Spring 2025

05 Transit Network II: Scheduling & Pricing

CIVIL-324 Urban public transport systems

École polytechnique fédérale de Lausanne

Overview

- Transit network design problems discussed in this course
  - Stop location and line planning
    - where to put transit stations and how to connect them into transit lines?
  - **Scheduling and pricing**
    - how to design the timetable and coordinate multiple transit lines?
    - how to price transit trips under different operational objectives?
  - Operations
    - how to design the vehicle schedules given the transit time table?
    - how to manage the potential delay during the operations?

Overview

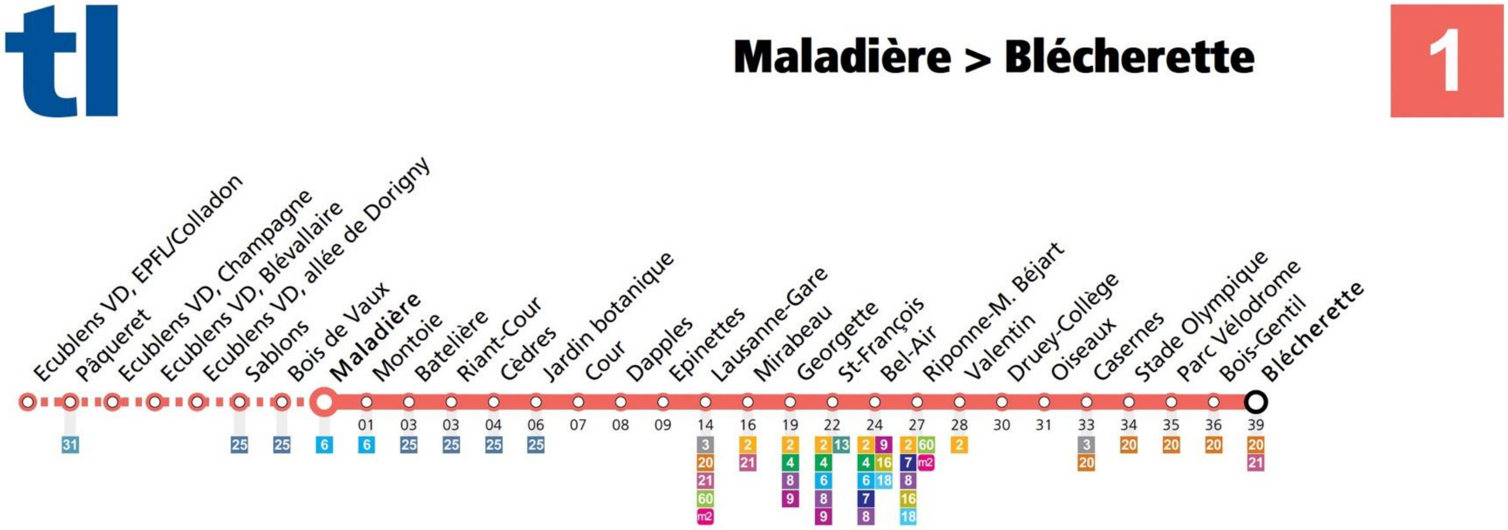
- Scheduling and pricing
  - How to design the timetable and coordinate multiple transit lines?
    - design the timetable of a single transit line
    - coordinate timetables of multiple lines
  - How to price transit to increase transit ridership?
    - in competition with other modes
    - jointly optimized with other design variable

Scheduling: Single line

- Scheduling and pricing
  - How to design the timetable and coordinate multiple transit lines?
    - **design the timetable of a single transit line**
    - coordinate timetables of multiple lines
  - How to price transit to increase transit ridership?
    - in competition with other modes
    - jointly optimized with other design variable

Scheduling: Single line

- Bus tl 1 between Lausanne Gare and EPFL



Scheduling: Single line

- Bus tl 1 between Lausanne Gare and EPFL

Lundi-Vendredi

Ecublens VD, EPFL/Colladon									06:30	▲7'	09:28				15:30	▲7'	18:49	19:00			
Ecublens VD, allée de Dorigny									06:34	▲7'	09:32				15:34	▲7'	18:53	19:04			
Sablons									06:37	▲7'	09:35				15:37	▲7'	18:56	19:07			
Maladière	05:10	05:25	05:40	05:53	06:05	06:17	06:29	06:35	06:41	▲7'	09:39	09:45	▲8'	15:33	15:41	▲7'	19:00	19:11	19:18	19:26	19:36
Montoie	05:12	05:27	05:42	05:55	06:07	06:19	06:31	06:37	06:42	▲7'	09:40	09:47	▲8'	15:35	15:42	▲7'	19:01	19:12	19:20	19:28	19:38
Jardin botanique	05:16	05:31	05:46	05:59	06:12	06:24	06:36	06:42	06:47	▲7'	09:45	09:52	▲8'	15:40	15:47	▲7'	19:06	19:17	19:25	19:33	19:43
Epinettes	05:18	05:33	05:48	06:01	06:14	06:26	06:38	06:44	06:49	▲7'	09:48	09:55	▲8'	15:43	15:50	▲7'	19:09	19:20	19:28	19:36	19:46
Lausanne-Gare	05:21	05:36	05:51	06:04	06:17	06:29	06:41	06:47	06:53	▲7'	09:52	09:59	▲8'	15:47	15:54	▲7'	19:13	19:24	19:32	19:39	19:49
Georgette	05:24	05:39	05:54	06:07	06:20	06:32	06:44	06:50	06:56	▲7'	09:55	10:02	▲8'	15:51	15:58	▲7'	19:17	19:27	19:35	19:42	19:52
St-François	05:26	05:41	05:56	06:09	06:22	06:34	06:46	06:53	06:59	▲7'	09:58	10:05	▲8'	15:54	16:01	▲7'	19:20	19:30	19:38	19:45	19:55
Bel-Air	05:28	05:43	05:58	06:11	06:24	06:36	06:48	06:55	07:01	▲7'	10:00	10:07	▲8'	15:56	16:03	▲7'	19:22	19:32	19:40	19:47	19:57
Riponne-M. Béjart	05:30	05:45	06:00	06:13	06:26	06:38	06:50	06:57	07:03	▲7'	10:02	10:09	▲8'	15:58	16:05	▲7'	19:24	19:34	19:42	19:49	19:59
Casernes	05:34	05:49	06:04	06:17	06:30	06:43	06:55	07:02	07:08	▲7'	10:08	10:15	▲8'	16:04	16:11	▲7'	19:29	19:39	19:47	19:54	20:04
Blécherette	05:38	05:53	06:09	06:22	06:35	06:48	07:00	07:07	07:13	▲7'	10:13	10:20	▲8'	16:10	16:17	▲7'	19:34	19:44	19:52	19:59	20:09

Ecublens VD, EPFL/Colladon																					
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Maladière	19:47	19:58	20:08	20:18	20:28	20:38	20:48	21:00	21:15	21:30	21:45	22:00	22:15	22:30	22:45	23:00	23:15	23:30	23:45	00:00	00:15
Montoie	19:49	20:00	20:10	20:20	20:30	20:40	20:50	21:02	21:17	21:32	21:47	22:02	22:17	22:32	22:47	23:02	23:17	23:32	23:47	00:02	00:17
Jardin botanique	19:54	20:04	20:14	20:24	20:34	20:44	20:54	21:06	21:21	21:36	21:51	22:06	22:21	22:36	22:51	23:06	23:21	23:36	23:51	00:06	00:21
Epinettes	19:57	20:07	20:17	20:27	20:37	20:47	20:57	21:08	21:23	21:38	21:53	22:08	22:23	22:38	22:53	23:08	23:23	23:38	23:53	00:08	00:23
Lausanne-Gare	20:00	20:10	20:20	20:30	20:40	20:50	21:00	21:11	21:26	21:41	21:56	22:11	22:26	22:41	22:56	23:11	23:26	23:41	23:56	00:11	00:26
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Bel-Air	20:07	20:17	20:27	20:37	20:47	20:57	21:07	21:18	21:32	21:47	22:02	22:17	22:32	22:47	23:02	23:17	23:32	23:47	00:02	00:17	00:32
Riponne-M. Béjart	20:09	20:19	20:29	20:39	20:49	20:59	21:09	21:20	21:34	21:49	22:04	22:19	22:34	22:49	23:04	23:19	23:34	23:49	00:04	00:19	00:34
Casernes	20:14	20:24	20:34	20:44	20:54	21:04	21:14	21:25	21:39	21:54	22:09	22:24	22:39	22:54	23:09	23:23	23:38	23:53	00:08	00:23	00:38
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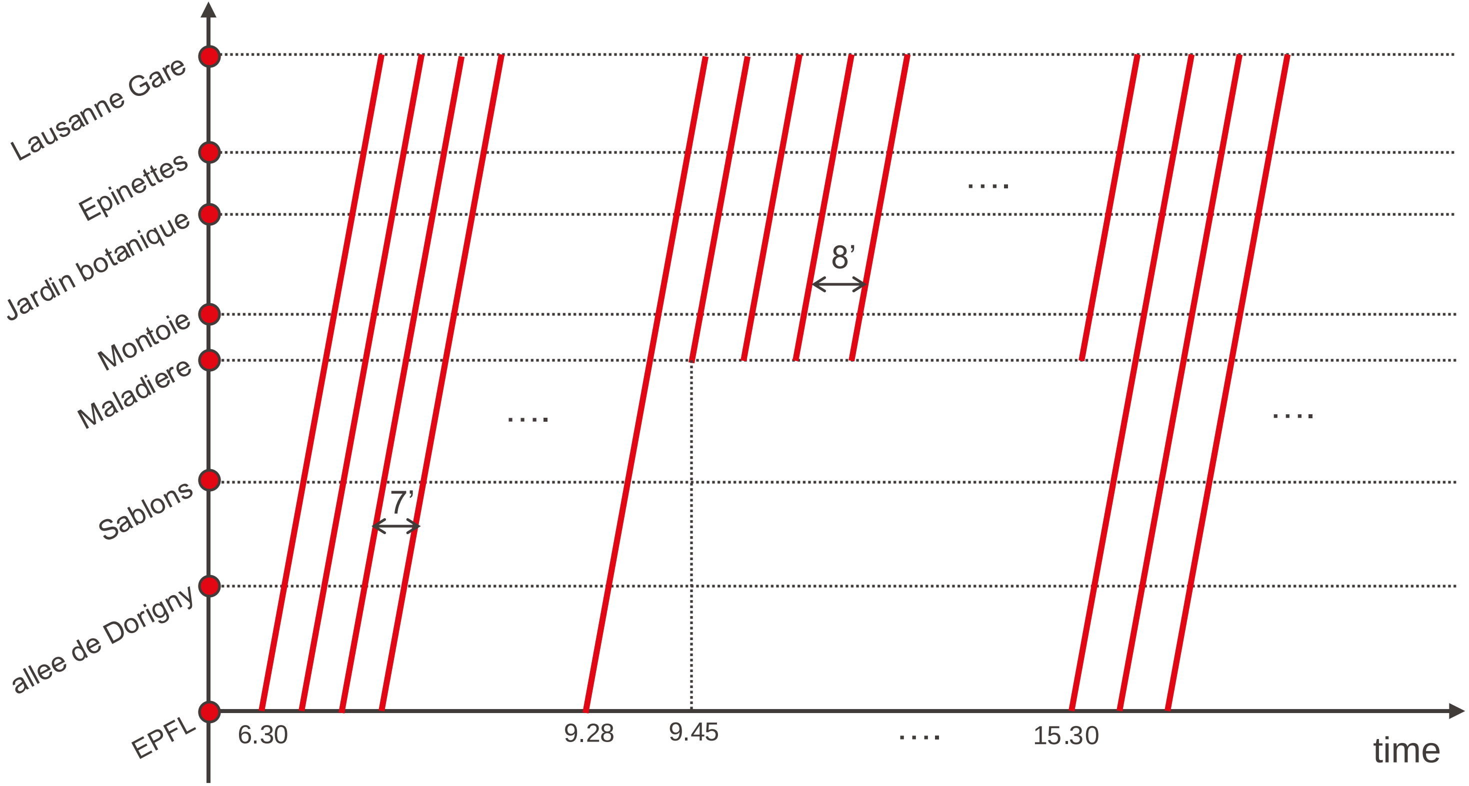
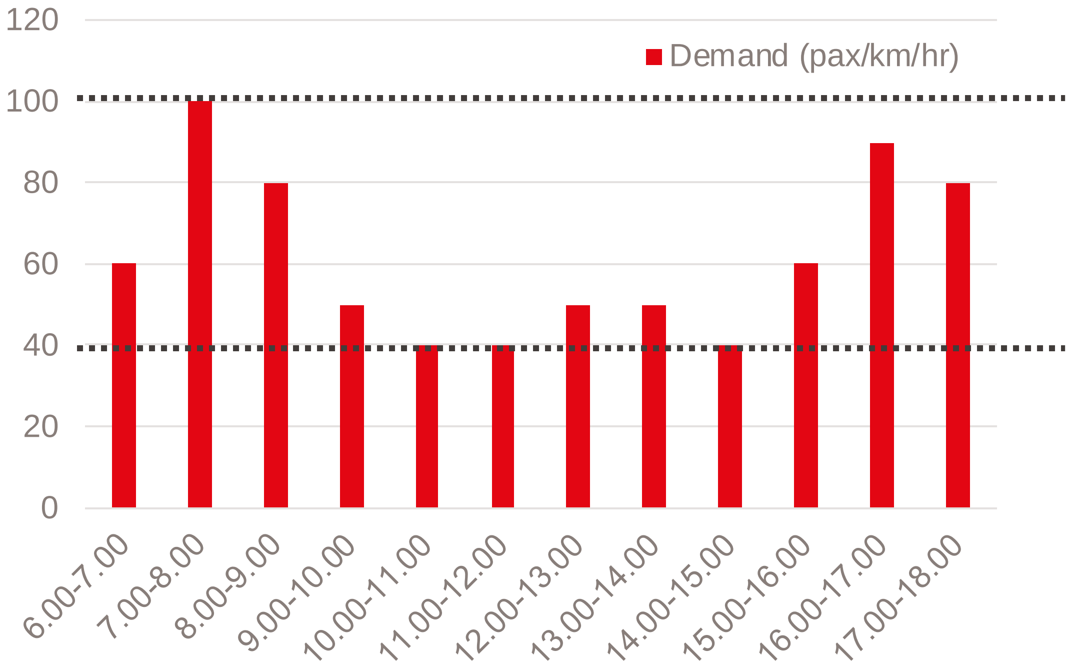
Scheduling: Single line

- Bus tl 1 between Lausanne Gare and EPFL



Scheduling: Single line

- Bus frequency under time-varying demand



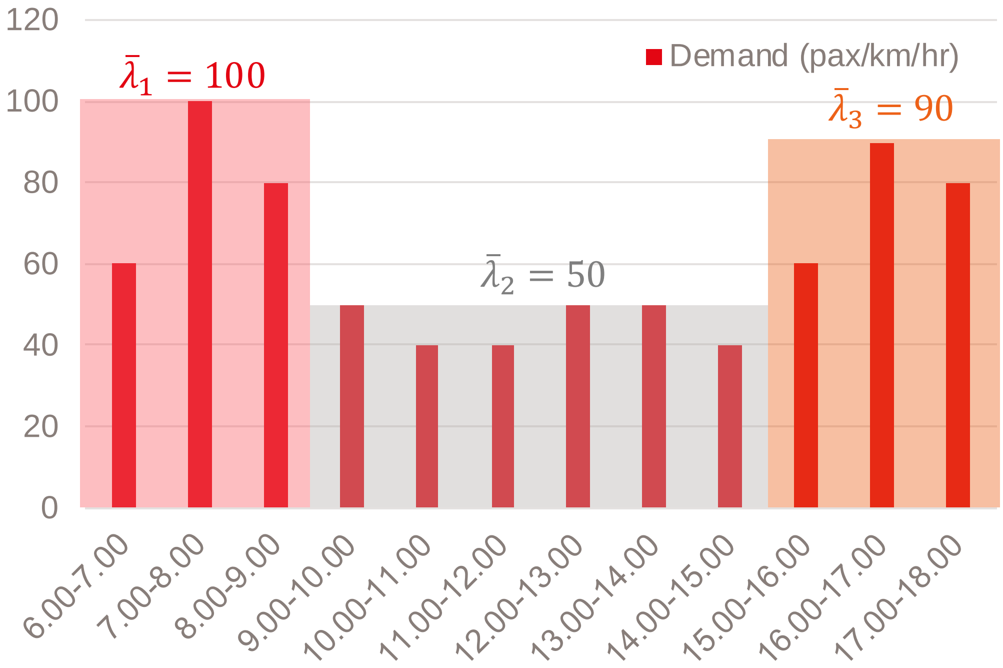
$$\lambda_1 = 100 \Rightarrow f^* = \sqrt{\frac{\beta\lambda_1}{2c}} = 5 \text{ /hr}$$

$$\lambda_2 = 40 \Rightarrow f^* = \sqrt{\frac{\beta\lambda_2}{2c}} \approx 3 \text{ /hr}$$

