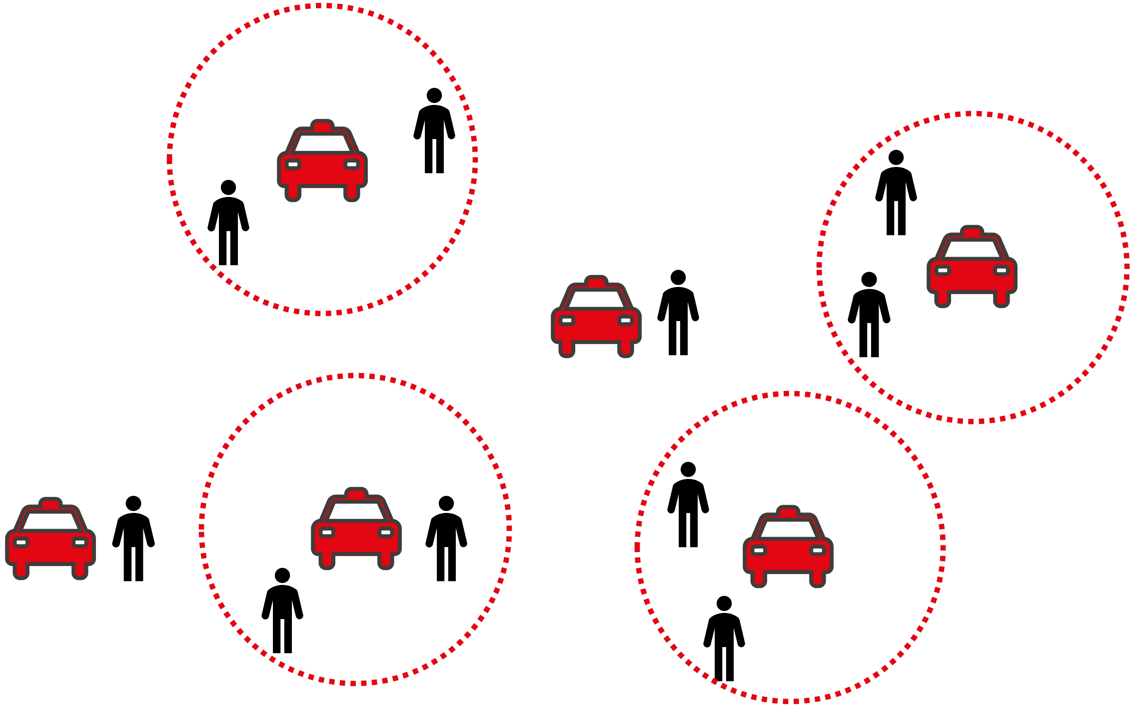
Collective vs individual transport

- An alternative is to provide on-demand mobility services
  - e.g., taxi



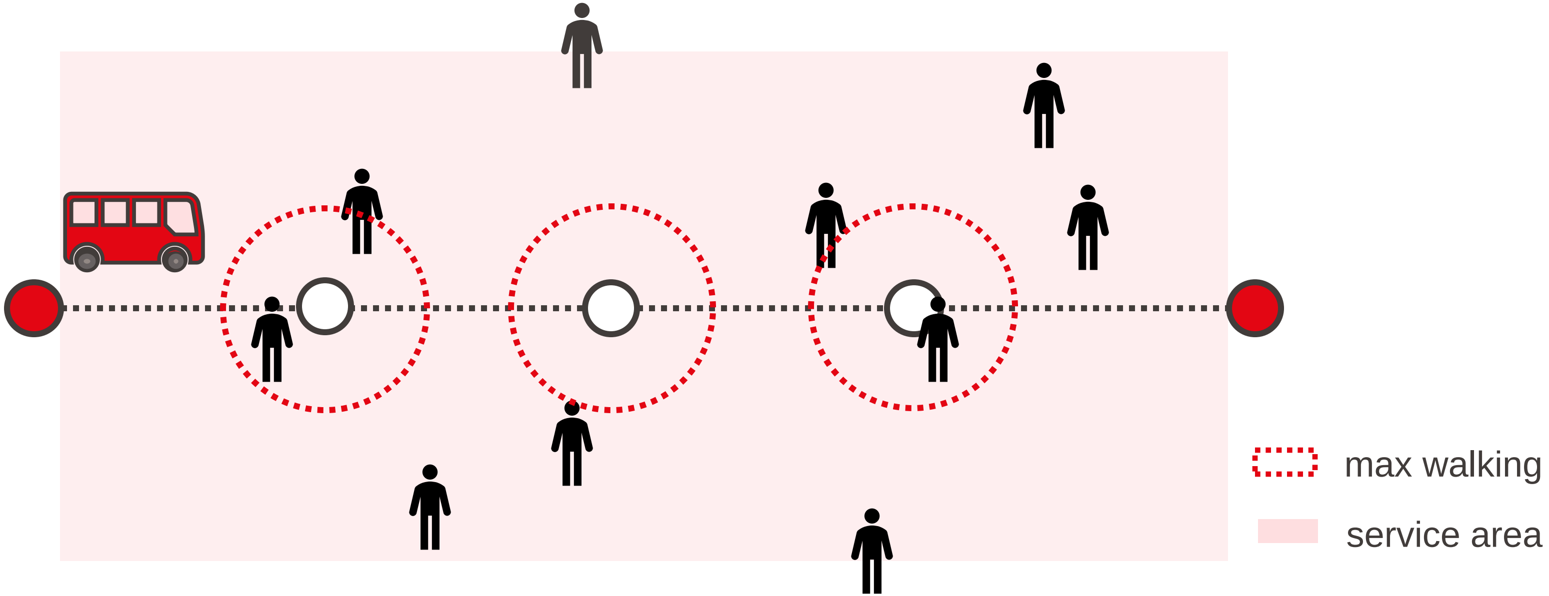
Collective vs individual transport

- An alternative is to provide on-demand mobility services
  - e.g., ride-pooling



Collective vs individual transport

- There also exist options in between that
  - retain the structure of fixed-route transit (e.g., stops, schedule)
    - passengers close to stops directly go there as in fixed-route transit
  - operate in a more flexible way (e.g., local routing)
    - passengers far away send trip requests as in on-demand mobility



Collective vs individual transport

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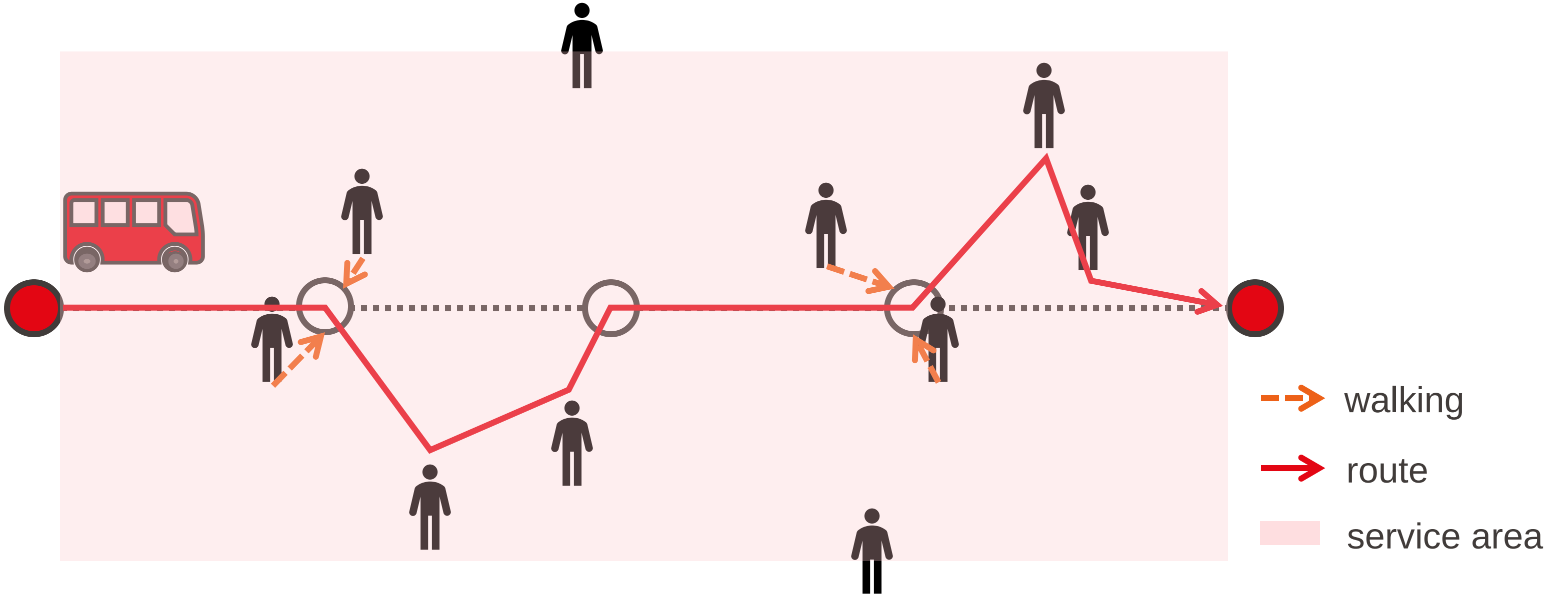


