EPFL

Spring 2025

02 Shuttle system

CIVIL-324 Urban public transport systems

École
polytechnique
fédérale
de Lausanne

Before we start

- Share this to your family and friends who regularly commute by car

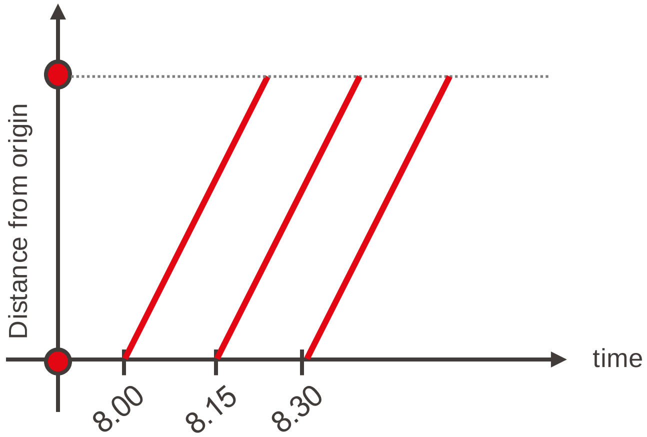
What is a shuttle system?

- The simplest transit system with a single origin and destination

Schedule

id	Departure	Arrival
1	8.00	8.20
2	8.15	8.35
3	8.30	8.50
…	…	…

Time-space diagram



What is a shuttle system?

- The simplest transit system with a single origin and destination

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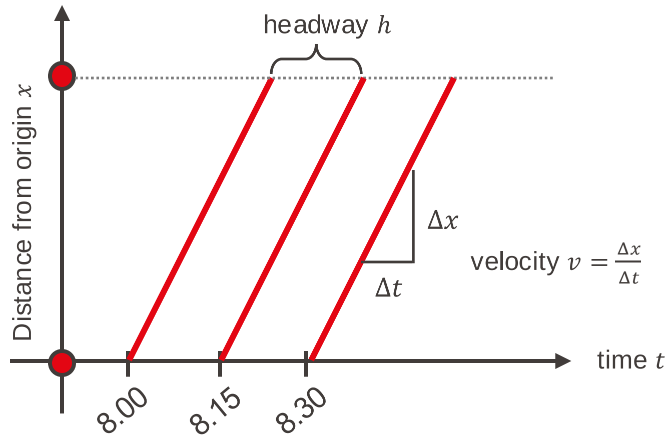


How to design a shuttle system?

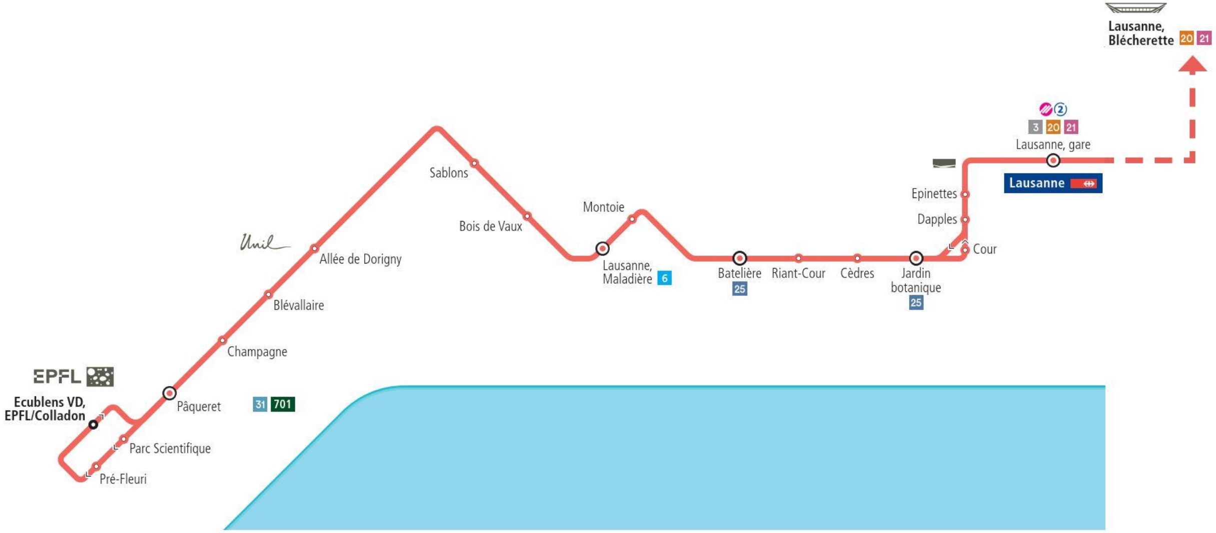
- Suppose we replace tl1 with an EPFL shuttle
  - Morning: Lausanne Gare → EPFL
  - Evening: EPFL → Lausanne Gare

- ***Q: What are the design variables and necessary inputs?***

How to design a shuttle system?

- Suppose we replace tl1 with an EPFL shuttle
  - Morning: Lausanne Gare → EPFL
  - Evening: EPFL → Lausanne Gare

