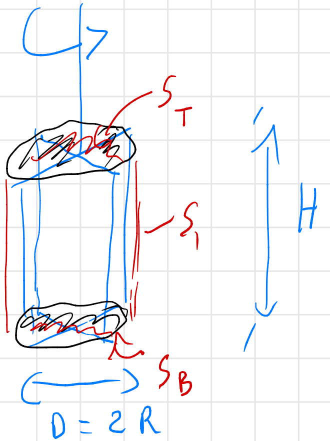
WEEK 6

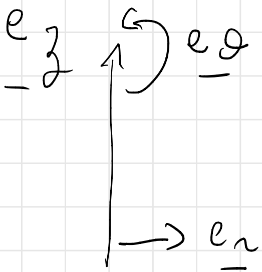
06 2020

SCISSOMÈTRE

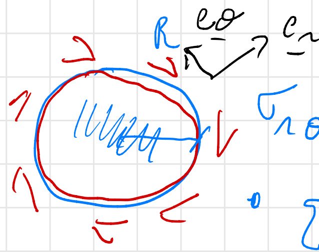


- Sols fins
- Réponse non-drainée

$$c_u = f(M_{max})$$



VUE DESSUS



$$\sigma_{r\theta} = c_u$$

- $[[\dot{u}_i]] = \dot{\psi}\, r\, \underline{e}_\theta$

APPROCHE CINÉMATIQUE

- PUISSANCE DES EFFORTS EXT.

$$P_{ext}(\dot{\psi}) = M \cdot \dot{\psi}$$

PUISSANCE DISSIPÉE MAX.

S_{TOP} / S_{BOT}.

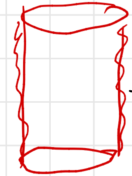
$$T_i = [\sigma_{rz}, \sigma_{\theta z}, \sigma_{zz}]$$

$$[[u_i]] = r\,\dot{\varphi}\,\underline{e}_\theta \qquad \sigma_{\theta z} = c_u$$

$$P^{S_{TOP}}_{dissipée} = \int_0^R \int_0^{2\pi} \underbrace{c_u\, r\, \dot{\varphi}}_{T_i \cdot [[\dot{u}_i]]} \; r\,dr\,d\theta$$

$$P^{S_{TOP}}_{dissipée}(\dot{\varphi}) = 2\pi\,\dot{\varphi}\,c_u\,\frac{R^3}{3} = P^{BOT}_{dissipée}(\dot{\varphi})$$

S_p sur le Perimètre



$\longrightarrow \underline{n} = \underline{e_r}$

$T_i \cdot [[v_i]] = \overset{Cu}{\sigma_{r\theta}} \times (R \cdot \dot{\varphi})$

$R\,d\theta\,dz$

$$P^{Sp}_{Dissipée}(\dot{\varphi}) = \int_0^H \int_0^{2\pi} C_u R \dot{\varphi} \; R\,d\theta\,dz$$

$$= 2\pi H C_u \dot{\varphi} R^2$$

$$P_{Dissipée}(\dot{\varphi}) = 2 P^{TOP}_{Diss.}(\dot{\varphi}) + P^{Sp}_{Dissipée}(\dot{\varphi})$$

$$= \frac{4}{3}\pi R^3 C_u \dot{\varphi} + 2\pi R^2 C_u \dot{\varphi} H$$

$$P_{Dissipée} = 2\pi R^2 C_u \dot{\varphi} \left(H + \frac{2R}{3} \right) = P_{ext}(\dot{\varphi}) = M\dot{\varphi}$$