Collective vs individual transport

Individual

Private driving

On-demand mobility service

Out of the scope

Collective

Flexible transit

Mass transit

Collective vs individual transport

Individual

Private driving

On-demand mobility service

Collective

Flexible transit

Mass transit

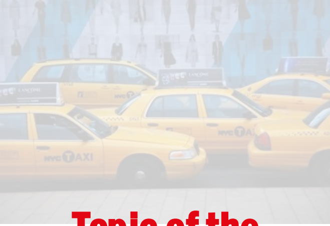
First half of this course

Collective vs individual transport

Individual

Private driving

On-demand mobility service



Topic of the next lecture

Collective

Flexible transit

Mass transit

Collective vs individual transport

Individual

Private driving

On-demand mobility service


Collective

Flexible transit

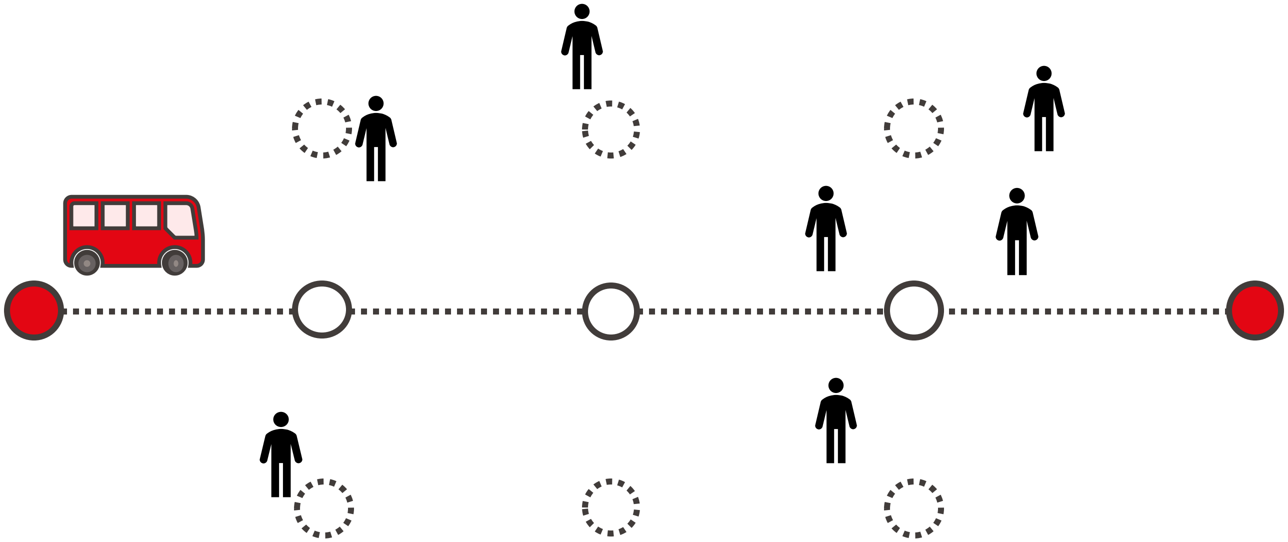
Mass transit

Topic of this lecture

Questions?

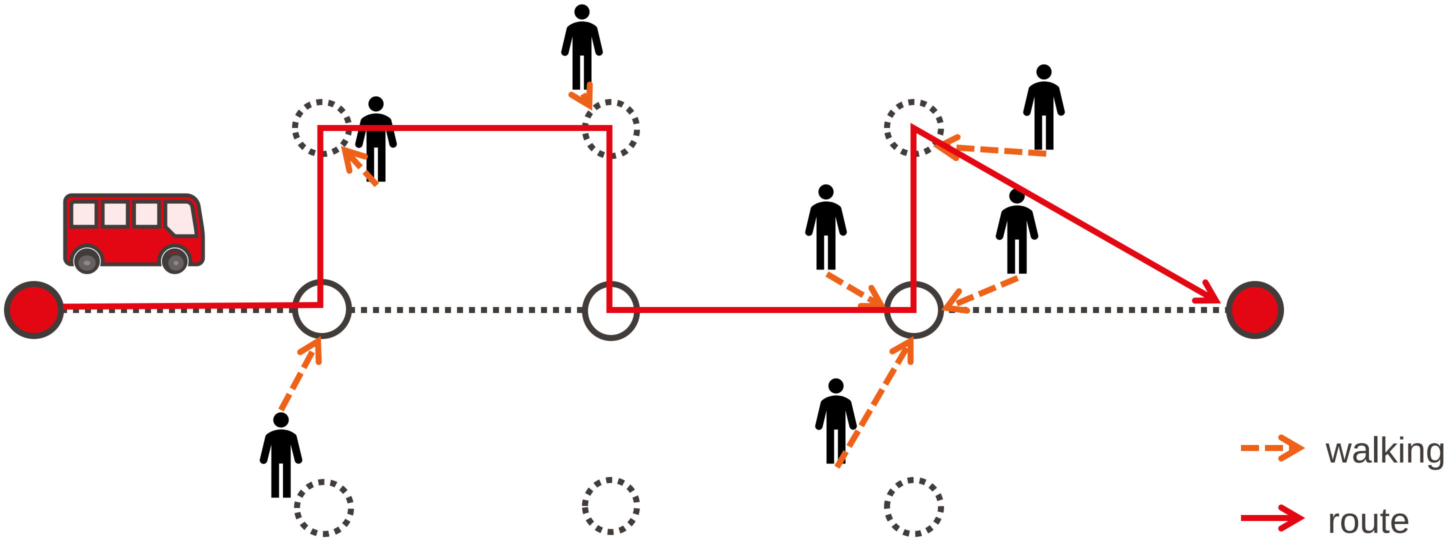
Flexible-route transit

- "Flexibility" at different levels
  - Skeleton route + optional stops



Flexible-route transit

- "Flexibility" at different levels
  - Skeleton route + optional stops
    - adaptive route connecting regular stops and optional stops with passengers