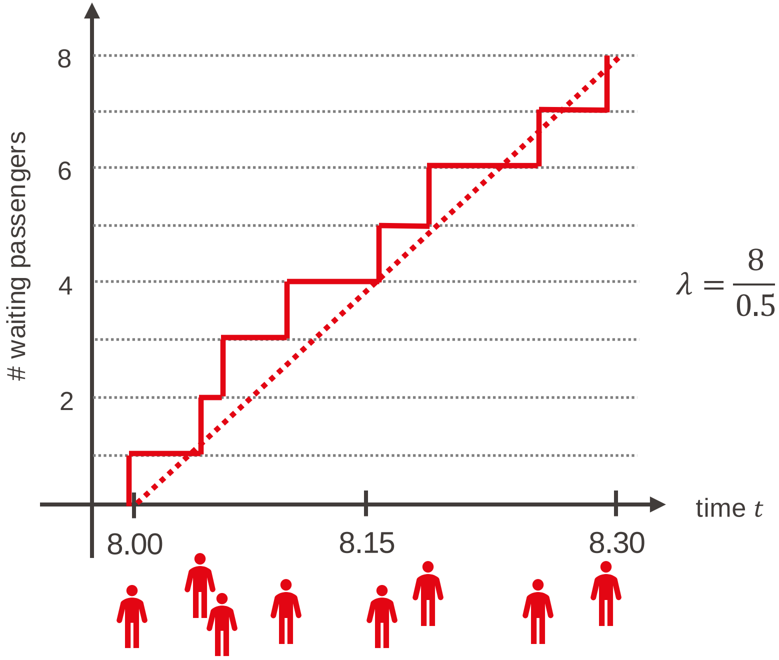
- Design variables
  - Timetable
  - Vehicle size
  - Fare
  - …
- Design inputs
  - Potential demand
  - Vehicle speed
  - Service sensitivity
  - …



Base model

- Assumptions
  - Travelers randomly arrival at the origin stop
  - Approximated by aggregate arrival rate λ (pax/hr)

Queue at origin

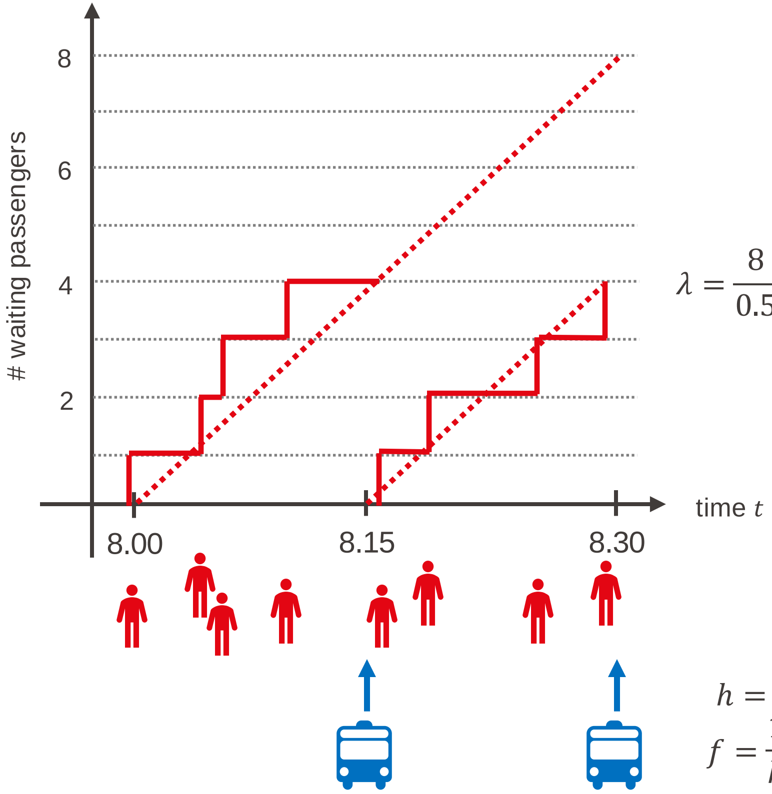


$$\lambda = \frac{8}{0.5} = 16$$

Base model

- Assumptions
  - Travelers randomly arrival at the origin stop
  - Approximated by aggregate arrival rate λ (pax/hr)
  - Shuttles depart from the origin stop at frequency f (/hr)
    - correspond to headway $h = 1/f$ (hr)
  - Shuttles have equal and sufficiently large capacity κ (pax)
    - queue restart after each shuttle

Queue at origin



$$\lambda = \frac{8}{0.5} = 16$$

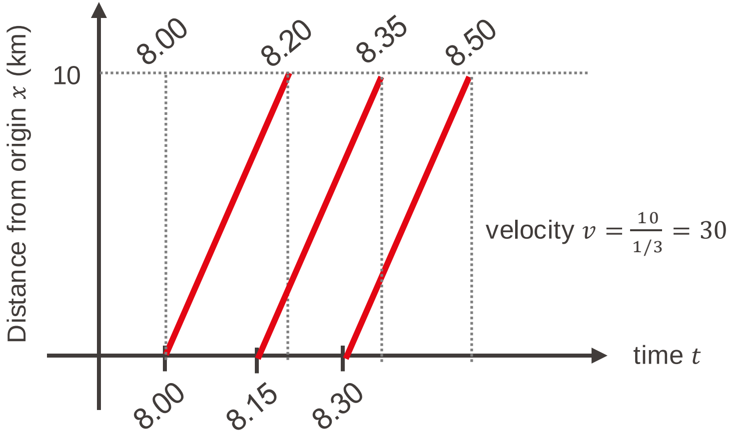
$$h = 0.25$$

$$f = \frac{1}{h} = 4$$

Base model

- Assumptions
  - Travelers randomly arrival at the origin stop
  - Approximated by aggregate arrival rate λ (pax/hr)
  - Shuttles depart from the origin stop at frequency f (/hr)
    - correspond to headway $h = 1/f$ (hr)
  - Shuttles have equal and sufficiently large capacity κ (pax)
    - queue restart after each shuttle
  - Shuttles run at speed v (km/hr)
    - constant travel time τ (hr)

Time-space diagram



Base model

- Design problem
  - Determine the shuttle frequency f to ***minimize*** the total system cost $TC(f)$

$$\min_{f} \; TC(f) = cf + \beta\lambda\,(w + \tau)$$

  - c: operation cost per shuttle (CHF)
  - f: shuttle frequency (1/hr)
  - β: value of time (CHF/hr)
  - λ: demand rate (pax/hr)
  - w: average waiting time (hr)
  - τ: shuttle travel time (hr)