$c = 100$ CHF, $\beta = 50$ CHF/hr

Scheduling: Single line

- Segmentation of period and envelope demand

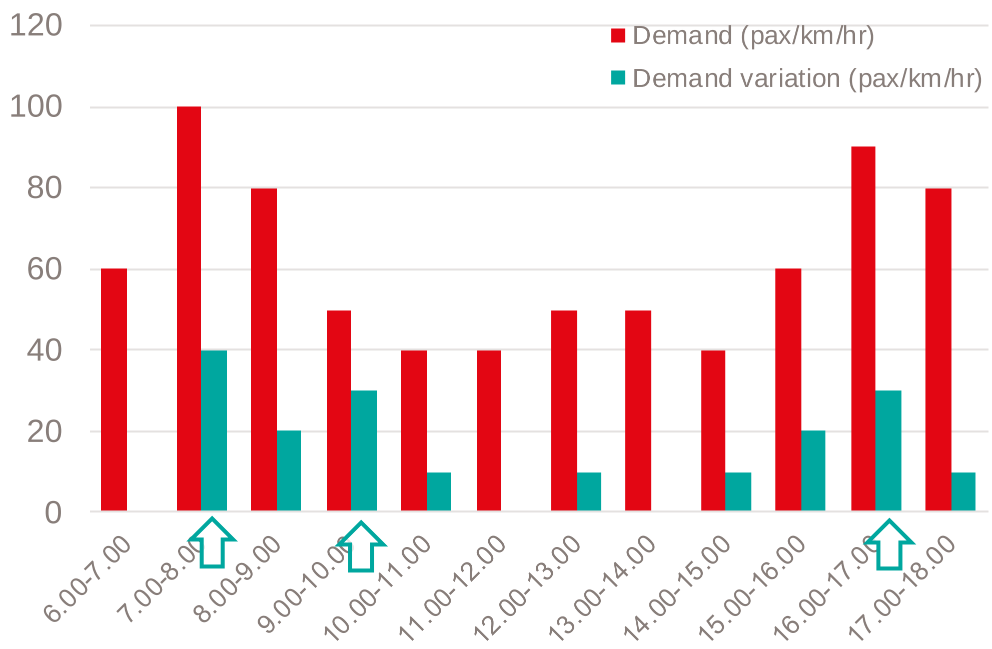


- Morning peak: 6.00-9.00
- Mid-of-day: 9.00-15.00
- Evening peak:15.00-18.00

- ***Q: How to separate demand periods?***

Scheduling: Single line

- Segmentation of period and envelope demand
  - Rule-based method: split time at largest demand changes

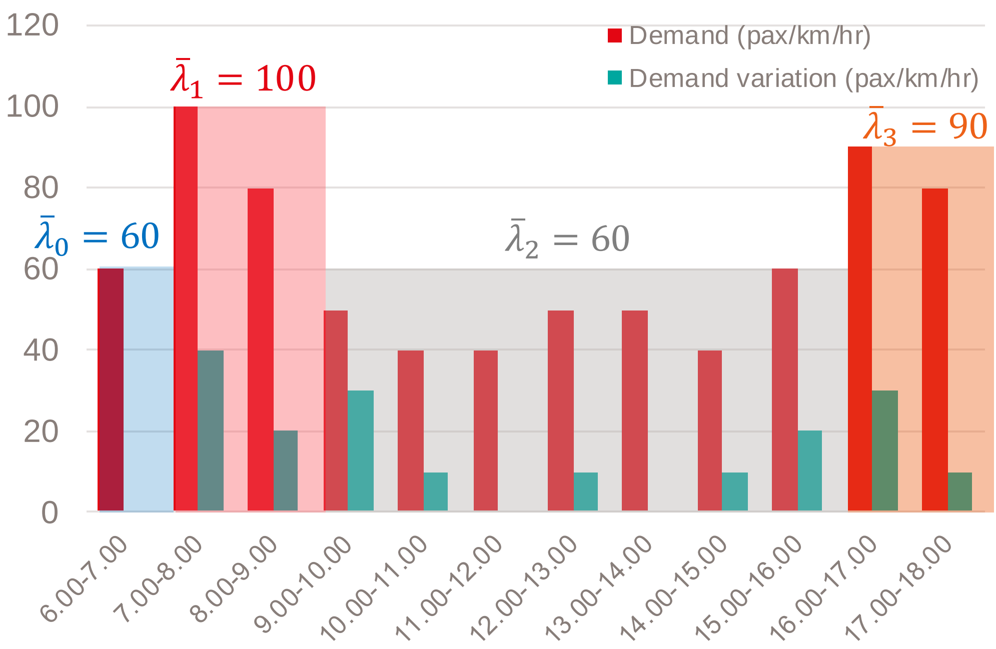


- Demand variation

$$\delta\lambda_i = |\lambda_i - \lambda_{i-1}|$$

Scheduling: Single line

- Segmentation of period and envelope demand
  - Rule-based method: split time at largest demand changes



- Demand variation

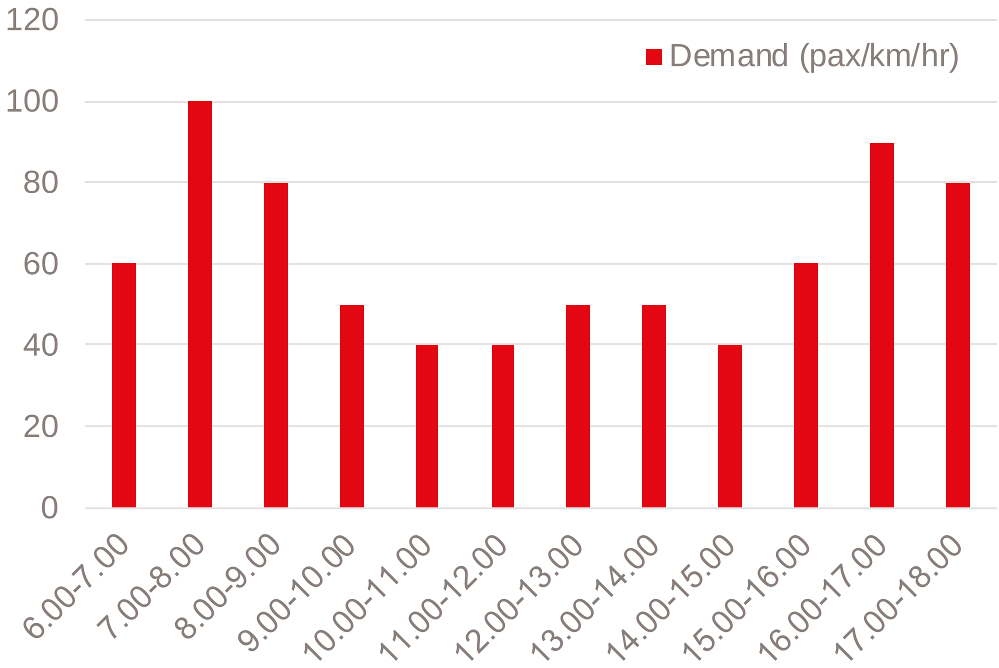
$$\delta\lambda_i = |\lambda_i - \lambda_{i-1}|$$

- Early-morning: 6.00-7.00
- Morning peak: 7.00-9.00
- Mid-of-day: 9.00-16.00
- Evening peak: 16.00-18.00

- ***Q: Is there a more systematic method to separate time periods?***

Scheduling: Single line

- Segmentation of period and envelope demand
  - Optimization-based method

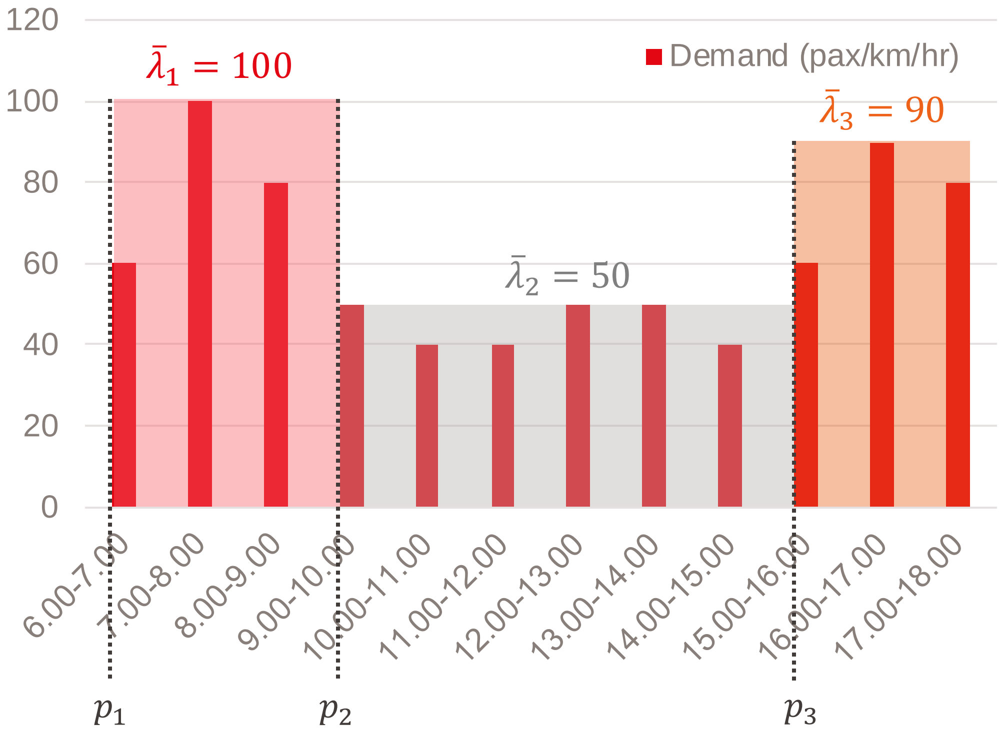


- Inputs
  - # of periods K
  - Demand $\{\lambda_i\}_{i=1}^{N}$ (pax/hr)
  - Time interval δ_t (hr)
  - Operating cost c (CHF/veh)
  - Value of time β (CHF/hr)

- Outputs
  - Period start time $\{p_k\}_{k=1}^{K}$, where $1 = p_1 < p_2 < \cdots < p_K < N$
  - Envelope demand $\{\bar{\lambda}_k\}_{k=1}^{K}$

Scheduling: Single line

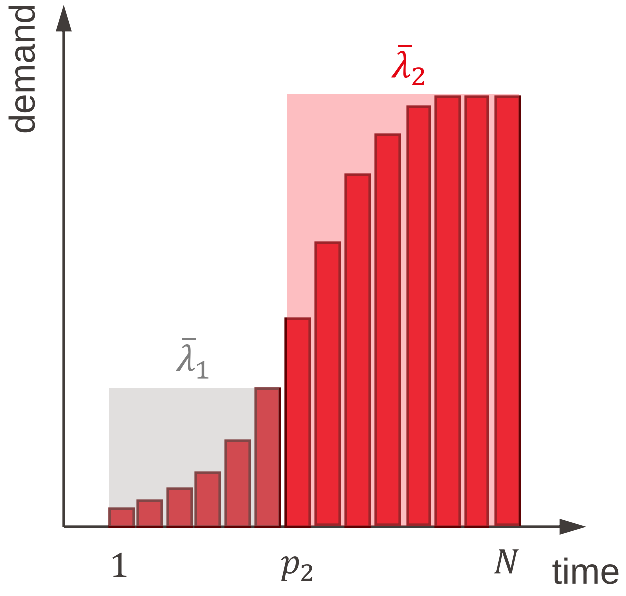
- Segmentation of period and envelope demand
  - Optimization-based method



- A feasible solution for $K = 3$
  - $p_1 = 1, p_2 = 4, p_3 = 10$

Scheduling: Single line

- Segmentation of period and envelope demand
  - Optimization-based method



- Formulation for $K = 2$

$$\min_{p_2} TC(p_2) = c[f_1(p_2 - 1)\delta_t + f_2(N - p_2)\delta_t]$$

$$+\beta\left(\frac{\delta_t}{2f_1}\sum_{i=1}^{p_2-1}\lambda_i + \frac{\delta_t}{2f_2}\sum_{i=p_2}^{N}\lambda_i\right)$$

$$s.t. \quad 1 < p_2 < N$$

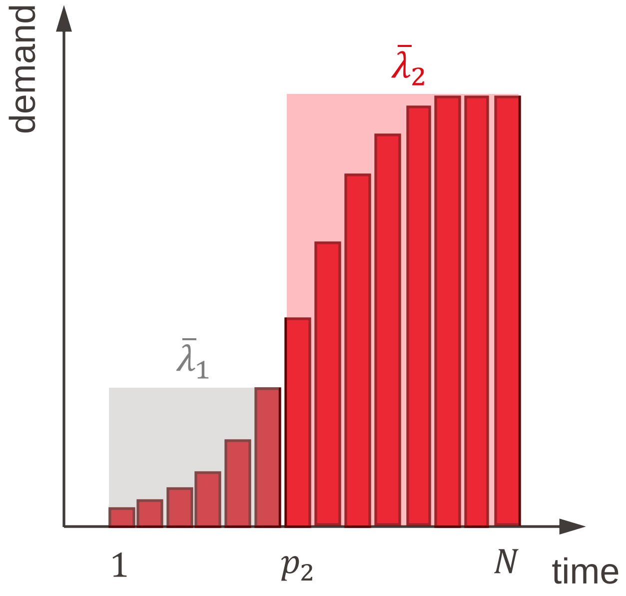
$$f_k = \sqrt{\frac{\beta\bar{\lambda}_k}{2c}}, \ k = 1,2,$$

$$\bar{\lambda}_1 = \max_{1\le i<p_2,} \lambda_i ,$$

$$\bar{\lambda}_2 = \max_{p_2\le i\le N,} \lambda_i ,$$

Scheduling: Single line

- Segmentation of period and envelope demand
  - Optimization-based method



- Formulation for $K = 2$

operating cost

$$\min_{p_2} TC(p_2) = c[f_1(p_2 - 1)\delta_t + f_2(N - p_2)\delta_t]$$

$$+\beta\left(\frac{\delta_t}{2f_1}\sum_{i=1}^{p_2-1}\lambda_i + \frac{\delta_t}{2f_2}\sum_{i=p_2}^{N}\lambda_i\right)$$

waiting cost

$$s.t. \quad 1 < p_2 < N$$

$$f_k = \sqrt{\frac{\beta\bar{\lambda}_k}{2c}},\ k = 1,2,$$

optimal frequency

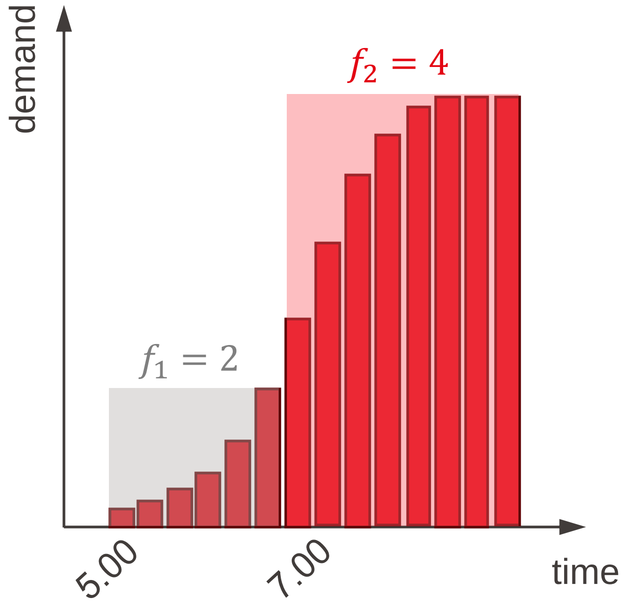
$$\bar{\lambda}_1 = \max_{1 \le i < p_2,} \lambda_i\,,$$

$$\bar{\lambda}_2 = \max_{p_2 \le i \le N,} \lambda_i\,,$$

envelope demand

Scheduling: Single line

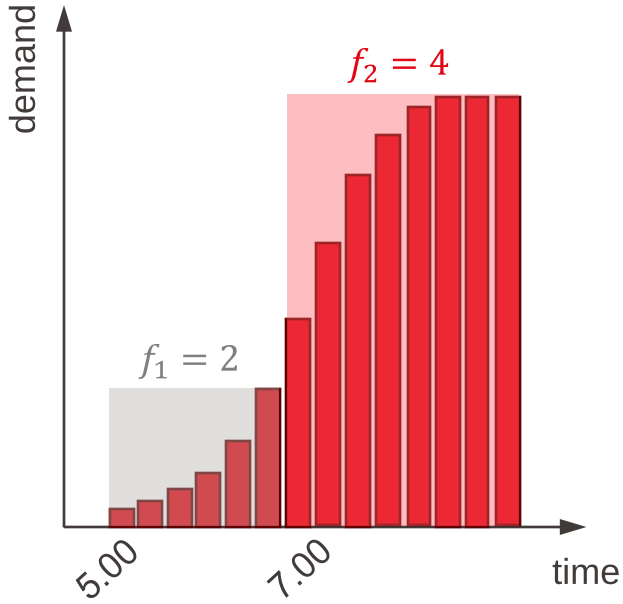
- Segmentation of period and envelope demand
  - ***Q: Do you find any issue with this schedule?***