Flexible-route transit

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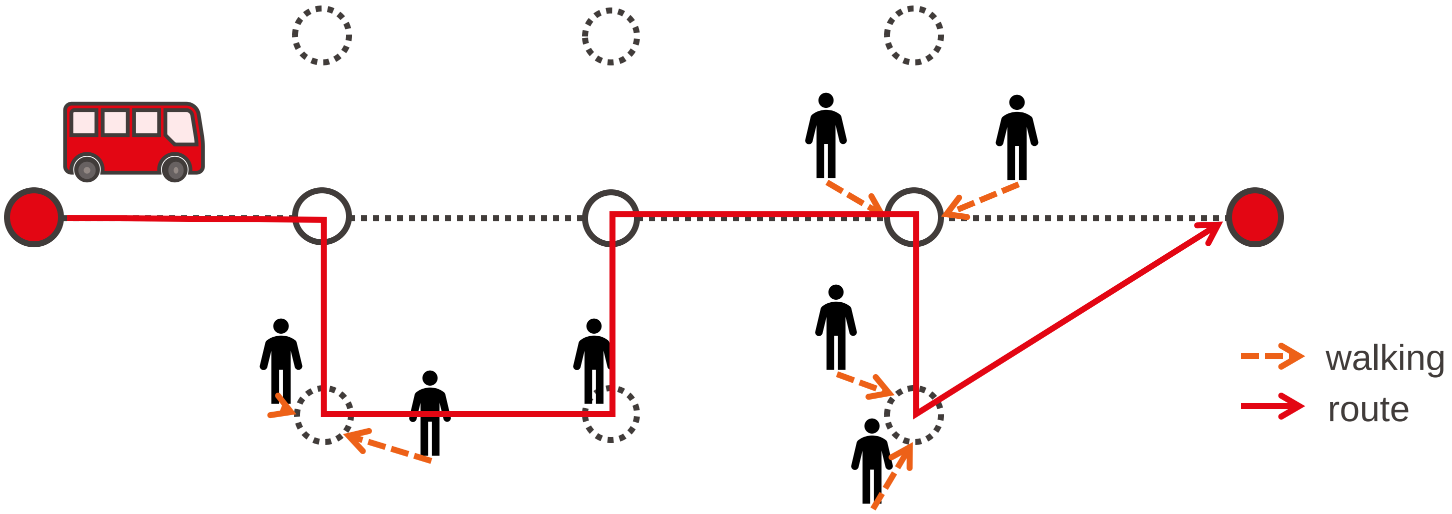
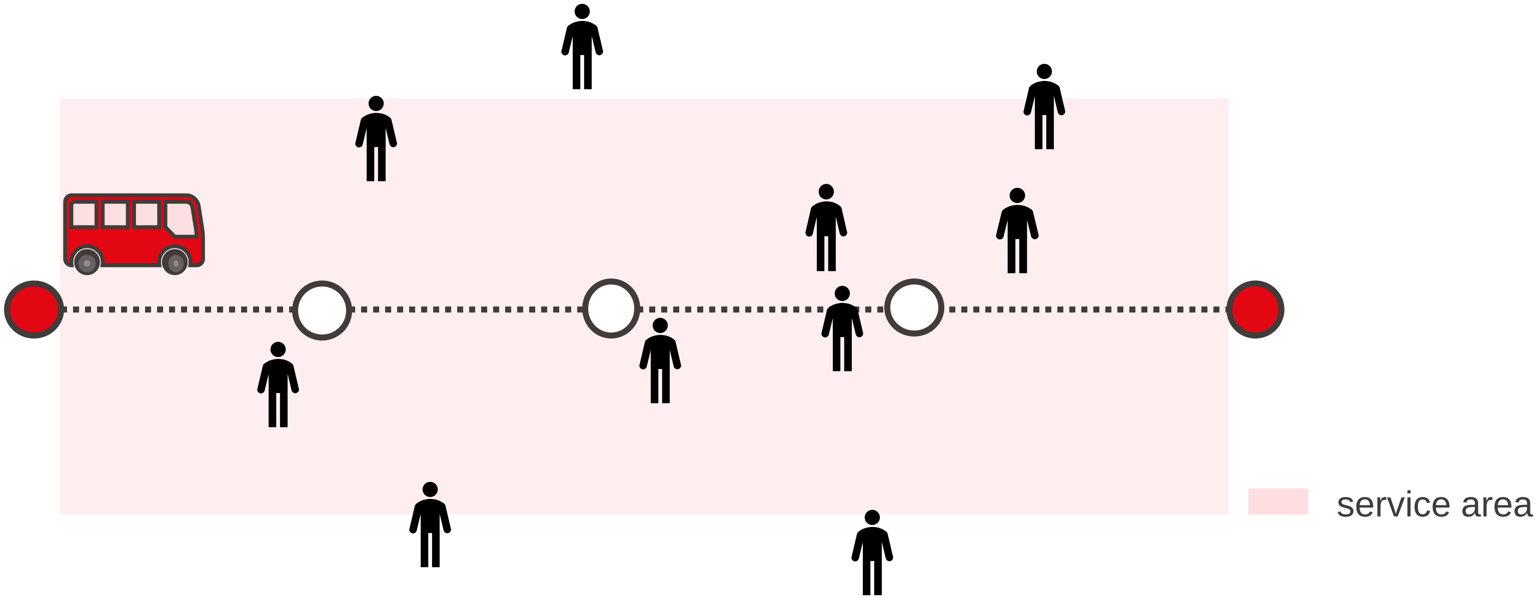


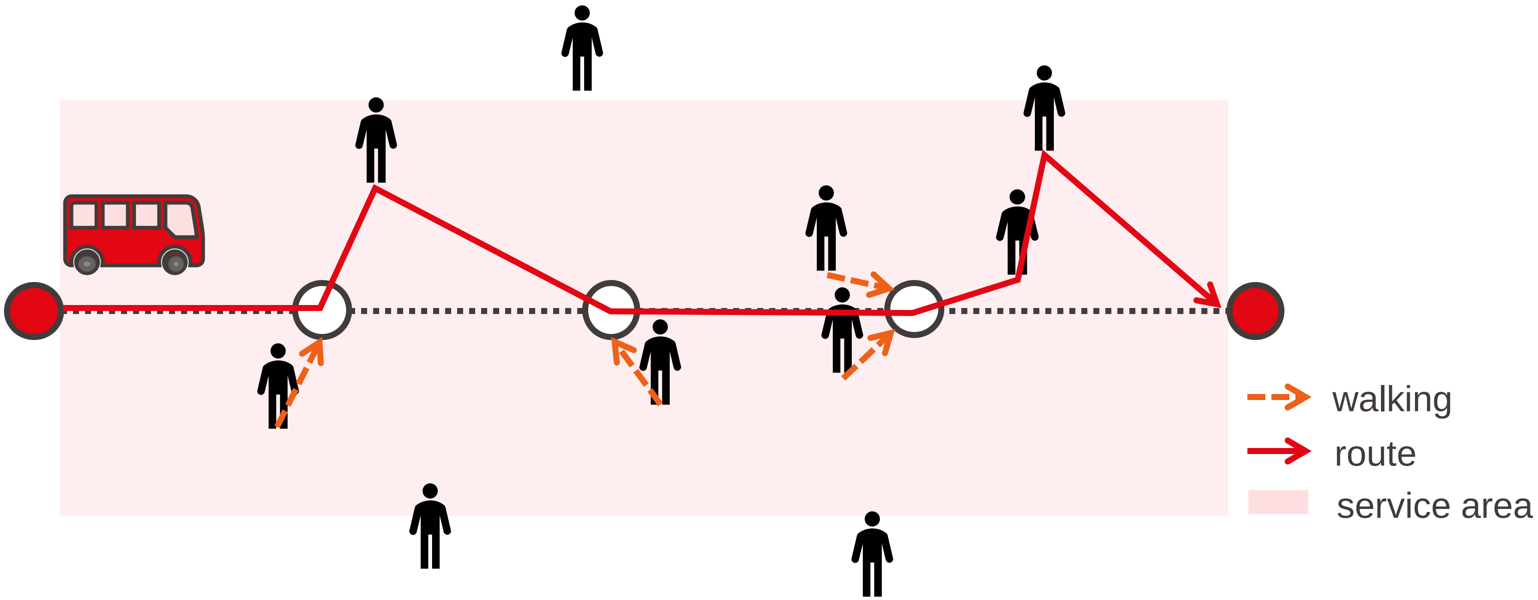
Flexible-route transit

- "Flexibility" at different levels
  - Skeleton route + optional stops
  - Skeleton route + service zone
    - pick up passengers within the service zone on-demand