- ***Q: Why shuttle fare is not considered?***

Base model

- Design problem
  - Determine the shuttle frequency f to ***minimize*** the total system cost $TC(f)$?

$$\min_{f} \; TC(f) = cf + \beta\lambda\,(w + \tau)$$

- c: operation cost per shuttle (CHF)
- f: shuttle frequency (1/hr)
- β: value of time (CHF/hr)
- λ: demand rate (pax/hr)
- w: average waiting time (hr)
- τ: shuttle travel time (hr)

- ***Q: How to estimate average waiting time w?***

Base model

- Design problem
  - Determine the shuttle frequency f to ***minimize*** the total system cost $TC(f)$?

$$\min_{f} \; TC(f) = cf + \beta\lambda\,(w + \tau)$$

    - c: operation cost per shuttle (CHF)
    - f: shuttle frequency (1/hr)
    - β: value of time (CHF/hr)
    - λ: demand rate (pax/hr)
    - w: average waiting time (hr)
    - τ: shuttle travel time (hr)

  - Assume passengers randomly arrive between two shuttles

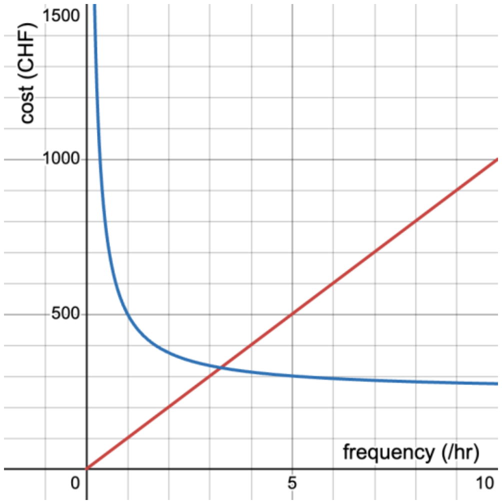
$$w = \frac{h}{2} = \frac{1}{2f}$$

Base model

- Design problem
  - Determine the shuttle frequency f to ***minimize*** the total system cost $TC(f)$?

$$\min_f TC(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right)$$

- ***Operation cost***
  - linearly increase with frequency
- ***User cost***
  - inverse-linearly decrease with frequency
- There is some frequency that min $TC(f)$



$\lambda = 100$ pax/hr, $c = 100$ CHF, $\beta = 50$ CHF/hr, $\tau = 0.5$ hr

Base model

- Design problem
  - Determine the shuttle frequency f to ***minimize*** the total system cost $TC(f)$?

$$\min_f TC(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right)$$

  - First-order condition

$$\frac{\partial TC(f)}{\partial f} = c - \frac{\beta\lambda}{2f^2} = 0$$

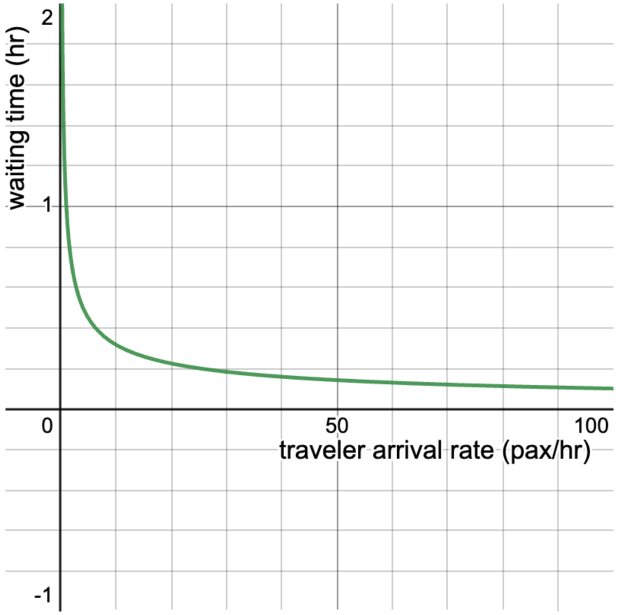
$$\Rightarrow \quad f^* = \sqrt{\frac{\beta\lambda}{2c}}$$

Base model

- Issue of the optimal frequency

$$w^* = \frac{1}{2f^*} = \sqrt{\frac{c}{2\beta\ \lambda}}$$

- ***Q: How does waiting time change with travel demand?***



$c = 100$ CHF, $\beta = 50$ CHF/hr, $\tau = 0.5$ hr

Base model

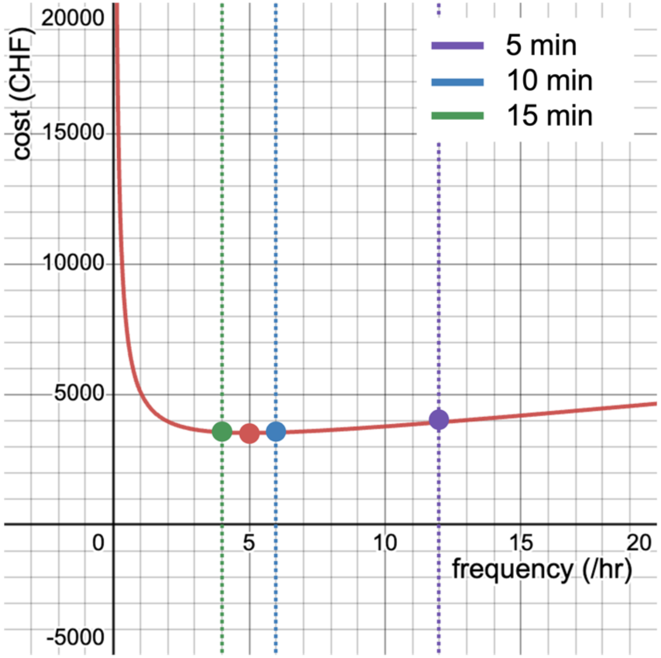
- Issue of the optimal frequency

$$w^* = \frac{1}{2f^*} = \sqrt{\frac{c}{2\beta\,\lambda}}$$

- Max waiting time $w_{\max}$
  - correspond to a min frequency $f_{\min} = \frac{1}{2w_{\max}}$

$$\min_f \quad TC(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right)$$

$$s.t. \qquad f \geq f_{\min}$$



$\lambda = 100$ pax/hr, $c = 100$ CHF, $\beta = 50$ CHF/hr, $\tau = 0.5$ hr

Questions?

