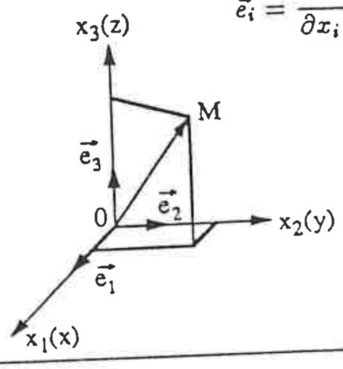
*Tableau de passage des coordonnées*

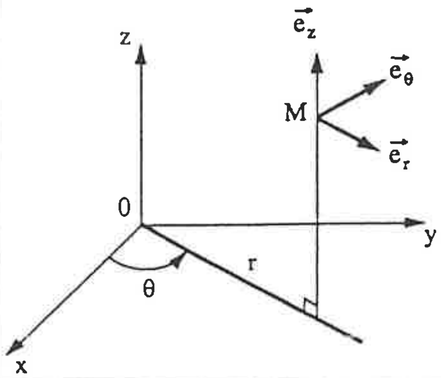
— Coordonnées cartésiennes orthonormées,

— Coordonnées cylindriques,

— Coordonnées sphériques,

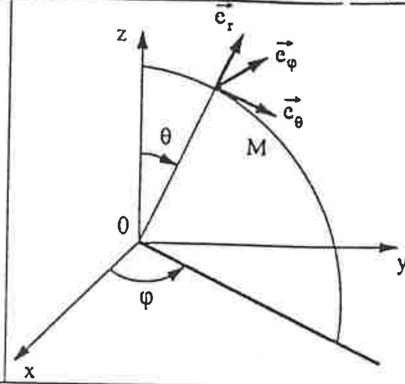
| | Coordonnées cartésiennes orthonormées |
|---|---|
| Définitions | $\vec{e}_i = \dfrac{\partial \vec{M}}{\partial x_i}$   |
| coordonnées | $\overrightarrow{OM} = x_i \vec{e}_i$ |
| champ de vecteurs $\vec{u}(M)$ | $\vec{u}(M) = u_i(M)\vec{e}_i$ |
| vecteurs unitaires de base | $d\vec{e}_i = 0$<br>$d\vec{M} = dx_i \vec{e}_i$ |
| gradient d'un scalaire $f$<br>$df = \overrightarrow{grad}\, f \cdot d\vec{M}$ | $\overrightarrow{grad}\, f = \dfrac{\partial f}{\partial x_i}\vec{e}_i = f_{,i}\vec{e}_i$ |
| gradient d'un champ de vecteurs $\vec{u}$<br>$d\vec{u} =(\text{grad } \vec{u}) \cdot d\vec{M}$<br><br>champ de déformations<br>$\epsilon = \dfrac{1}{2}(\text{grad } \vec{u} + {}^t\text{grad}\vec{u})$ | $\text{grad } \vec{u} = \dfrac{\partial u_i}{\partial x_j}\vec{e}_i \otimes \vec{e}_j$<br>$= u_{i,j}\, \vec{e}_i \otimes \vec{e}_j$<br><br>$\epsilon_{ij} = \dfrac{1}{2}(u_{i,j} + u_{j,i})$ |

| | Coordonnées cartésiennes orthonormées |
|---|---|
| divergence d'un champ de vecteurs div $\vec{u}$ =Tr(**grad** $\vec{u}$) | div $\vec{u} = u_{k,k}$ |
| laplacien d'un scalaire $\Delta f$ =div ($\overrightarrow{\text{grad}}\ f$) | $\Delta f = \sum_i \frac{\partial^2 f}{\partial x_i^2} = f_{,ii}$ |
| laplacien d'un champ de vecteurs $\vec{\Delta}\vec{u}$ =$\overrightarrow{\text{div}}$ (**grad** $\vec{u}$) | $\vec{\Delta}\vec{u} = u_{i,kk}\vec{e_i}$ |
| champ de tenseurs du second ordre **T** | $\mathbf{T} = T_{ij}\vec{e_i} \otimes \vec{e_j}$ |
| divergence d'un champ de tenseurs du second ordre symétriques $\overrightarrow{\text{div}}\ \boldsymbol{\sigma}$ | $\overrightarrow{\text{div}}\ \boldsymbol{\sigma} = \sigma_{ij,j}\vec{e_i}$ |
| changement de coordonnées pour les vecteurs de base | |
| changement de coordonnées pour un vecteur $\vec{u}$ | $\vec{u} = u_i\vec{e_i}$ |
| changement de coordonnées pour un tenseur du second ordre symétrique $\epsilon$ | $\epsilon = \epsilon_{ij}\vec{e_i} \otimes \vec{e_j}$ |