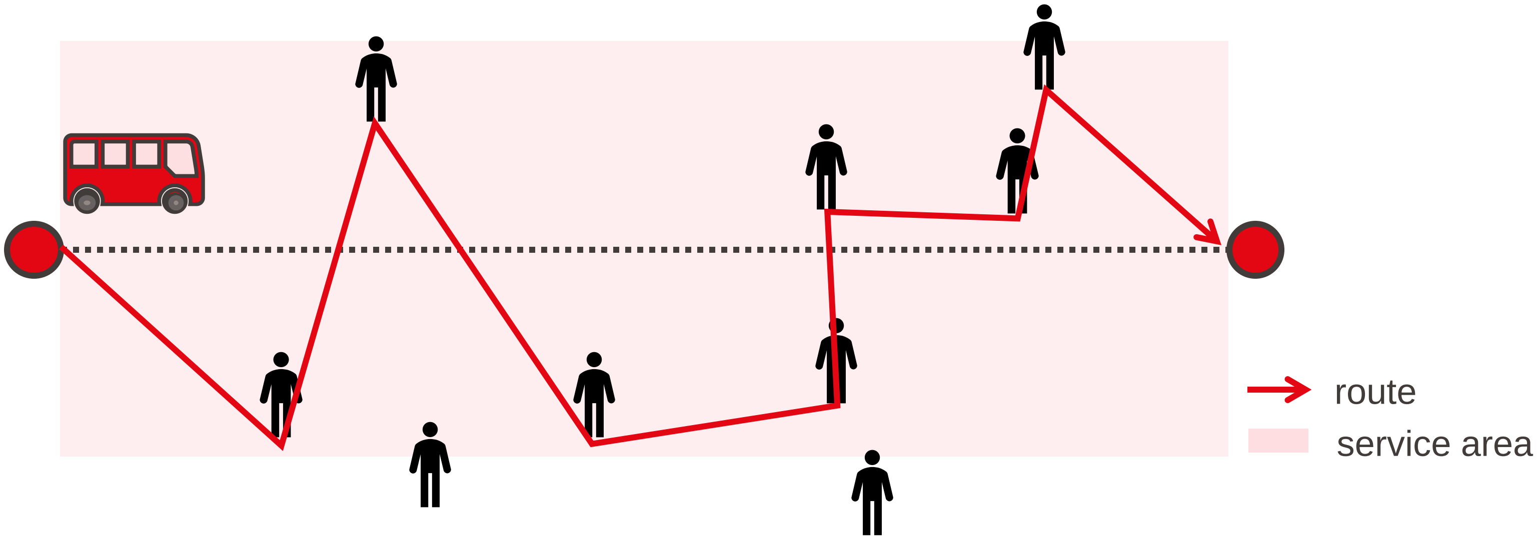
Flexible-route transit

- “Flexibility” at different levels
  - Skeleton route + optional stops
  - Skeleton route + service zone
    - visit regular stops and pick up passengers off-route on-demand



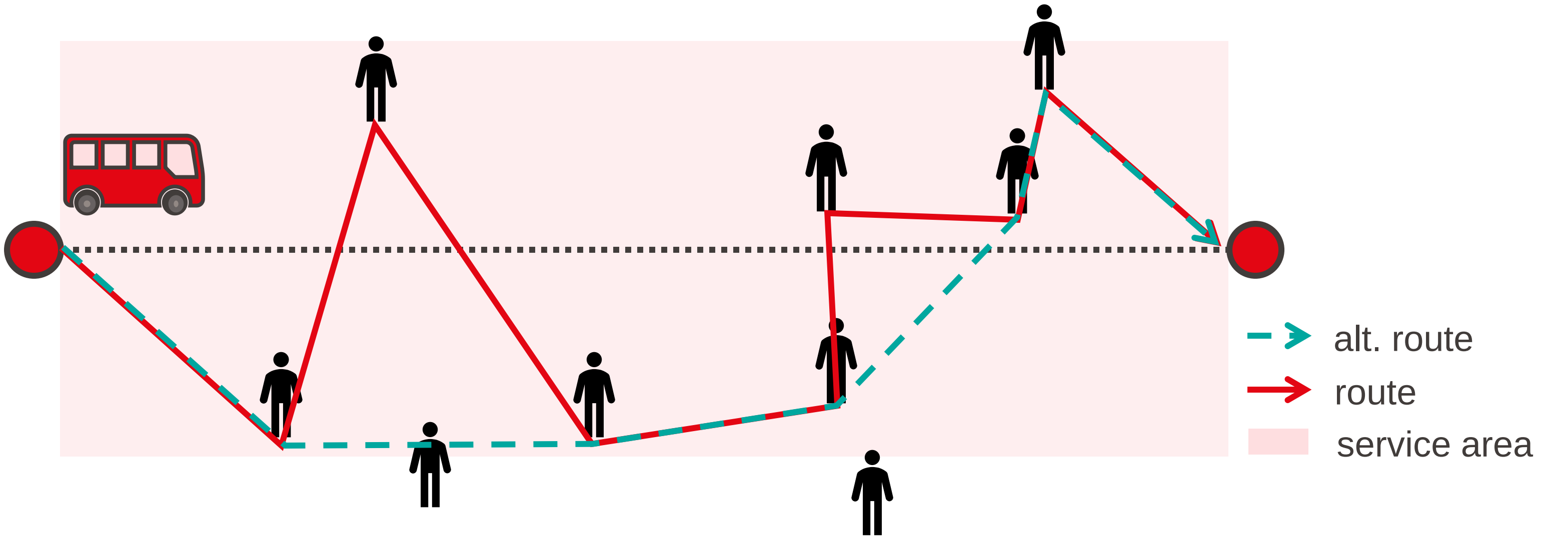
Flexible-route transit

- "Flexibility" at different levels
  - Skeleton route + optional stops
  - Skeleton route + service zone
  - Service zone
    - pick up passengers inside the service zone on-demand



Flexible-route transit

- “Flexibility” at different levels
  - Skeleton route + optional stops
  - Skeleton route + service zone
  - Service zone
    - pick up passengers inside the service zone on-demand
    - some passengers may be rejected to reduce total detour



Flexible-route transit

- “Demand responsive” at different levels
  - Reservation-based
    - e.g., Dial-a-ride (DAR)

Flexible-route transit

- "Demand responsive" at different levels
  - Reservation-based
    - e.g., Dial-a-ride (DAR)

  - Real-time
    - e.g., demand responsive transit (DRT)

Flexible-route transit

- "Flexibility" at different levels
  - Skeleton route + optional stops
  - Skeleton route + service zone
  - Service zone

service design

- "Demand responsive" at different levels
  - Reservation-based
  - On-demand

service operations

Flexible-route transit

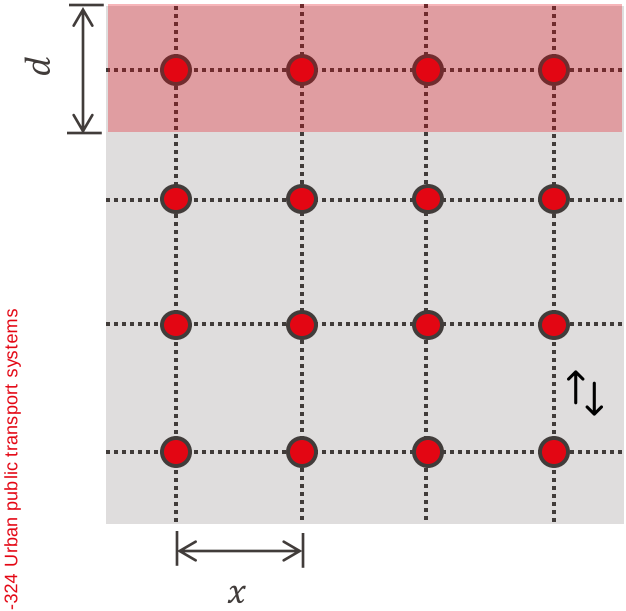
- "Flexibility" at different levels
  - Skeleton route + optional stops
  - Skeleton route + service zone
  - Service zone

stylized model with skeleton route

- "Demand responsive" at different levels
  - Reservation-based
  - On-demand

Questions?

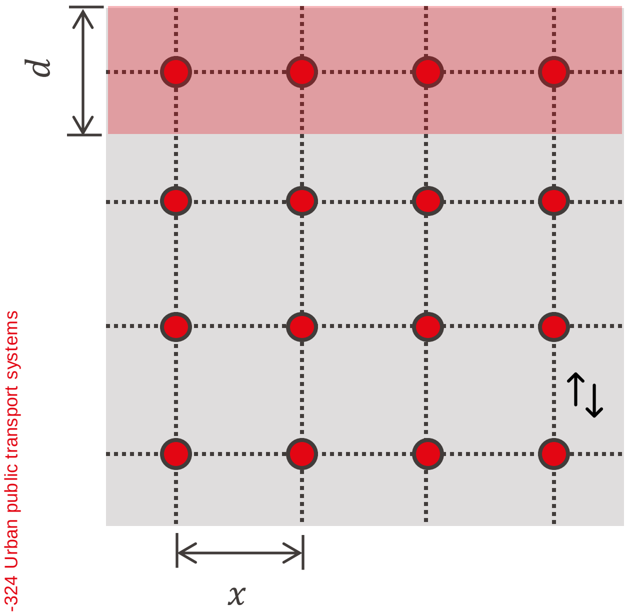
Model setting



- A grid network of skeleton routes
  - Infinite service region with bidirectional lines
  - Constant line spacing x (km)