Extension I: Service-sensitive demand

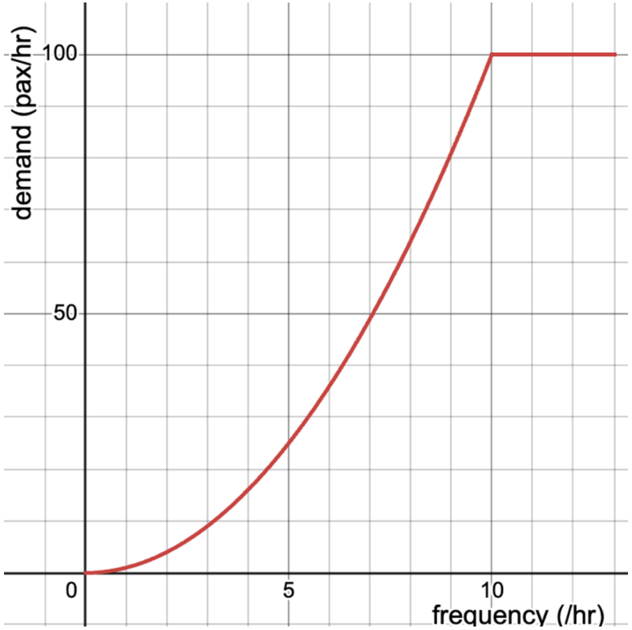
- Elastic demand
  - Total travel demand λ_0 (pax/hr)

  - The longer travel time, the fewer passengers
    - if $f \searrow$, then $w \nearrow$ and $\lambda \searrow$

  - Backup option with fixed total travel time τ_0 (hr) and operation cost c_0 (CHF/pax)
    - with demand $\lambda_0 - \lambda$ (pax/hr)

  - Simple demand function

$$\lambda(f) = \begin{cases} \alpha f^2, & 0 \le f \le \sqrt{\lambda_0/\alpha} \\ \lambda_0, & \text{otherwise} \end{cases}$$



$\alpha = 1$ pax·hr, $\lambda_0 = 100$ pax/hr

Extension I: Service-sensitive demand

- Problem I: min shuttle system cost

$$\min_{f} \quad TC_{\text{shuttle}}(f) = cf + \beta\lambda(f)\left(\frac{1}{2f} + \tau\right)$$

- Problem II: min total system cost

$$\min_{f} \quad TC_{\text{all}}(f) = TC_{\text{shuttle}}(f) + \big(\lambda_0 - \lambda(f)\big)(\beta\tau_0 + c_0)$$

- ***Q: What are solutions to these two problems?***

Extension I: Service-sensitive demand

- Problem I: min shuttle system cost

$$\min_{f} \quad TC_{\text{shuttle}}(f) = cf + \beta\lambda(f)\left(\frac{1}{2f} + \tau\right)$$

$$\frac{\partial TC_{\text{shuttle}}(f)}{\partial f} = c + \frac{\alpha\beta}{2} + 2\alpha\beta\tau f > 0 \Rightarrow \quad f^* = 0$$

- ***Q: What is the physical meaning?***

Extension I: Service-sensitive demand

- Problem II: min total system cost

$$\min_f \quad TC_{\text{all}}(f) = TC_{\text{shuttle}}(f) + \big(\lambda_0 - \lambda(f)\big)(\beta\tau_0 + c_0)$$

- Case 1: all demand is served by shuttles, i.e., $\lambda(f) = \lambda_0 \Rightarrow f \geq \sqrt{\lambda_0/\alpha}$

$$\min_f \quad TC_{\text{all}}(f) = cf + \beta\lambda_0\left(\frac{1}{2f} + \tau\right)$$

$$\frac{\partial TC_{\text{all}}(f)}{\partial f} = c - \frac{\beta\lambda_0}{2f^2} = 0 \Rightarrow \quad f = \sqrt{\beta\lambda_0/2c}$$

  - Case 1.1: $\sqrt{\beta\lambda_0/2c} < \sqrt{\lambda_0/\alpha}$
    - possible minimizer $f^* = \sqrt{\lambda_0/\alpha}$
  - Case 1.2: $\sqrt{\beta\lambda_0/2c} \geq \sqrt{\lambda_0/\alpha}$
    - possible minimizer $f^* = \sqrt{\beta\lambda_0/2c}$

Extension I: Service-sensitive demand

- Problem II: min total system cost

$$\min_{f} \quad TC_{\text{all}}(f) = TC_{\text{shuttle}}(f) + \big(\lambda_0 - \lambda(f)\big)(\beta\tau_0 + c_0)$$

- Case 2: partial demand is served by shuttles, i.e., $f < \sqrt{\lambda_0/\alpha}$, and unserved demand is not costly, i.e., $\beta\tau_0 + c_0 < \beta\tau$

$$\min_{f} \quad TC_{\text{all}}(f) = cf + \beta\lambda(f)\left(\frac{1}{2f} + \tau\right) + \big(\lambda_0 - \lambda(f)\big)(\beta\tau_0 + c_0)$$

$$\frac{\partial TC_{\text{all}}(f)}{\partial f} = c + \frac{\alpha\beta}{2} - 2\alpha[\beta(\tau_0 - \tau) + c_0]f > 0$$