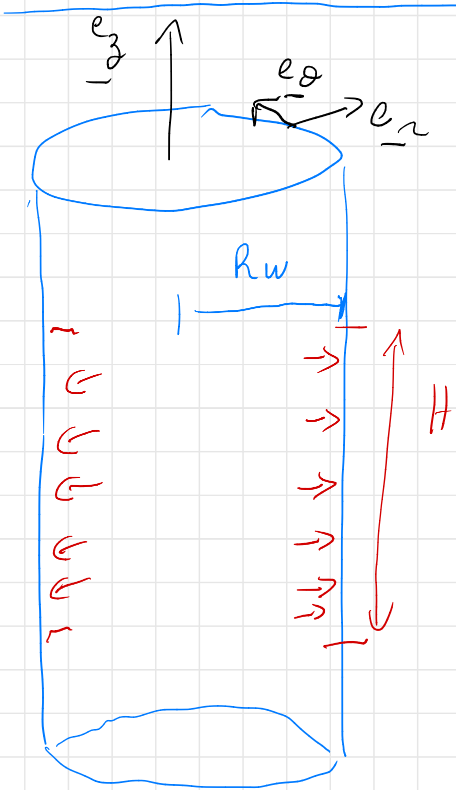
ON OBTIENT LA BORNE SUP.

$$\hookrightarrow \quad M = \frac{\pi D^2}{2} C_u \left(H + \frac{D}{3} \right)$$

$$D = 2R$$

$$\hookrightarrow \quad \boxed{C_u = \frac{6M}{\pi D^2 (3H + D)}}$$

PRESSIOMETRE



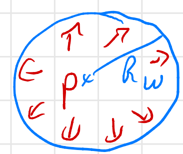
PAs à l'echelle

HYPOTHESES

1- Axisymmetrie (INDEPENDANT de θ)

2- $H \gg R_w \Rightarrow$ DEF. PLANE selon $\underline{e}_z$ (INDEPENDANT de z)

VUE de DESSUS



C.L. en Traction en $r = R_w$

$t_i = \sigma_{ij} n_j$, $\underline{n} = \underline{e}_r$

$$\begin{cases} \sigma_{rr} = p \\ \sigma_{r\theta} = 0 \end{cases}$$

C.L à l'∞ $\lim_{r \to \infty} u_i = 0$

Forme de $U_i = \begin{cases} u_r = u(r) \\ u_\theta = 0 \end{cases}$

*) TENSEUR de DEF.

$$\varepsilon_{rr} = \frac{du}{dr} \qquad \varepsilon_{\theta\theta} = \frac{u}{r} \qquad \varepsilon_{r\theta} = 0$$

*) EQUATIONS D'EQUILIBRE EN POLAIRE

Avec indépendance selon θ

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \qquad \text{selon } \underline{e_r}$$

$$\checkmark \; \frac{\partial \sigma_{\theta r}}{\partial r} - \frac{2\sigma_{\theta r}}{r} = 0 \qquad \text{selon } \underline{e_\theta}$$

$$\hookrightarrow \sigma_{\theta r} = C r^2 = 0$$

la C.L. en $r = R_w$ $\quad \sigma_{\theta r}(r = R_w) = 0$

$$\hookrightarrow C = 0$$

ÉLASTICITÉ DU SOL

On note $\Delta\sigma_{ij} = \sigma_{ij} - \sigma_{ij}^0$

$$\Delta\sigma_{rr} = \sigma_{rr} - \sigma_h^0 \qquad \Delta\sigma_{\theta\theta} = \sigma_{\theta\theta} - \sigma_h^0$$

$\Delta P = P - \underset{\sigma_h^0}{\underset{\shortparallel}{P^0}}$ ⟵ PRESSION DANS LA CHAMBRE

On écrit les DEF par RAPPORT à l'ÉTAT 0

$$\Delta\sigma_{rr} = \frac{E}{1+\nu}\varepsilon_{rr} + \frac{\nu E}{(1+\nu)(1-2\nu)}\left(\varepsilon_{rr} + \varepsilon_{\theta\theta}\right)$$

$$\Delta\sigma_{\theta\theta} = \text{〃}\ \varepsilon_{\theta\theta} + \text{〃}\ \left(\varepsilon_{rr} + \varepsilon_{\theta\theta}\right)$$