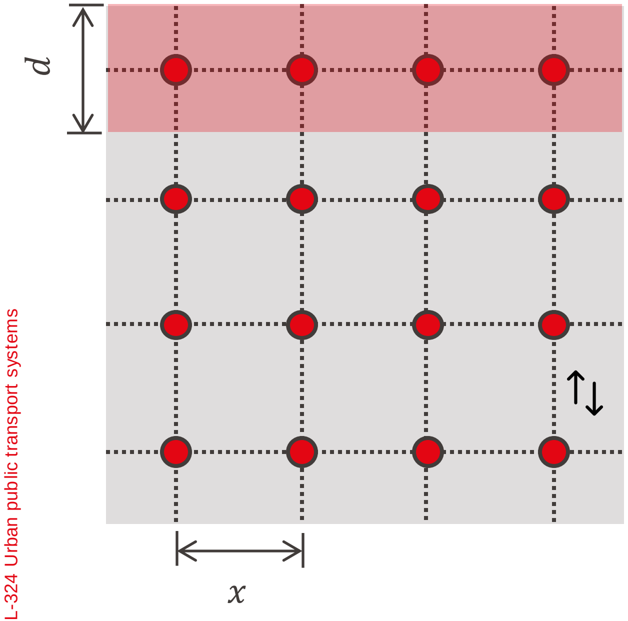
- ***Q: How many lines in total in a unit area? What is the width of service region?***

Model setting



- A grid network of skeleton routes
  - Infinite service region with bidirectional lines
  - Constant line spacing x (km)
  - Each line serves a "tube" of width $d = x$ (km)
- Homogeneous travel demand
  - Uniformly distributed with rate λ (pax/hr/km^2)
  - Average trip distance ℓ (km) with single transfer

Model setting



- A grid network of skeleton routes
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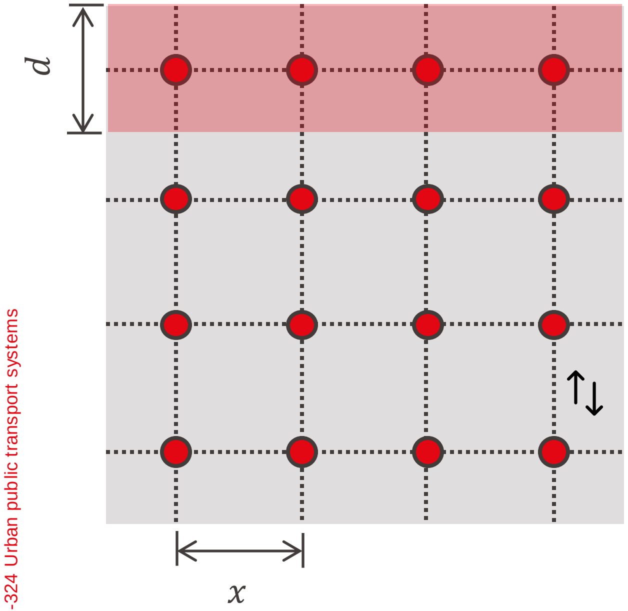
  - Each line serves a "tube" of width $d = x$ (km)
- Homogeneous travel demand
  - Uniformly distributed with rate λ (pax/hr/km^2)
  - Average trip distance ℓ (km) with single transfer
- Flexible-route transit service
  - Uniform frequency f (/hr) without coordination
  - Average vehicle speed v_b (km/hr)
  - Stop time ω_0 (hr) and transfer time ω_1 (hr)
  - Unlimited vehicle capacity
  - Pick up and drop off passengers within service tube between each two transfer stops

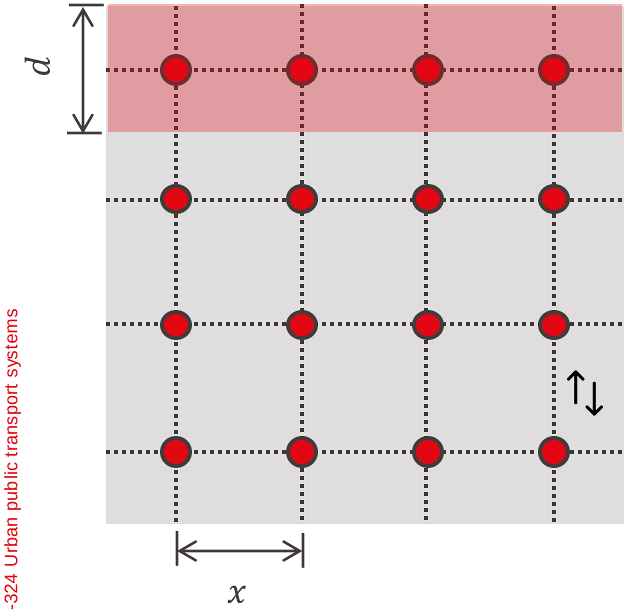
Model setting



- Design variables
  - Line frequency f (/hr)
  - Line spacing x (km)

- Design trade-offs
  - Higher frequency
    - more vehicle dispatches
    - shorter waiting time
  - Shorter line spacing
    - more lines and transfer stops
    - ~~shorter access time~~

Model setting



- Design variables
  - Line frequency f (/hr)
  - Line spacing x (km)

- Design trade-offs
  - Higher frequency
    - more vehicle dispatches
    - shorter waiting time
    - shorter headway
  - Shorter line spacing
    - more lines and transfer stops
    - ~~shorter access time~~
    - smaller service region

fewer off-route requests
⇒ shorter detour
⇒ shorter vehicle distance

Design problem

- Determine frequency f and line spacing x to mini ***minimize*** the total system cost $TC(f,x)$ per hour per unit area

$$\min_{f,x} \; TC(f,x) = \frac{4cf}{x} + \beta\lambda\,(w + \tau_b + \delta)$$

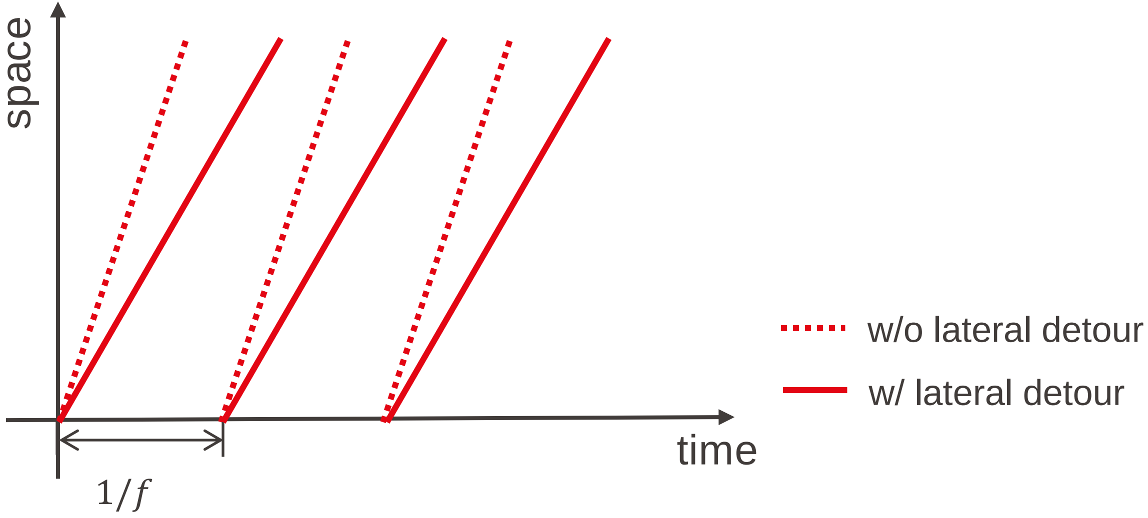
  - f: line frequency (/hr)
  - x: line spacing (km)
  - c: operation cost per line (CHF/km)
  - β: value of time (CHF/hr)
  - λ: demand rate (pax/km^2/hr)
  - w: average waiting time (hr)
  - τ_b: vehicle travel time (hr)
  - δ: stopping time (hr)

- ***Q: How to approximate average waiting time, in-vehicle travel time and stopping time?***