  - possible minimizer $f^* = 0$

Extension I: Service-sensitive demand

- Problem II: min total system cost

$$\min_{f} \quad TC_{\text{all}}(f) = TC_{\text{shuttle}}(f) + \big(\lambda_0 - \lambda(f)\big)(\beta\tau_0 + c_0)$$

- Case 3: partial demand is served by shuttles, i.e., $f < \sqrt{\lambda_0/\alpha}$, and unserved demand is costly, i.e., $\beta\tau_0 + c_0 > \beta\tau$

$$\min_{f} \quad TC_{\text{all}}(f) = cf + \beta\lambda(f)\left(\frac{1}{2f} + \tau\right) + \big(\lambda_0 - \lambda(f)\big)(\beta\tau_0 + c_0)$$

$$\frac{\partial TC_{\text{all}}(f)}{\partial f} = c + \frac{\alpha\beta}{2} - 2\alpha[\beta(\tau_0 - \tau) + c_0]f$$

- local maximizer $\frac{\partial TC_{\text{all}}(f)}{\partial f} = 0 \Rightarrow f^{+} = \frac{2c+\alpha\beta}{4\alpha[\beta(\tau_0-\tau)+c_0]} > 0$

Extension I: Service-sensitive demand

- Problem II: min total system cost

$$\min_f \quad TC_{\text{all}}(f) = TC_{\text{shuttle}}(f) + \big(\lambda_0 - \lambda(f)\big)(\beta\tau_0 + c_0)$$

- Case 3: partial demand is served by shuttles, i.e., $f < \sqrt{\lambda_0/\alpha}$, and unserved demand is costly, i.e., $\beta\tau_0 + c_0 > \beta\tau$

$$\min_f \quad TC_{\text{all}}(f) = cf + \beta\lambda(f)\left(\frac{1}{2f} + \tau\right) + \big(\lambda_0 - \lambda(f)\big)(\beta\tau_0 + c_0)$$

$$\frac{\partial TC_{\text{all}}(f)}{\partial f} = c + \frac{\alpha\beta}{2} - 2\alpha[\beta(\tau_0 - \tau) + c_0]f$$

  - Case 3.1: $f^+ \geq \sqrt{\lambda_0/\alpha}$
    - possible minimizer $f^* = 0$
  - Case 3.2: $0 < f^+ < \sqrt{\lambda_0/\alpha}$
    - possible minimizer $f^* = \arg\min\{TC_{\text{all}}(0), TC_{\text{all}}\big(\sqrt{\lambda_0/\alpha}\big)\}$

Extension I: Service-sensitive demand

- Problem II: min total system cost

$$\min_f \quad TC_{\text{all}}(f) = TC_{\text{shuttle}}(f) + \big(\lambda_0 - \lambda(f)\big)(\beta\tau_0 + c_0)$$

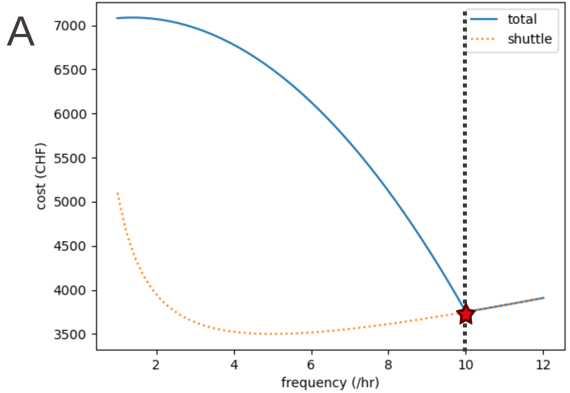
- Summary

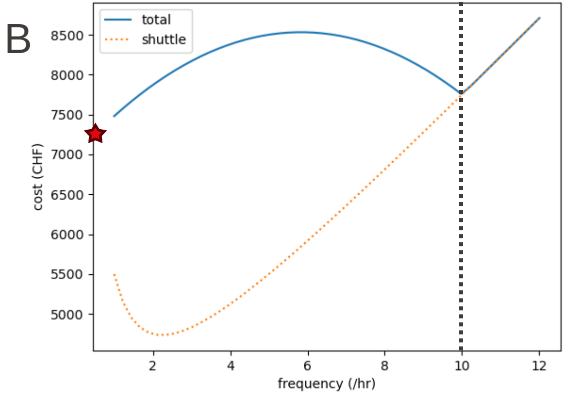
Case	Possible f^*
1.1	$\sqrt{\lambda_0/\alpha}$
1.2	$\sqrt{\beta\lambda_0/2c}$
2	0
2.1	0
2.2	0 or $\sqrt{\lambda_0/\alpha}$

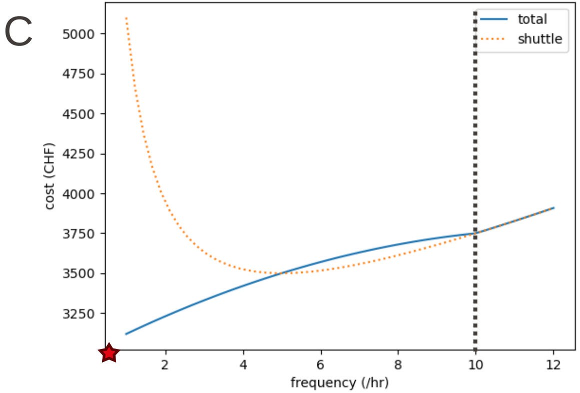
- Condition 1 (**C1**): $\sqrt{\beta\lambda_0/2c} < \sqrt{\lambda_0/\alpha}$
  - the optimal frequency of base model with demand λ_0 is lower than the threshold frequency
- Condition 2 (**C2**): $0 < \frac{2c+\alpha\beta}{4\alpha[\beta(\tau_0-\tau)+c_0]} < \sqrt{\lambda_0/\alpha}$
  - the frequency that yields the max cost is smaller than the threshold frequency
- Condition 3 (**C3**): $TC_{\text{all}}(0) < TC_{\text{all}}\left(\sqrt{\lambda_0/\alpha}\right)$
  - the system cost without the shuttle service is lower than that at the threshold frequency