| | Coordonnées cylindriques |
|---|---|
| Définitions |  $\begin{cases} \vec{e}_r = \dfrac{\partial \vec{M}}{\partial r} \\ \vec{e}_\theta = \dfrac{1}{r}\dfrac{\partial \vec{M}}{\partial \theta} \\ \vec{e}_z = \dfrac{\partial \vec{M}}{\partial z} \end{cases}$ |
| coordonnées | $\overline{OM} = r\vec{e}_r + z\vec{e}_z$ |
| champ de vecteurs $\vec{u}(M)$ | $\vec{u}(M) = u_r\vec{e}_r + u_\theta\vec{e}_\theta + u_z\vec{e}_z$ |
| vecteurs unitaires de base | $d\vec{e}_r = d\theta\vec{e}_\theta \quad d\vec{e}_\theta = -d\theta\vec{e}_r \quad d\vec{e}_z = 0$ <br> $d\vec{M} = dr\vec{e}_r + rd\theta\vec{e}_\theta + dz\vec{e}_z$ |
| gradient d'un scalaire $f$ <br> $df = \overrightarrow{grad}\, f \cdot d\vec{M}$ | $\overrightarrow{grad}\, f = \dfrac{\partial f}{\partial r}\vec{e}_r + \dfrac{1}{r}\dfrac{\partial f}{\partial \theta}\vec{e}_\theta + \dfrac{\partial f}{\partial z}\vec{e}_z$ |
| gradient d'un champ de vecteurs $\vec{u}$ <br> $d\vec{u} =(\text{grad } \vec{u}) \cdot d\vec{M}$ | $\text{grad } \vec{u} = \begin{bmatrix} \dfrac{\partial u_r}{\partial r} & \dfrac{1}{r}(\dfrac{\partial u_r}{\partial \theta} - u_\theta) & \dfrac{\partial u_r}{\partial z} \\ \dfrac{\partial u_\theta}{\partial r} & \dfrac{1}{r}(\dfrac{\partial u_\theta}{\partial \theta} + u_r) & \dfrac{\partial u_\theta}{\partial z} \\ \dfrac{\partial u_z}{\partial r} & \dfrac{1}{r}\dfrac{\partial u_z}{\partial \theta} & \dfrac{\partial u_z}{\partial z} \end{bmatrix}$ |
| champ de déformations <br> $\epsilon = \dfrac{1}{2}(\text{grad } \vec{u} + {}^t\text{grad}\vec{u})$ | $\epsilon = \begin{bmatrix} \dfrac{\partial u_r}{\partial r} & \dfrac{1}{2}\dfrac{\partial u_\theta}{\partial r} + \dfrac{1}{2r}(\dfrac{\partial u_r}{\partial \theta} - u_\theta) & \dfrac{1}{2}(\dfrac{\partial u_r}{\partial z} + \dfrac{\partial u_z}{\partial r}) \\ & \dfrac{1}{r}(\dfrac{\partial u_\theta}{\partial \theta} + u_r) & \dfrac{1}{2}(\dfrac{\partial u_\theta}{\partial z} + \dfrac{1}{r}\dfrac{\partial u_z}{\partial \theta}) \\ \text{sym} & & \dfrac{\partial u_z}{\partial z} \end{bmatrix}$ |

