

■ Autrice des dias (2023)
Prof. Katrin Beyer

■ Earthquake
Engineering and
Structural Dynamics
Laboratory

■ Dias supplémentaires
Prof. E. Denarié (2025)

■ Laboratoire de
comportement et
conception des
structures en béton



Statique



Prof. E. Denarié



Objectif du cours

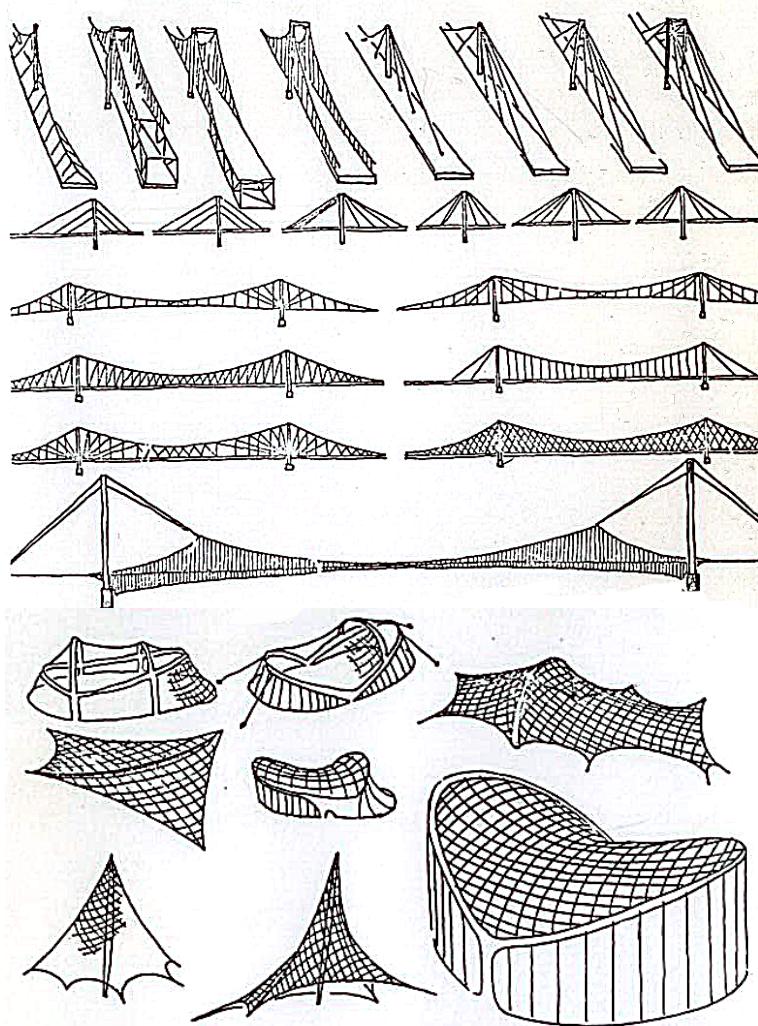
A la fin de ce cours, vous saurez :

- Quelles sont les caractéristiques statiques d'un câble
- Comment déterminer la forme d'un câble sous charges arbitraires en utilisant les équations différentielles d'équilibre ou l'équilibre des fragments
- Comment déterminer la forme d'un câble sous charges concentrées
- Quelles sont les similitudes entre un câble et un arc

1. Dérivation de l'équation différentielle des câbles
2. Câble sous une charge uniformément répartie sur l'horizontale
 - Analyse par l'équation différentielle
 - Analyse par l'équilibre des fragments
3. Câble sous une charge uniformément repartie le long de l'axe du câble
4. Câble sous charges concentrées
5. Comparaison câble - arc

Types de câbles

- Câbles quasi-rectilignes
 - Câbles qui ne sont presque pas chargés entre leurs extrémités
 - Haubans, suspentes et tirants
- Câbles courbes
 - Charges distribuées
 - Câbles de grande portée
- Réseaux (ou nappes)
 - Câbles croisés
 - → Surface courbe
- Autres
 - Tenségrités
 - Toiles
 - Structures pneumatiques





<https://www.danvanderzwalm.com/album/el-suizo-toni-ruttimann-the-bridge-builder>

Tony «El Suizo» Rüttimman

Constructeur de ponts, bénévole, autodidacte
Recyclage de câbles usagés de téléphériques

https://www.youtube.com/watch?v=H7_V3wvsnZA

<https://www.danvanderzwalm.com/album/el-suizo-toni-ruttimann-the-bridge-builder>

https://en.wikipedia.org/wiki/Toni_Rüttimann



<https://www.aguaplan.org/en/toni-el-suizo-4/>



Voir [https://fr.wikiarquitectura.com/bâtiment/stade-olympique-de-munich/](https://fr.wikiarquitectura.com/b%C3%A2timent/stade-olympique-de-munich/)







- **Elément structural qui n'est résistant qu'à l'effort normal de traction (tension du câble)**
 - Le câble représente une liaison unilatérale
 - Le câble n'offre de résistance ni à la compression, ni à l'effort tranchant, ni à la flexion, ni à la torsion (le diamètre est très faible par rapport à sa longueur)
- **Matériaux**
 - Acier à très haute résistance (acier pour câbles)
 - Fibres de carbone
 - Fibres naturelles
- **Géométrie du câble**
 - Câble non chargé : lâche, sans forme définie
 - Les charges sur le câble définissent sa configuration
 - But des analyses des câbles : trouver pour une charge donnée la forme qui permet d'avoir l'équilibre du câble
 - Types de charges : poids propre, charges concentrées ou distribuées, pré-tension
 - L'hypothèse des petites déformations n'est plus valable

Dérivation de l'équation différentielle des câbles

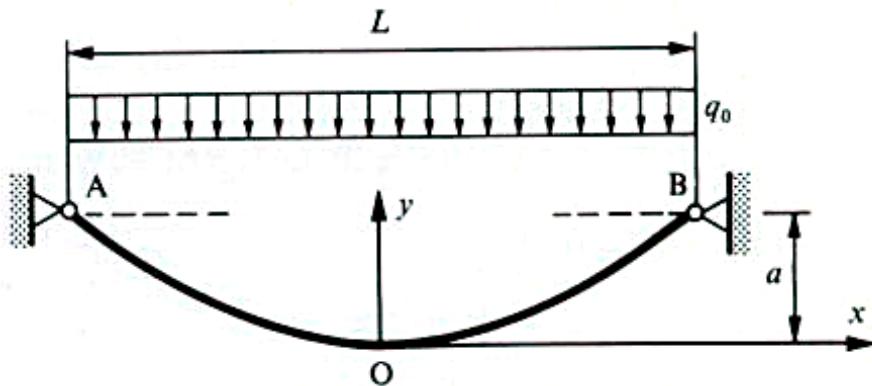
Câble sous charge répartie arbitraire (cas plan)

- Hypothèses :

- Le câble est inextensible (la forme change mais pas la longueur)
- La force répartie $q(x)$ agit vers le bas (effet de la pesanteur)

- Objectif :

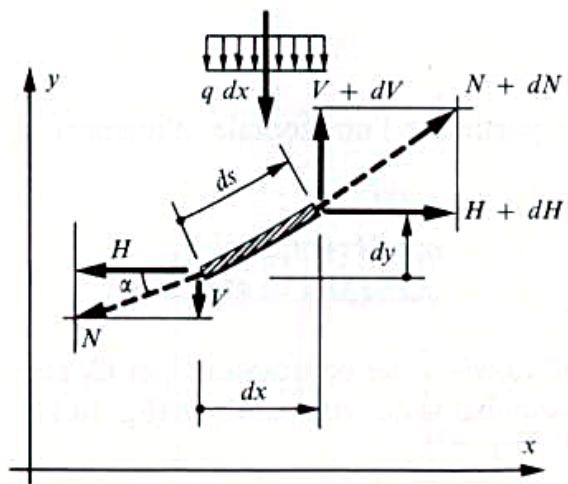
Trouver $y(x)$: la forme qui permet l'équilibre du câble avec la charge $q(x)$



L : la portée du câble
 a : la flèche du câble
 s : la longueur du câble

Dérivation de l'équation différentielle des câbles

Equilibre d'un petit tronçon ds d'un câble sous des charges verticales :

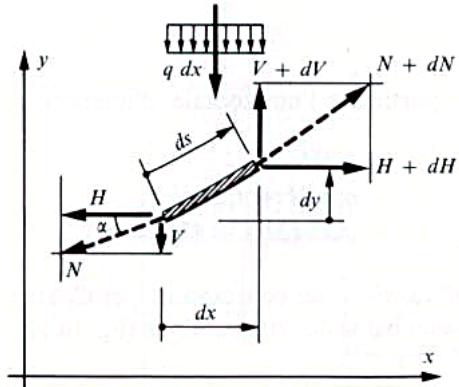


N : effort normal (tension) qui agit dans la direction du câble (N a la même pente que le câble)

Décomposition de N en 2 inconnues V et H :

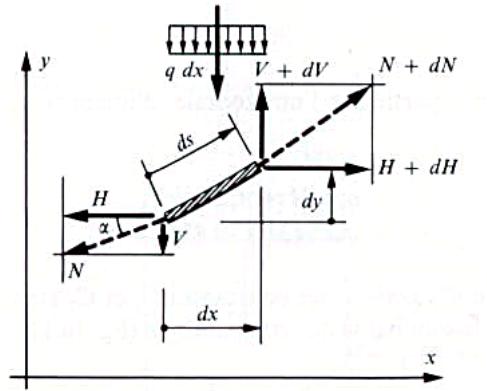
Dérivation de l'équation différentielle des câbles

Equilibre d'un petit tronçon ds d'un câble sous des charges verticales :



Longueur du câble :

Equation différentielle d'équilibre d'un câble



N Effort normal

V Composante verticale

H Composante horizontale

q Charge répartie verticale (positive vers le bas)

y Configuration du câble / forme en équilibre

s Longueur du câble

$$V = Hy'$$

$$V'(x) = q(x)$$

$$H(x) = \text{const.}$$

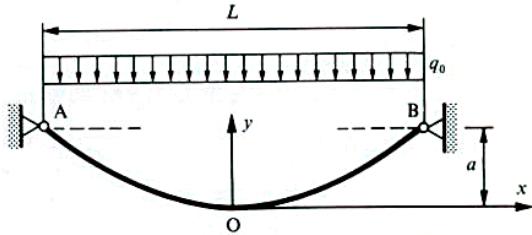
$$\boxed{y''(x) = \frac{q(x)}{H}}$$

$$ds = dx \sqrt{1 + y'(x)^2}$$

$$s = \int_{x_1}^{x_2} ds = \int_{x_1}^{x_2} \sqrt{1 + y'(x)^2} dx$$

$$N = \sqrt{H^2 + V^2} = H \sqrt{1 + y'(x)^2}$$

Exemple : câble sous une charge uniformément répartie sur l'horizontale

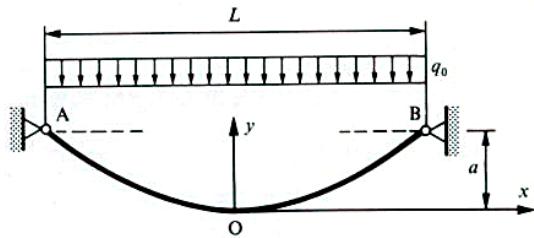


$$y''(x) = \frac{q(x)}{H}$$

Calculer avec l'équation différentielle

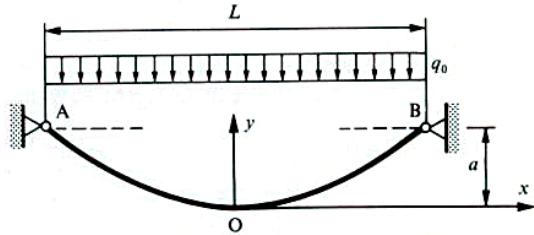
- La composante horizontale H si la flèche est égale à a
- L'effort axial minimal et maximal dans le câble (N_{min} et N_{max})

Exemple (suite)



$$y''(x) = \frac{q(x)}{H}$$

Exemple (suite)



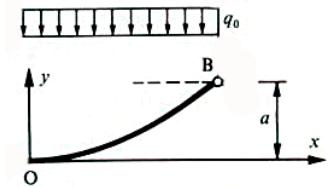
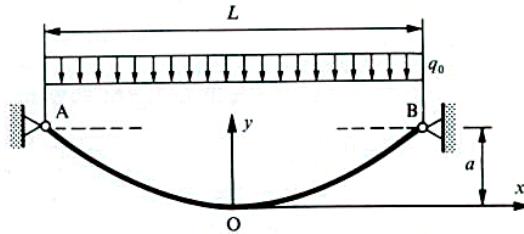
$$y''(x) = \frac{q(x)}{H}$$



A votre tour!

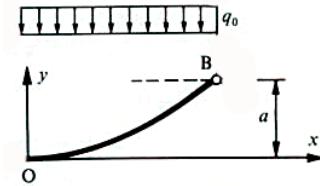
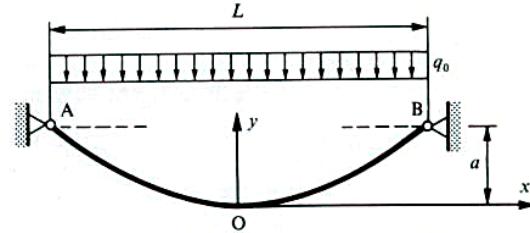
Calculer avec l'équilibre par tronçon

- La composante horizontale H si la flèche est égale à a
- L'effort axial minimal et maximal dans le câble (N_{min} et N_{max})

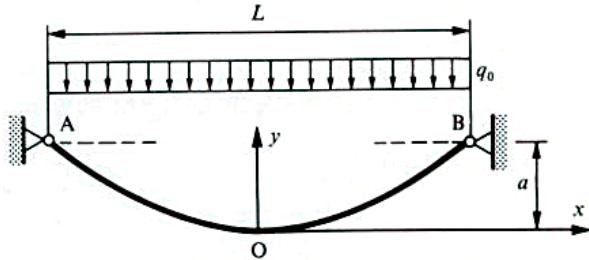




A votre tour! (suite)



Résumé : câble sous une charge uniformément répartie sur l'horizontale



$$y''(x) = \frac{q(x)}{H}$$

- 2 supports sur la même hauteur :

$$y = \frac{q_0 x^2}{2H} \quad -\frac{L}{2} \leq x \leq \frac{L}{2}$$

$$a = \frac{q_0 L^2}{8H} \quad H = \frac{q_0 L^2}{8a}$$

$$N = q_0 \sqrt{x^2 + \left(\frac{L^2}{8a}\right)^2} = H \sqrt{1+k^2}$$

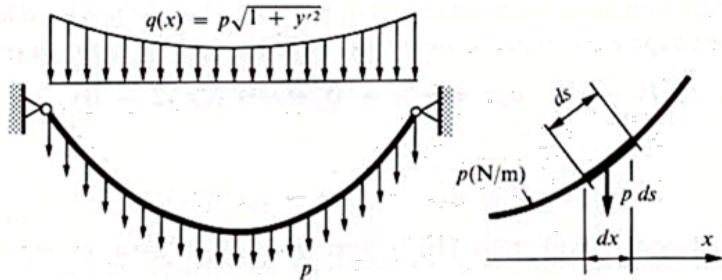
$$s = 2 \int_0^{L/2} ds = \frac{L}{2} \left[\sqrt{1+k^2} + \frac{1}{k} \ln \left(k + \sqrt{1+k^2} \right) \right]$$

- La configuration du câble est une parabole.