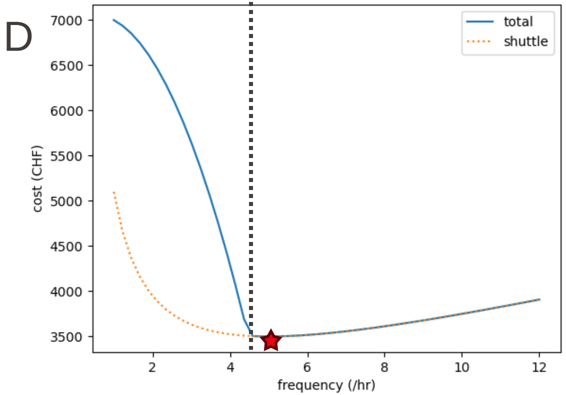
Extension I: Service-sensitive demand

- Graphical illustration of optimal solution









	C1	C2	C3	f^*
A	Y	Y	N	$\sqrt{\lambda_0/\alpha}$
B	Y	Y	Y	0
C	Y	N	Y	0
D	N			$\sqrt{\beta\lambda_0/2c}$

★ optimal

TC Ext. 1

TC Base

Extension II: Limited vehicle capacity

- Unserved demand
  - Passenger abandon queue if not able to board the first shuttle
  - Unserved passengers take backup option with travel time τ_0
  - Penalty for each unserved traveler

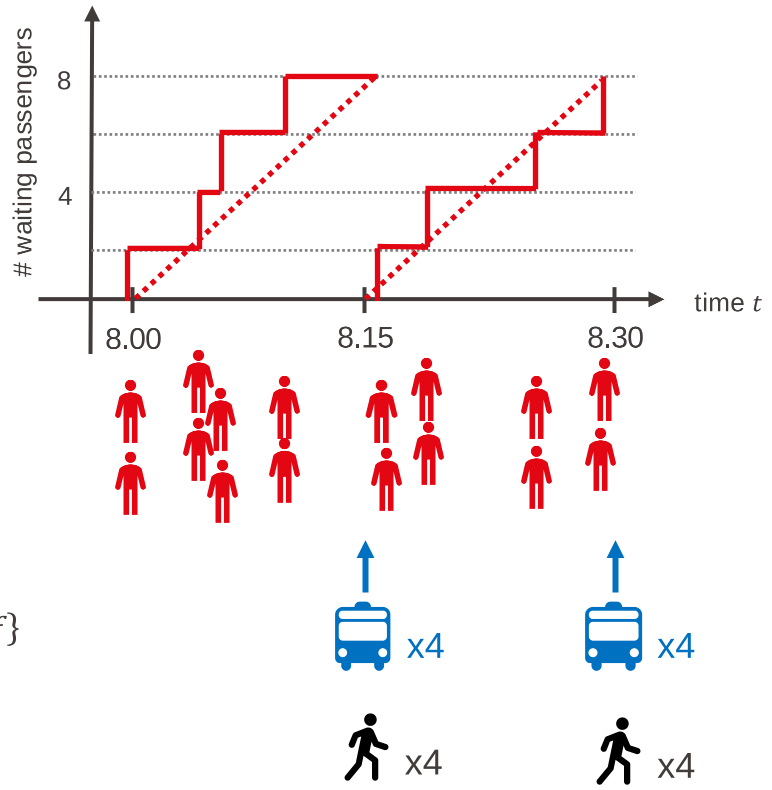
$$\rho = \beta\left(\frac{1}{2f} + \tau_0\right) - \beta\left(\frac{1}{2f} + \tau\right) = \beta(\tau_0 - \tau)$$

  - Penalty of each shuttle and each hour

$$P_{\text{shuttle}}(f,\kappa) = \rho \max\left\{0, \frac{\lambda}{f} - \kappa\right\}$$

$$P(f,\kappa) = f P_{\text{shuttle}}(f,\kappa) = \rho \max\{0, \lambda - \kappa f\}$$

Queue at origin



Extension II: Limited vehicle capacity

- Problem I: fixed vehicle capacity

$$\min_{f} \quad TC_{\text{cap}}(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f, \kappa)$$

- Problem II: jointly design frequency and vehicle capacity

$$\min_{f,\kappa} \quad TC_{\text{cap}}(f, \kappa) = c(\kappa)f + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f, \kappa)$$

$$c(\kappa) = k_0 + k_1\kappa$$

  - k_0: fixed cost (CHF)
  - k_0: variable cost (CHF/veh)

- ***Q: What are solutions to these two problems?***

Extension II: Limited vehicle capacity

- Problem I: fixed vehicle capacity

$$\min_f \quad TC_{\text{cap}}(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f, \kappa)$$

- Case 1: capacity is sufficient, i.e., $\kappa f \geq \lambda$

$$\min_f \quad TC_{\text{cap}}(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right)$$

$$\frac{\partial TC_{\text{cap}}(f)}{\partial f} = c - \frac{\beta\lambda}{2f^2} = 0 \;\Rightarrow\; f = \sqrt{\frac{\beta\lambda}{2c}}$$