| | Coordonnées cylindriques |
|---|---|
| divergence d'un champ de vecteurs div $\vec{u}$ =Tr(**grad**$\vec{u}$) | $\mathrm{div}\ \vec{u} = \frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}$ |
| laplacien d'un scalaire $\Delta f$ =div($\overrightarrow{\mathrm{grad}}\ f$) | $\Delta f = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$ |
| laplacien d'un champ de vecteurs $\vec{\Delta}\vec{u}$ =$\overrightarrow{\mathrm{div}}$ (**grad** $\vec{u}$) | $\vec{\Delta}\vec{u} = (\Delta u_r - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r^2})\vec{e}_r$ $+(\Delta u_\theta + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2})\vec{e}_\theta + \Delta u_z \vec{e}_z$ |
| champ de tenseurs du second ordre **T** | $\mathbf{T} = T_{rr}\vec{e}_r \otimes \vec{e}_r + T_{r\theta}\vec{e}_r \otimes \vec{e}_\theta + T_{rz}\vec{e}_r \otimes e_z$ $+T_{\theta r}\vec{e}_\theta \otimes \vec{e}_r + T_{\theta\theta}\vec{e}_\theta \otimes \vec{e}_\theta + T_{\theta z}\vec{e}_\theta \otimes \vec{e}_z$ $+T_{zr}\vec{e}_z \otimes \vec{e}_r + T_{z\theta}\vec{e}_z \otimes \vec{e}_\theta + T_{zz}\vec{e}_z \otimes \vec{e}_z$ |
| divergence d'un champ de tenseurs du second ordre symétriques $\overrightarrow{\mathrm{div}}\ \boldsymbol{\sigma}$ | $\overrightarrow{\mathrm{div}}\boldsymbol{\sigma} = \left(\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r}\right)\vec{e}_r$ $+\left(\frac{\partial \sigma_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{r\theta}}{r}\right)\vec{e}_\theta$ $+\left(\frac{\partial \sigma_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{z\theta}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{zr}}{r}\right)\vec{e}_z$ |
| changement de coordonnées pour les vecteurs de base | $\vec{e}_r = \cos\theta\vec{e}_1 + \sin\theta\vec{e}_2$ $\vec{e}_\theta = -\sin\theta\vec{e}_1 + \cos\theta\vec{e}_2$ $\vec{e}_z = \vec{e}_3$ $\vec{e}_1 = \cos\theta\vec{e}_r - \sin\theta\vec{e}_\theta$ $\vec{e}_2 = \sin\theta\vec{e}_r + \cos\theta\vec{e}_\theta$ $\vec{e}_3 = \vec{e}_z$ |

| | Coordonnées cylindriques |
|---|---|
| changement de coordonnées pour un vecteur $\vec{u}$ | $u_1 = u_r \cos\theta - u_\theta \sin\theta$<br>$u_2 = u_r \sin\theta + u_\theta \cos\theta$<br>$u_3 = u_z$<br><br>$u_r = u_1 \cos\theta + u_2 \sin\theta$<br>$u_\theta = -u_1 \sin\theta + u_2 \cos\theta$<br>$u_z = u_3$ |
| changement de coordonnées pour un tenseur du second ordre symétrique $\epsilon$ | $\epsilon_{11} = \epsilon_{rr}\cos^2\theta + \epsilon_{\theta\theta}\sin^2\theta - 2\epsilon_{r\theta}\sin\theta\cos\theta$<br>$\epsilon_{22} = \epsilon_{rr}\sin^2\theta + \epsilon_{\theta\theta}\cos^2\theta + 2\epsilon_{r\theta}\sin\theta\cos\theta$<br>$\epsilon_{33} = \epsilon_{zz}$<br>$\epsilon_{12} = (\epsilon_{rr} - \epsilon_{\theta\theta})\sin\theta\cos\theta + \epsilon_{r\theta}(\cos^2\theta - \sin^2\theta)$<br>$\epsilon_{13} = \epsilon_{rz}\cos\theta - \epsilon_{\theta z}\sin\theta$<br>$\epsilon_{23} = \epsilon_{rz}\sin\theta + \epsilon_{\theta z}\cos\theta$<br><br>$\epsilon_{rr} = \epsilon_{11}\cos^2\theta + \epsilon_{22}\sin^2\theta + 2\epsilon_{12}\sin\theta\cos\theta$<br>$\epsilon_{\theta\theta} = \epsilon_{11}\sin^2\theta + \epsilon_{22}\cos^2\theta - 2\epsilon_{12}\sin\theta\cos\theta$<br>$\epsilon_{zz} = \epsilon_{33}$<br>$\epsilon_{r\theta} = (\epsilon_{22} - \epsilon_{11})\sin\theta\cos\theta + \epsilon_{12}(\cos^2\theta - \sin^2\theta)$<br>$\epsilon_{rz} = \epsilon_{13}\cos\theta + \epsilon_{23}\sin\theta$<br>$\epsilon_{\theta z} = -\epsilon_{13}\sin\theta + \epsilon_{23}\cos\theta$ |