- Plus la composante horizontale est grande, plus la flèche du câble est petite
- $k = \frac{4a}{L}$
- Longueur :

Câble sous une charge uniformément répartie le long de l'axe du câble

La configuration d'un câble sous une charge uniformément répartie le long de l'axe du câble est appelée chaînette



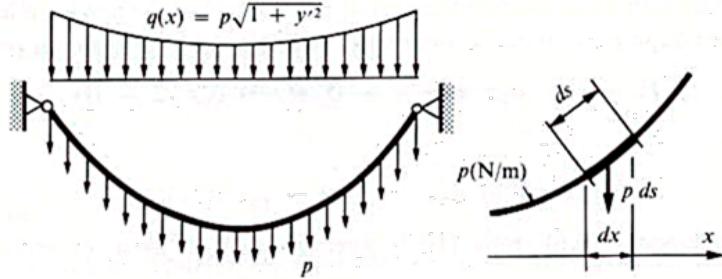
p : poids propre du câble par mètre de longueur du câble

$$q(x)dx = p \cdot ds = p\sqrt{1+y'^2}dx$$

$$y''(x) = \frac{q(x)}{H} = \frac{p}{H}\sqrt{1+y'^2}$$

Câble sous une charge uniformément répartie le long de l'axe du câble

$$y''(x) = \frac{q(x)}{H} = \frac{p}{H} \sqrt{1 + y'^2}$$



On peut montrer que la solution de cette équation différentielle est la chaînette :

$$y(x) = \frac{H}{p} \cosh\left(\frac{p}{H}x + C_1\right) + C_2$$

Axes passant par le sommet :

$$y(x) = \frac{H}{p} \left(\cosh \frac{p}{H}x - 1 \right)$$

Câble sous une charge uniformément répartie le long de l'axe du câble

$$y''(x) = \frac{q(x)}{H} = \frac{p}{H} \sqrt{1 + y'^2}$$

$$y(x) = \frac{H}{p} \left(\cosh \frac{p}{H} x - 1 \right)$$

Longueur du câble :

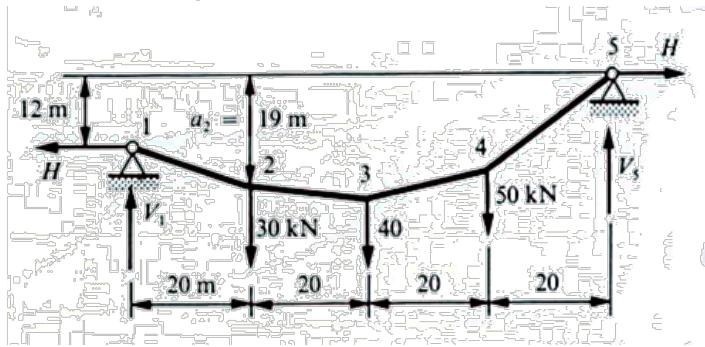
Effort axial du câble :



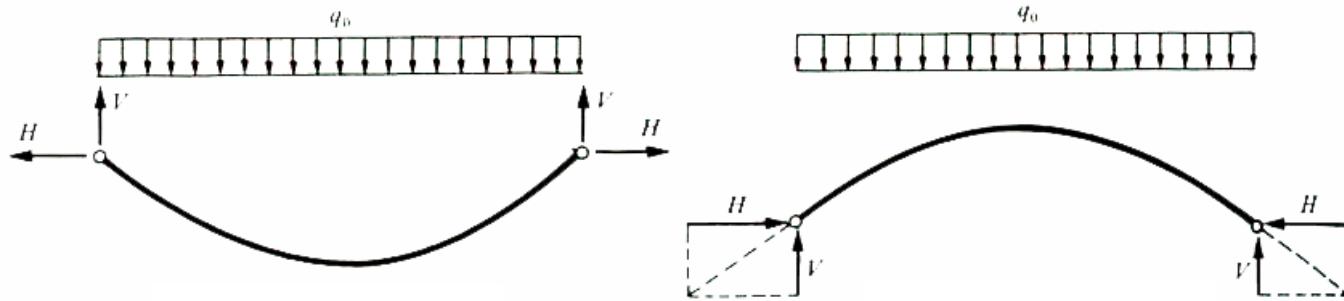
A votre tour!

Câble sous forces concentrées

- Configuration du câble: ligne brisée (poids propre négligé)
- Objectif: trouver H