Period	Time
1	…
	6.00
	6.30
	7.00
2	7.15
	7.30
	…

Scheduling: Single line

- Transitional frequency
  - Real demand does not jump but gradually changes
  - Add transitional frequency to avoid overcrowding or underutilization



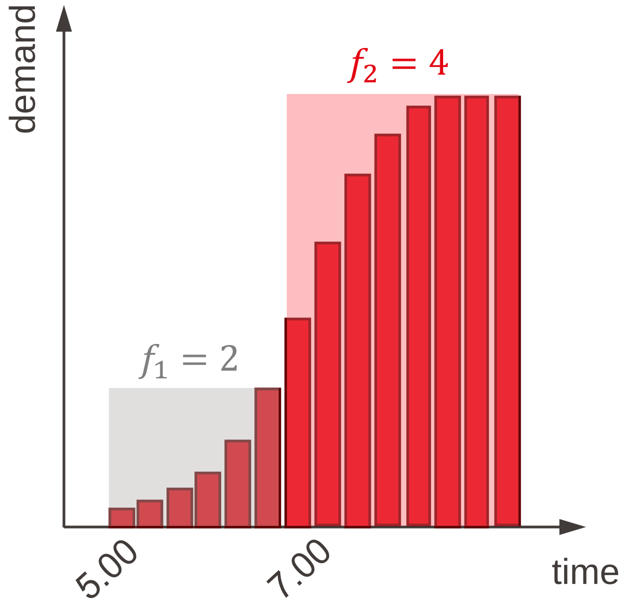
Period	Time
1	…
	6.00
	6.30
	~~7.00~~ *6.50*
2	~~7.15~~ *7.10*
	~~7.30~~ *7.25*
	…

30 min
20 min
20 min
15 min

- A possible schedule

Scheduling: Single line

- Transitional frequency
  - Rule-based method: use average headway in the transition



Period	Time
1	…
	6.00
	6.30
	~~7.00~~ *6.53*
2	~~7.15~~ *7.16*
	~~7.30~~ *7.31*
	…

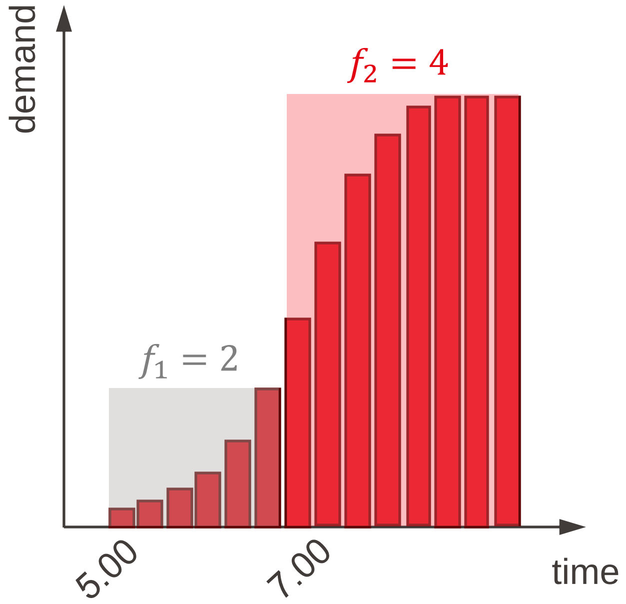
30 min (6.00 → 6.30)
23 min (6.30 → 6.53)
23 min (6.53 → 7.16)
15 min (7.16 → 7.31)

- Transitional headway

$$h = \frac{1/f_1 + 1/f_2}{2} \approx 23 \text{ min}$$

Scheduling: Single line

- Transitional frequency
  - Optimization-based method

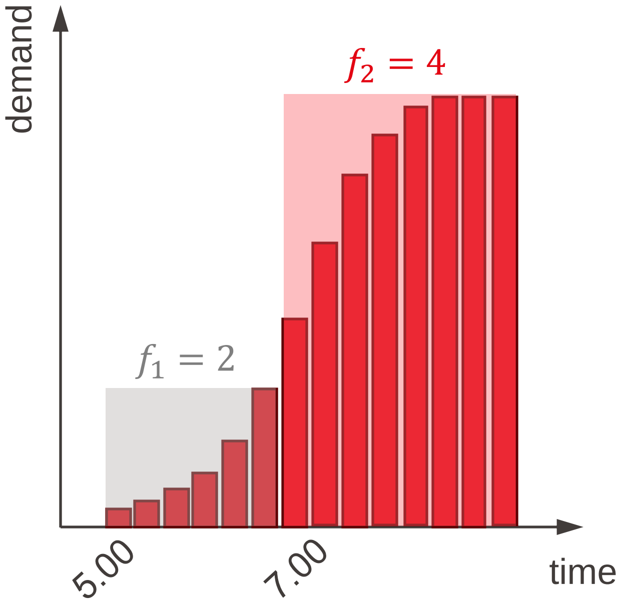


- Inputs
  - Transition time t_s
  - Frequencies before and after transition f_1, f_2 (/hr)
  - Envelope demand before and after transition $\bar{\lambda}_1, \bar{\lambda}_2$ (pax/hr)
  - Demand during transition $\lambda_i, i \in [t_s - 1/f_1, t_s + 1/f_2]$ (pax/hr)
  - Time interval δ_t (hr)
  - Operating cost c (CHF/veh)
  - Value of time β (CHF/hr)

- Outputs
  - Transitional frequency f_s

Scheduling: Single line

- Transitional frequency
  - Optimization-based method



$$\min_{f_s} TC(f_s) = cf_s\left(\frac{1}{f_1}+\frac{1}{f_2}\right)\delta_t + \frac{\beta\delta_t}{2f_s}\sum_{i=t_s-1/f_1}^{t_s+1/f_2}\lambda_i$$

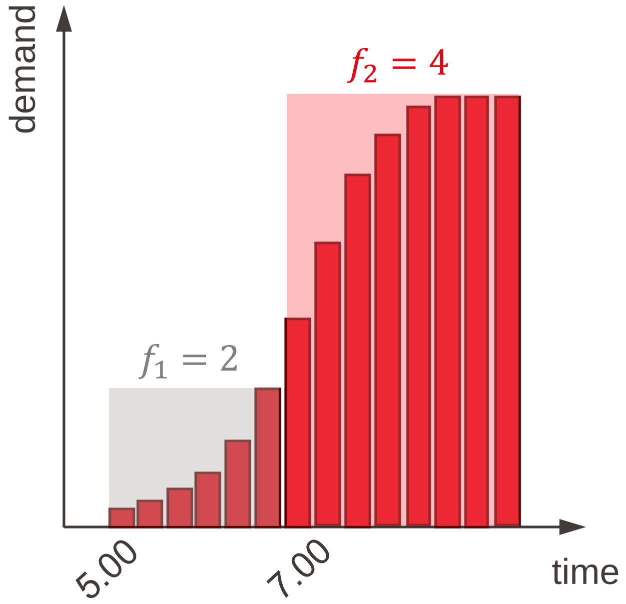
$$= cf_s\left(\frac{1}{f_1}+\frac{1}{f_2}\right)\delta_t + \frac{\beta\delta_t}{2f_s}\bar{\lambda}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$$

- $\bar{\lambda}$: average demand during transition

- ***Q: What is the optimal solution?***

Scheduling: Single line

- Transitional frequency
  - Optimization-based method



$$\min_{f_s} TC(f_s) = c f_s \left(\frac{1}{f_1} + \frac{1}{f_2}\right)\delta_t + \frac{\beta\delta_t}{2f_s} \sum_{i=t_s-1/f_1}^{t_s+1/f_2} \lambda_i$$

$$= c f_s \left(\frac{1}{f_1} + \frac{1}{f_2}\right)\delta_t + \frac{\beta\delta_t}{2f_s} \bar{\lambda} \left(\frac{1}{f_1} + \frac{1}{f_2}\right)$$

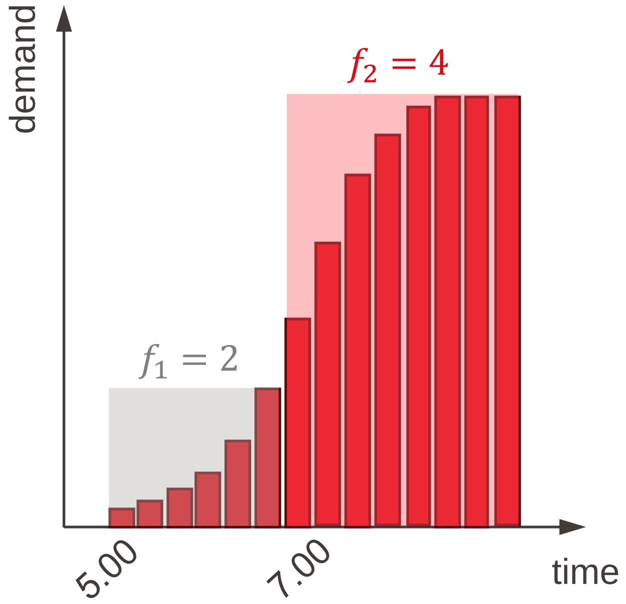
$$\frac{\partial TC(f_s)}{\partial f_s} = c \left(\frac{1}{f_1} + \frac{1}{f_2}\right)\delta_t - \frac{\beta\delta_t}{2f_s^2} \bar{\lambda} \left(\frac{1}{f_1} + \frac{1}{f_2}\right) = 0$$

$$\Rightarrow f_s^* = \sqrt{\frac{\beta\bar{\lambda}}{2c}}$$

- ***Q: How does f_s^* relate to f_1, f_2?***

Scheduling: Single line

- Transitional frequency
  - Optimization-based method



- Total demand over the transition

$$\frac{\bar{\lambda}_1}{f_1} + \frac{\bar{\lambda}_2}{f_2} = \bar{\lambda}\left(\frac{1}{f_1} + \frac{1}{f_2}\right)$$

- Optimal frequencies before/after transition

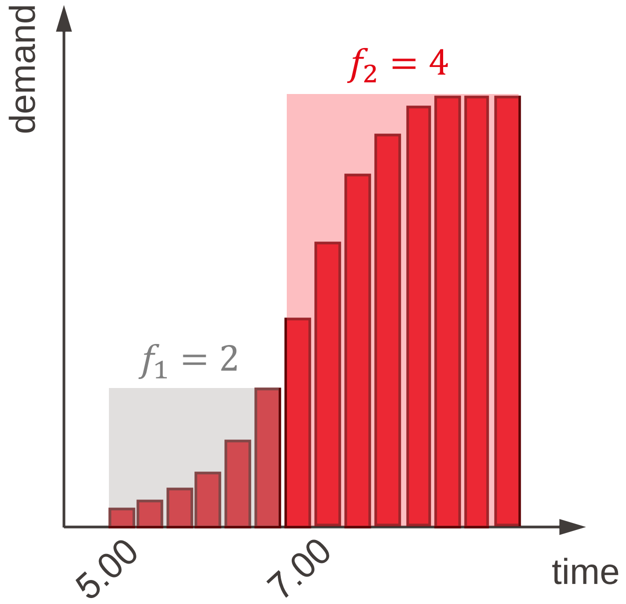
$$f_k = \sqrt{\frac{\beta \bar{\lambda}_k}{2c}} \Rightarrow \bar{\lambda}_k = \frac{2c}{\beta} f_k^2 \quad k = 1,2$$

- Optimal transitional frequency

$$f_s^* = \sqrt{\frac{\beta \bar{\lambda}}{2c}} = \sqrt{\frac{\beta}{2c} \frac{\frac{\bar{\lambda}_1}{f_1} + \frac{\bar{\lambda}_2}{f_2}}{\frac{1}{f_1} + \frac{1}{f_2}}} = \sqrt{\frac{\beta}{2c} \frac{\bar{\lambda}_1 f_2 + \bar{\lambda}_2 f_1}{f_1 + f_2}} = \sqrt{f_1 f_2}$$

Scheduling: Single line

- Transitional frequency
  - Optimization-based method



Period	Time
1	…
	6.00
	6.30
	~~7.00~~ *6.51*
2	~~7.15~~ *7.12*
	~~7.30~~ *7.27*
	…

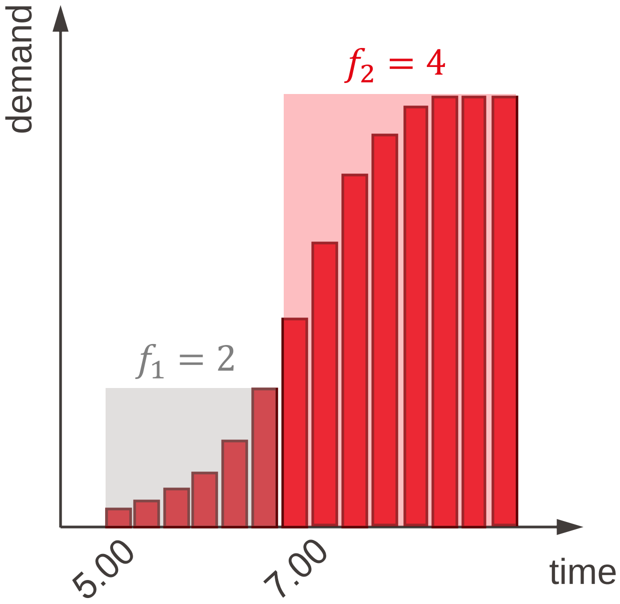
30 min, *21 min*, *21 min*, *15 min*

- Transitional headway

$$h = \frac{1}{\sqrt{f_1 f_2}} \approx 21 \text{ min}$$

Scheduling: Single line

- Transitional frequency



- Rule-based method

$$\frac{1}{f_s} = \frac{1/f_1 + 1/f_2}{2} \Rightarrow f_s = \frac{2f_1 f_2}{f_1 + f_2}$$