| | Coordonnées sphériques |
|---|---|
| **Définitions** |  $\vec e_r = \frac{\partial \vec M}{\partial r}$, $\vec e_\theta = \frac{1}{r}\frac{\partial \vec M}{\partial \theta}$, $\vec e_\varphi = \frac{1}{r\sin\theta}\frac{\partial \vec M}{\partial \varphi}$ |
| coordonnées | $\overrightarrow{OM} = r\vec e_r$ |
| champ de vecteurs $\vec u(M)$ | $\vec u(M) = u_r\vec e_r + u_\theta\vec e_\theta + u_\varphi\vec e_\varphi$ |
| vecteurs unitaires de base | $d\vec e_r = d\theta\vec e_\theta + \sin\theta d\varphi\vec e_\varphi$<br>$d\vec e_\theta = -d\theta\vec e_r + \cos\theta d\varphi\vec e_\varphi$<br>$d\vec e_\varphi = -\sin\theta d\varphi\vec e_r - \cos\theta d\varphi\vec e_\theta$<br>$d\vec M = dr\vec e_r + rd\theta\vec e_\theta + r\sin\theta d\varphi\vec e_\varphi$ |
| gradient d'un scalaire $f$<br>$df = \overrightarrow{grad}\, f\cdot d\vec M$ | $\overrightarrow{grad}\, f = \frac{\partial f}{\partial r}\vec e_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec e_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\vec e_\varphi$ |
| gradient d'un champ de vecteurs $\vec u$<br>$d\vec u = (\mathbf{grad}\ \vec u)\cdot d\vec M$ | $\mathbf{grad}\ \vec u = \begin{bmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{r}(\frac{\partial u_r}{\partial \theta} - u_\theta) & \frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial u_r}{\partial \varphi} - u_\varphi) \\ \frac{\partial u_\theta}{\partial r} & \frac{1}{r}(\frac{\partial u_\theta}{\partial \theta} + u_r) & \frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial u_\theta}{\partial \varphi} - \frac{u_\varphi}{\mathrm{tg}\theta}) \\ \frac{\partial u_\varphi}{\partial r} & \frac{1}{r}\frac{\partial u_\varphi}{\partial \theta} & \frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\theta}{\mathrm{tg}\theta} + u_r) \end{bmatrix}$ |
| champ de déformations<br>$\epsilon = \frac{1}{2}(\text{grad } \vec u + {}^t\text{grad} \vec u)$ | $\epsilon = \begin{bmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{2}\frac{\partial u_\theta}{\partial r} + \frac{1}{2r}(\frac{\partial u_r}{\partial \theta} - u_\theta) & \frac{1}{2}\frac{\partial u_\varphi}{\partial r} + \frac{1}{2r}(\frac{1}{\sin\theta}\frac{\partial u_r}{\partial \varphi} - u_\varphi) \\ & \frac{1}{r}(\frac{\partial u_\theta}{\partial \theta} + u_r) & \frac{1}{2r}\frac{\partial u_\varphi}{\partial \theta} + \frac{1}{2r}(\frac{1}{\sin\theta}\frac{\partial u_\theta}{\partial \varphi} - \frac{u_\varphi}{\mathrm{tg}\theta}) \\ \text{sym} & & \frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\theta}{\mathrm{tg}\theta} + u_r) \end{bmatrix}$ |