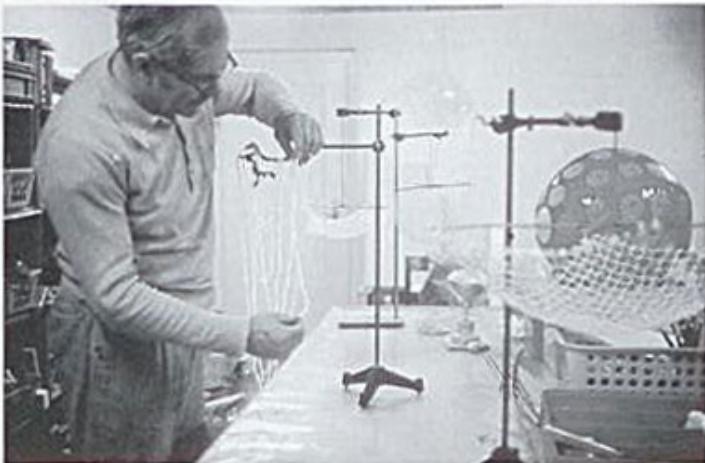
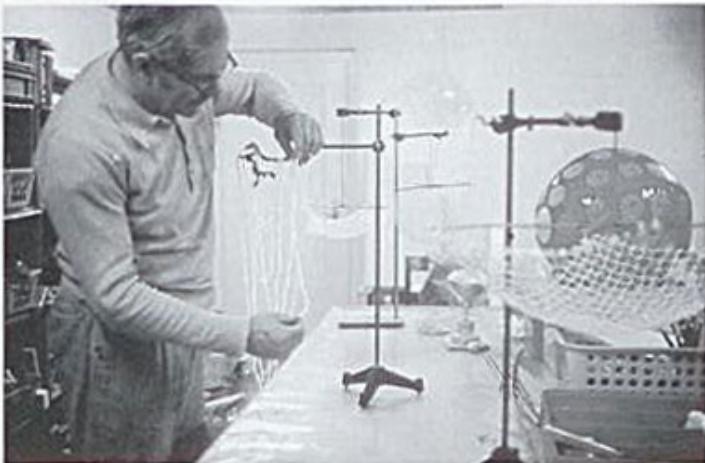
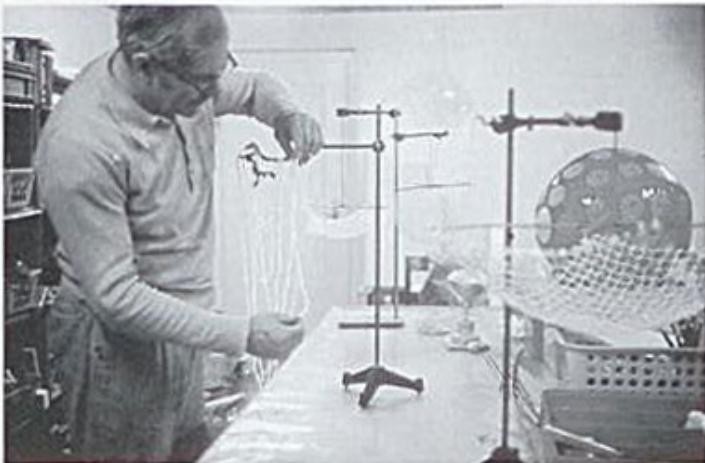
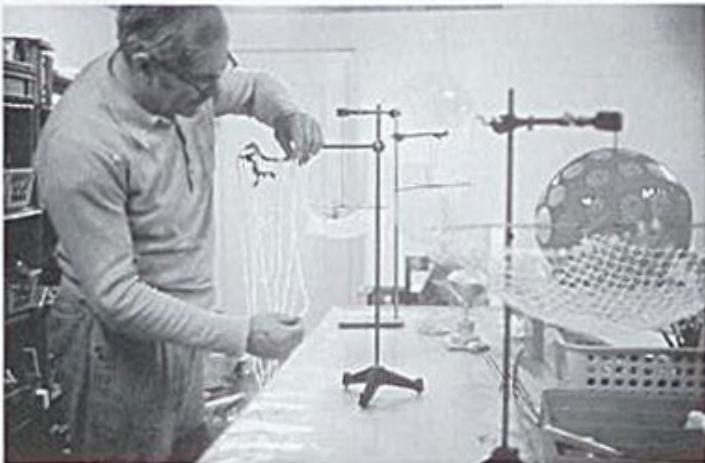
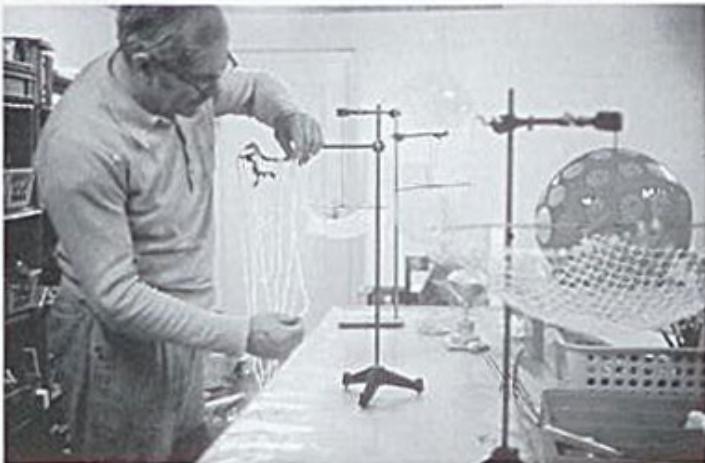
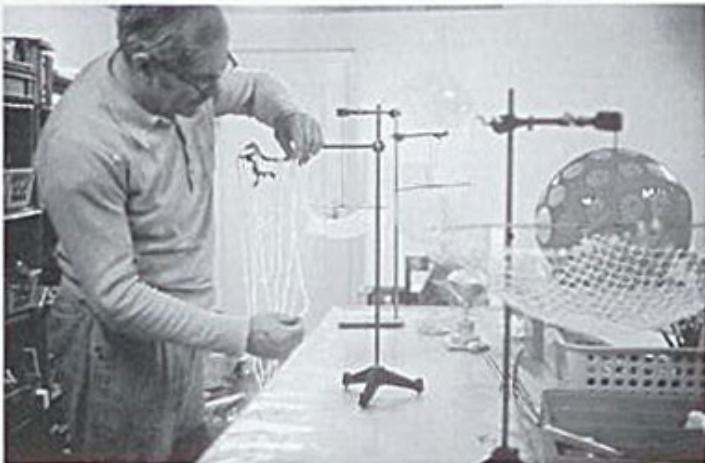
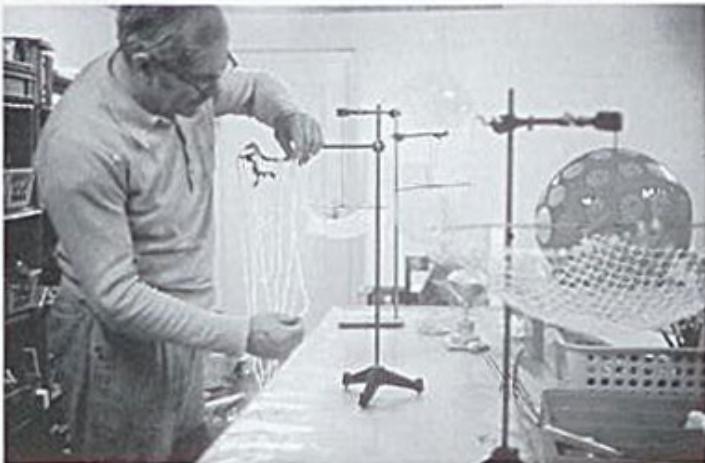
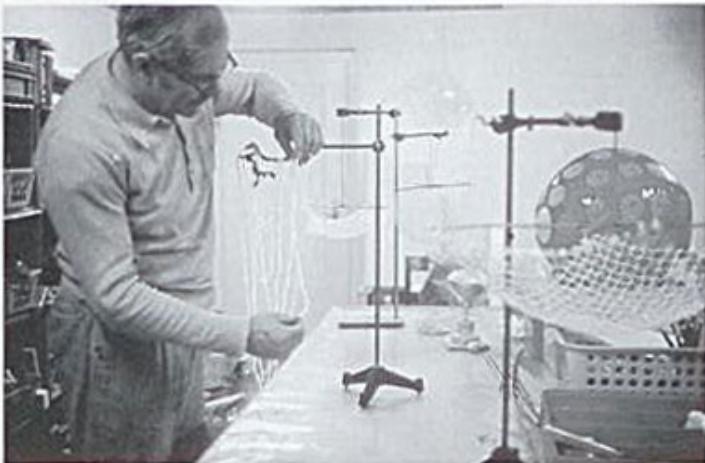
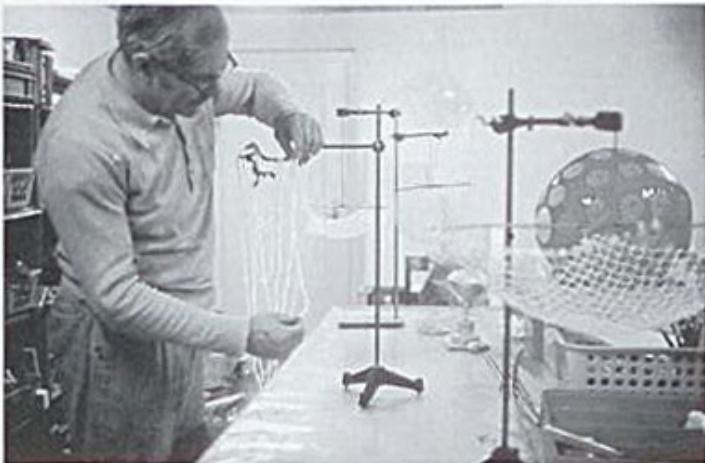
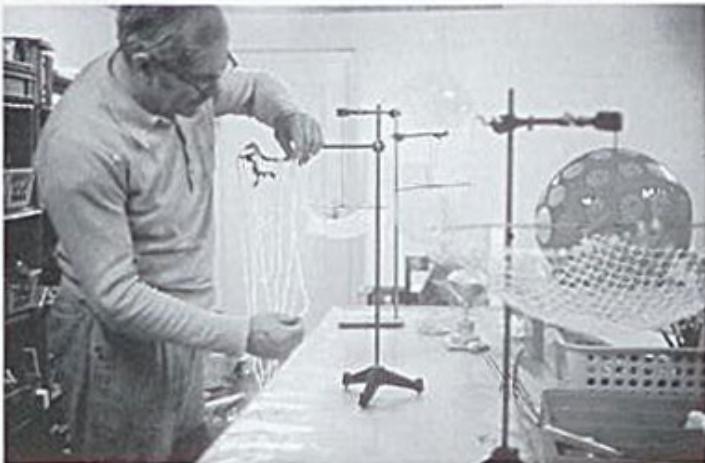
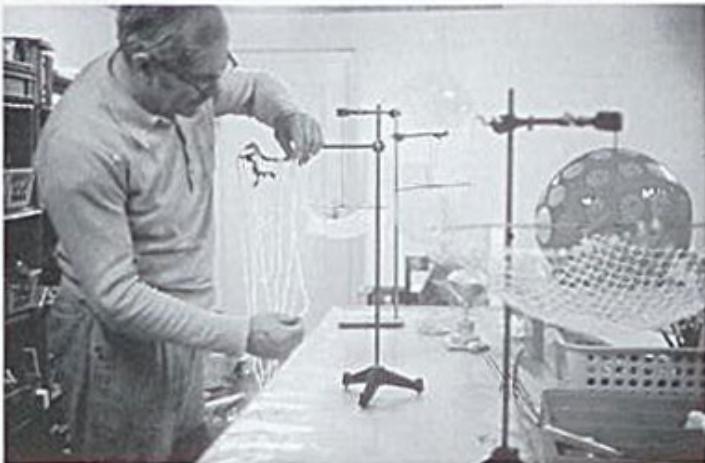
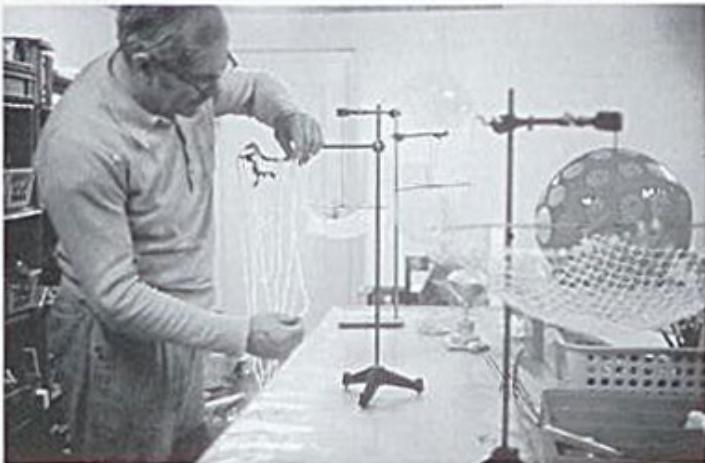
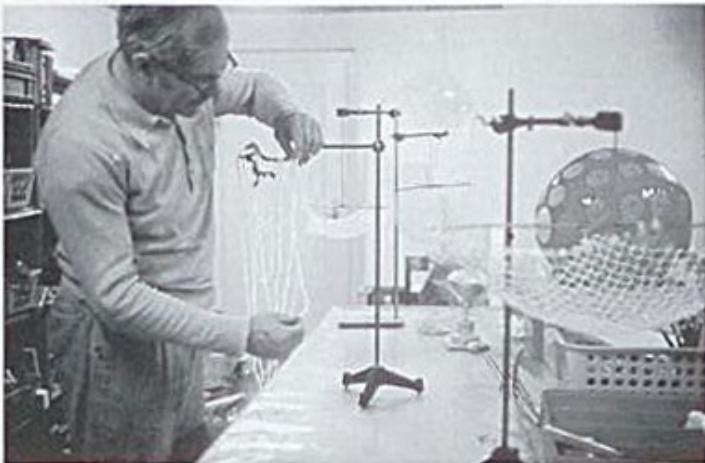
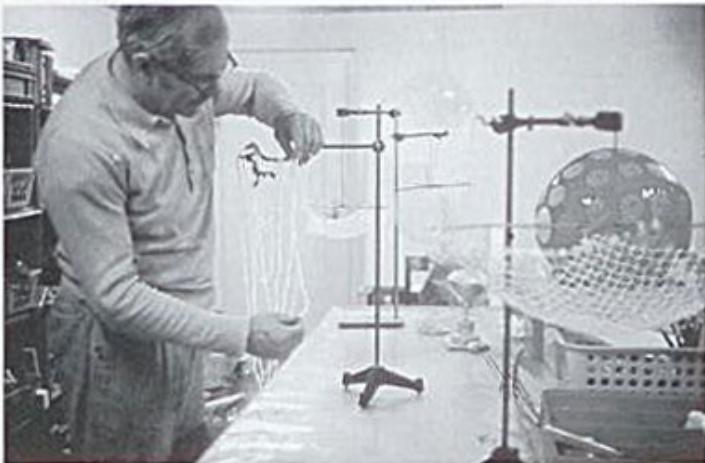
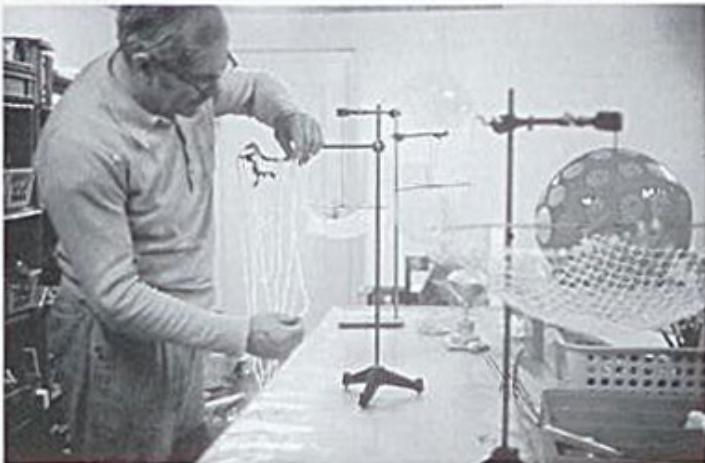
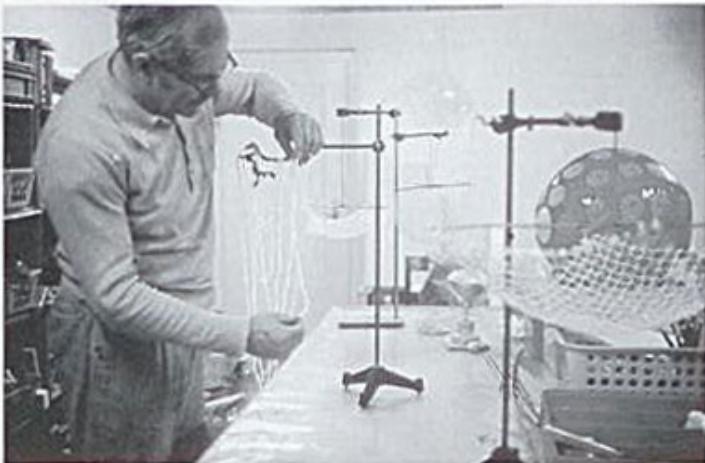
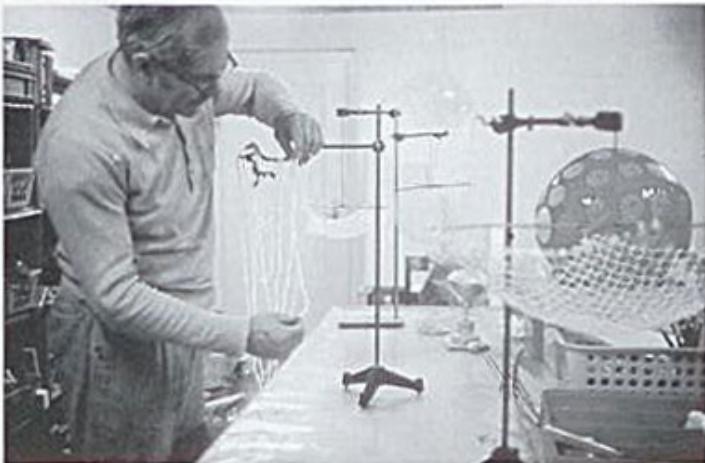