Cauchy inequality

$a + b \geq 2\sqrt{ab}$

$$\leq \frac{2f_1 f_2}{2\sqrt{f_1 f_2}} = \sqrt{f_1 f_2}$$

- Optimization-based method

$$f_s = \sqrt{f_1 f_2}$$

  - always higher than rule-based solution

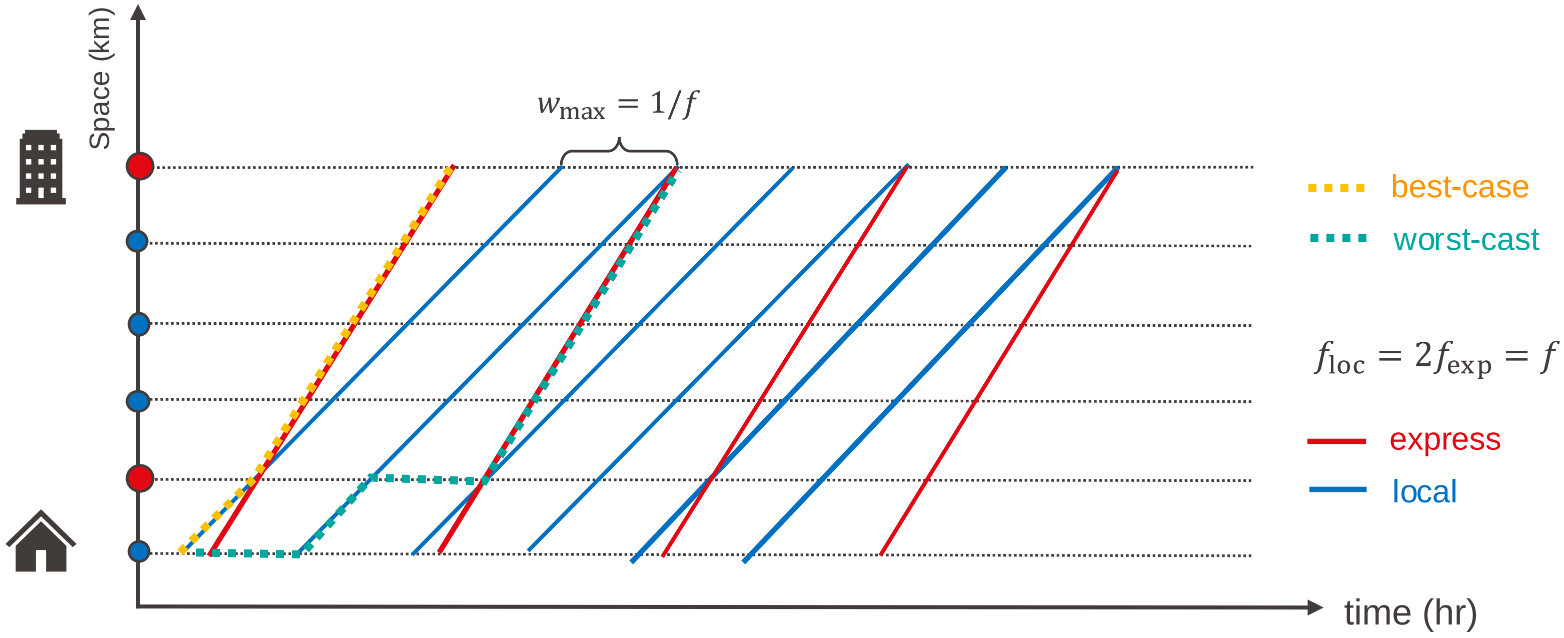
Questions?

Scheduling: Multiple lines

- Scheduling and pricing
  - How to design the timetable and coordinate multiple transit lines?
    - design the timetable of a single transit line
    - **coordinate timetables of multiple lines**
  - How to price transit to increase transit ridership?
    - in competition with other modes
    - jointly optimized with other design variable

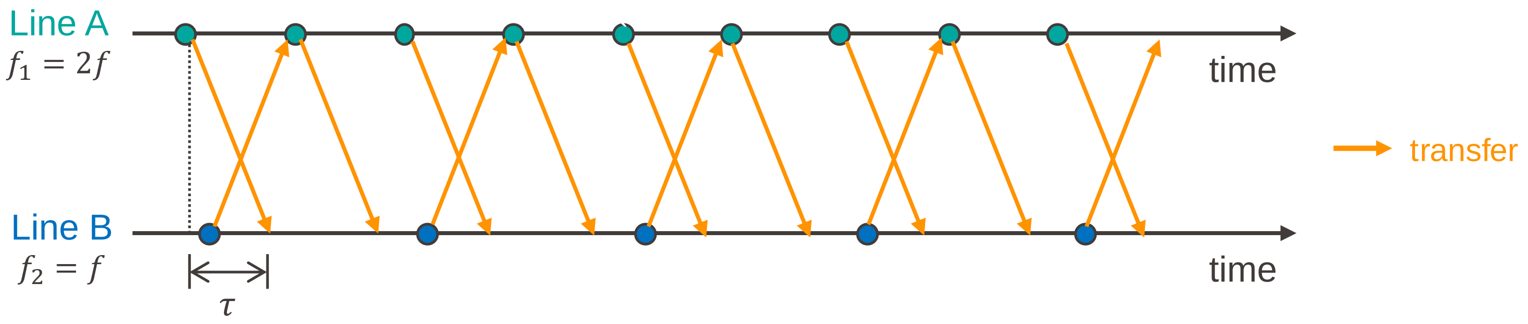
Scheduling: Multiple lines

- Hierarchical corridor with synchronized schedule
  - ***Q: How to solve schedule synchronization in general?***



Scheduling: Multiple lines

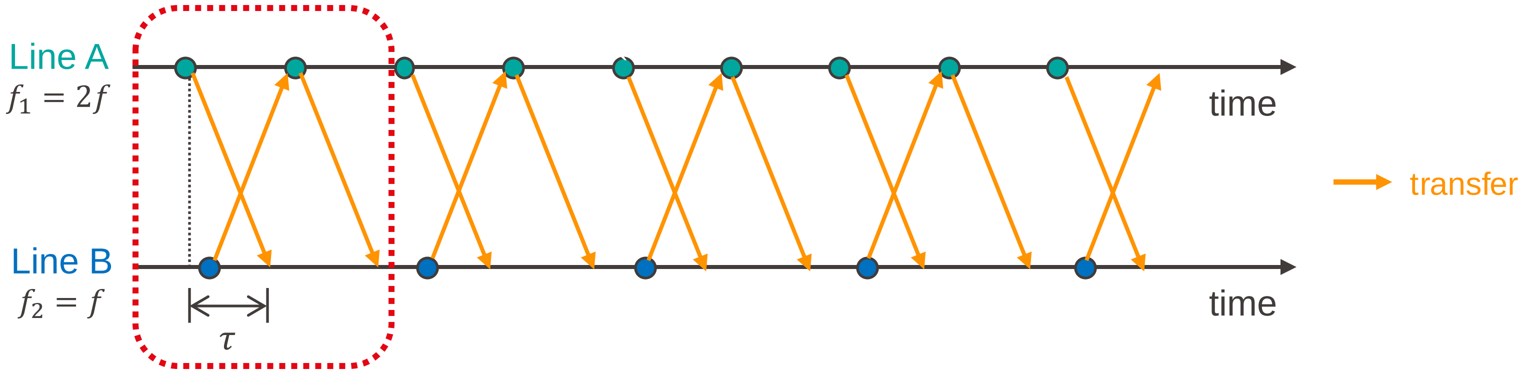
- Coordinate two uni-directional transit lines
  - With proportional frequency $f_1 = \alpha f_2$
  - Non-zero transfer time τ



- ***Q: Can we focus on a short period instead of analyzing full schedule?***

Scheduling: Multiple lines

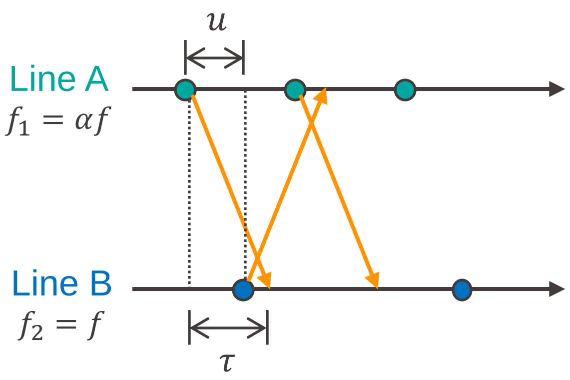
- Coordinate two uni-directional transit lines
  - With proportional frequency $f_1 = \alpha f_2$
  - Non-zero transfer time τ



- ***Q: Can we focus on a short period instead of analyzing full schedule?***
  - Yes, we only need to analyze a cycle of $\frac{1}{f_1} = \frac{\alpha}{f_2}$ with $\alpha + 1$ possible transfers
    - e.g., two transfers from A to B, and one transfer from B to A

Scheduling: Multiple lines

- Coordinate two uni-directional transit lines



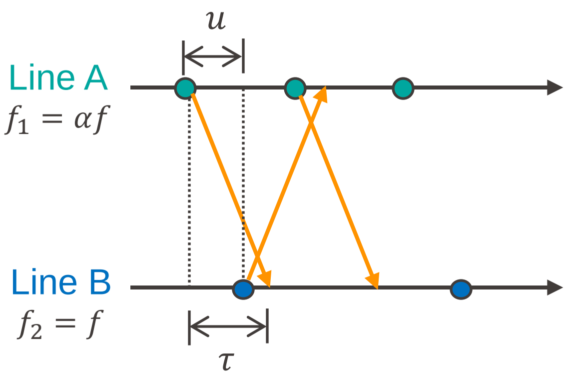
- Assume
  - Travelers do not know the schedules
    - arrive at each stop at same rate
  - Stop time is sufficiently small $\omega \to 0$

- Inputs
  - Transfer demand $\lambda_{12}, \lambda_{21}$
  - Base frequency f and ratio α
  - Transfer time $\tau < 1/\alpha f$

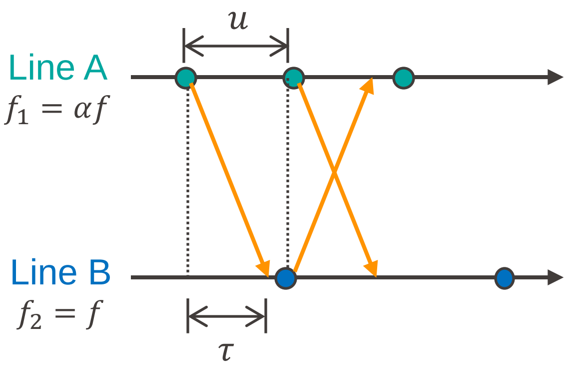
- Outputs
  - Offset of Line B $u \in [0, 1/f)$

Scheduling: Multiple lines

- Coordinate two uni-directional transit lines



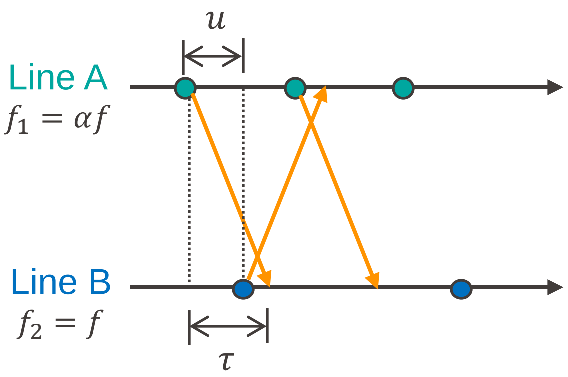
- Transfer from A to B with $\alpha = 2$
  - If $u \leq \tau$,
    - Case 1: wait for $\frac{1}{f} - (\tau - u)$ till next arrival of B
    - Case 2: wait for $\frac{1}{f} - (\tau - u) - \frac{1}{2f}$ till next arrival of B



  - Otherwise,
    - Case 1: wait for $u - \tau$ till next arrival of B
    - Case 2: wait for $(u - \tau) + \frac{1}{f} - \frac{1}{2f}$ till next arrival of B

Scheduling: Multiple lines

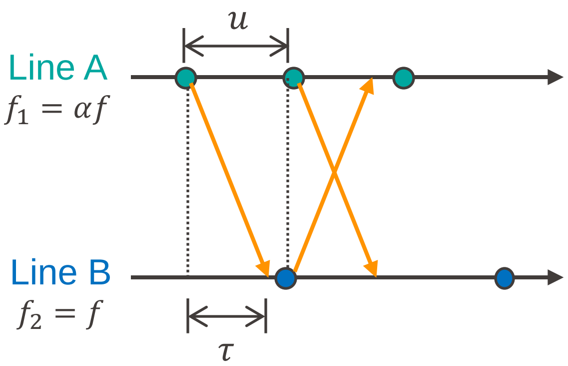
- Coordinate two uni-directional transit lines



- Transfer from A to B with $\alpha = 2$
  - If $u \leq \tau$, on average wait for

$$\frac{1}{f} - (\tau - u) - \frac{1}{4f} = u - \tau + \frac{3}{4f}$$

till next arrival of B



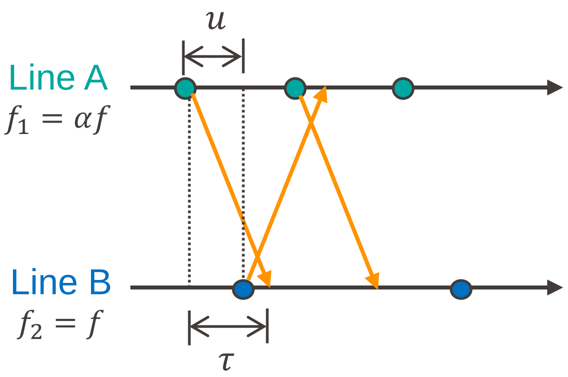
  - Otherwise, on average wait for

$$u - \tau + \frac{1}{4f}$$

till next arrival of B

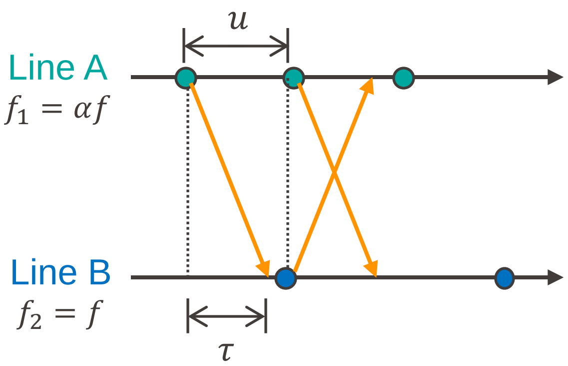
Scheduling: Multiple lines

- Coordinate two uni-directional transit lines



- Transfer from A to B with α
  - If $u \leq \tau$, average wait time till next arrival of B

$$w_{12}(u) = u - \tau + \frac{1}{f} - \frac{1}{\alpha}\sum_{i=0}^{\alpha-1}\frac{i}{\alpha f} = u - \tau + \frac{\alpha+1}{2\alpha f}$$

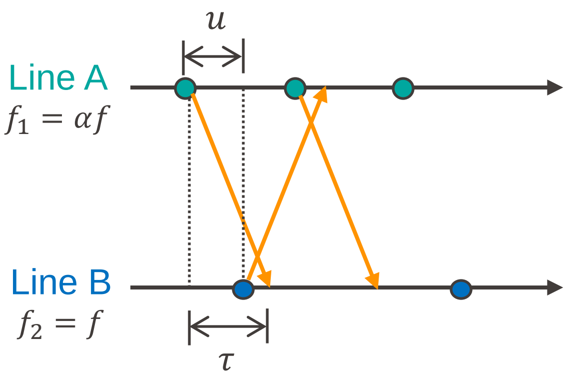


  - Otherwise, average wait time till next arrival of B

$$w_{12}(u) = u - \tau + \left(\frac{1}{f} - \frac{1}{\alpha f}\right) - \frac{1}{\alpha}\sum_{i=0}^{\alpha-1}\frac{i}{\alpha f} = u - \tau + \frac{\alpha-1}{2\alpha f}$$