ON INTRODUIT DANS L'ÉQUATION D'ÉQUILIBRE

$$\Delta\sigma_{rr} - \Delta\sigma_{\theta\theta} = \frac{E}{1+\nu}\left(\varepsilon_{rr} - \varepsilon_{\theta\theta}\right)$$

$$\left(\frac{du}{dr} - \frac{u}{r}\right)$$

$$\Delta\sigma_{rr} = \frac{E}{1+\nu}\left[\frac{du}{dr} + \frac{\nu}{(1+\nu)(1-2\nu)}\left[\frac{du}{dr} + \frac{u}{r}\right]\right]$$

$$\frac{\partial\Delta\sigma_{rr}}{\partial r} + \frac{\Delta\sigma_{rr} - \Delta\sigma_{\theta\theta}}{r} = 0$$

$$\underbrace{\frac{d^2u}{dr^2} - \frac{u}{r^2} + \frac{du}{dr}}_{0} + \underbrace{\frac{\nu}{(1+\nu)(1-2\nu)}\left[\frac{d^2u}{dr^2} + \frac{du}{dr} - \frac{u}{r^2}\right]}_{0} = 0$$

$$\frac{d^2u}{dr^2}+\frac{du}{dr}-\frac{u}{r^2}=0 \;\rightarrow\; \boxed{u(r)=\frac{C_1}{r}}+\underbrace{C_2\, r}$$

$$C_2=0$$

$$u(r\to\infty)=0$$

C. L. $\quad \Delta\sigma_{rr}(r=R_w)=\Delta P \quad (*)$

$$\Delta\sigma_{rr}=\frac{E}{1+\nu}\left(-\frac{C_1}{r^2}\right)+\frac{\nu E}{(1+\nu)(1-2\nu)}\underbrace{\left(\begin{array}{c}\varepsilon_{rr}+\varepsilon_{\theta\theta}\\ -\frac{C_1}{r^2}+\frac{C_1}{r^2}\end{array}\right)}_{\shortparallel\; 0}$$

$$\underline{C_1=-\frac{R_w^2\,\Delta P\,(1+\nu)}{E}}$$

$$2G=\frac{E}{1+\nu}$$

$$\Delta\sigma_{rr}=\Delta P\left(\frac{R_w}{r}\right)^2$$

$$\Delta\sigma_{\theta\theta}=-\Delta P\left(\frac{R_w}{r}\right)^2$$

$$\Delta\sigma_{rr}=-\Delta\sigma_{\theta\theta}$$

$$\Delta V = V - V_0$$

$$V_0 = H \pi R_w^2$$

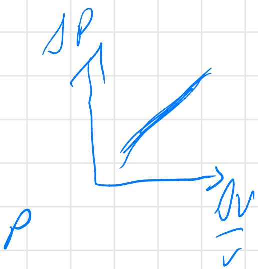
$$V = H \pi \left(R_w (1+\delta)\right)^2$$