  - Case 1.1: $\sqrt{\beta\lambda/2c} < \lambda/\kappa$
    - possible minimizer $f^* = \lambda/\kappa$
  - Case 1.2: $\sqrt{\beta\lambda/2c} \geq \lambda/\kappa$
    - possible minimizer $f^* = \sqrt{\beta\lambda/2c}$

Extension II: Limited vehicle capacity

- Problem I: fixed vehicle capacity

$$\min_{f} \quad TC_{\text{cap}}(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f, \kappa)$$

  - Case 2: capacity is insufficient, i.e., $\kappa f < \lambda$, and unserved demand is more costly than unit vehicle capacity, i.e., $\beta(\tau_0 - \tau)\kappa > c$

$$\min_{f} \quad TC_{\text{cap}}(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right) + \beta(\tau_0 - \tau)(\lambda - \kappa f)$$

$$\frac{\partial TC_{\text{cap}}(f)}{\partial f} = c - \beta(\tau_0 - \tau)\kappa - \frac{\beta\lambda}{2f^2} < 0$$

    - possible minimizer $f^* = \lambda/\kappa$

Extension II: Limited vehicle capacity

- Problem I: fixed vehicle capacity

$$\min_f \quad TC_{\text{cap}}(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f, \kappa)$$

- Case 3: capacity is insufficient, i.e., $\kappa f < \lambda$, and unserved demand is less costly than unit vehicle capacity, i.e., $\beta(\tau_0 - \tau)\kappa \leq c$

$$\min_f \quad TC_{\text{cap}}(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right) + \beta(\tau_0 - \tau)(\lambda - \kappa f)$$

$$\frac{\partial TC_{\text{cap}}(f)}{\partial f} = c - \beta(\tau_0 - \tau)\kappa - \frac{\beta\lambda}{2f^2} = 0 \Rightarrow \quad f = \sqrt{\frac{\beta\lambda}{2[c - \beta(\tau_0 - \tau)\kappa]}}$$

  - Case 3.1: $\sqrt{\frac{\beta\lambda}{2[c-\beta(\tau_0-\tau)\kappa]}} < \lambda/\kappa$
    - possible minimizer $f^* = \sqrt{\frac{\beta\lambda}{2[c-\beta(\tau_0-\tau)\kappa]}}$
  - Case 3.2: $\sqrt{\frac{\beta\lambda}{2[c-\beta(\tau_0-\tau)\kappa]}} \geq \lambda/\kappa$
    - possible minimizer $f^* = \lambda/\kappa$

Extension II: Limited vehicle capacity

- Problem I: fixed vehicle capacity

$$\min_{f} \quad TC_{\text{cap}}(f) = cf + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f, \kappa)$$

- Summary

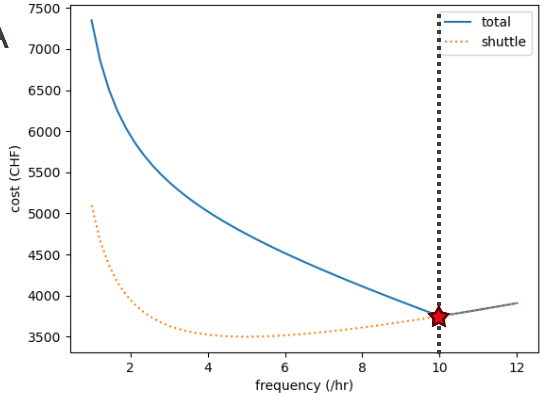
Case	Possible f^*
1.1	λ/κ
1.2	$\sqrt{\beta\lambda/2c}$
2	λ/κ
2.1	$\sqrt{\frac{\beta\lambda}{2[c-\beta(\tau_0-\tau)\kappa]}}$
2.2	λ/κ

- Condition 1 (**C1**): $\sqrt{\beta\lambda/2c} < \lambda/\kappa$
  - the optimal frequency of base model is lower than the threshold frequency
- Condition 2 (**C2**): $\beta(\tau_0 - \tau) > c/\kappa$
  - unserved demand is more costly than unit vehicle capacity
- Condition 3 (**C3**): $\sqrt{\frac{\beta\lambda}{2[c-\beta(\tau_0-\tau)\kappa]}} < \lambda/\kappa$
  - the optimal frequency considering unserved model is lower than the threshold frequency

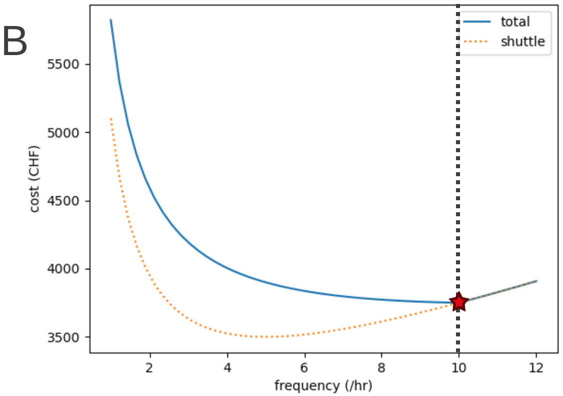
Extension II: Limited vehicle capacity

- Graphical illustration of optimal solution

A



B



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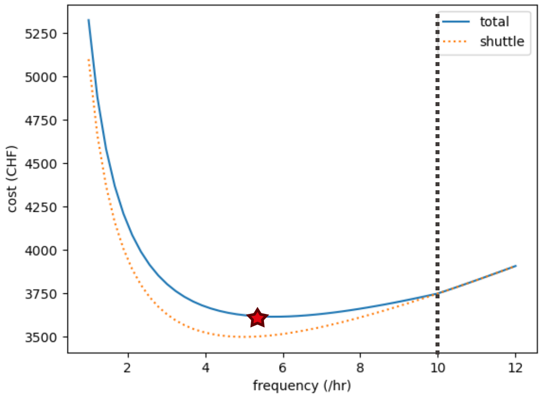