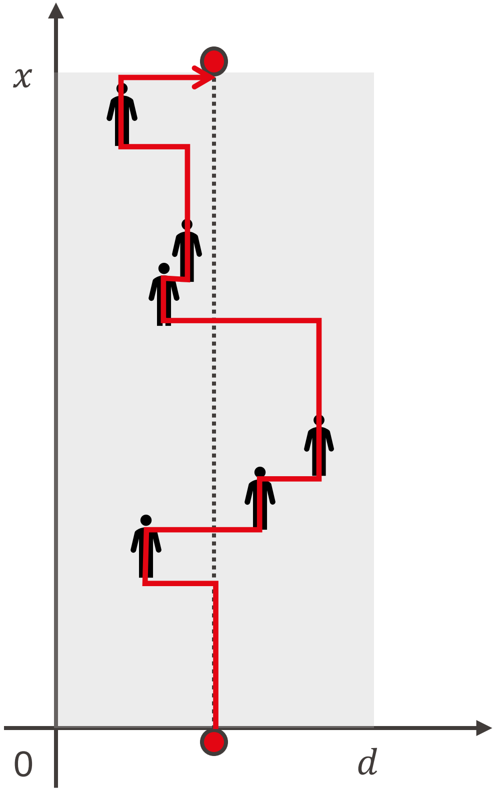
Travel time approximation

- Waiting time with a single transfer
  - Due to uniform demand, each dispatch has the same average lateral detour
  - Headway remain the same as fixed-route transit

$$w = 2\left(\frac{1}{2f}\right) = \frac{1}{f}$$

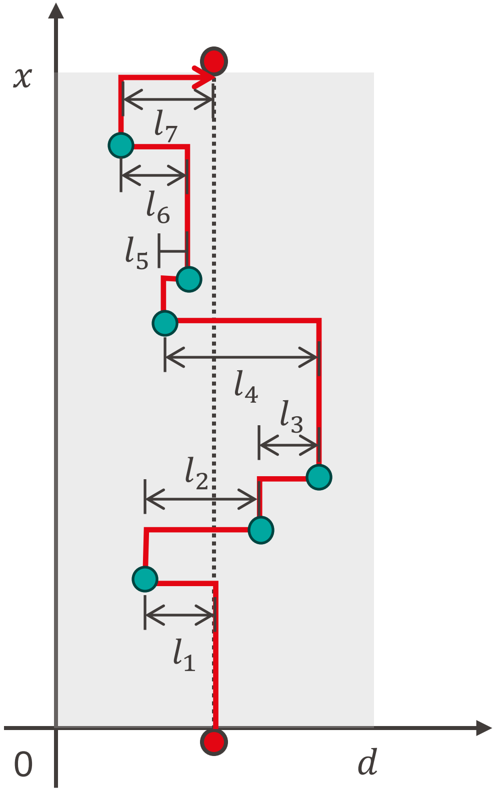


Travel time approximation



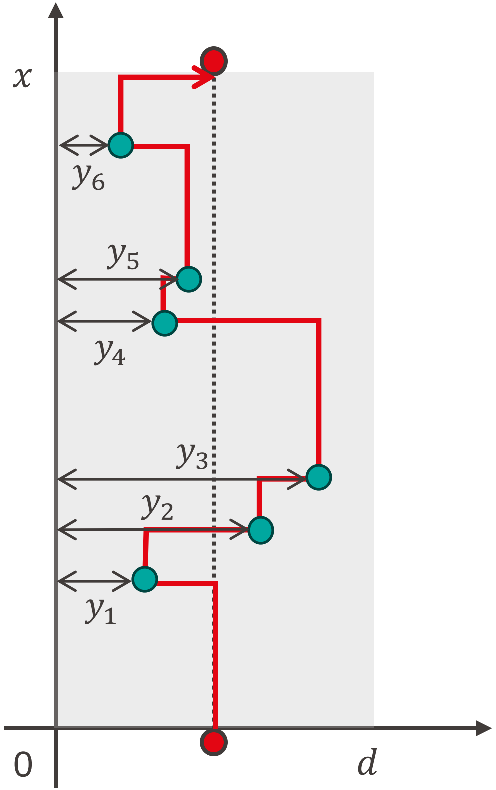
- Flexible route between two transfer stops
  - Travel either vertically or horizontally
  - Always move forward to the next transfer stop
  - Visit all demand points (pickup/dropoff)

Travel time approximation



- Flexible route between two transfer stops
  - Travel either vertically or horizontally
  - Always move forward to the next transfer stop
  - Visit all demand points (pickup/dropoff)
- Vehicle travel distance
  - Central $L_c = x$ (km)
  - Lateral $L_a = \sum_{i=1}^{n+1} l_i$ (km)

Travel time approximation



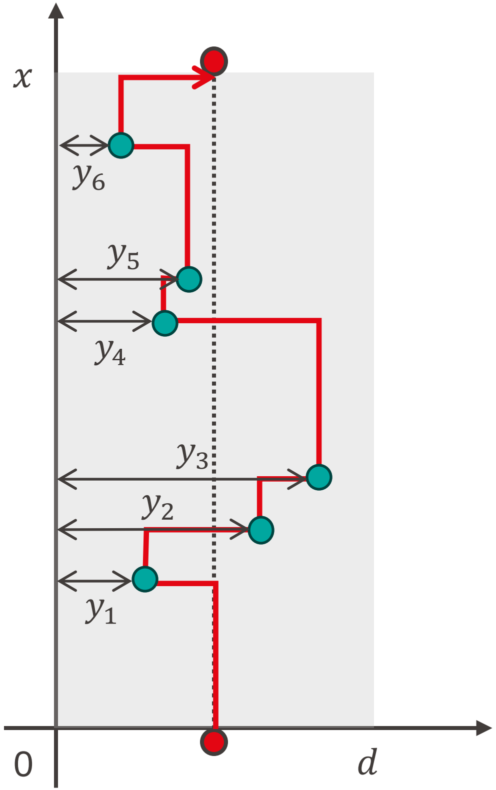
- Flexible route between two transfer stops
  - Travel either vertically or horizontally
  - Always move forward to the next transfer stop
  - Visit all demand points (pickup/dropoff)
- Vehicle travel distance
  - Central $L_c = x$ (km)
  - Lateral

$$L_a = \sum_{i=1}^{n+1} l_i = \left|\frac{d}{2} - y_1\right| + \sum_{i=2}^{n} |y_i - y_{i-1}| + |\frac{d}{2} - y_n|$$

$$\mathbb{E}[L_a] = \mathbb{E}\left[\sum_{i=2}^{n} |y_i - y_{i-1}|\right] \approx \mathbb{E}[n]\ \mathbb{E}[|y_i - y_j|]$$

  - ***Q: What is the meaning of $\mathbb{E}[n]$ and $\mathbb{E}[|y_i - y_j|]$?***

Travel time approximation



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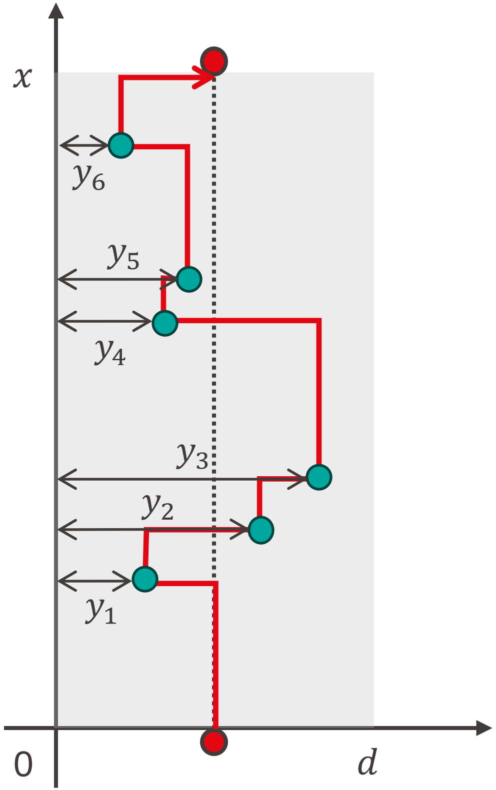
$$\mathbb{E}[L_a] = \mathbb{E}\left[\sum_{i=2}^{n} |y_i - y_{i-1}|\right] \approx \mathbb{E}[n]\,\mathbb{E}\left[|y_i - y_j|\right]$$