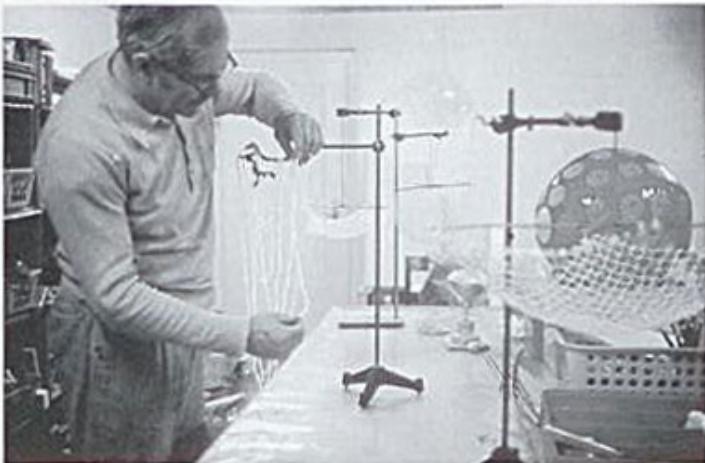
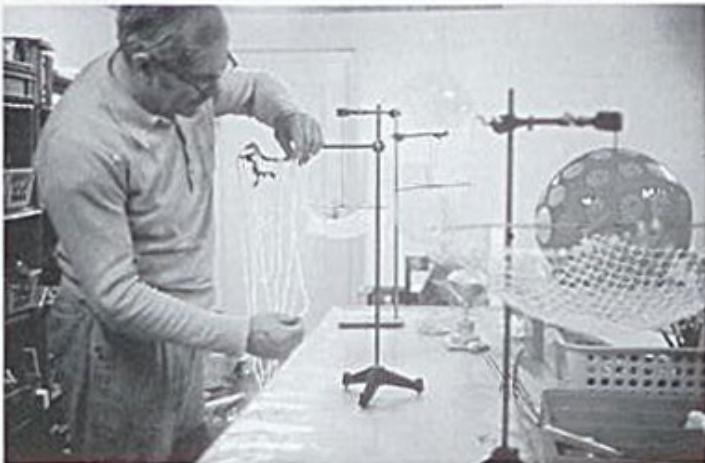
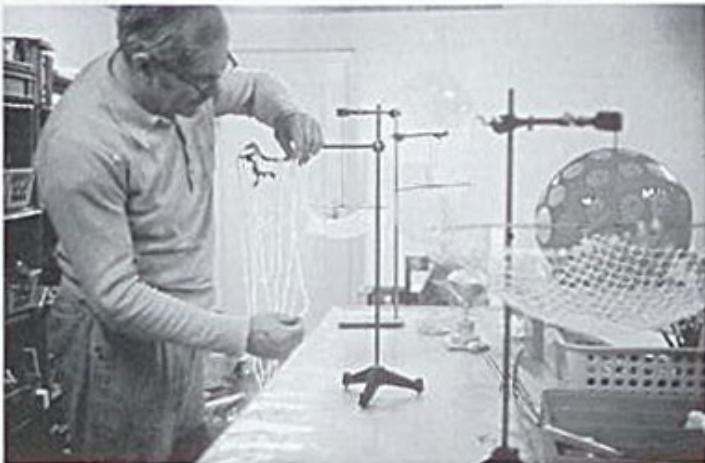
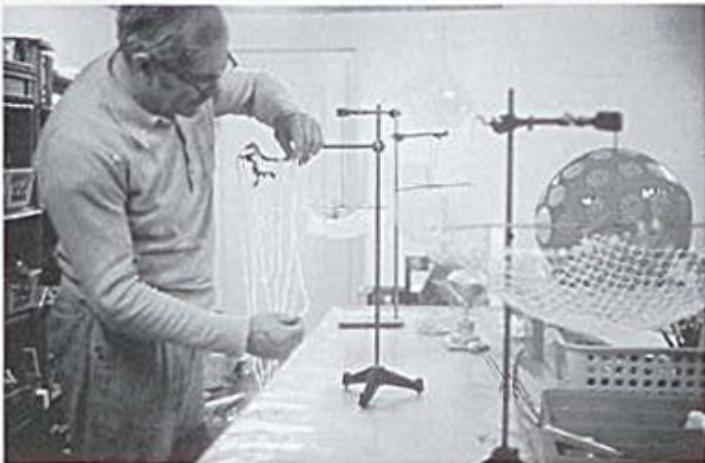
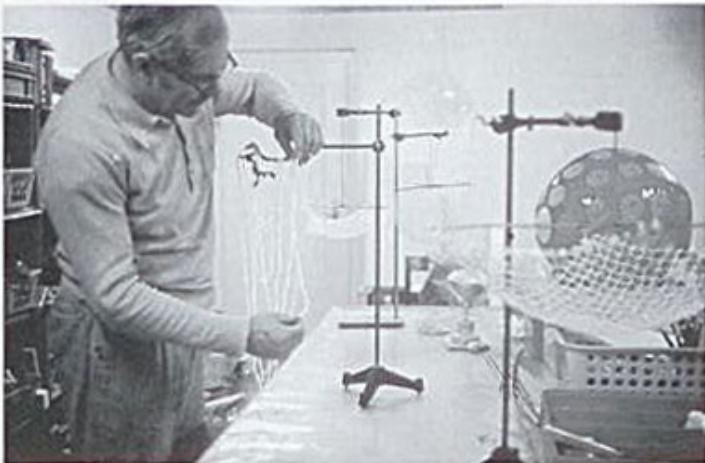
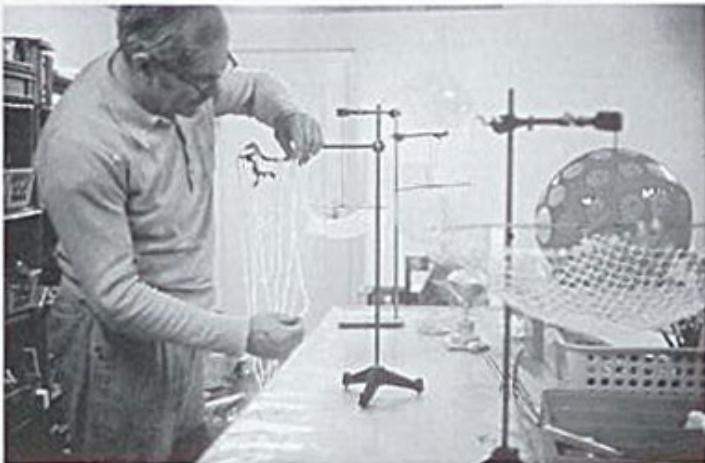
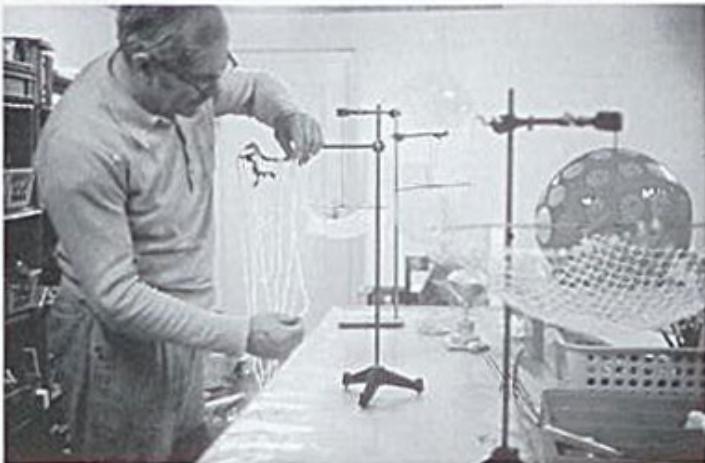
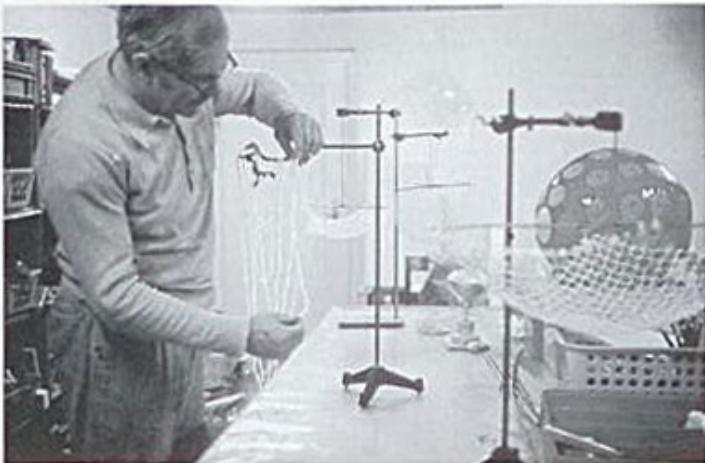
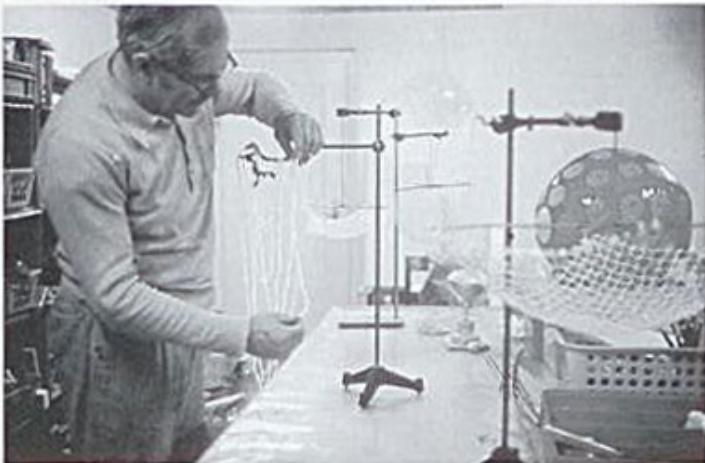
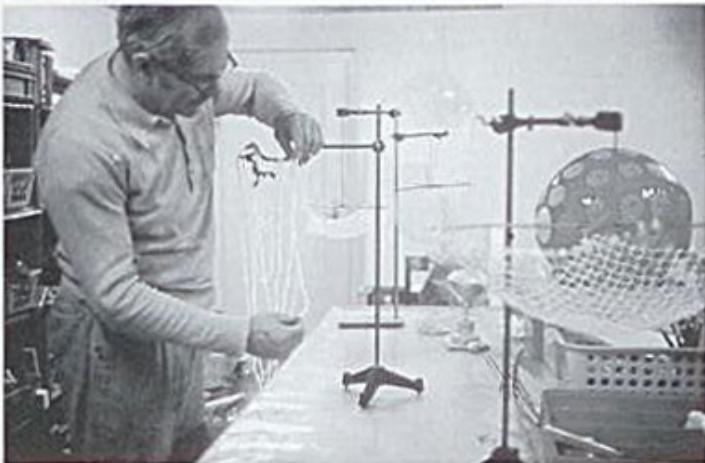
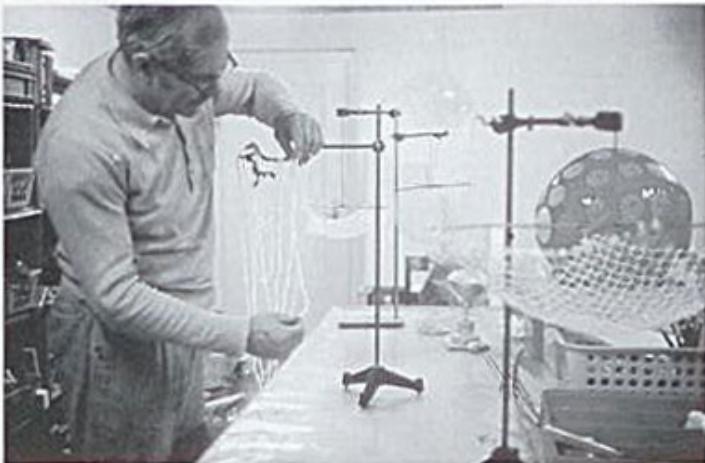
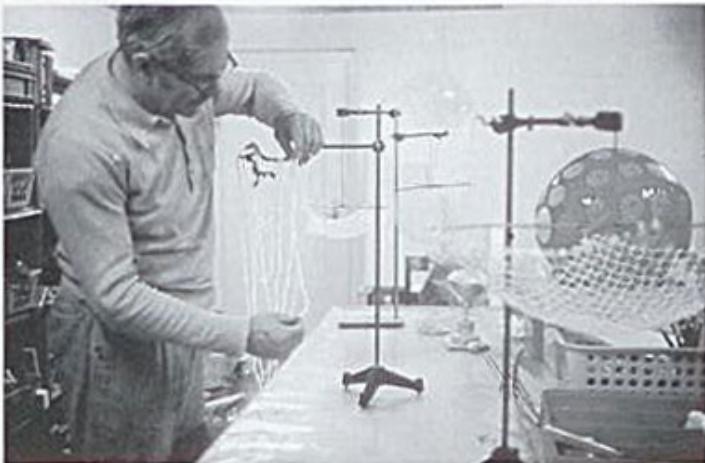
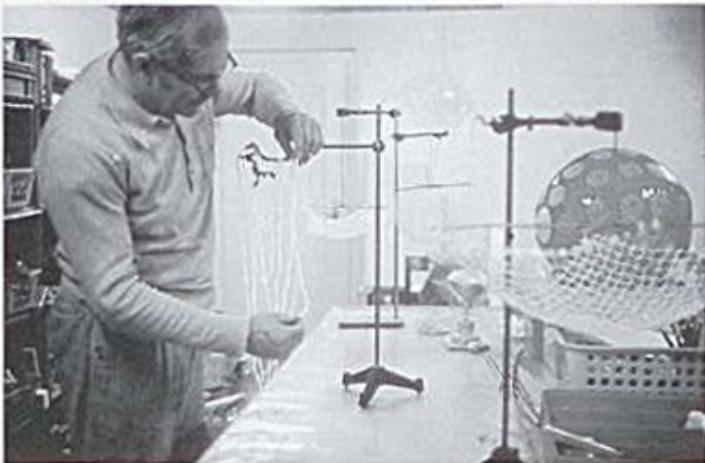
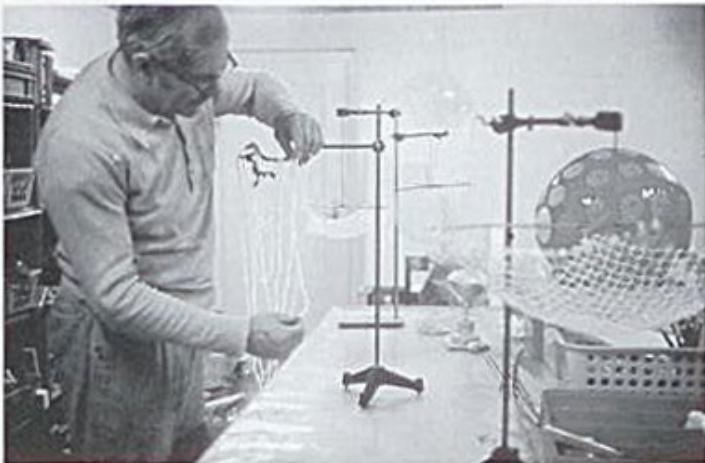
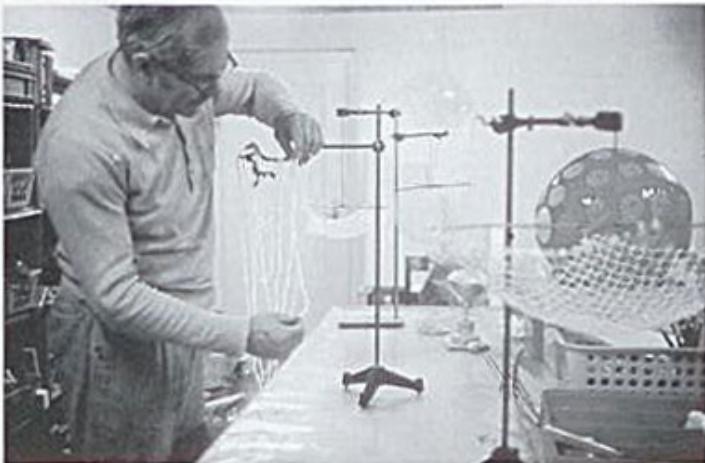
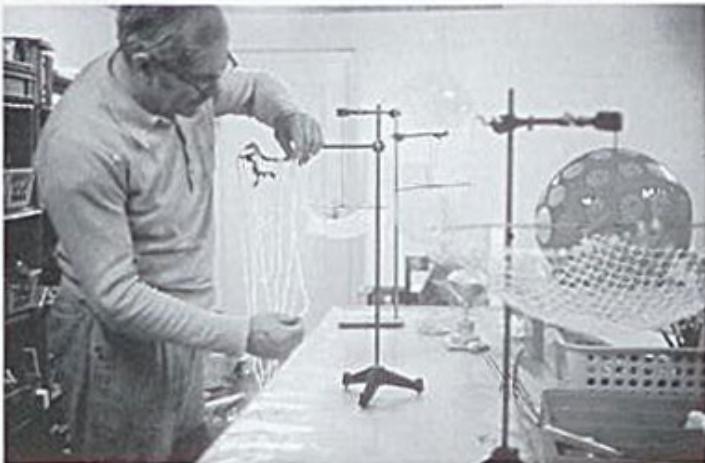
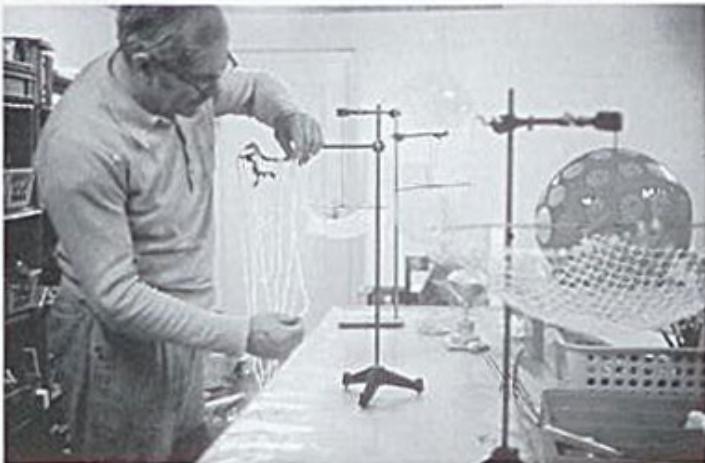