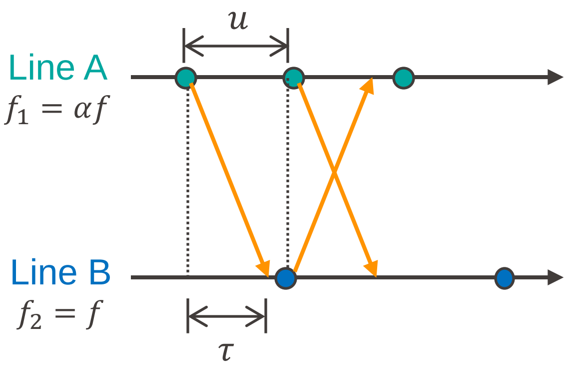
Scheduling: Multiple lines

- Coordinate two uni-directional transit lines



- Transfer from A to B with α
  - If $u \leq \tau$, total travel time for transfer

$$TT_{12}(u) = \lambda_{12}(\tau + w_{12}(u)) = \lambda_{12}\left(u + \frac{\alpha + 1}{2\alpha f}\right)$$

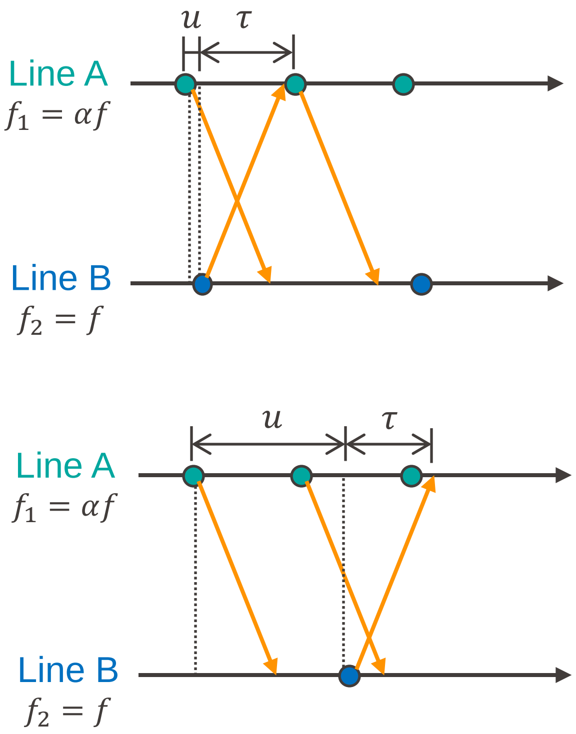


  - Otherwise, total travel time for transfer

$$TT_{12}(u) = \lambda_{12}(\tau + w_{12}(u)) = \lambda_{12}\left(u + \frac{\alpha - 1}{2\alpha f}\right)$$

Scheduling: Multiple lines

- Coordinate two uni-directional transit lines



- Transfer from B to A with α
  - Average wait time till next arrival of A

$$w_{21}(u) = \sum_{i=1}^{\alpha+1} \mathbf{1}\left\{\frac{i-1}{\alpha f} < u + \tau \leq \frac{i}{\alpha f}\right\}\left(\frac{i}{\alpha f} - u - \tau\right)$$

  - Total travel time for transfer

$$TT_{21}(u) = \lambda_{21}\big(\tau + w_{21}(u)\big)$$

$$= \lambda_{21}\left(\sum_{i=1}^{\alpha+1} \mathbf{1}\left\{\frac{i-1}{\alpha f} - \tau < u \leq \frac{i}{\alpha f} - \tau\right\}\frac{i}{\alpha f} - u\right)$$

Scheduling: Multiple lines

- Coordinate two uni-directional transit lines
  - Total travel time for transfer
    - If $u \leq \tau$,

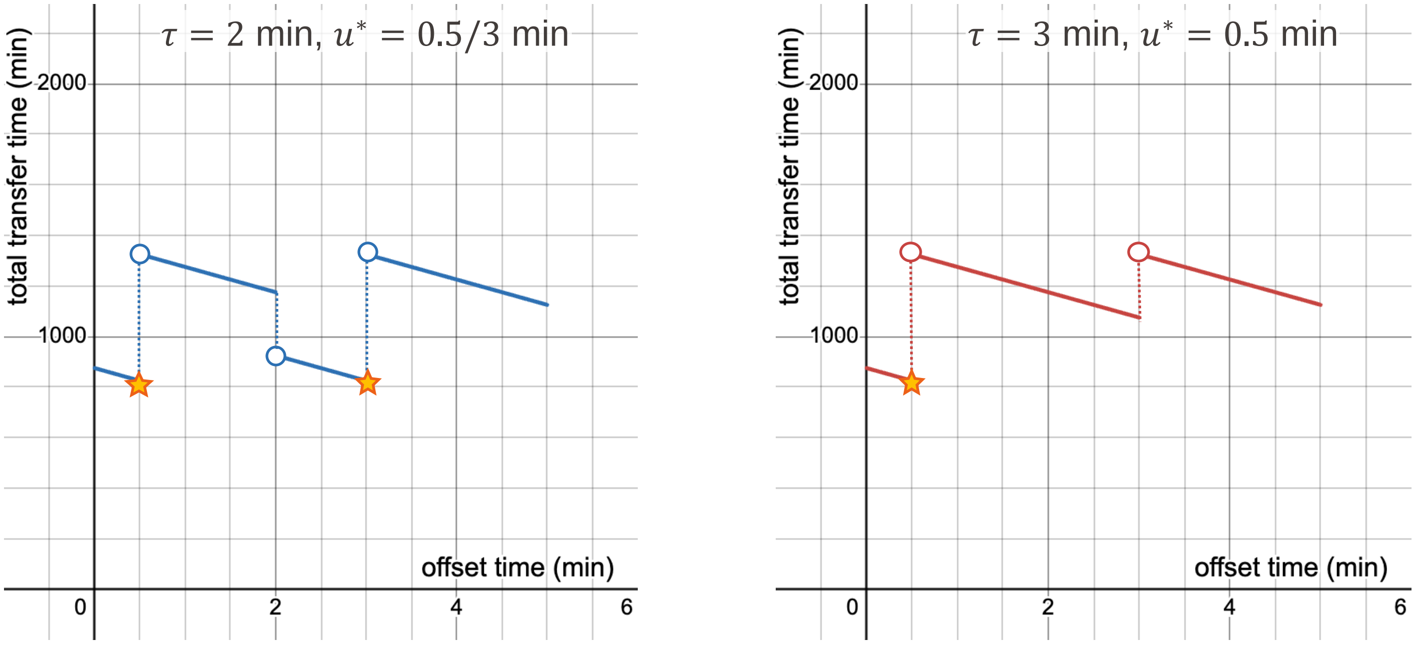
$$TT(u) = TT_{12}(u) + TT_{21}(u)$$
$$= \lambda_{12}\left(u + \frac{\alpha + 1}{2\alpha f}\right) + \lambda_{21}\left(\sum_{i=1}^{\alpha+1} \mathbf{1}\left\{\frac{i-1}{\alpha f} - \tau < u \leq \frac{i}{\alpha f} - \tau\right\}\frac{i}{\alpha f} - u\right)$$

    - Otherwise,

$$TT(u) = TT_{12}(u) + TT_{21}(u)$$
$$= \lambda_{12}\left(u + \frac{\alpha - 1}{2\alpha f}\right) + \lambda_{21}\left(\sum_{i=1}^{\alpha+1} \mathbf{1}\left\{\frac{i-1}{\alpha f} - \tau < u \leq \frac{i}{\alpha f} - \tau\right\}\frac{i}{\alpha f} - u\right)$$

Scheduling: Multiple lines

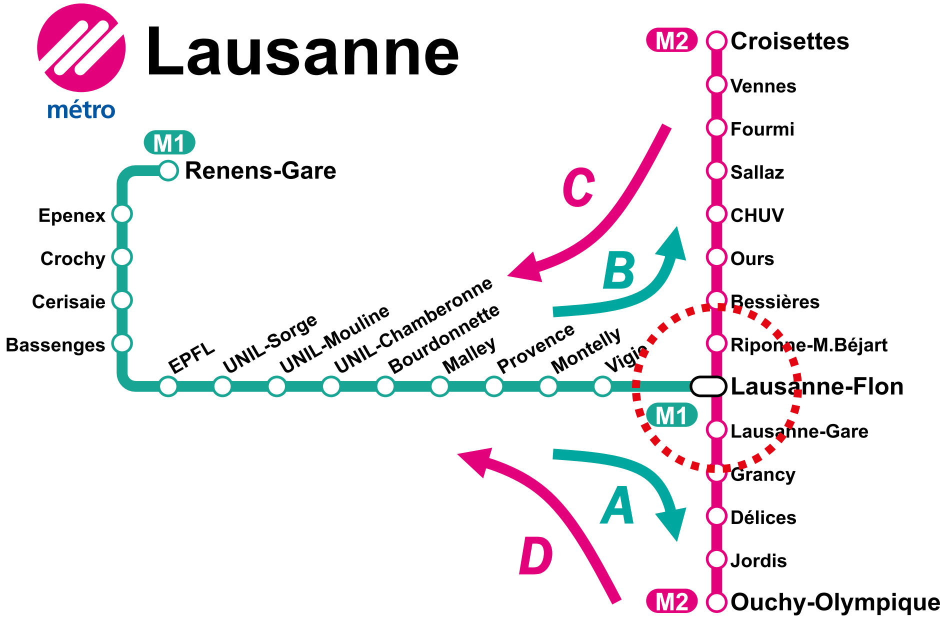
- Coordinate two uni-directional transit lines
  - Total transfer time



$1/f = 5$ min, $\lambda_{12} = 100$ pax/hr, $\lambda_{21} = 200$ pax/hr, $\alpha = 2$

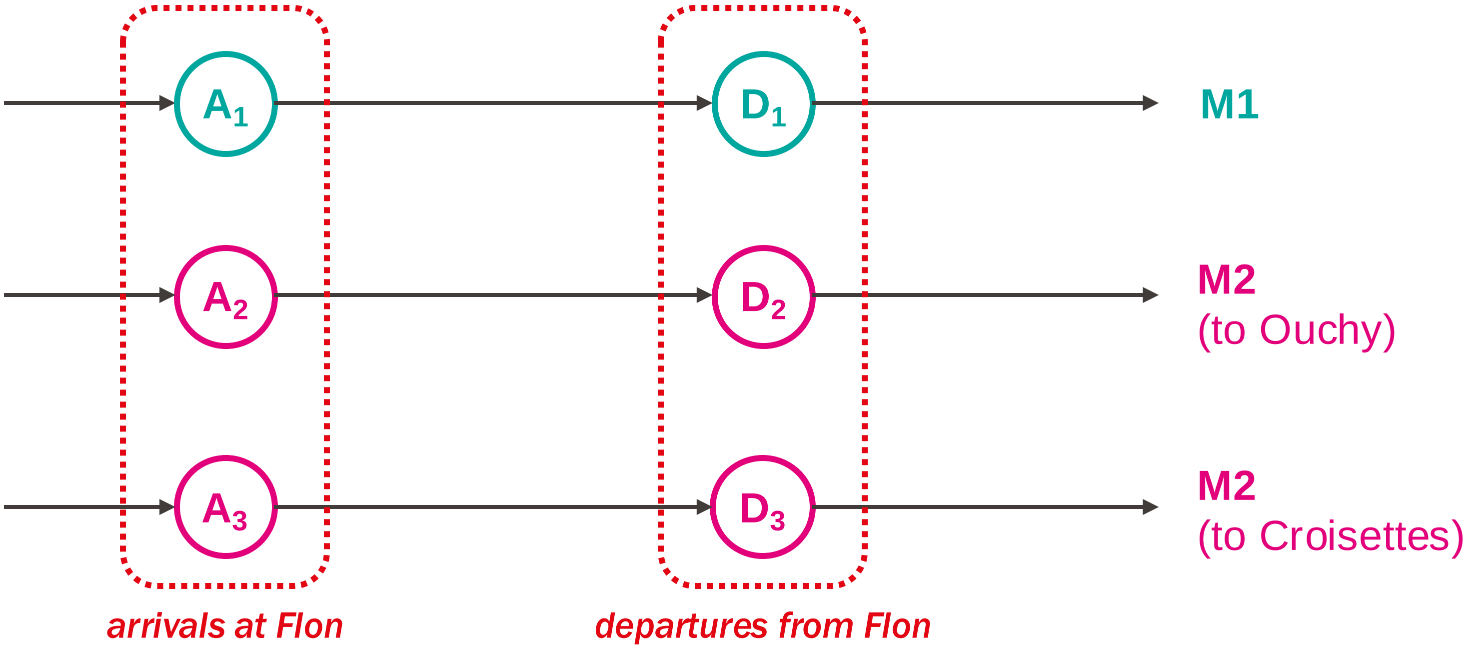
Scheduling: Multiple lines

- Coordinate multiple transfer flows
  - Transfer directions at Flon
    - A: From M1 to M2 (to Ouchy)
    - B: From M1 to M2 (to Croisettes)
    - C: From M2 (to Ouchy) to M1
    - D: From M2 (to Croisettes) to M1



Scheduling: Multiple lines

- Event-activity network
  - Node as events (e.g., arrival/departure)
  - Link as activity (e.g., movement, stop, transfer)



Scheduling: Multiple lines

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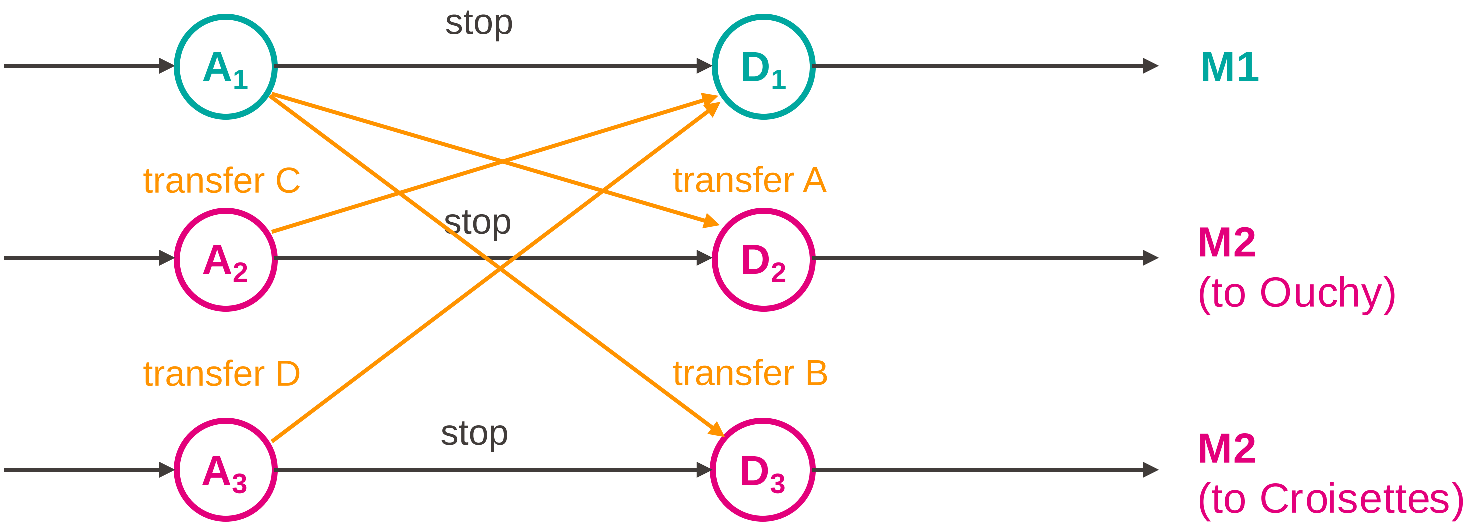
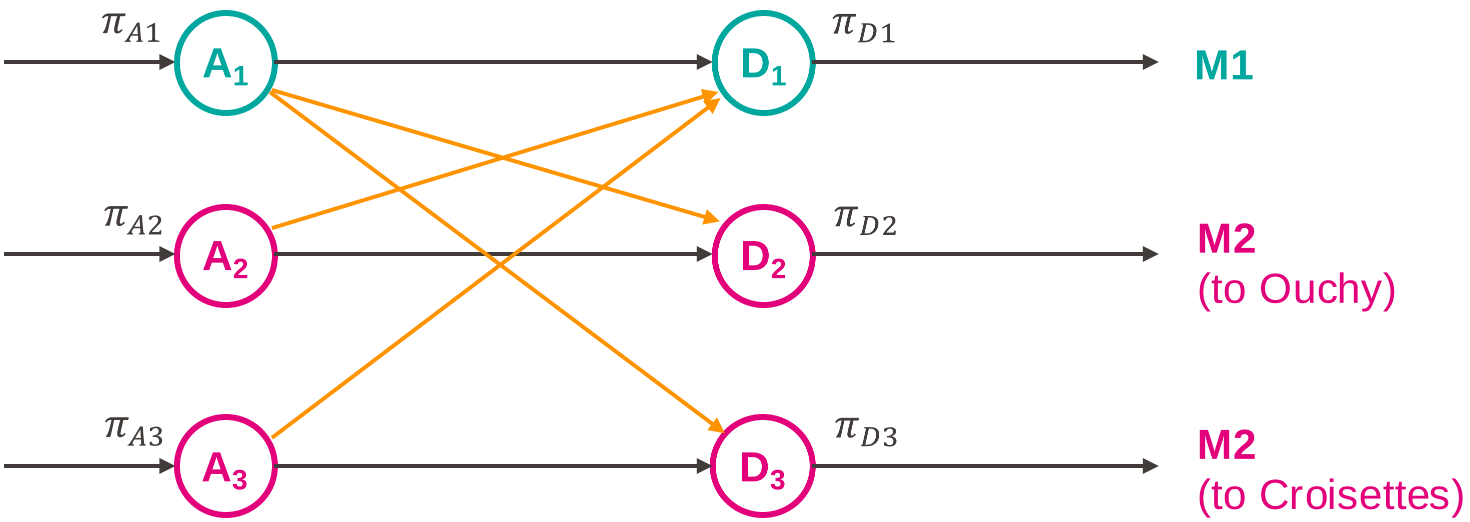