$$u = -\frac{\Delta P}{2G} \frac{R_w^2}{r}$$

$$\delta = \frac{|u(r=R_w)|}{R_w} = \frac{\Delta P}{2G}$$

$$\frac{\Delta V}{V_0} = \frac{\Delta P}{2G}$$

la courbe

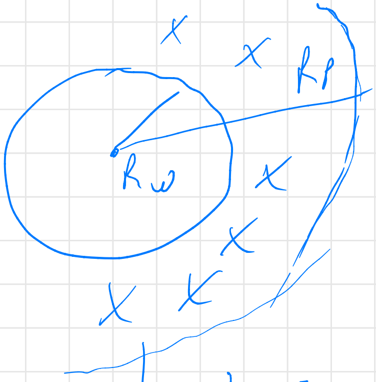


La PENTE $\frac{\Delta V}{V_0} - \Delta P$

Nous donne $2G = E_M$

Le PRESSIOMETRE MESURE $\underline{G}$

Si $\Delta P \nearrow$ ON ARRIVE A LA PLASTIFICATION

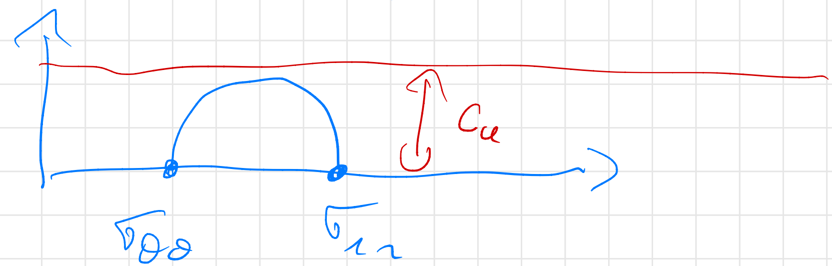


ZONE ELASTIQUE

ON A VU QUE

$\underline{e_r}$ & $\underline{e_\theta}$

DIRECTION PRINCIPALES



ZONE PLASTIQUE

$$\sigma_{rr} - \sigma_{\theta\theta} = 2 C_u$$

$$\Delta\sigma_{rr} - \Delta\sigma_{\theta\theta} = 2 C_u$$

DANS LA ZONE PLASTIQUE

$$r \in [R_w, R_p] \quad \frac{\partial \Delta\sigma_{rr}}{\partial r} + \frac{2 C_u}{r} = 0 \Rightarrow \Delta\sigma_{rr} = C_1 \ln\frac{1}{r} + C_2$$

C.2 $\Delta\sigma_{rr} = \Delta P$ en $r = R_w$

$\Delta\sigma_{rr} = c_u$ $\quad r = R_p$

$\Delta\sigma_{rr} = -\Delta\sigma_{\theta\theta}$ dans la zone élastique

$$\Delta\sigma_{rr} = c_u + 2c_u \ln\frac{R_p}{r}$$

$$\Delta\sigma_{\theta\theta} = -c_u + 2c_u \ln\frac{R_p}{r}$$

car dans la zone plastique on a $\Delta\sigma_{rr} - \Delta\sigma_{\theta\theta} = 2c_u$

On a $\Delta\sigma_{rr}(r = R_w) = \Delta P$

$$\Rightarrow \Delta P - c_u = 2c_u \ln\frac{R_p}{R_w}$$

$$\Rightarrow \boxed{\frac{R_p}{R_w} = e^{\frac{\Delta P - c_u}{2c_u}}}$$

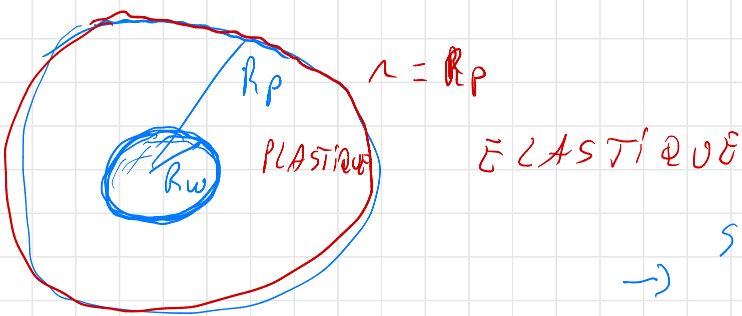
RELATION $\frac{\Delta V}{V}$ avec C_u ?

en $r = R_p$ Transition zone élastique / PLASTIQUE

$$\frac{u}{R_p} = -\frac{C_u}{2G}$$

PAS de VARIATION de volume / d'aire

$$\pi\left(R_p^2 - R_w^2\right) = \pi\left(\left(R_p - \frac{C_u}{2G}\right)^2 - R_w^2\right)$$



$$\hookrightarrow \frac{\Delta V}{V} = \frac{C_u}{G}\left(\frac{R_p}{R_w}\right)^2$$

→ Si on connait G de la phase élastique

→ On peut déterminer C_u