| | Coordonnées sphériques |
|---|---|
| divergence d'un champ de vecteurs<br>div $\vec{u}$ =Tr(**grad** $\vec{u}$) | $\text{div}\ \vec{u} = \frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + 2\frac{u_r}{r} + \frac{1}{r\sin\theta}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\theta}{r\text{tg}\theta}$ |
| laplacien d'un scalaire<br>$\Delta f$ =div ($\overrightarrow{\text{grad}}\ f$) | $\Delta f = \frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{1}{r^2\text{tg}\theta}\frac{\partial f}{\partial \theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$ |
| laplacien d'un champ de vecteurs<br>$\vec{\Delta}\vec{u}$ =$\overrightarrow{\text{div}}$ (**grad** $\vec{u}$) | $\vec{\Delta}\vec{u} = (\Delta u_r - \frac{2u_r}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial(u_\theta\sin\theta)}{\partial\theta} - \frac{2}{r^2\sin\theta}\frac{\partial u_\varphi}{\partial\varphi})\vec{e}_r$<br>$+(\Delta u_\theta + \frac{2}{r^2}\frac{\partial u_r}{\partial\theta} - \frac{u_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\varphi}{\partial\varphi})\vec{e}_\theta$<br>$+(\Delta u_\varphi + \frac{2}{r^2\sin^2\theta}\frac{\partial u_r}{\partial\varphi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\theta}{\partial\varphi} - \frac{u_\varphi}{r^2\sin^2\theta})\vec{e}_\varphi$ |
| champ de tenseurs du second ordre **T** | $\mathbf{T}=T_{rr}\vec{e}_r\otimes\vec{e}_r + T_{r\theta}\vec{e}_r\otimes\vec{e}_\theta + T_{r\varphi}\vec{e}_r\otimes e_\varphi$<br>$+T_{\theta r}\vec{e}_\theta\otimes\vec{e}_r + T_{\theta\theta}\vec{e}_\theta\otimes\vec{e}_\theta + T_{\theta\varphi}\vec{e}_\theta\otimes\vec{e}_\varphi$<br>$+T_{\varphi r}\vec{e}_\varphi\otimes\vec{e}_r + T_{\varphi\theta}\vec{e}_\varphi\otimes\vec{e}_\theta + T_{\varphi\varphi}\vec{e}_\varphi\otimes\vec{e}_\varphi$ |
| divergence d'un champ de tenseurs du second ordre symétriques<br>$\overrightarrow{\text{div}}\ \boldsymbol{\sigma}$ | $\overrightarrow{\text{div}}\boldsymbol{\sigma} = \left[\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\sigma_{r\varphi}}{\partial\varphi} + \frac{1}{r}(2\sigma_{rr} - \sigma_{\theta\theta}\right.$<br>$\left.-\sigma_{\varphi\varphi} + \sigma_{r\theta}\text{cotg}\theta)\right]\vec{e}_r + \left[\frac{\partial\sigma_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\sigma_{\theta\varphi}}{\partial\varphi}\right.$<br>$\left.+\frac{1}{r}(\sigma_{\theta\theta}\text{cotg}\theta - \sigma_{\varphi\varphi}\text{cotg}\theta + 3\sigma_{r\theta}\right]\vec{e}_\theta$<br>$+\left[\frac{\partial\sigma_{\varphi r}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\varphi\theta}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\sigma_{\varphi\varphi}}{\partial\varphi} + \frac{1}{r}(3\sigma_{r\varphi} + \frac{2\sigma_{\theta\varphi}}{\text{tg}\theta})\right]\vec{e}_\varphi$ |
| changement de coordonnées pour les vecteurs de base | $\vec{e}_r = \sin\theta\cos\varphi\vec{e}_1 + \sin\theta\sin\varphi\vec{e}_2 + \cos\theta\vec{e}_3$<br>$\vec{e}_\theta = \cos\theta\cos\varphi\vec{e}_1 + \cos\theta\sin\varphi\vec{e}_2 - \sin\theta\vec{e}_3$<br>$\vec{e}_\varphi = -\sin\varphi\vec{e}_1 + \cos\varphi\vec{e}_2$<br><br>$\vec{e}_1 = \sin\theta\cos\varphi\vec{e}_r + \cos\theta\cos\varphi\vec{e}_\theta - \sin\varphi\vec{e}_\varphi$<br>$\vec{e}_2 = \sin\theta\sin\varphi\vec{e}_r + \cos\theta\sin\varphi\vec{e}_\theta + \cos\varphi\vec{e}_\varphi$<br>$\vec{e}_3 = \cos\theta\vec{e}_r - \sin\theta\vec{e}_\theta$ |