Scheduling: Multiple lines

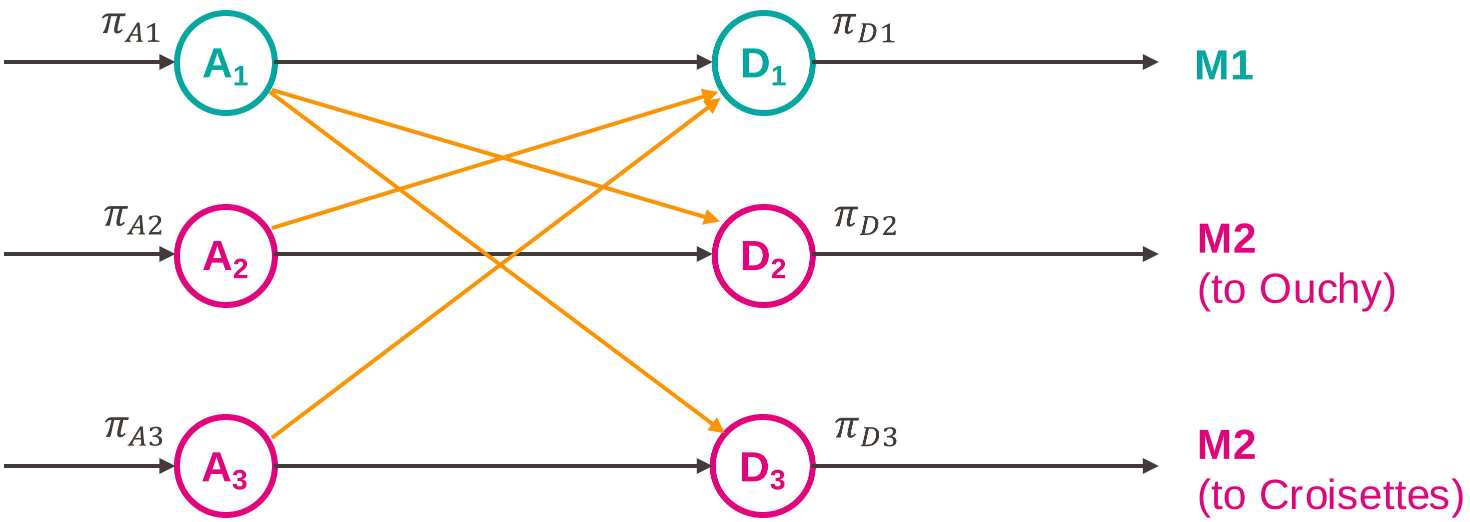
- Event-activity network
  - Node as events (e.g., arrival/departure)
    - event time π_i
  - Link as activity (e.g., stop, transfer)
    - activity duration $x_{ij} = \pi_j - \pi_i$



- ***Q: What is the stop time of M2 (to Ouchy) at Flon?***

Scheduling: Multiple lines

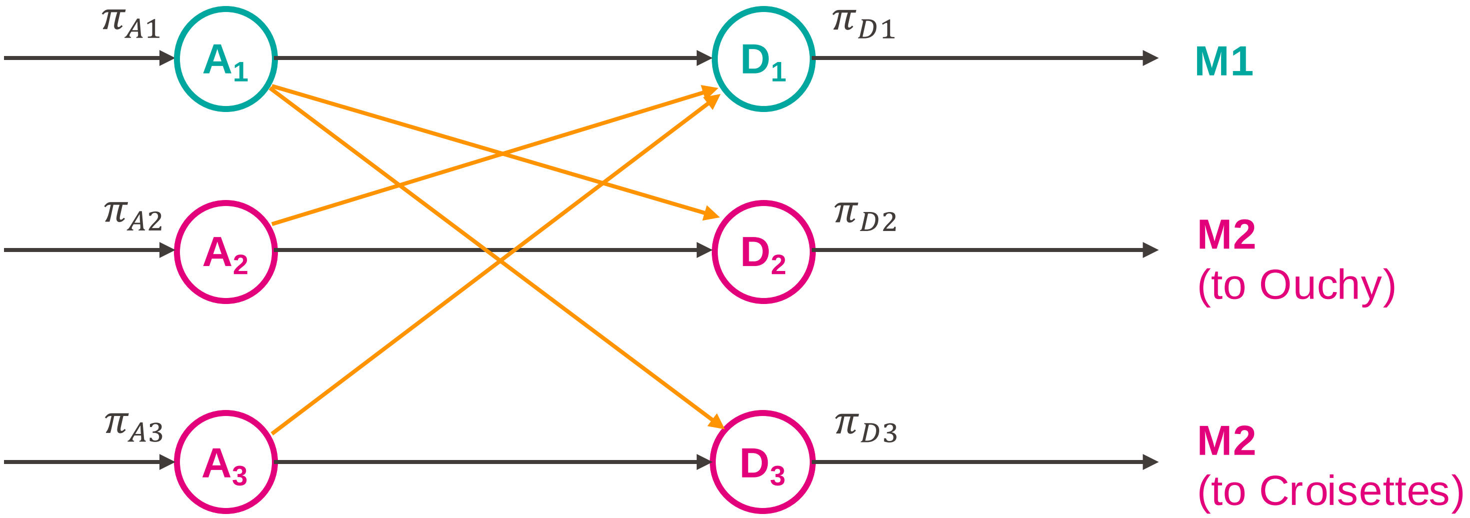
- Event-activity network
  - Node as events (e.g., arrival/departure)
    - event time π_i
  - Link as activity (e.g., stop, transfer)
    - activity duration $x_{ij} = \pi_j - \pi_i$



- ***Q: What is the transfer time from M1 to M2 (to Croisettes) at Flon?***

Scheduling: Multiple lines

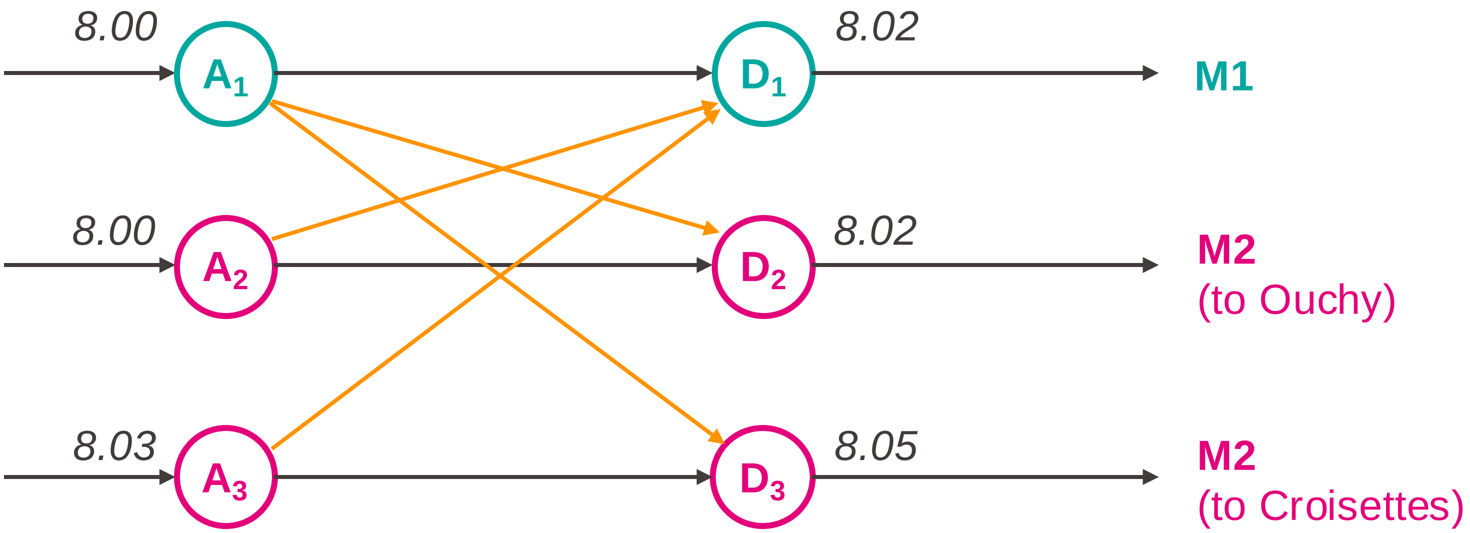
- Event-activity network
  - M1 arrives at 8.00, 8.05, … and departs at 8.02, 8.07, …
    - headway h = 5 min, stop time ω = 2 min
  - Min transfer time is τ = 3 min



- ***Q: How to represent periodic events and transfer constraint?***

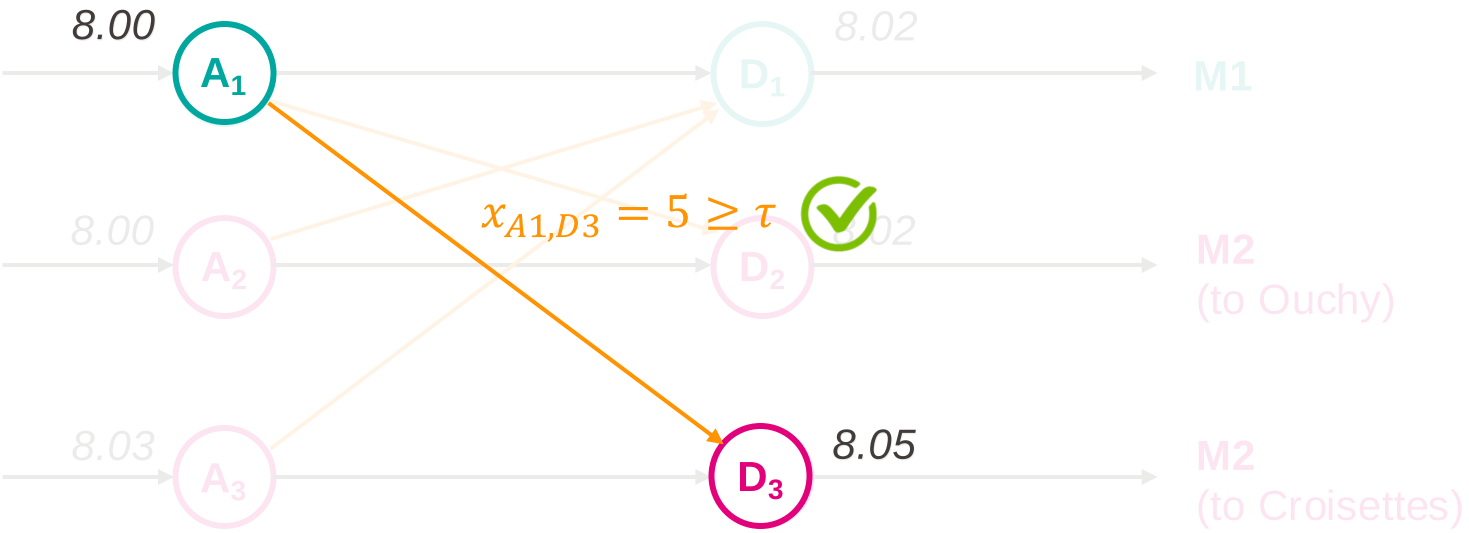
Scheduling: Multiple lines

- Event-activity network
  - Headway $h = 5$ min
  - Stop time $\omega = 2$ min
  - Min transfer time $\tau = 3$ min



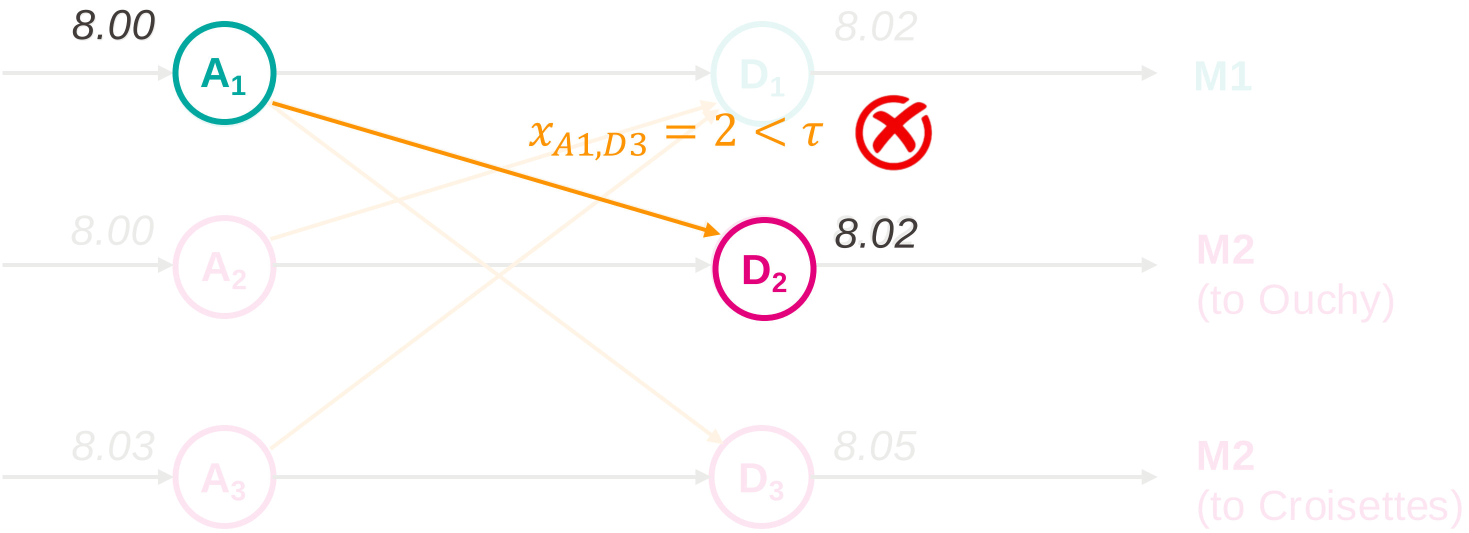
Scheduling: Multiple lines

- Event-activity network
  - Stop time $\omega = 2$ min
  - Transfer time $\tau = 3$ min
  - Headway $h = 5$ min