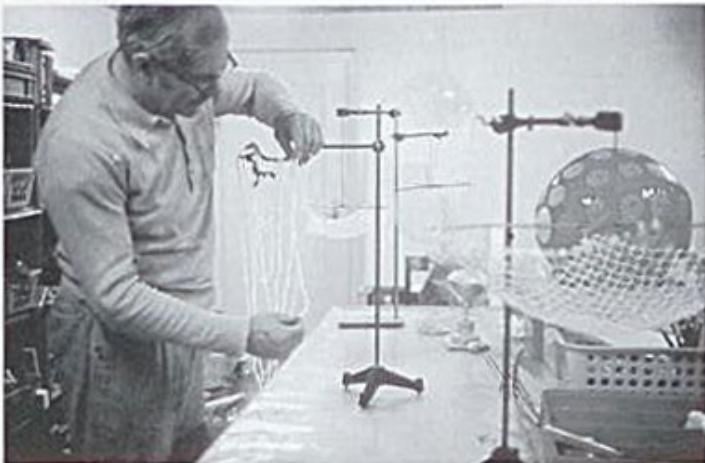
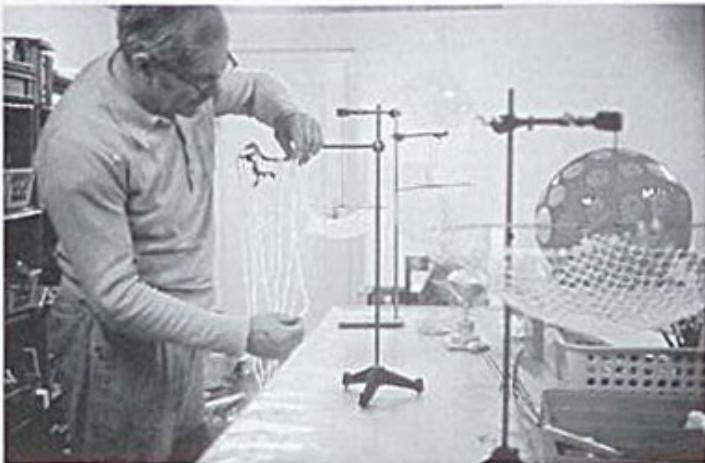
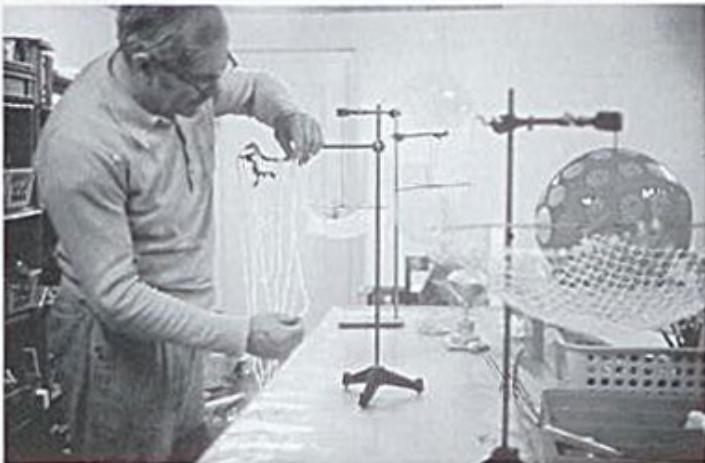
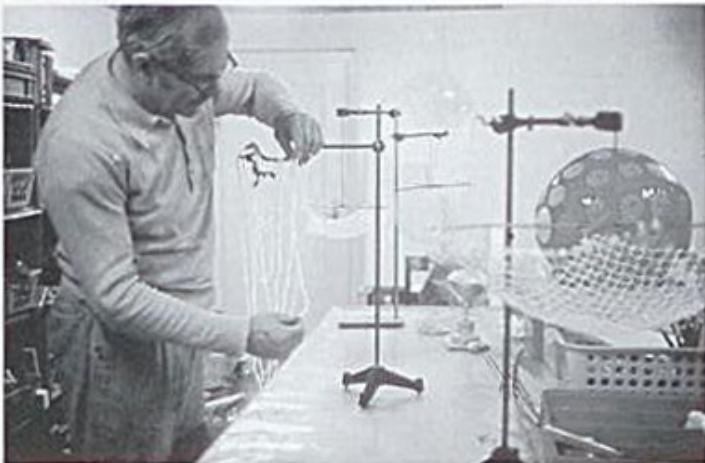
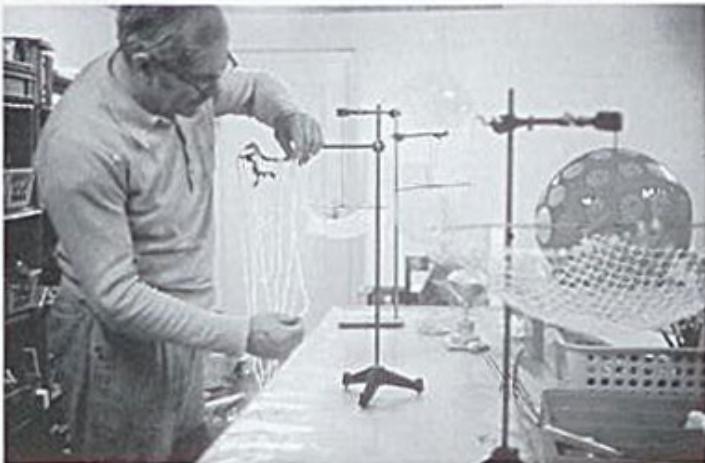
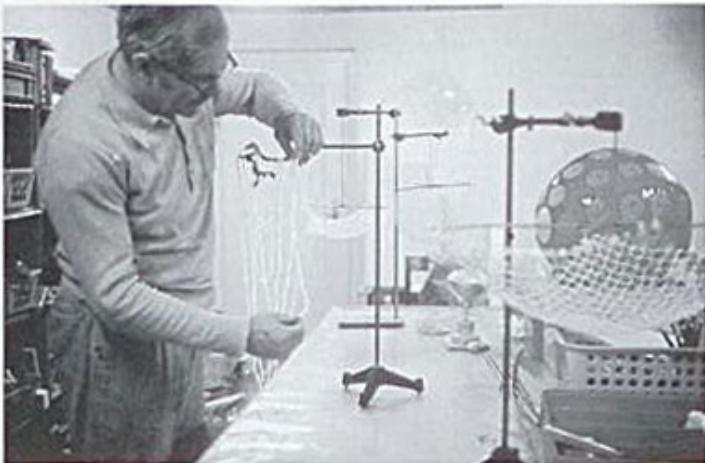
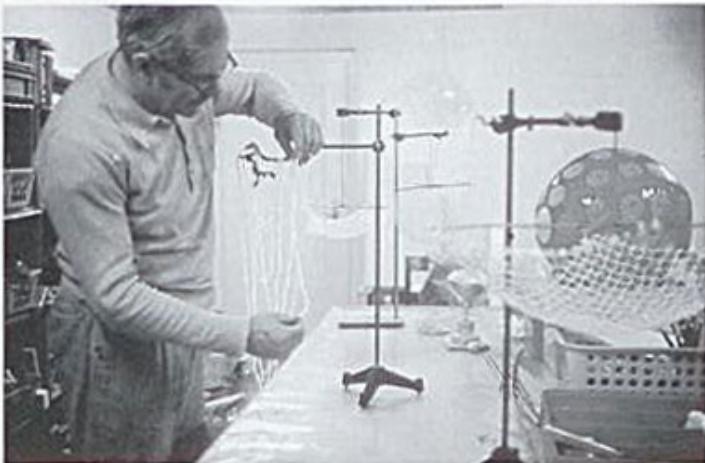
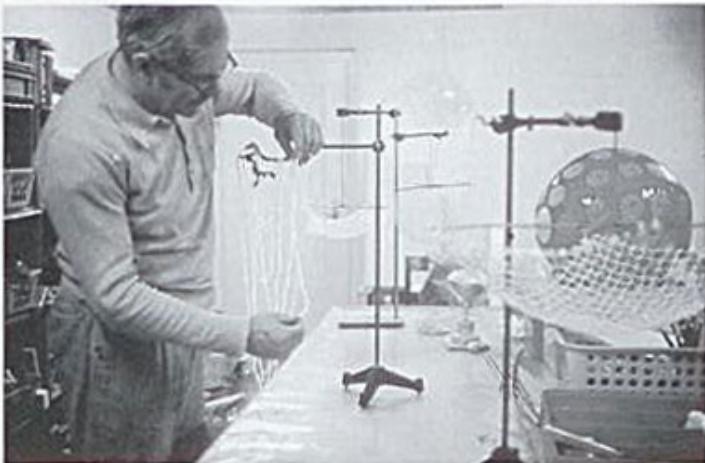
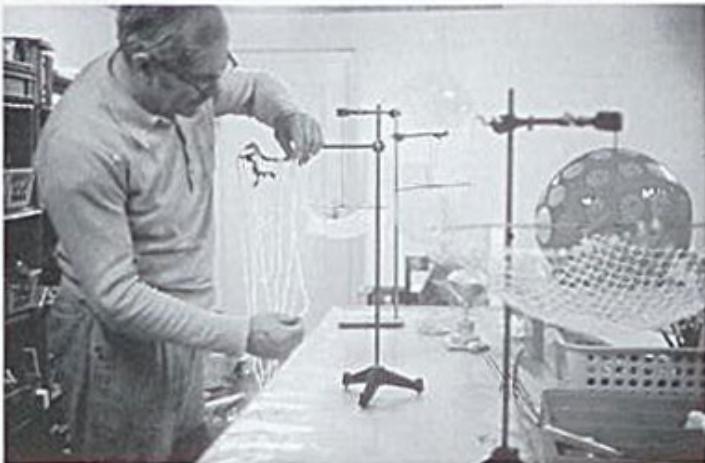
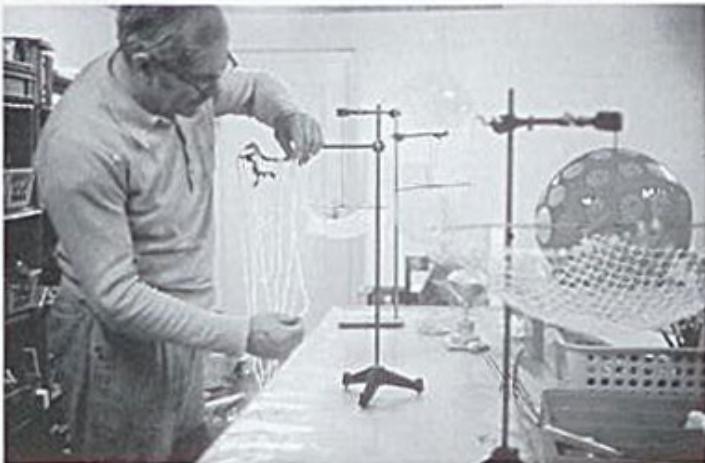
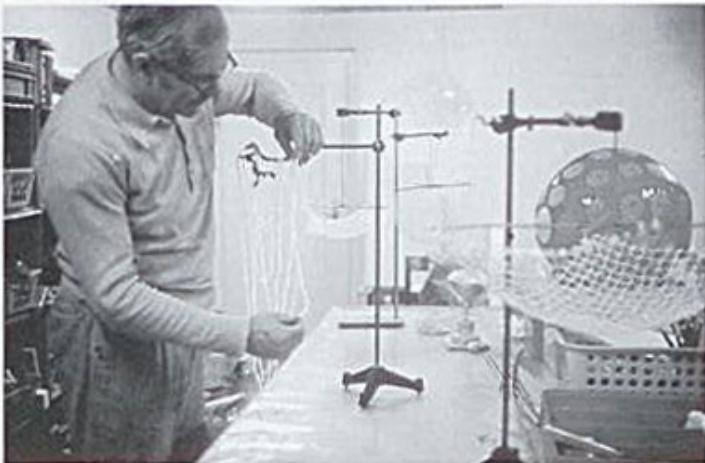
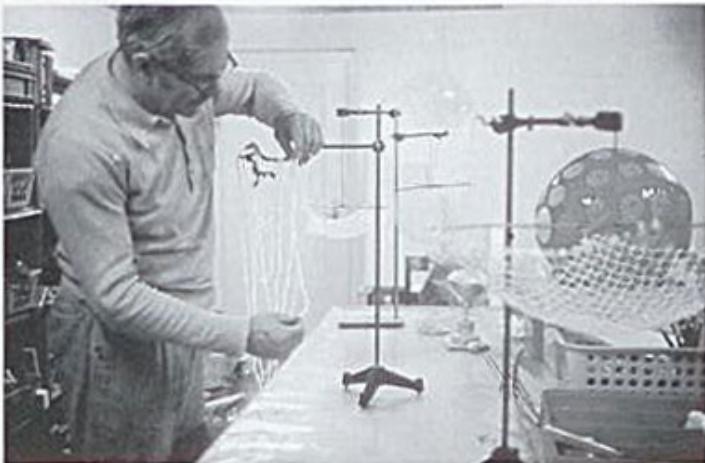
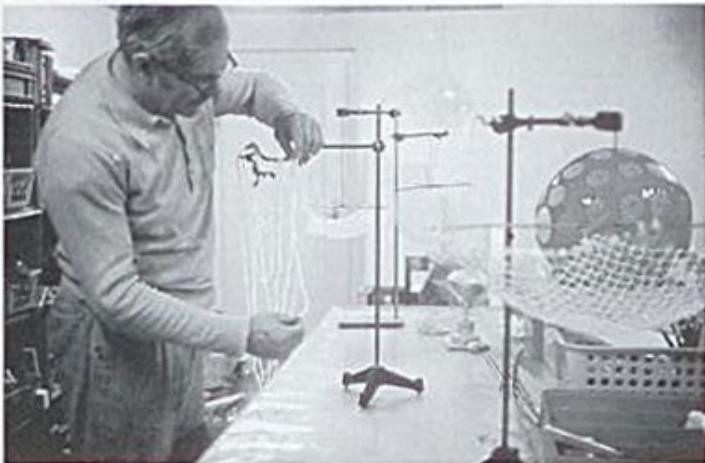
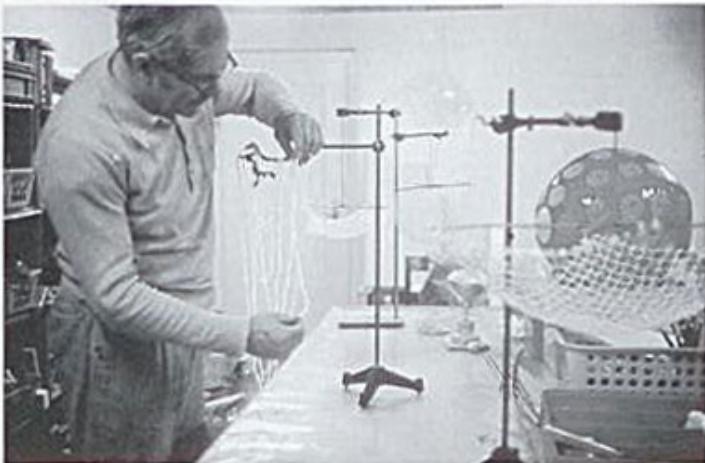
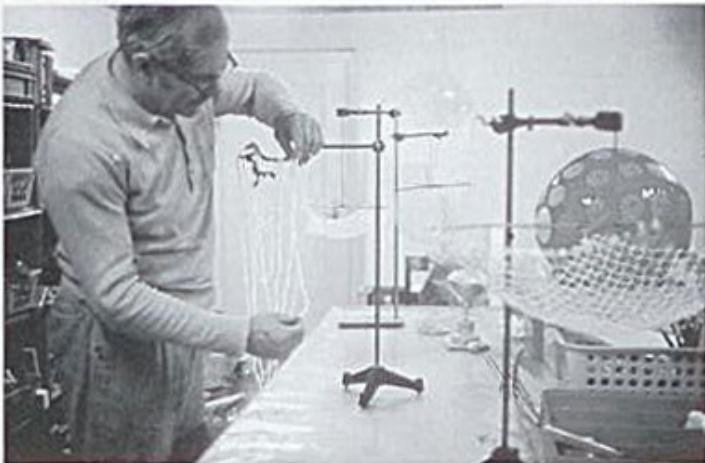
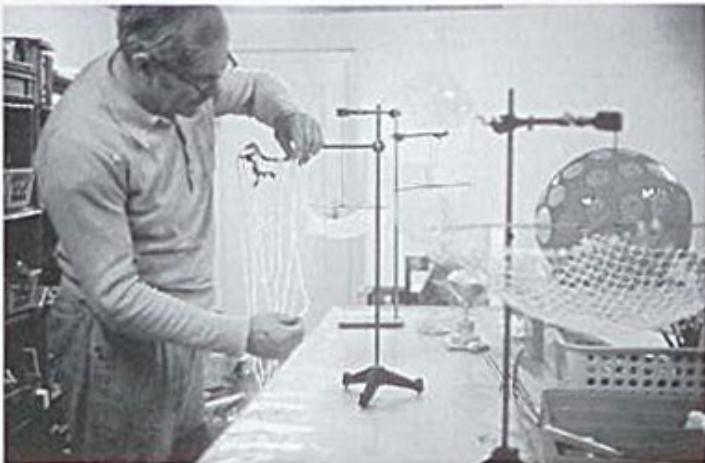
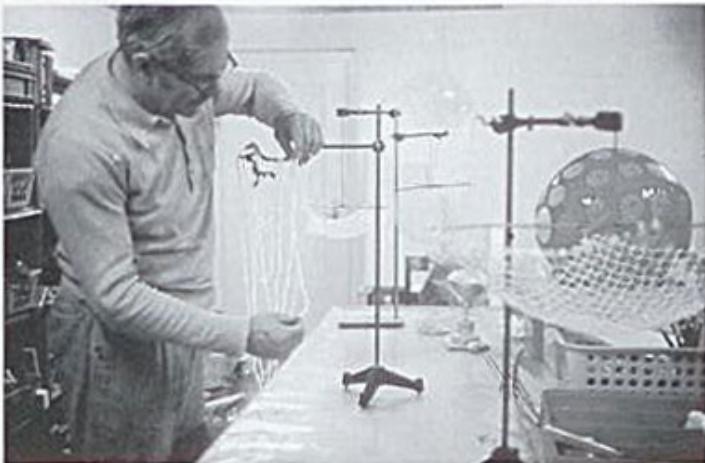