| | Coordonnées sphériques |
|---|---|
| changement de coordonnées | $u_1 = u_r \sin\theta\cos\varphi + u_\theta\cos\theta\cos\varphi - u_\varphi\sin\varphi$<br>$u_2 = u_r\sin\theta\sin\varphi + u_\theta\cos\theta\sin\varphi + u_\varphi\cos\varphi$<br>$u_3 = u_r\cos\theta - u_\theta\sin\theta$ |
| pour un vecteur $\vec{u}$ | $u_r = u_1\sin\theta\cos\varphi + u_2\sin\theta\sin\varphi + u_3\cos\theta$<br>$u_\theta = u_1\cos\theta\cos\varphi + u_2\cos\theta\sin\varphi - u_3\sin\theta$<br>$u_\varphi = -u_1\sin\varphi + u_2\cos\varphi$ |
| changement de coordonnées pour un tenseur du second ordre symétrique $\epsilon$ | $\epsilon_{11} = \epsilon_{rr}\sin^2\theta\cos^2\varphi + \epsilon_{\theta\theta}\cos^2\theta\cos^2\varphi + \epsilon_{\varphi\varphi}\sin^2\varphi$<br>$+2\epsilon_{r\theta}\sin\theta\ cos\theta\cos^2\varphi - 2\epsilon_{r\varphi}\sin\theta\sin\varphi\cos\varphi - 2\epsilon_{\theta\varphi}\cos\theta\sin\varphi\cos\varphi$<br>$\epsilon_{22} = \epsilon_{rr}\sin^2\theta\sin^2\varphi + \epsilon_{\theta\theta}\cos^2\theta\sin^2\varphi + \epsilon_{\varphi\varphi}\cos^2\varphi$<br>$+2\epsilon_{r\theta}\sin\theta\cos\theta\sin^2\varphi + 2\epsilon_{r\varphi}\sin\theta\sin\varphi\cos\varphi + 2\epsilon_{\theta\varphi}\cos\theta\sin\varphi\cos\varphi$<br>$\epsilon_{33} = \epsilon_{rr}\cos^2\theta + \epsilon_{\theta\theta}\sin^2\theta - 2\epsilon_{r\theta}\sin\theta\cos\theta$<br>$\epsilon_{12} = \epsilon_{rr}\sin^2\theta\sin\varphi\cos\varphi + \epsilon_{\theta\theta}\cos^2\theta\sin\varphi\cos\varphi - \epsilon_{\varphi\varphi}\sin\varphi\cos\varphi$<br>$+2\epsilon_{r\theta}\sin\theta\cos\theta\sin\varphi\cos\varphi + (\epsilon_{r\varphi}\sin\theta + \epsilon_{\theta\varphi}\cos\theta)(\cos^2\varphi - \sin^2\varphi)$<br>$\epsilon_{13} = (\epsilon_{rr} - \epsilon_{\theta\theta})\sin\theta\cos\theta\cos\varphi + \epsilon_{r\theta}(\cos^2\theta - \sin^2\theta)\cos\varphi$<br>$-\epsilon_{r\varphi}\cos\theta\sin\varphi + \epsilon_{\theta\varphi}\sin\theta\sin\varphi$<br>$\epsilon_{23} = (\epsilon_{rr} - \epsilon_{\theta\theta})\sin\theta\cos\theta\sin\varphi + \epsilon_{r\theta}(\cos^2\theta - \sin^2\theta)\sin\varphi$<br>$+\epsilon_{r\varphi}\cos\theta\cos\varphi - \epsilon_{\theta\varphi}\sin\theta\cos\varphi$ |
| | $\epsilon_{rr} = \epsilon_{11}\sin^2\theta\cos^2\varphi + \epsilon_{22}\sin^2\theta\sin^2\varphi + \epsilon_{33}\cos^2\theta$<br>$+2\epsilon_{12}\sin^2\theta\sin\varphi\cos\varphi + 2\epsilon_{13}\sin\theta\cos\theta\cos\varphi + 2\epsilon_{23}\sin\theta\cos\theta\sin\varphi$<br>$\epsilon_{\theta\theta} = \epsilon_{11}\cos^2\theta\cos^2\varphi + \epsilon_{22}\cos^2\theta\sin^2\varphi + \epsilon_{33}\sin^2\theta$<br>$+2\epsilon_{12}\cos^2\theta\sin\varphi\cos\varphi - 2\epsilon_{13}\sin\theta\cos\theta\cos\varphi - 2\epsilon_{23}\sin\theta\cos\theta\sin\varphi$<br>$\epsilon_{\varphi\varphi} = \epsilon_{11}\sin^2\varphi + \epsilon_{22}\cos^2\varphi - 2\epsilon_{12}\sin\varphi\cos\varphi$<br>$\epsilon_{r\theta} = \epsilon_{11}\sin\theta\cos\theta\cos^2\varphi + \epsilon_{22}\sin\theta\cos\theta\sin^2\varphi - \epsilon_{33}\sin\theta\cos\theta$<br>$+2\epsilon_{12}\sin\theta\cos\theta\sin\varphi\cos\varphi + (\epsilon_{13}\cos\varphi + \epsilon_{23}\sin\varphi)(\cos^2\theta - \sin^2\theta)$<br>$\epsilon_{r\varphi} = (\epsilon_{22} - \epsilon_{11})\sin\theta\sin\varphi\cos\varphi + \epsilon_{12}\sin\theta(\cos^2\varphi - \sin^2\varphi)$<br>$-\epsilon_{13}\cos\theta\sin\varphi + \epsilon_{23}\cos\theta\cos\varphi$<br>$\epsilon_{\theta\varphi} = (\epsilon_{22} - \epsilon_{11})\cos\theta\sin\varphi\cos\varphi + \epsilon_{12}\cos\theta(\cos^2\varphi - \sin^2\varphi)$<br>$+\epsilon_{13}\sin\theta\sin\varphi - \epsilon_{23}\sin\theta\cos\varphi$ |