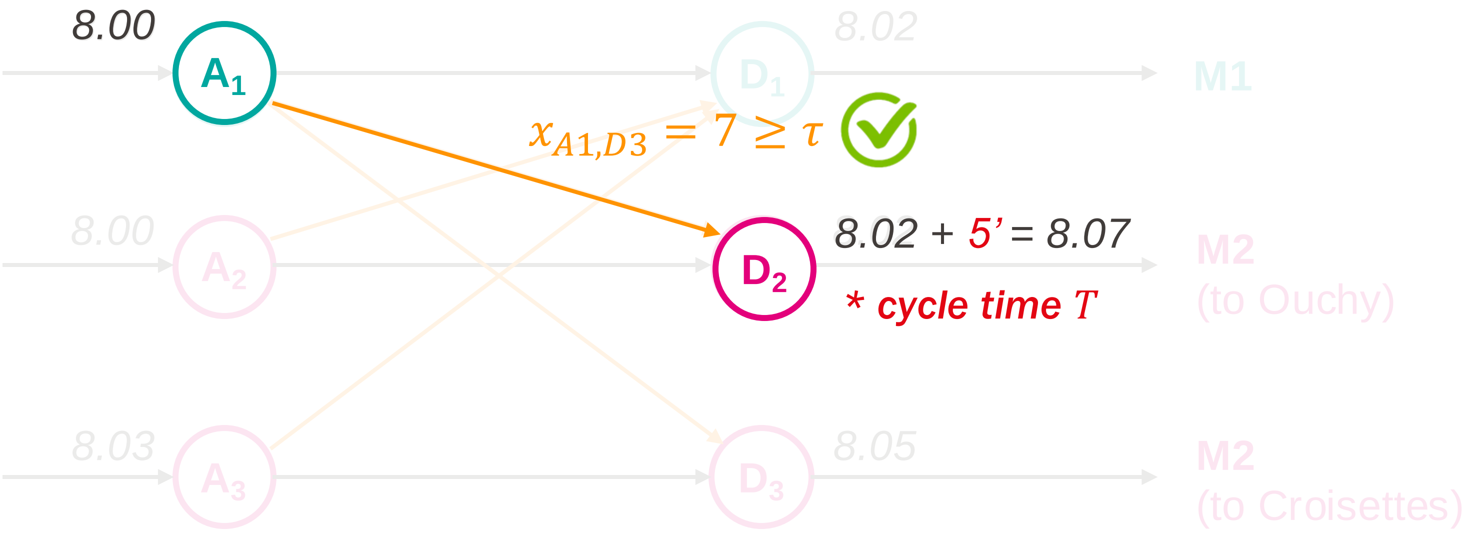
Scheduling: Multiple lines

- Event-activity network
  - Stop time $\omega = 2$ min
  - Transfer time $\tau = 3$ min
  - Headway $h = 5$ min



Scheduling: Multiple lines

- Event-activity network
  - Stop time $\omega = 2$ min
  - Transfer time $\tau = 3$ min
  - Headway $h = 5$ min



Scheduling: Multiple lines

- Schedule feasibility
  - Event times π_i, π_j is considered feasible if there exists $z \in \mathbb{Z}_+$ such that

$$\pi_j - \pi_i + zT \in [L_{ij}, U_{ij}]$$

    where L_{ij}, U_{ij} are lower and upper bounds of activity duration (e.g., transfer time)
    - e.g., suppose stop time is fixed to 2 min, then $L = U = 2$ min
    - e.g., suppose min transfer time is 3 min, then $L = 3$ min, $U = \infty$ min

- Cycle time
  - For L transit lines with different headways $h_1, h_2, \dots, h_N$, the cycle time T is set to be ***greatest common divisor***

$$T = \gcd(h_1, \dots, h_N),$$

    - e.g., if M1 has headway 3 min and M2 has headway 2 min, the cycle time is 1 min

Scheduling: Multiple lines

- Periodic event scheduling problem (PESP)
  - Assume
    - ***Schedules are announced***
      - all travelers choose trip plans with min travel time
    - No uncertainties in stop and transfer times

  - Inputs
    - Transfer demand $\{\lambda_{rs}\}_{r \in R, s \in S}$, with transfer origin set R and destination set S
    - Headways $\{h_k\}_{k=1}^{N}$, of N lines
    - Stop time ω
    - Min transfer time τ

  - Outputs
    - Event times $\{\pi_{rs}\}_{r \in R, s \in S}$
    - Event cycles $\{z_{ij}\}_{\forall (i,j)}$

Scheduling: Multiple lines

- Periodic event scheduling problem (PESP)
  - Step 1: Build event-activity network
    - specify activity duration bounds $\{L_{ij}, U_{ij}\}_{\forall(i,j)}$
    - determine cycle time T
  - Step 2: Solve a mixed integer linear program (MILP)

$$\min_{\pi,z} TT(\pi, z) = \sum_{r,s} \lambda_{rs}(\pi_s - \pi_r + z_{rs}T)$$

$$s.t. \quad L_{ij} \leq \pi_j - \pi_i + z_{ij}T \leq U_{ij}, \qquad \forall(i,j),$$

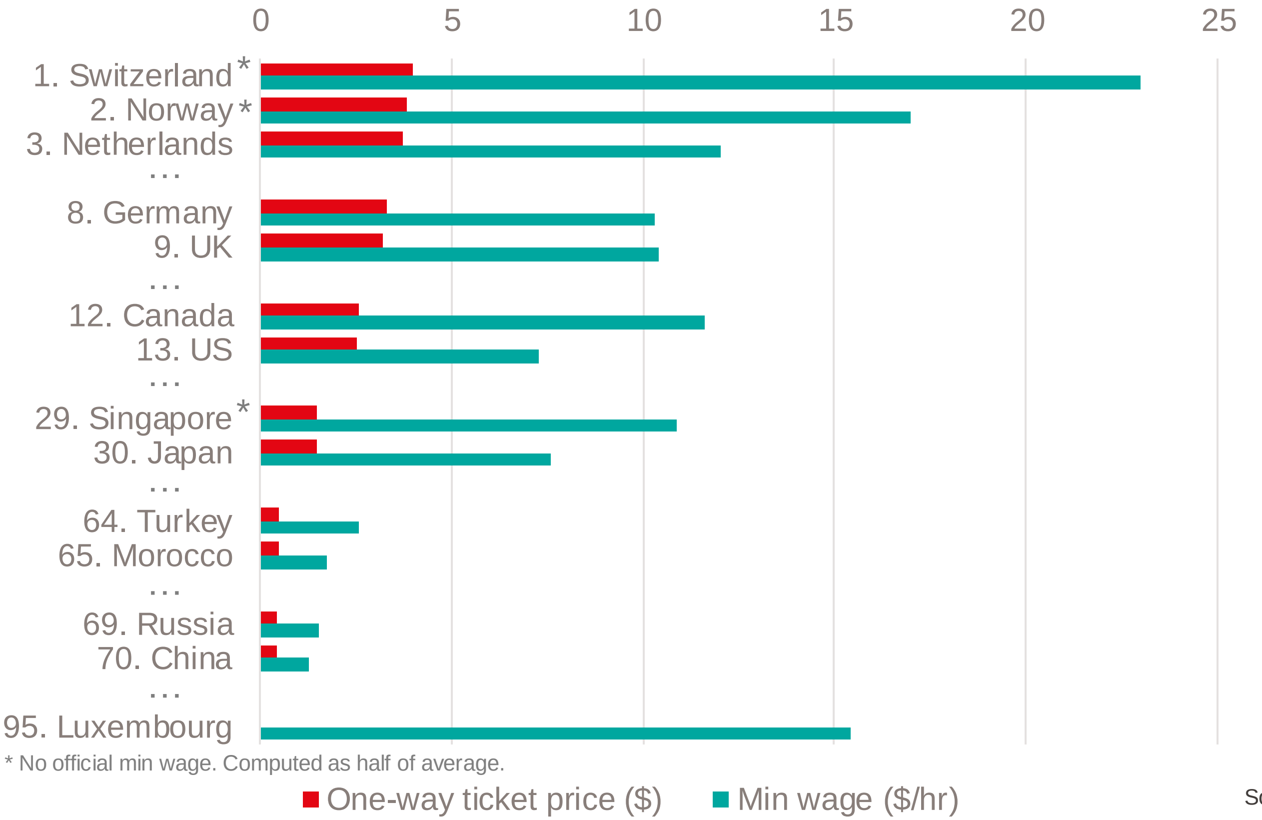
$$\pi_i \in \mathbb{R}_+, \qquad \forall\, i,$$

$$z_{ij} \in \mathbb{Z}_+, \qquad \forall\, (i,j).$$

Questions?

Pricing

- Transit price over the world

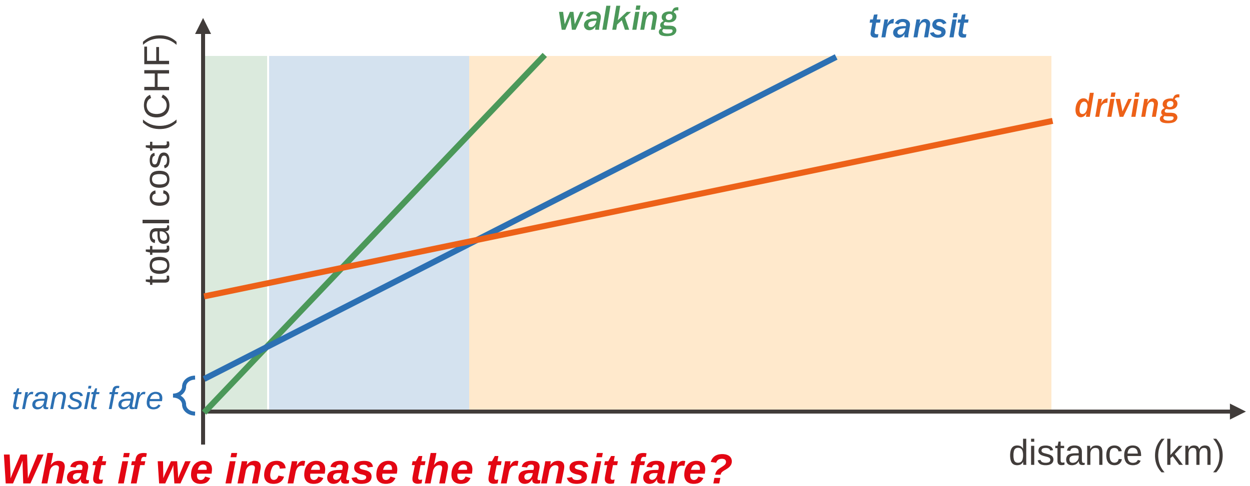


Source: Numbeo, Wikipedia

Pricing

- Higher in Europe and developed countries
  - also where transit provides the best services

- Relatively cheap with respect to income level
  - because transit is considered a public service
  - a higher price can hardly attract sufficient ridership

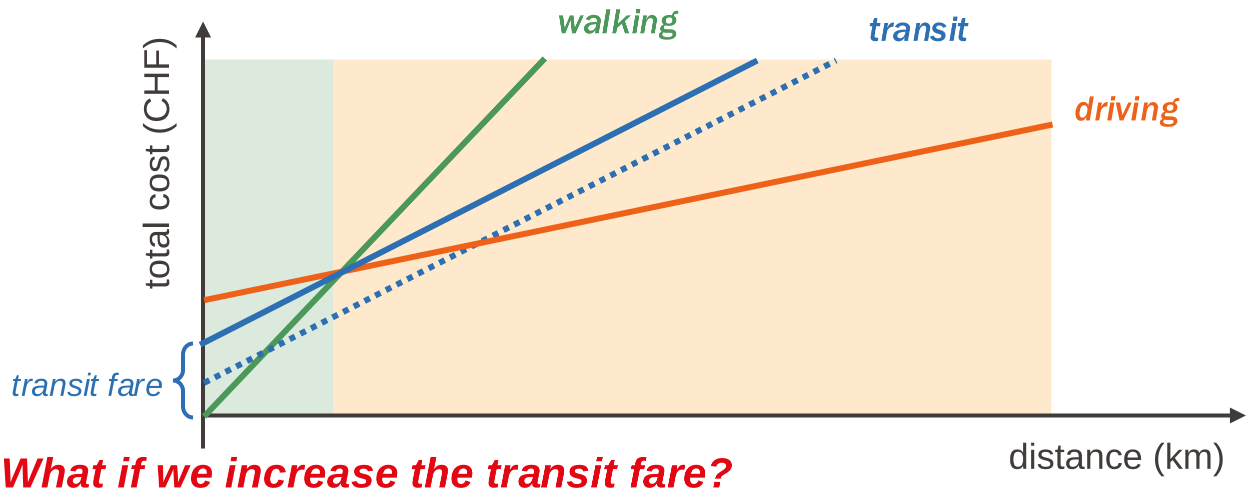


- ***Q: What if we increase the transit fare?***

Pricing

- Higher in Europe and developed countries
  - also where transit provides the best services

- Relatively cheap with respect to income level
  - because transit is considered a public service
  - a higher price can hardly attract sufficient ridership



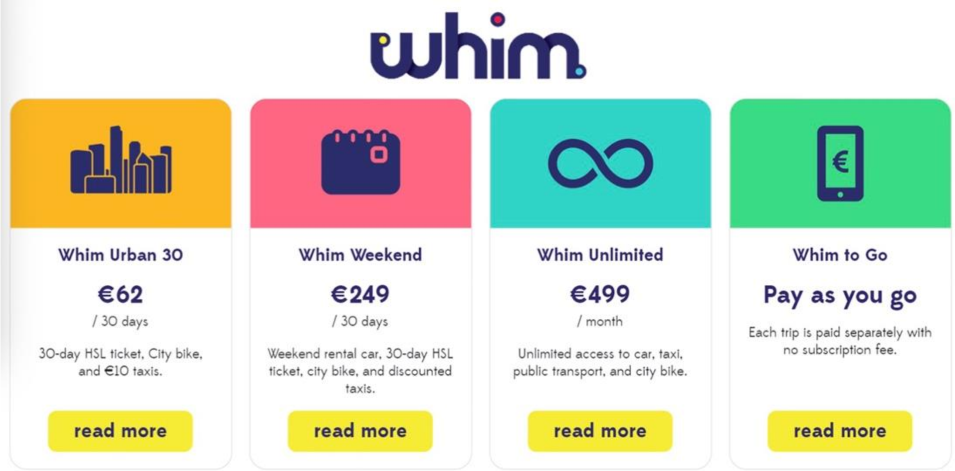
- ***Q: What if we increase the transit fare?***

Pricing

- Common pricing schemes
  - Flat rate
    - New York City subway/bus: $ 2.90
    - Paris metro/bus: € 2.15
  - Distance-based
    - SBB + half-fare
      - EPFL <> Geneva (60 km): CHF 12.90 (CHF 0.215/km)
      - EPFL <> Zurich (225 km): CHF 41.90 (CHF 0.186/km)
  - Subscription
    - tl Grand Lausanne: CHF 78 /month
    - SBB GA: CHF 3995 /year (CHF 333 /month)

Pricing

- Common pricing schemes
  - Bundle
    - A combination of tickets of/access to multiple modes
    - Commonly used by mobility-as-a-service (MaaS) platforms



Pricing

- Pricing objectives
  - Maximize profit
    - fare revenue – operating cost
  - Maximize social welfare
    - travel utility – system cost (– environmental impact)
  - Minimize system cost
    - operation cost + user cost
  - Maximize ridership
- ***Q: What would you choose, and why?***

Pricing

- Pricing objectives
  - Maximize profit
    - fare revenue – operating cost

chosen by private operators

  - Maximize social welfare
    - travel utility – system cost (– environmental impact)
  - Minimize system cost
    - operation cost + user cost

chosen by public operators

  - Maximize ridership

Pricing

- Pricing constraints
  - Service quality
    - e.g., max waiting time, min accessibility, …
  - Profitability
    - e.g., profit neutral
  - Demand sensitivity
    - e.g., mode choice between transit and other transport options

- ***Q: What are other possible constraints?***

Pricing

- A simple parallel network

- A total number of λ (pax/hr) commuters travel from **Home** to **City**
  - Two options: driving on **highway** vs. taking **train**
  - Travelers make mode choice based on the total cost of each mode
  - Transit fare is flat and travelers do not know the schedule
  - No walking is needed to take the train