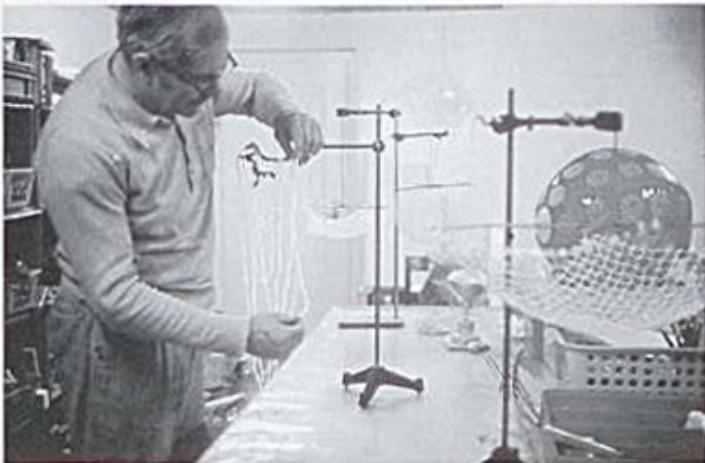
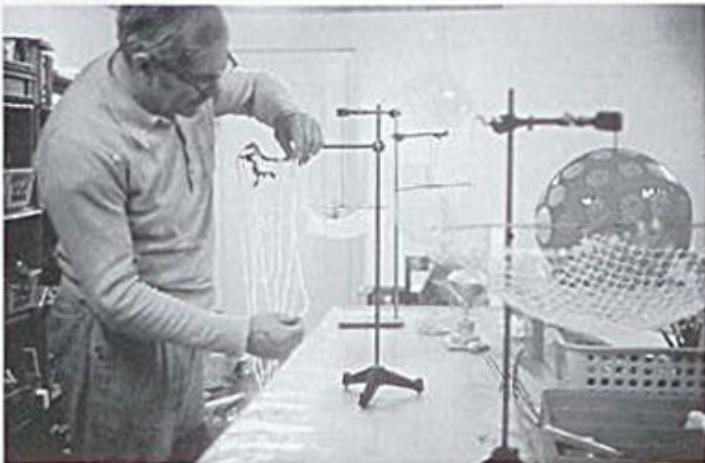
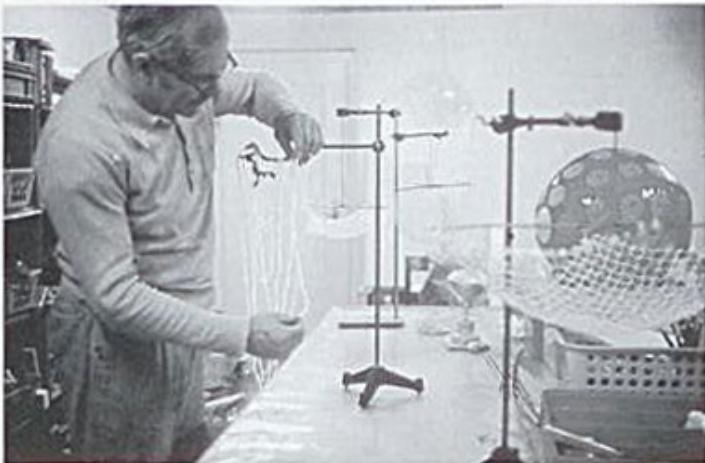
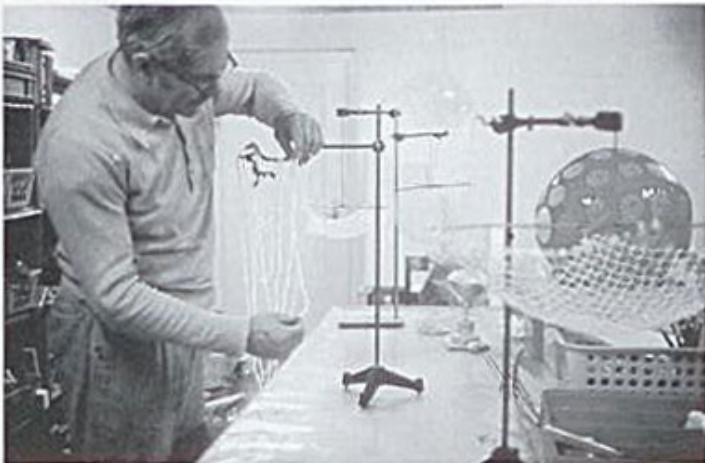
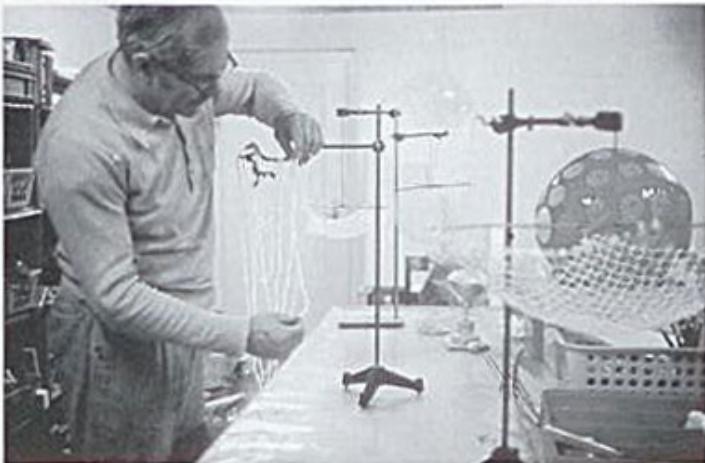
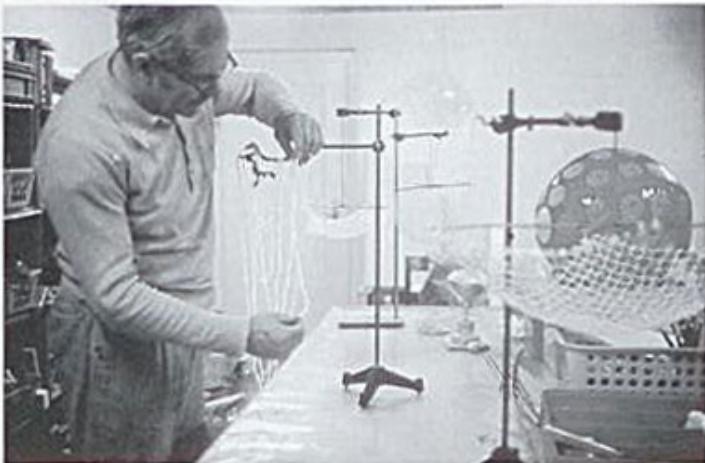
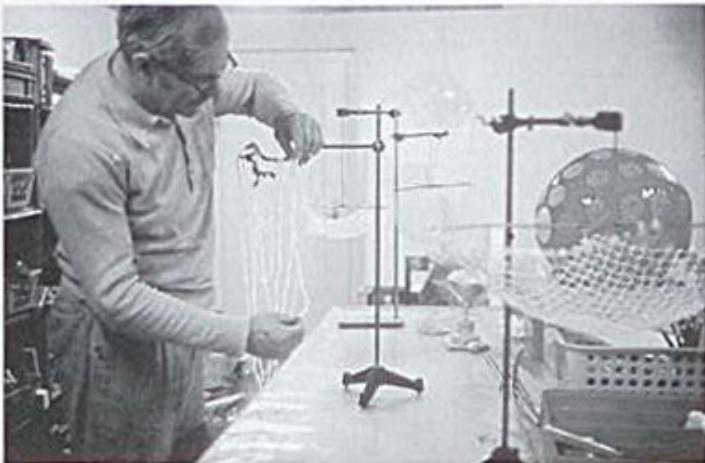
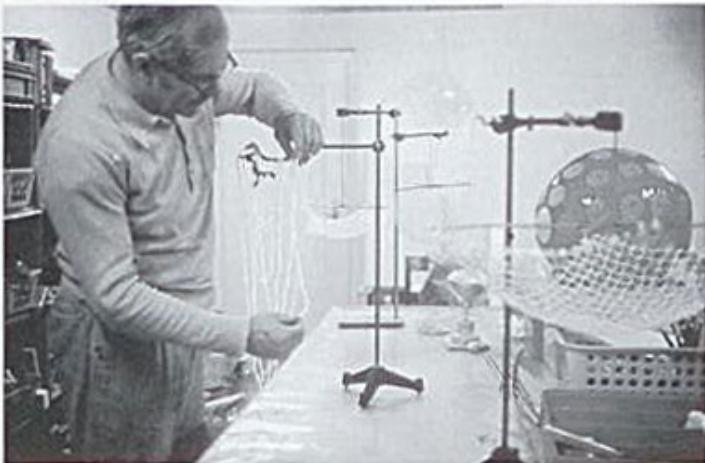
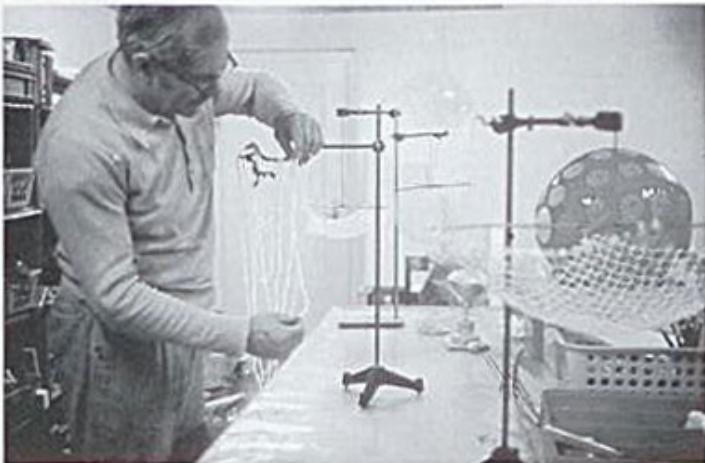
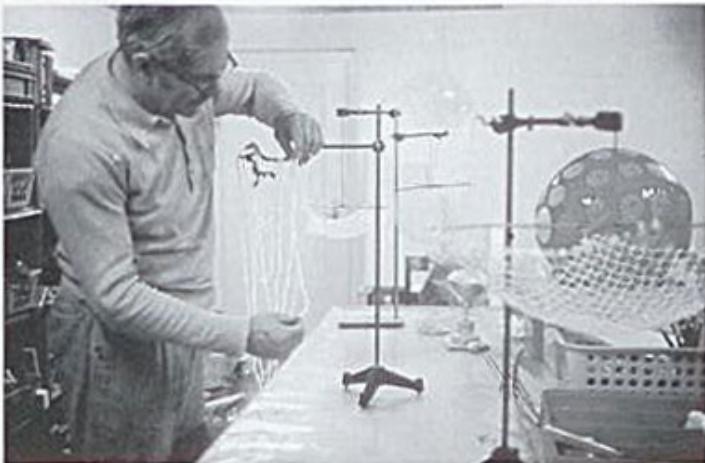
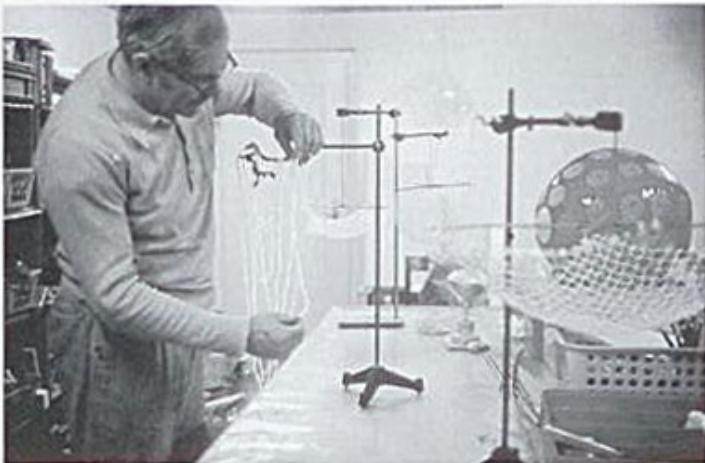
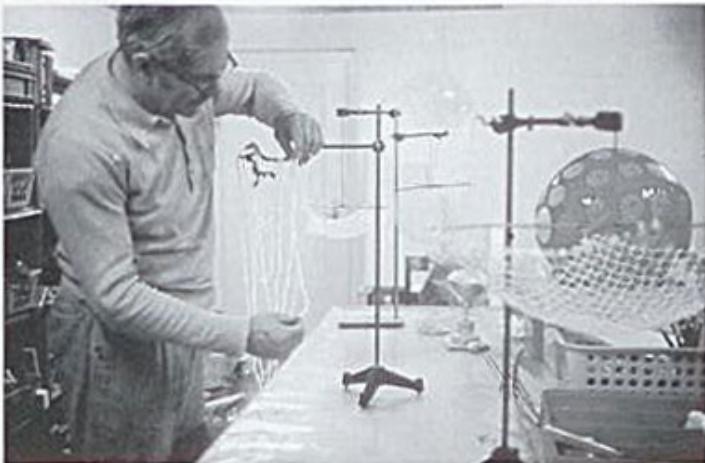
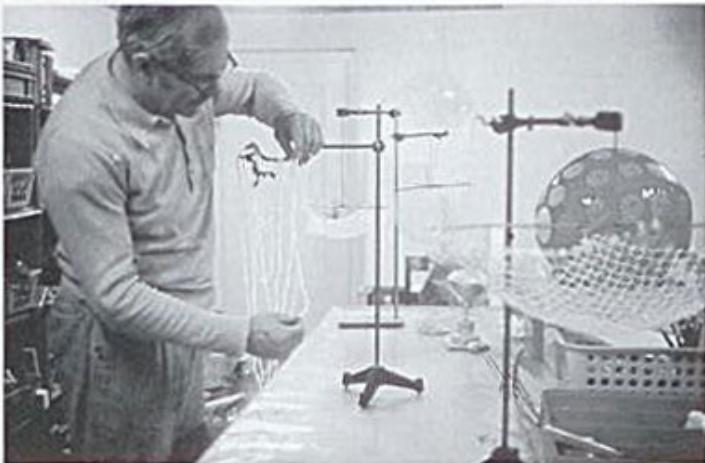
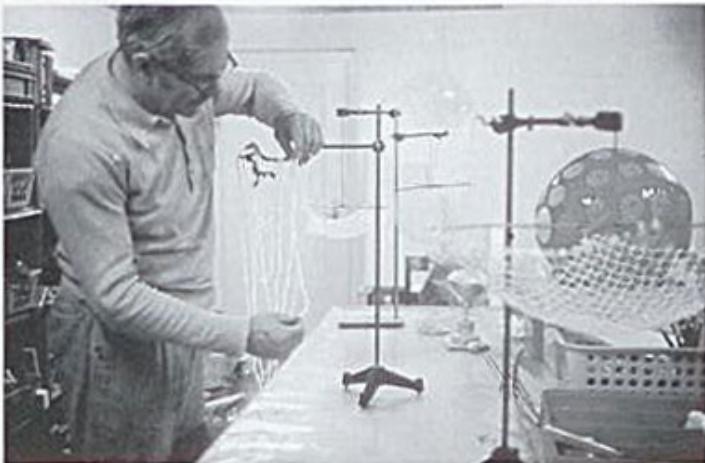
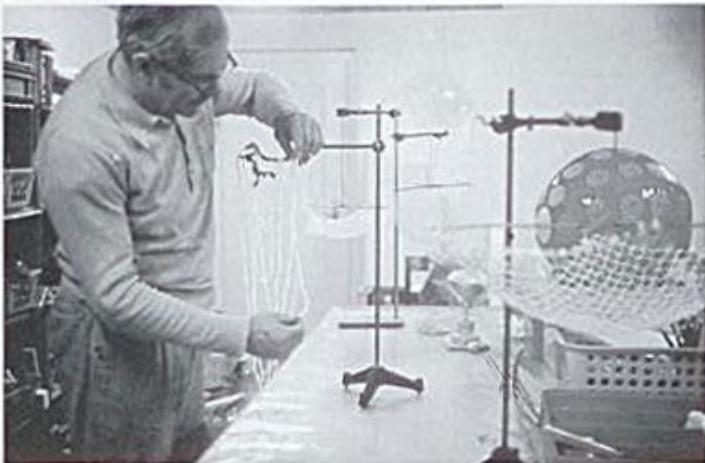
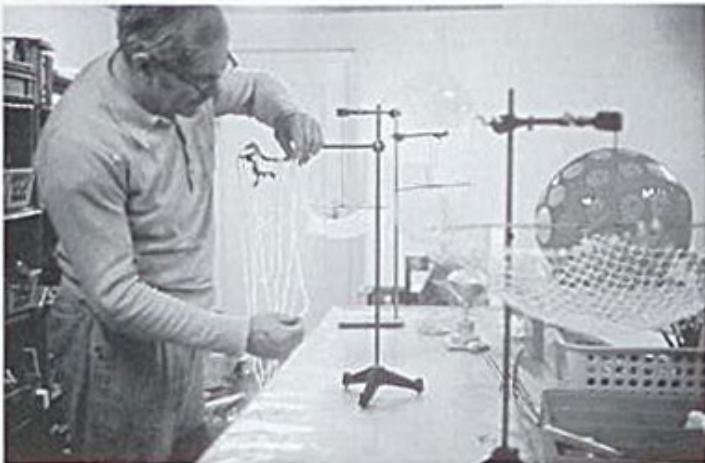
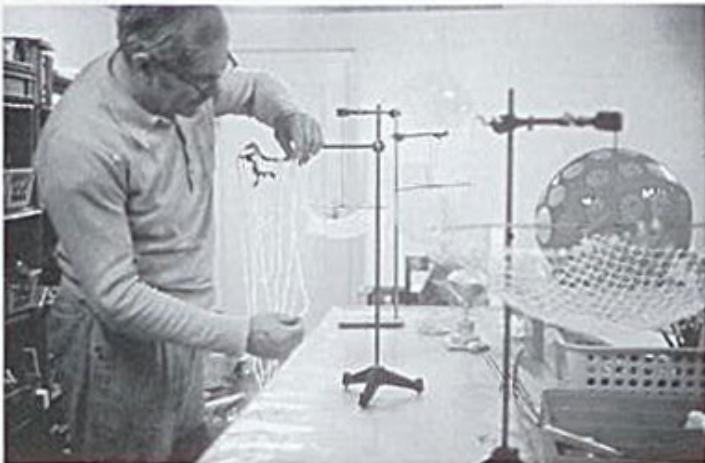