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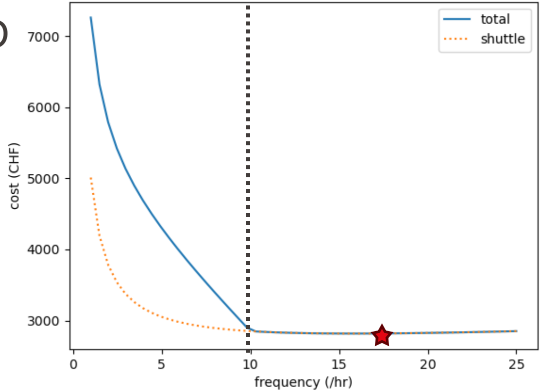

	C1	C2	C3	f^*
A	Y	N		λ/κ
B	Y	Y	N	λ/κ
C	Y	Y	Y	$\sqrt{\dfrac{\beta\lambda}{2[c-\beta(\tau_0-\tau)\kappa]}}$
D	N			$\sqrt{\beta\lambda_0/2c}$

★ optimal
— TC Ext. II
··· TC Base

Extension II: Limited vehicle capacity

- Problem II: jointly design frequency and vehicle capacity

$$\min_{f,\kappa} \quad TC_{\text{cap}}(f,\kappa) = c(\kappa)f + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f,\kappa)$$

- Case 1: capacity is sufficient, i.e., $\kappa f \geq \lambda$

$$\min_{f,\kappa} \quad TC_{\text{cap}}(f,\kappa) = (k_0 + k_1\kappa)f + \beta\lambda\left(\frac{1}{2f} + \tau\right)$$

$$\frac{\partial TC_{\text{cap}}(f,\kappa)}{\partial \kappa} = k_1 f > 0 \Rightarrow \quad \kappa^* = \lambda/f$$ plugging in $TC_{\text{cap}}(f,\kappa)$

$$\min_{f} \quad TC_{\text{cap}}(f) = k_0 f + k_1\lambda + \beta\lambda\left(\frac{1}{2f} + \tau\right)$$

- Possible minimizer $f^* = \sqrt{\frac{\beta\lambda}{2k_0}}, \kappa^* = \sqrt{\frac{2k_0\lambda}{\beta}}$

Extension II: Limited vehicle capacity

- Problem II: jointly design frequency and vehicle capacity

$$\min_{f,\kappa} \quad TC_{\text{cap}}(f,\kappa) = c(\kappa)f + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f,\kappa)$$

- Case 2: capacity is insufficient, i.e., $\kappa f < \lambda$, and unserved demand is more costly than unit vehicle capacity, i.e., $\beta(\tau_0 - \tau) \geq k_1$

$$\min_{f,\kappa} \quad TC_{\text{cap}}(f,\kappa) = (k_0 + k_1\kappa)f + \beta\lambda\left(\frac{1}{2f} + \tau\right) + \beta(\tau_0 - \tau)(\lambda - \kappa f)$$

$$\frac{\partial TC_{\text{cap}}(f,\kappa)}{\partial \kappa} = [k_1 - \beta(\tau_0 - \tau)]f < 0 \;\Rightarrow\; \kappa^* = \lambda/f$$ plugging in $TC_{\text{cap}}(f,\kappa)$

$$\min_{f} \quad TC_{\text{cap}}(f) = k_0 f + k_1\lambda + \beta\lambda\left(\frac{1}{2f} + \tau\right)$$

  - Possible minimizer $f^* = \sqrt{\frac{\beta\lambda}{2k_0}}, \kappa^* = \sqrt{\frac{2k_0\lambda}{\beta}}$

Extension II: Limited vehicle capacity

- Problem II: jointly design frequency and vehicle capacity

$$\min_{f,\kappa} \quad TC_{\text{cap}}(f,\kappa) = c(\kappa)f + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f,\kappa)$$

  - Case 3: capacity is insufficient, i.e., $\kappa f < \lambda$, and unserved demand is less costly than unit vehicle capacity, i.e., $\beta(\tau_0 - \tau) < k_1$

$$\min_{f,\kappa} \quad TC_{\text{cap}}(f,\kappa) = (k_0 + k_1\kappa)f + \beta\lambda\left(\frac{1}{2f} + \tau\right) + \beta(\tau_0 - \tau)(\lambda - \kappa f)$$

$$\frac{\partial TC_{\text{cap}}(f,\kappa)}{\partial \kappa} = [k_1 - \beta(\tau_0 - \tau)]f > 0 \;\Rightarrow\; \kappa^* = 0 \text{ plugging in } TC_{\text{cap}}(f,\kappa)$$

$$\min_{f} \quad TC_{\text{cap}}(f) = k_0 f + \beta\lambda\left(\frac{1}{2f} + \tau\right) + \lambda\beta(\tau_0 - \tau)$$

    - Possible minimizer $f^* = \sqrt{\frac{\beta\lambda}{2k_0}}, \kappa^* = 0$

- ***Q: Is this solution feasible? Why is it mathematically correct?***

Extension II: Limited vehicle capacity

- Problem II: jointly design frequency and vehicle capacity

$$\min_{f,\kappa} \quad TC_{\text{cap}}(f,\kappa) = c(\kappa)f + \beta\lambda\left(\frac{1}{2f} + \tau\right) + P(f,\kappa)$$

- Summary

Case	Possible f^*	Possible κ^*
1	$\sqrt{\beta\lambda/2k_0}$	$\sqrt{2k_0\lambda/\beta}$
2	$\sqrt{\beta\lambda/2k_0}$	$\sqrt{2k_0\lambda/\beta}$
3	0	0

- Condition: $\beta(\tau_0 - \tau) < k_1$
  - unserved demand is more costly than unit vehicle capacity

Questions?