Pricing

- Total travel cost for a trip of distance ℓ
  - Driving

$$C_d = p_d \ell + \beta t_d$$

    - unit driving cost p_d (CHF/km)
    - driving time $t_d = \ell / v_d$

  - Transit

$$C_t = p_t + \beta(t_w + t_b)$$

    - transit fare p_t
    - waiting time $t_w = 1/2f$
    - riding time $t_b = \ell / v_b$

- ***Q: Suppose you’re the transit operator. What are the design variables? Which variables can be considred fixed?***

Pricing

- Total travel cost for a trip of distance ℓ
  - Driving

$$C_d = p_d \ell + \beta t_d$$

    - unit driving cost p_d (CHF/km) $\Rightarrow$ ***fixed***
    - driving time $t_d = \ell / v_d$ $\Rightarrow$ ***not necessarily fixed***

  - Transit

$$C_t = p_t + \beta(t_w + t_b)$$

    - transit fare p_t $\Rightarrow$ ***design variable***
    - waiting time $t_w = 1/2f$ $\Rightarrow$ ***could be design variable***
    - riding time $t_b = \ell / v_b$ $\Rightarrow$ ***fixed***

Pricing

- Demand model
  - Deterministic
    - *"I'll definitely take the train if it is less costly."*

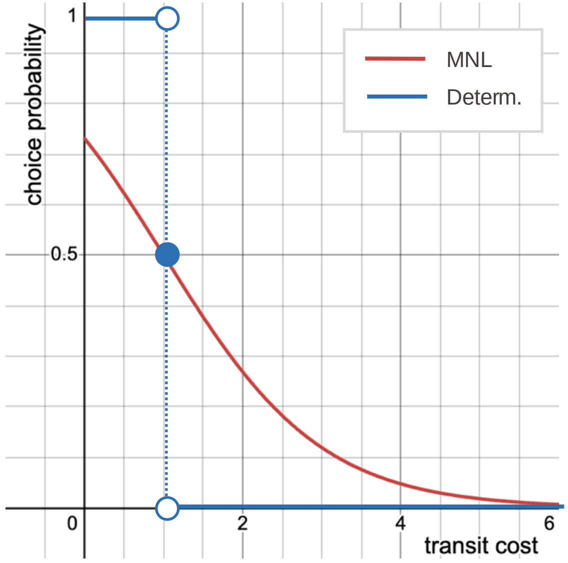
$$\mathbb{P}[\text{train}] = \begin{cases} 0, & C_t > C_d \\ 0.5, & C_t = C_d \\ 1, & C_t < C_d \end{cases}$$

  - Stochastic
    - *"I'll be more likely to take train if it is less costly."*

$$\mathbb{P}[\text{train}] = D(C_t, C_d; \theta)$$

    - Multinomial Logit (MNL) Model

$$\mathbb{P}[\text{train}] = \frac{\exp(-\theta C_t)}{\exp(-\theta C_t) + \exp(-\theta C_d)}$$



$C_d = 1, \theta = 1$

Pricing

- Transit pricing in a parallel network

- Problem 1: Design the transit fare to max ridership and ensure profit neutral
  - fixed train operations and highway travel time

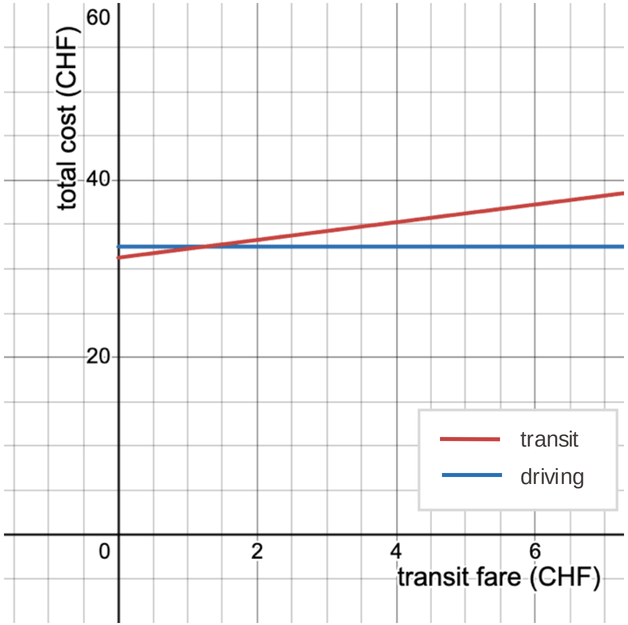
$$\max_{p_t} \lambda \mathbb{P}[\text{train}] \quad \text{transit ridership}$$

$$s.t. \quad \Pi(p_t) = p_t \lambda \mathbb{P}[\text{train}] - cf \geq 0$$

fare revenue *operating cost*

Pricing

- Transit pricing in a parallel network
  - Case 1: Deterministic demand

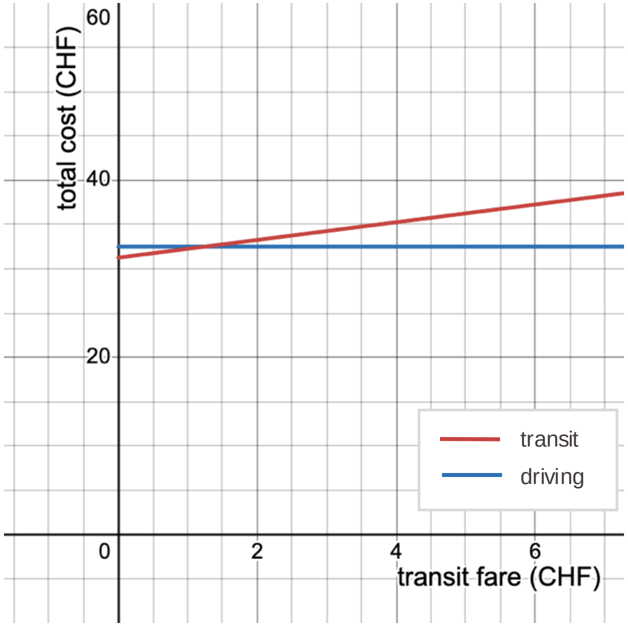


$\beta = 50$ CHF/hr, $\ell = 10$ km, $p_d = 2$ CHF/km, $v_d = 40$ km/hr, $v_b = 20$ km/hr, $f = 4$ /hr, $c = 100$ CHF/veh, $\lambda = 1000$ pax/hr

- if $C_t < C_d$,
  - $\mathbb{P}[\text{train}] = 1$
  - $\Pi(p_t) = \lambda p_t - cf$
- else if $C_t = C_d$,
  - $\mathbb{P}[\text{train}] = 0.5$
  - $\Pi(p_t) = \frac{\lambda}{2} p_t - cf$
- otherwise,
  - $\mathbb{P}[\text{train}] = 0$
  - $\Pi(p_t) = -cf$

Pricing

- Transit pricing in a parallel network
  - Case 1: Deterministic demand



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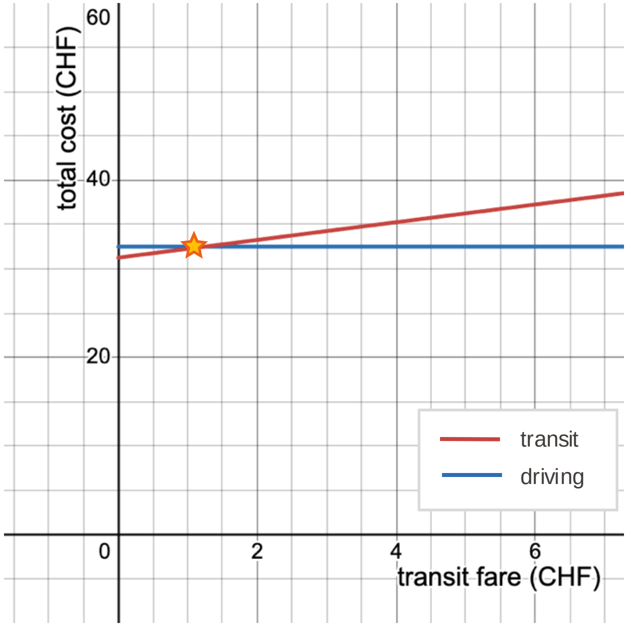
- if $C_t < C_d$, $\Rightarrow p_t + \beta(t_w + t_b) < p_d\ell + \beta t_d$
  - $\mathbb{P}[\text{train}] = 1$
  - $\Pi(p_t) = \lambda p_t - cf \geq 0 \Rightarrow p_t \geq cf/N$
- else if $C_t = C_d$, $\Rightarrow p_t + \beta(t_w + t_b) = p_d\ell + \beta t_d$
  - $\mathbb{P}[\text{train}] = 0.5$
  - $\Pi(p_t) = \frac{\lambda}{2} p_t - cf \geq 0 \Rightarrow p_t \geq 2cf/N$
- otherwise,
  - $\mathbb{P}[\text{train}] = 0$
  - $\Pi(p_t) = -cf < 0$

Pricing

- Transit pricing in a parallel network
  - Case 1: Deterministic demand

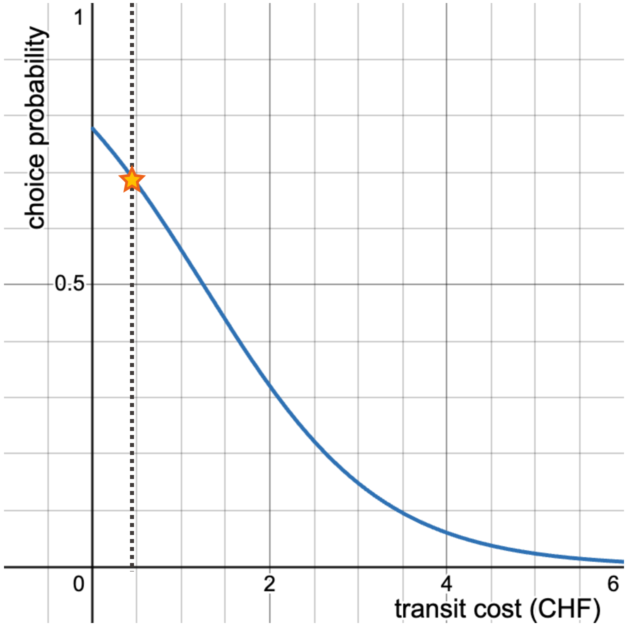


$\beta = 50$ CHF/hr, $\ell = 10$ km, $p_d = 2$ CHF/km, $v_d = 40$ km/hr, $v_b = 20$ km/hr,
$f = 4$ /hr, $c = 100$ CHF/veh, $\lambda = 1000$ pax/hr

- if $p_t < p_d\ell + \beta t_d - \beta(t_w + t_b)$ =1.25 CHF
  - $\mathbb{P}[\text{train}] = 1$
  - $p_t \geq cf/\lambda$ = 0.4 CHF
- else if $p_t = p_d\ell + \beta t_d - \beta(t_w + t_b)$ =1.25 CHF
  - $\mathbb{P}[\text{train}] = 0.5$
  - $p_t \geq 2cf/\lambda$ = 0.8 CHF
- otherwise,
  - $\mathbb{P}[\text{train}] = 0$
  - $\Pi(p_t) = -cf$
- ***Q: What is the optimal transit trip fare?***

Pricing

- Transit pricing in a parallel network
  - Case 2: MNL ($\theta = 1$)



$\beta = 50$ CHF/hr, $\ell = 10$ km, $p_d = 2$ CHF/km, $v_d = 40$ km/hr, $v_b = 20$ km/hr,
$f = 4$ /hr, $c = 100$ CHF/veh, $\lambda = 1000$ pax/hr

$$\mathbb{P}[\text{train}] = \frac{\exp(-\theta C_t)}{\exp(-\theta C_t) + \exp(-\theta C_d)}$$

$$= \frac{\exp[-\theta(p_t + \beta(t_w + t_b))]}{\exp[-\theta(p_t + \beta(t_w + t_b))] + \exp[-\theta(p_d + \beta t_d)]}$$

$$\Pi(p_t) = \lambda p_t - cf \geq 0 \Rightarrow p_t \geq cf/N = 0.4 \text{ CHF}$$

- ***Q: What is the optimal transit trip fare?***

Pricing

- Transit pricing in a parallel network

  - Problem 1: Design the transit fare to max ridership and ensure profit neutral
    - fixed train operations and highway travel time

$$\max_{p_t} \ \lambda \mathbb{P}[\text{train}]$$

$$s.t. \quad \Pi(p_t) = p_t \lambda \mathbb{P}[\text{train}] - cf \geq 0$$

- ***Q: What are the strongest assumptions to be relaxed?***

Pricing

- Transit pricing in a parallel network

- Problem 2: Design the transit fare to max ridership and ensure profit neutral
  - fixed train operations and ***flow-dependent highway speed***

$$v_d(\lambda_d) = \bar{v}_d \left(1 + \frac{\lambda_d}{\kappa_d}\right)^{-2}$$

where

- λ_d: driving demand
- $\bar{v}_d$: free-flow speed
- κ_d: saturation flow rate

Pricing

- Transit pricing in a parallel network

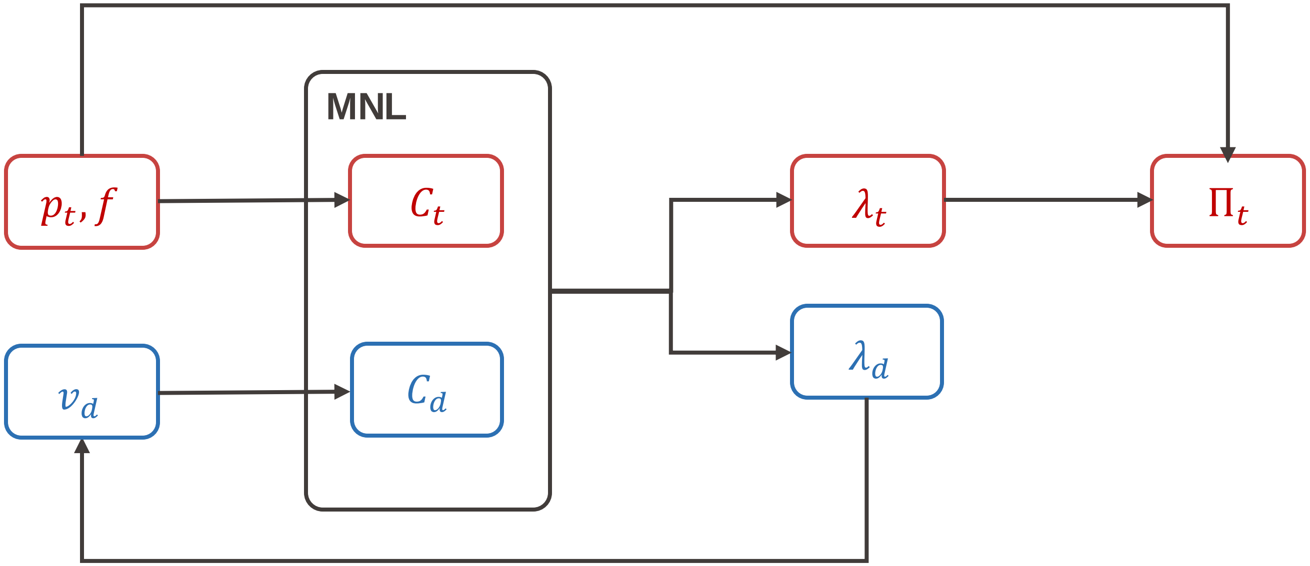
- Problem 3: Jointly design the transit fare and frequency to max ridership and ensure profit neutral
  - fixed train operating cost and flow-dependent highway speed

$$\max_{p_t, f} \ \lambda \mathbb{P}[\text{train}]$$

$$s.t. \quad \Pi(p_t, f) = p_t \lambda \mathbb{P}[\text{train}] - cf \geq 0$$

Pricing

- Transit pricing in a parallel network
  - Iterative solution procedure of equilibrium with MNL demand



Questions?