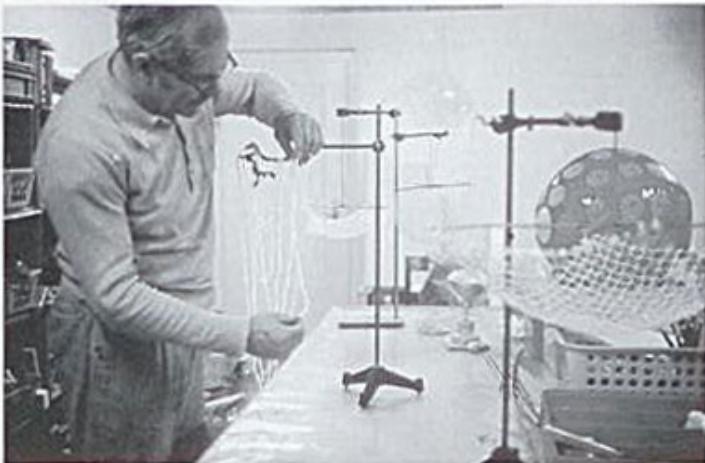
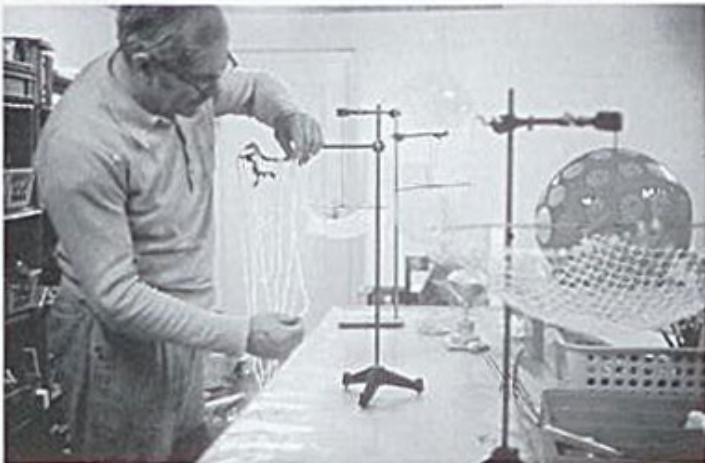
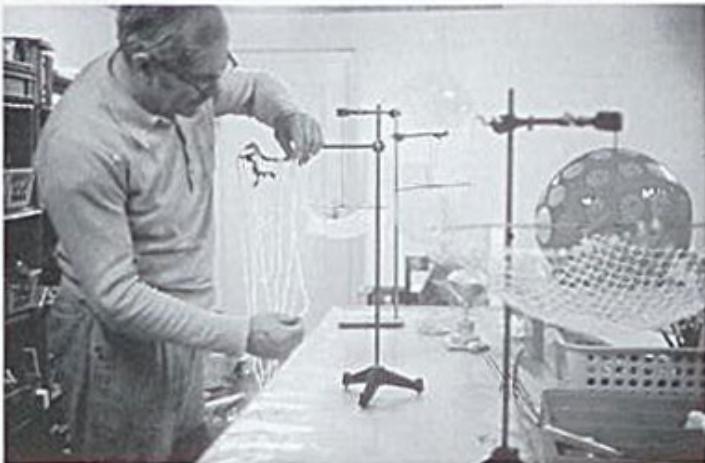
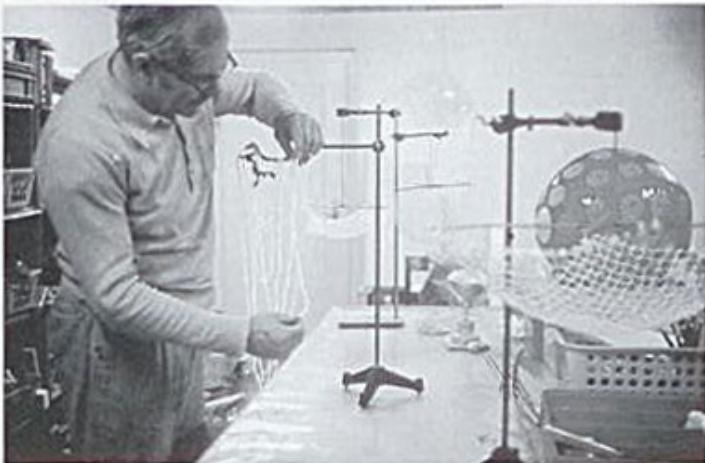
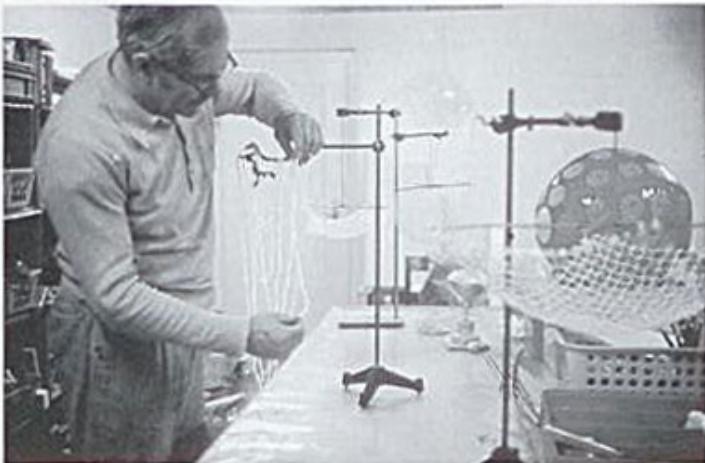
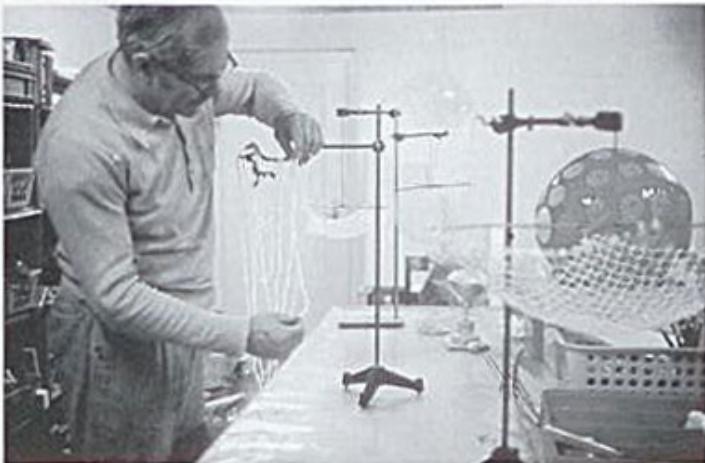
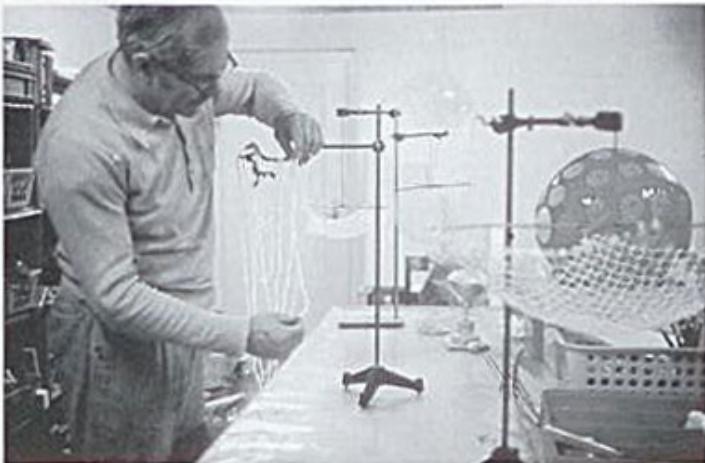
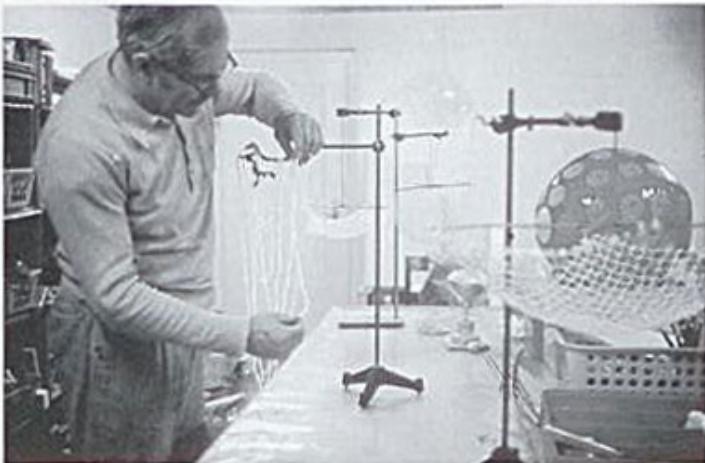
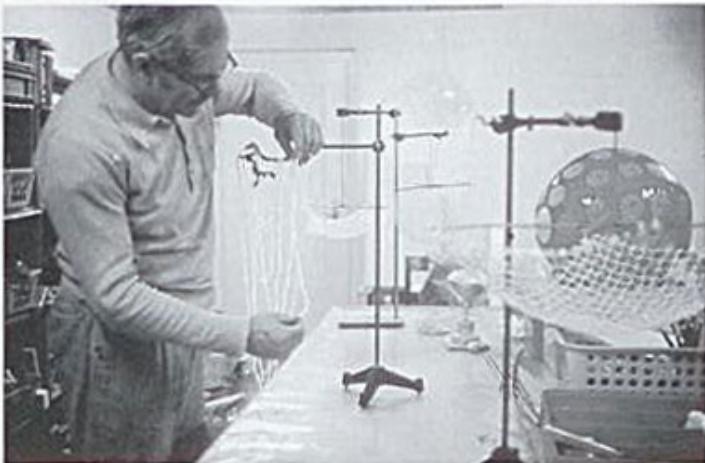
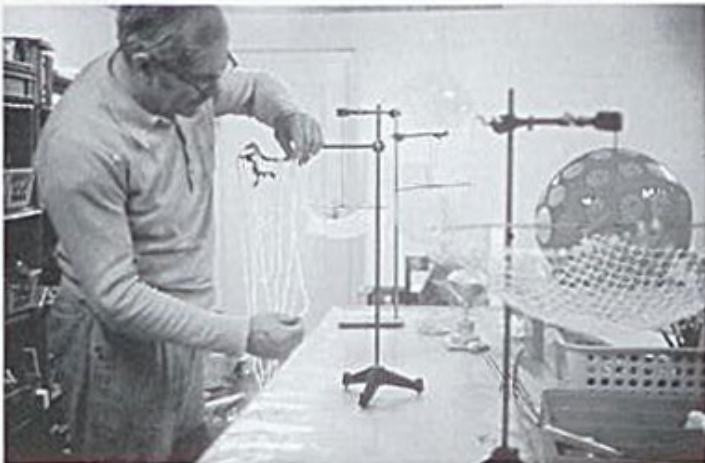
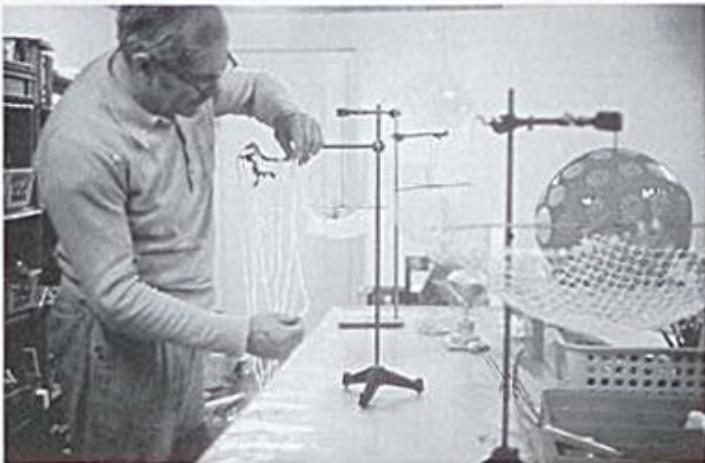
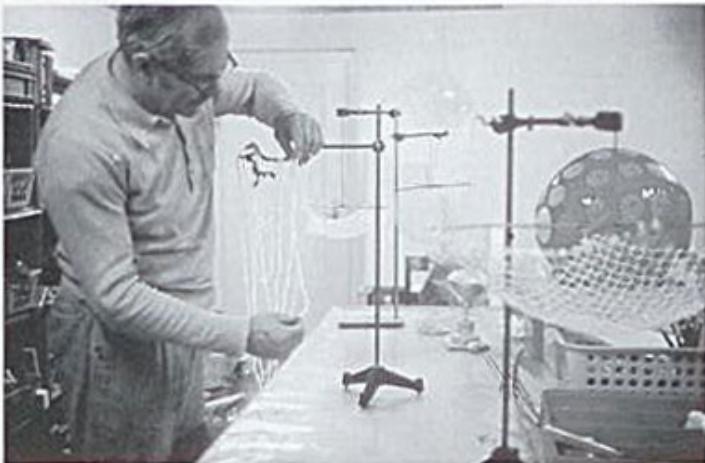
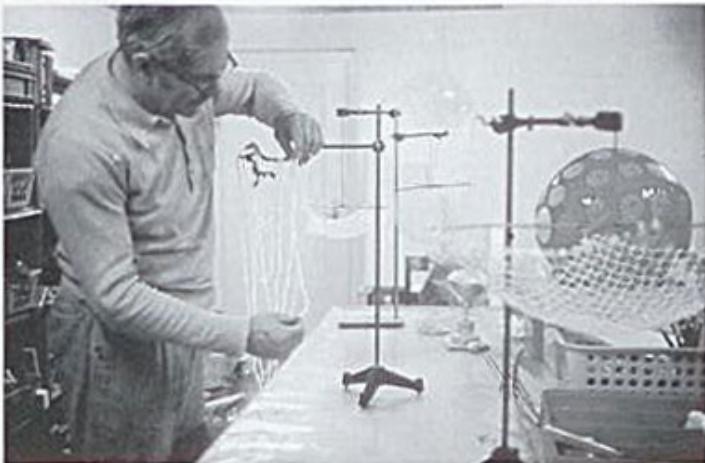
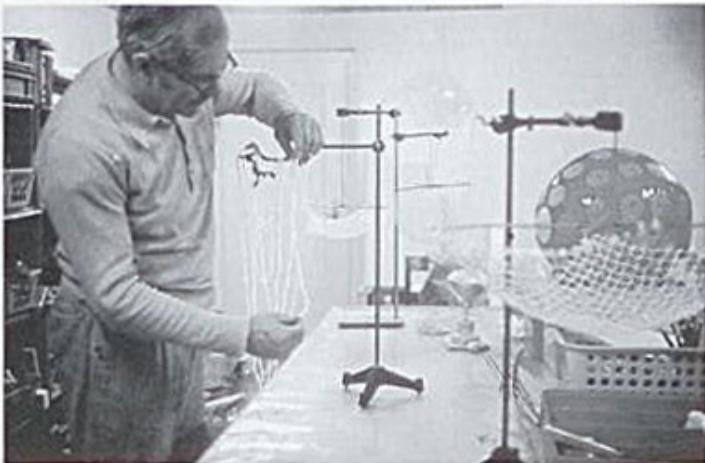
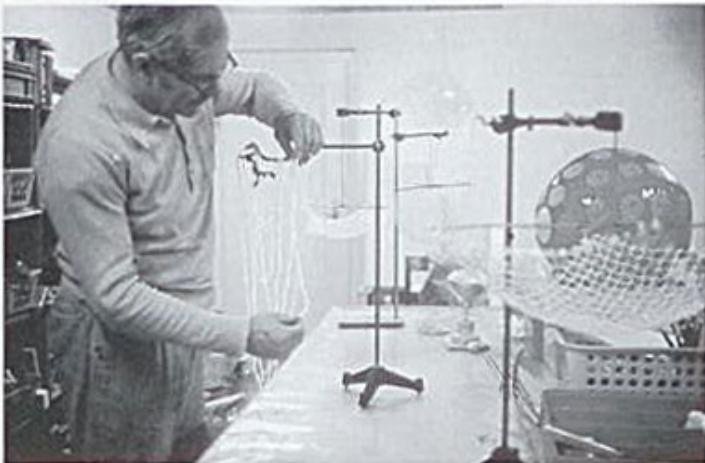
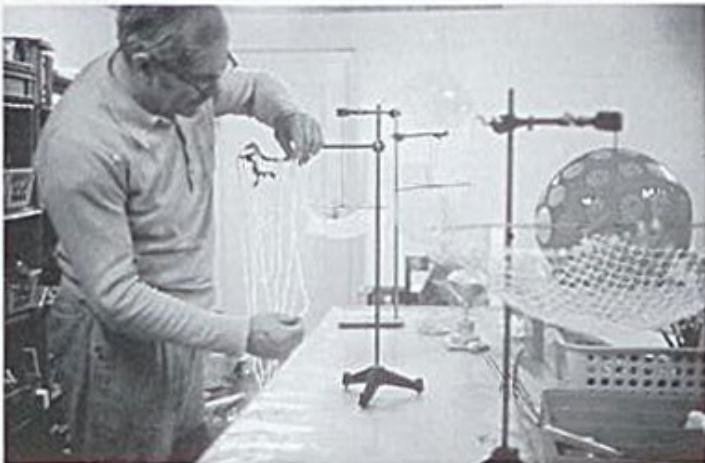
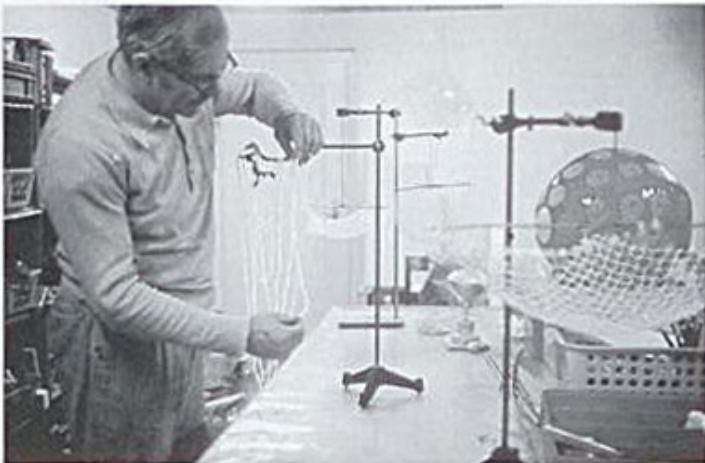