Von Mises

$$f(\underline{\sigma}) = \frac{1}{6}\left((\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{yy}-\sigma_{33})^2 + (\sigma_{xx}-\sigma_{33})^2\right) + \sigma_{xy}^2 + \sigma_{y3}^2 + \sigma_{x3}^2$$

$\mu(3\lambda+2\mu)$

| | $E,\nu$ | $E,\mu$ | $k,\nu$ | $k,\mu$ | $\lambda,\mu$ |
|---|---|---|---|---|---|
| $E$ | $E$ | $E$ | $3(1-2\nu)k$ | $\dfrac{9k}{1+3k/\mu}$ | $\dfrac{\mu(3+2\mu/\lambda)}{1+\mu/\lambda}$ |
| $\nu$ | $\nu$ | $-1+E/2\mu$ | $\nu$ | $\dfrac{1-2\mu/3k}{2+2\mu/3k}$ | $\dfrac{1}{2(1+\mu/\lambda)}$ |
| $\mu$ | $\dfrac{E}{2(1+\nu)}$ | $\mu$ | $\dfrac{3(1-2\nu)k}{2(1+\nu)}$ | $\mu$ | $\mu$ |
| $k$ | $\dfrac{E}{3(1-2\nu)}$ | $\dfrac{E}{9-3E/\mu}$ | $k$ | $k$ | $\lambda+2\mu/3$ |
| $\lambda$ | $\dfrac{E\nu}{(1+\nu)(1-2\nu)}$ | $\dfrac{E(1-2\mu/E)}{3-E/\mu}$ | $\dfrac{3k\nu}{1+\nu}$ | $k-2\mu/3$ | $\lambda$ |

$g, \nu$

$\dfrac{2g(1+\nu)}{3(1-2\nu)}$

module de Young $E$
coefficient de Poisson $\nu$
module d'élasticité en cisaillement (ou de Coulomb) $\mu$
module d'incompressibilité $k$
modules de Lamé $\lambda$. $\mu$

$$3\lambda + 2\mu = 3k = \frac{E}{1-2\nu} \qquad 9k = \lambda + \frac{2}{3}\mu + \frac{\mu}{3}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} = \mu = \frac{E}{2(1+\nu)}$$

$$\Rightarrow 2\nu = (1-2\nu) \Rightarrow$$

$$2\nu + 2\nu = 1$$

$$4\nu = 1 \qquad \nu = 1/4$$