 # pax **dist between two paxs**

Travel time approximation



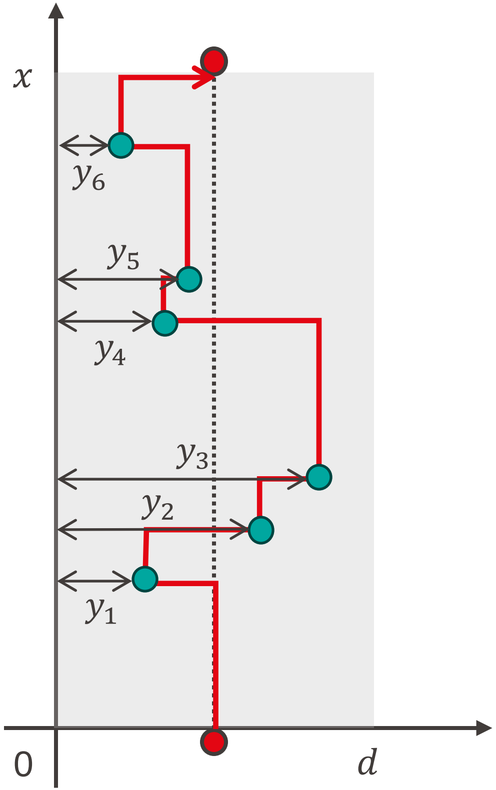
- Aggregate demand between two transfer stops and two dispatches

$$\mathbb{E}[n] = \lambda d x (1/f) = \frac{\lambda x^2}{f}$$

- Distance between two pickup/dropoff points

$$\mathbb{E}[|y_i - y_j|] = \frac{1}{d^2}\int_0^d \int_0^d |y_i - y_j|\,\mathrm{d}y_i\,\mathrm{d}y_j$$
$$= \frac{1}{d^2}\int_0^d \int_0^d |y_i - y_j|\,\mathrm{d}y_i\,\mathrm{d}y_j$$
$$= \frac{1}{d^2}\left[\int_0^d \int_0^{y_j} (y_j - y_i)\,\mathrm{d}y_i\,\mathrm{d}y_j + \int_0^d \int_{y_j}^d (y_i - y_j)\,\mathrm{d}y_i\,\mathrm{d}y_j\right]$$
$$= \frac{1}{d^2}\left[\int_0^d \left(\frac{1}{2}y_j^2\right)\mathrm{d}y_j + \int_0^d \left(\frac{1}{2}y_j^2\right)\mathrm{d}y_j\right] = \frac{d}{3} = \frac{x}{3}$$

Travel time approximation



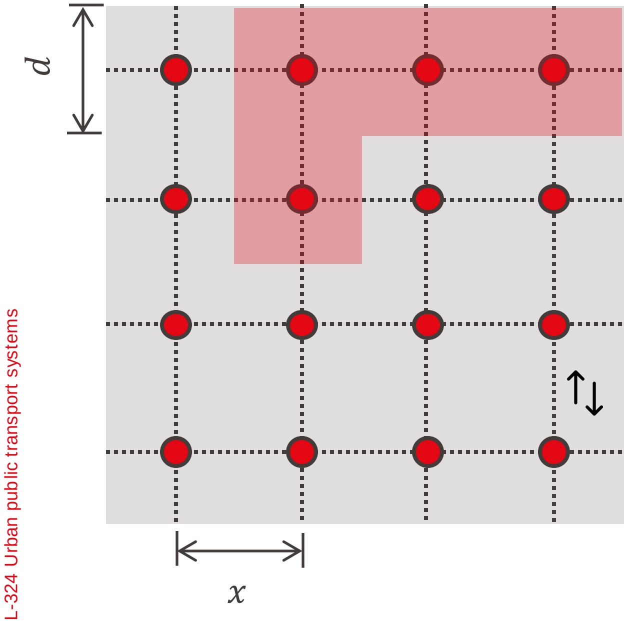
- Flexible route between two transfer stops
  - Travel either vertically or horizontally
  - Always move forward to the next transfer stop
  - Visit all demand points (pickup/dropoff)
- Vehicle travel distance
  - Central $L_c = x$ (km)
  - Lateral $L_a = \left(\frac{\lambda x^2}{f}\right)\left(\frac{x}{3}\right) = \frac{\lambda x^3}{3f}$
- Equivalent vehicle speed along route

$$\bar{v}_b = \frac{L_c}{L_c + L_a} v_b = \left(1 + \frac{\lambda x^2}{3f}\right)^{-1} v_b$$

- Vehicle travel time

$$\tau_b = \frac{\ell}{\bar{v}_b} = \left(1 + \frac{\lambda x^2}{3f}\right)\frac{\ell}{v_b}$$

Travel time approximation



- Stopping time with a single transfer
  - Regular stops

$$n_{\text{reg}} = \frac{\ell}{x}$$

  - Expected off-route stops

$$n_{\text{off}} = \lambda d \ell (1/f) = \frac{\lambda x \ell}{f}$$

  - Total stopping time

$$\delta = (n_{\text{reg}} + n_{\text{off}})\omega_0 + \omega_1$$

$$= \left(\frac{\ell}{x} + \frac{\lambda x \ell}{f}\right)\omega_0 + \omega_1$$

Design problem

- Determine frequency f and line spacing x to mini ***minimize*** the total system cost $TC(f,x)$ per hour per unit area

$$\min_{f,x} \; TC(f,x) = \frac{4cf}{x} + \beta\lambda \left[\frac{1}{f} + \left(1 + \frac{\lambda x^2}{3f}\right)\frac{\ell}{v_b} + \left(\frac{\ell}{x} + \frac{\lambda x \ell}{f}\right)\omega_0 + \omega_1\right]$$

- f: line frequency (/hr)
- x: line spacing (km)
- c: operation cost per line (CHF/km)
- β: value of time (CHF/hr)
- λ: demand rate (pax/km^2/hr)
- ℓ: average trip distance (km)
- v_b: vehicle speed (km/hr)
- ω_0, ω_1: stopping and transfer time (hr)

Design problem

- Determine frequency f and line spacing x to mini **minimize** the total system cost $TC(f,x)$ per hour per unit area

$$\min_{f,x} \; TC(f,x) = \frac{4cf}{x} + \beta\lambda \left[\left(\frac{1}{f} + \frac{\ell}{v_b} + \frac{\omega_0 \ell}{x} + \omega_1 \right) + \frac{\lambda x \ell}{f} \left(\frac{x}{3v_b} + \omega_0 \right) \right]$$

Fixed-route part   On-demand part

- Higher frequency
  - larger operation cost
  - shorter waiting time
  - shorter vehicle time in the on-demand part
- Shorter line spacing
  - larger operation cost
  - shorter vehicle time in the on-demand part
  - longer stopping time in the fixed-route part
  - shorter stopping time in the on-demand part

Design problem

- Determine frequency f and line spacing x to mini ***minimize*** the total system cost $TC(f,x)$ per hour per unit area

$$\min_{f,x}\ TC(f,x)=\frac{4cf}{x}+\beta\lambda\left[\left(\frac{1}{f}+\frac{\lambda x\ell}{f}\left(\frac{x}{3v_b}+\omega_0\right)\right)+\frac{\omega_0\ell}{x}+\frac{\ell}{v_b}+\omega_1\right]$$

  - First-order conditions

$$\frac{\partial TC(f,x)}{\partial f}=\frac{4c}{x}-\frac{\beta\lambda}{f^2}\left[1+\lambda x\ell\left(\frac{x}{3v_b}+\omega_0\right)\right]=0$$

$$\frac{\partial TC(f,x)}{\partial x}=-\frac{4cf}{x^2}+\beta\lambda\left(\frac{2\lambda x\ell}{3v_bf}+\frac{\lambda\omega_0\ell}{f}-\frac{\omega_0\ell}{x^2}\right)=0$$

- ***Q: How to solve the joint design problem?***

Design problem

- Determine frequency f and line spacing x to mini ***minimize*** the total system cost $TC(f,x)$ per hour per unit area

$$\min_{f,x} \; TC(f,x) = \frac{4cf}{x} + \beta\lambda\left[\left(\frac{1}{f} + \frac{\lambda x \ell}{f}\left(\frac{x}{3v_b} + \omega_0\right)\right) + \frac{\omega_0 \ell}{x} + \frac{\ell}{v_b} + \omega_1\right]$$

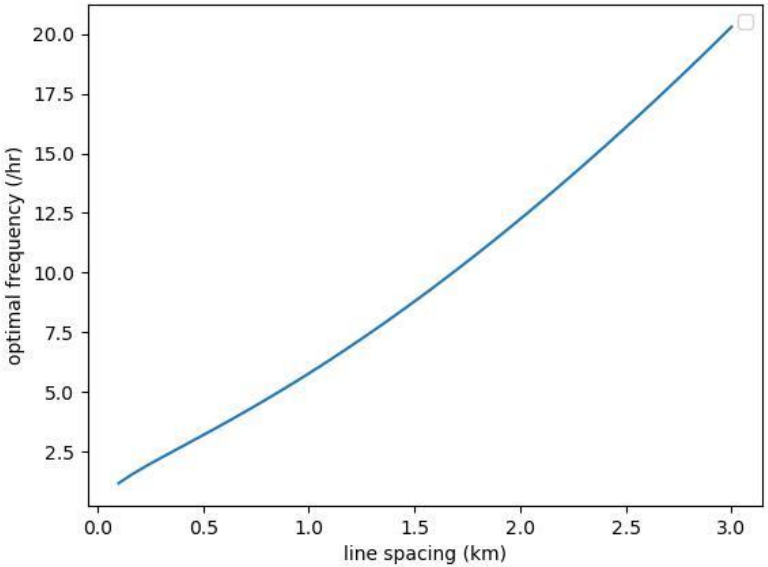
- First-order conditions

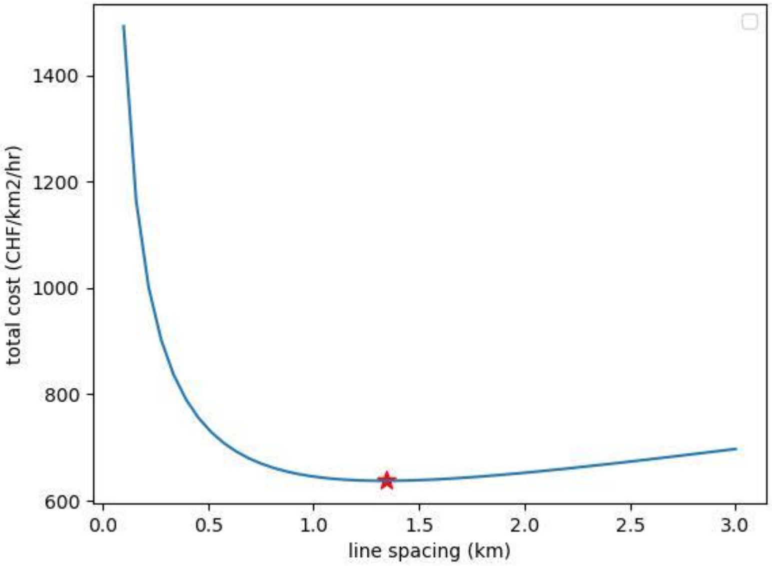
$$\frac{\partial TC(f,x)}{\partial f} = 0 \Rightarrow f^*(x) = \sqrt{\frac{\beta\lambda x}{4c}\left[1 + \lambda x \ell\left(\frac{x}{3v_b} + \omega_0\right)\right]}$$

- search over feasible x to find x^* that minimize $TC(f^*(x),x)$

Design problem

- Optimal frequency $f^*(x)$ and total cost $TC(f^*(x), x)$

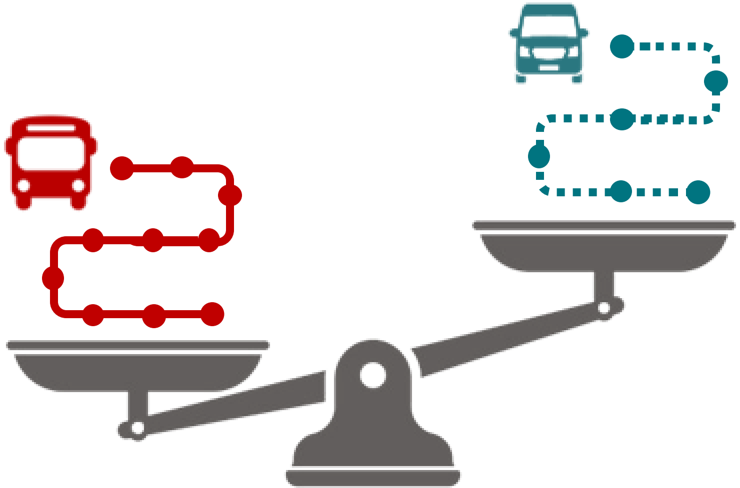




$c = 10$ [CHF/km], $v_b = 20$ [km/hr], $\lambda = 10$ [pax/hr/km^2], $\beta = 50$ [CHF/hr], $\ell = 5$ [km], $\omega_0 = 1$ [min], $\omega_1 = 2$ [min].

Case study

- Fixed-route vs flexible-route transit
  - Suppose a city wish to design its transit system as a grid network
    - how to choose between fixed-route and flexible-route transit?
    - what are the main factors of the decision?



Case study

- Fixed-route

$$\min_{f,x,s} \; TC(f,x,s) = \frac{4cf}{x} + \beta\lambda\left(\frac{x+s}{2v_a} + \frac{1}{f} + \frac{\ell}{v_b} + \frac{\omega_0 \ell}{s} + \omega_1\right)$$

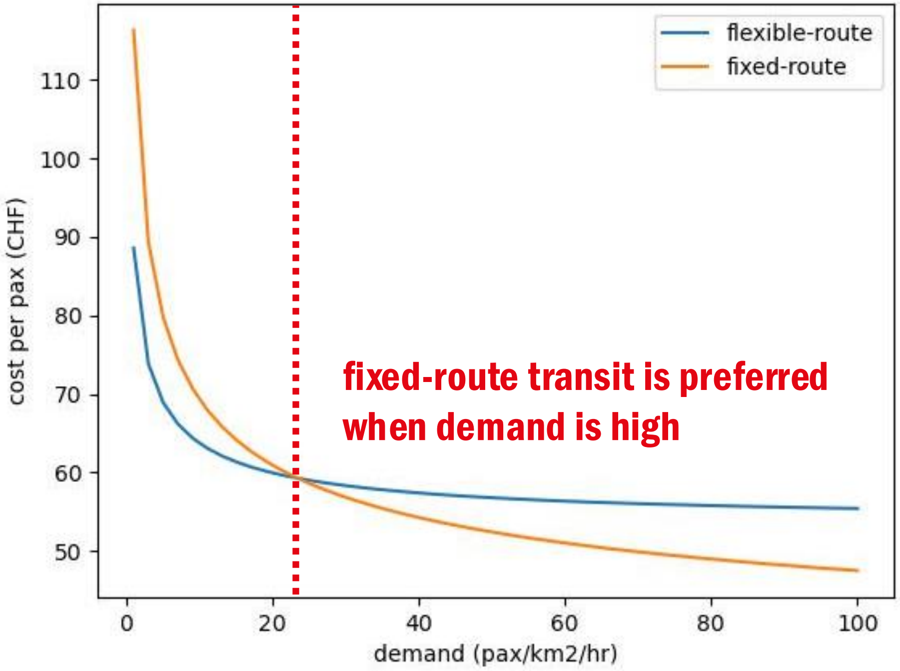
- Flexible-route

$$\min_{f,x} \; TC(f,x) = \frac{4cf}{x} + \beta\lambda\left[\left(\frac{1}{f} + \frac{\ell}{v_b} + \frac{\omega_0 \ell}{x} + \omega_1\right) + \frac{\lambda x \ell}{f}\left(\frac{x}{3v_b} + \omega_0\right)\right]$$

  - f: line frequency (/hr)
  - x, s: line and stop spacing (km)
  - c: operation cost per line (CHF/km)
  - β: value of time (CHF/hr)
  - λ: demand rate (pax/km^2/hr)
  - ℓ: average trip distance (km)
  - v_a, v_b: walking and vehicle speed (km/hr)
  - ω_0, ω_1: stopping and transfer time (hr)

Case study

- Cost per passenger TC^*/λ (CHF)



$c = 10$ [CHF/km], $v_b = 20$ [km/hr], $v_a = 2$ [km/hr], $\beta = 50$ [CHF/hr],
$\ell = 5$ [km], $\omega_0 = 1$ [min], $\omega_1 = 2$ [min].

Questions?