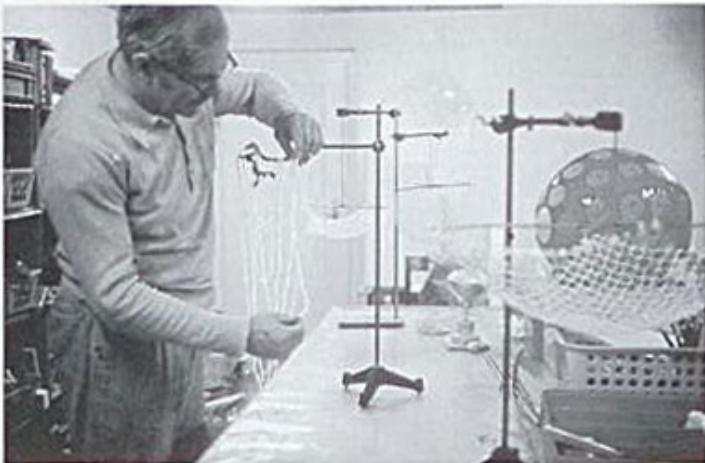
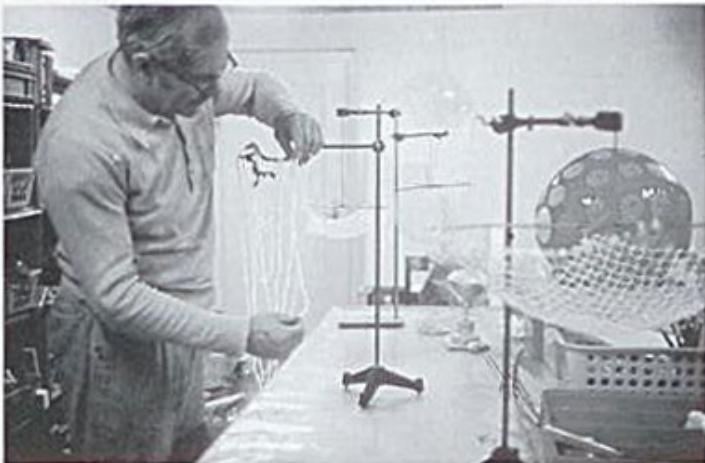
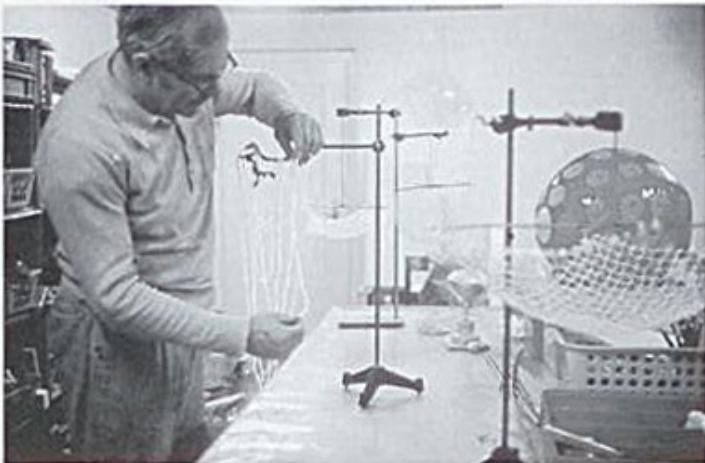
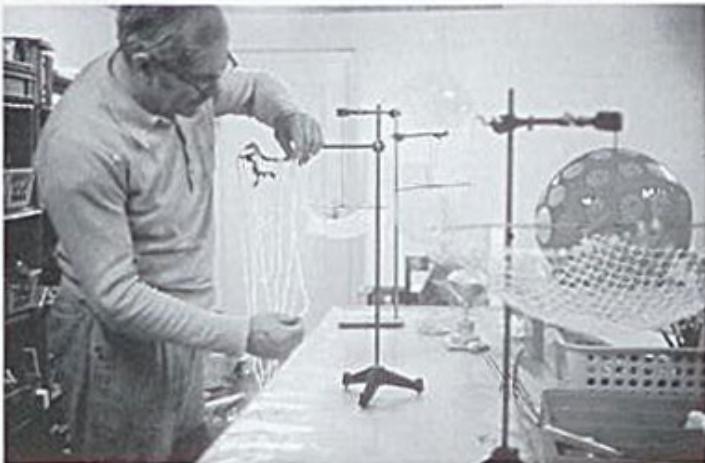
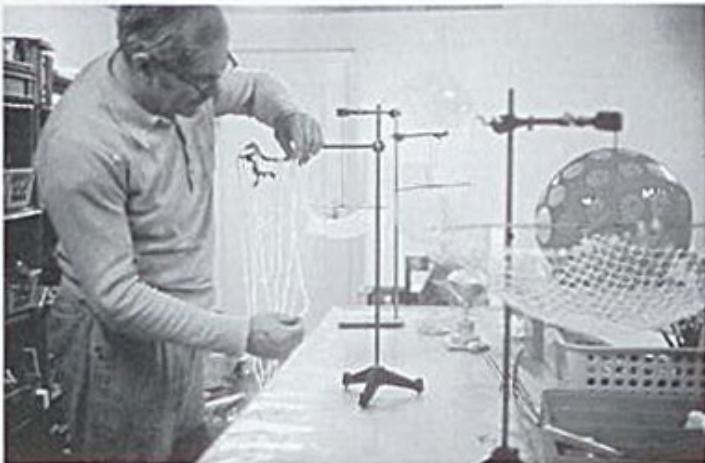
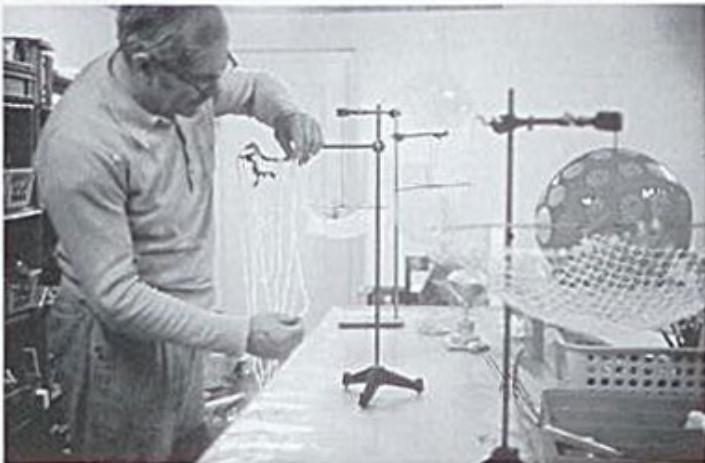
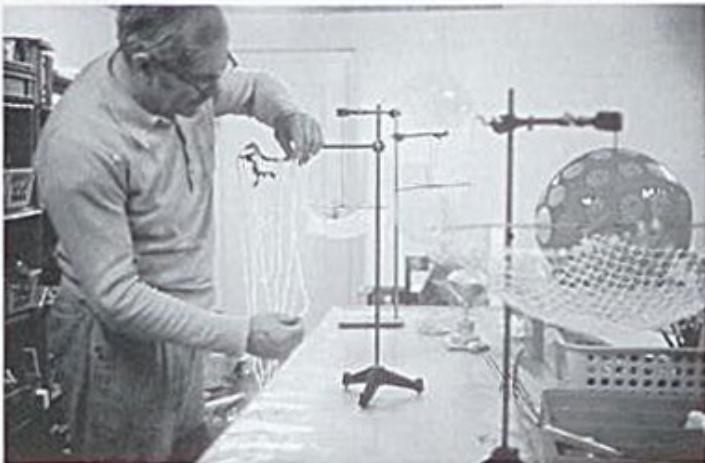
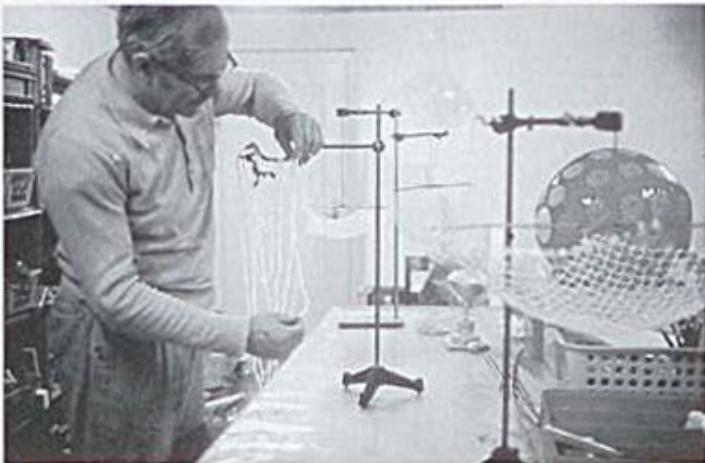
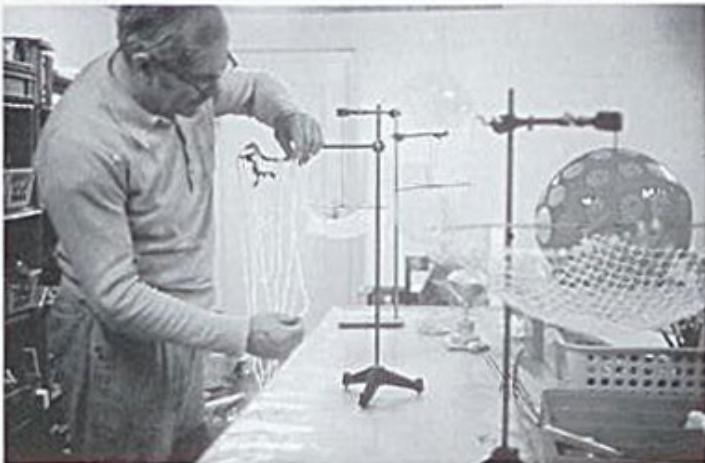
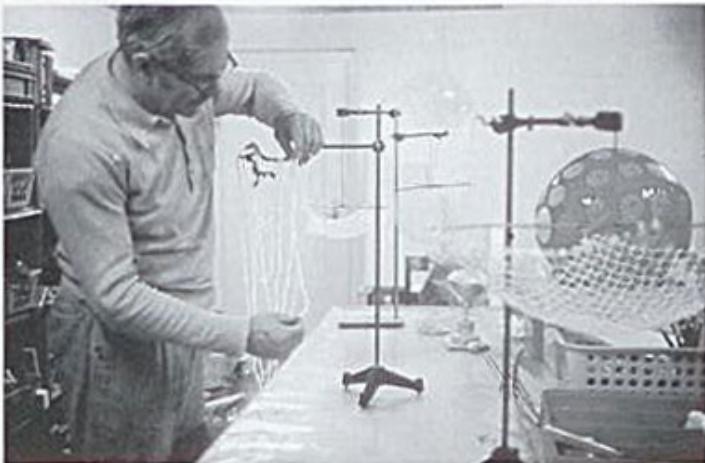
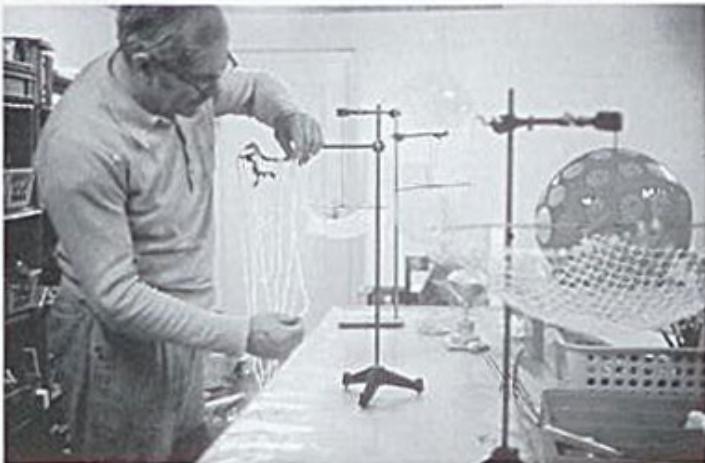
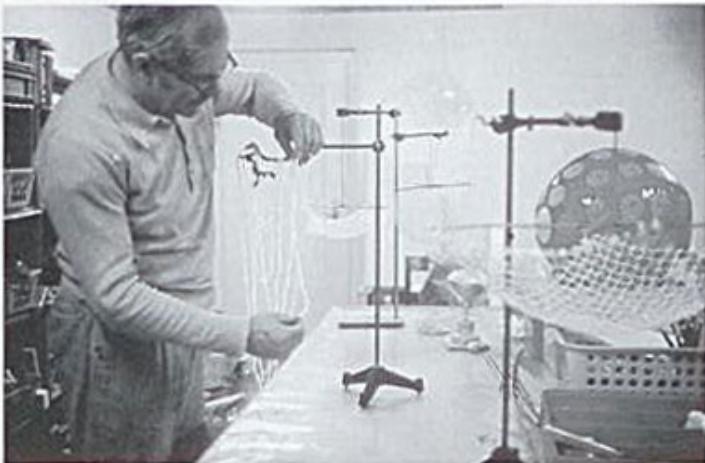
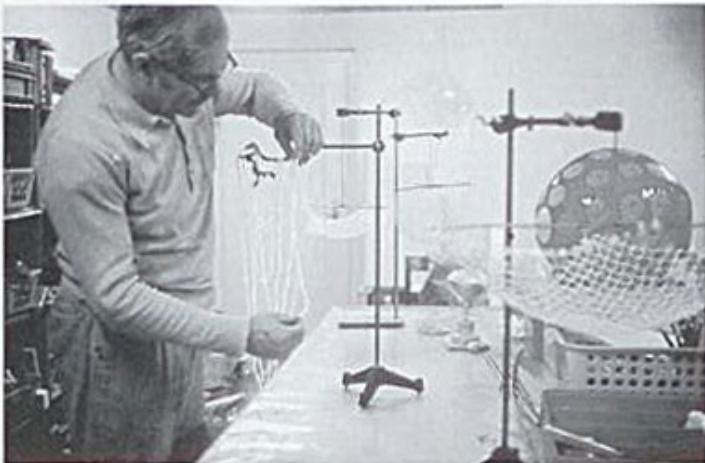
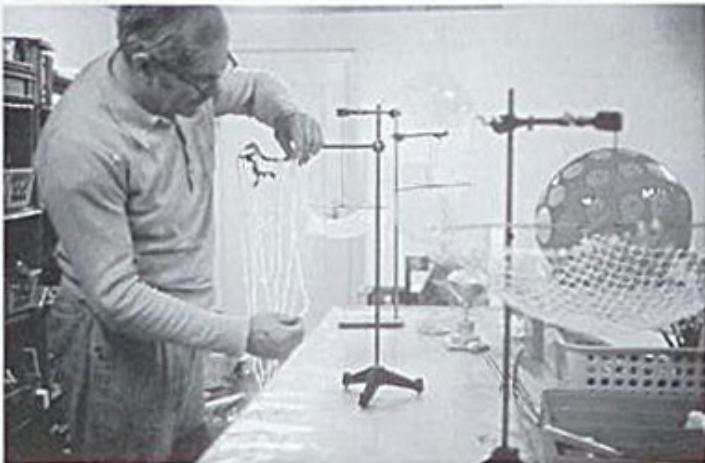
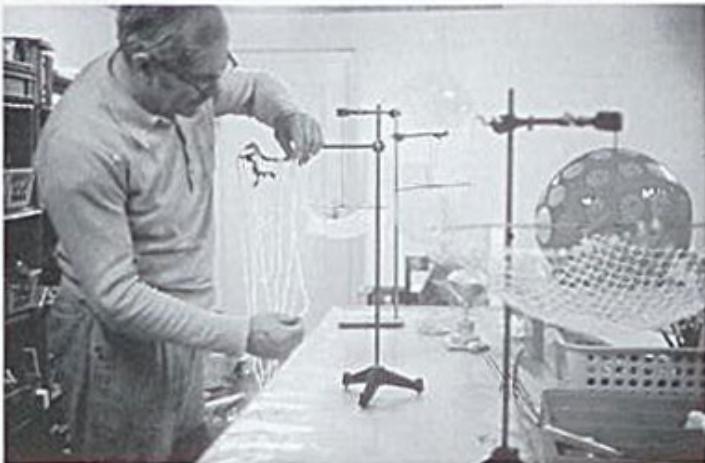
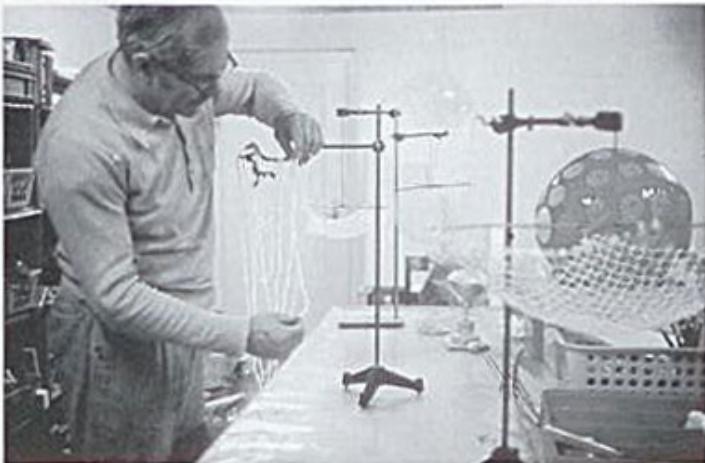
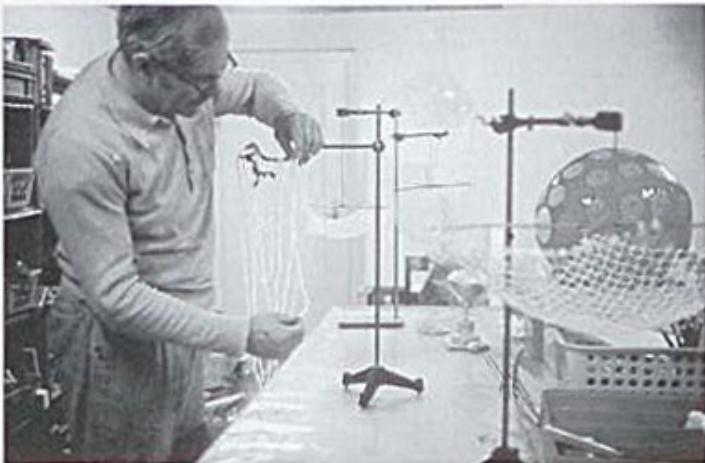
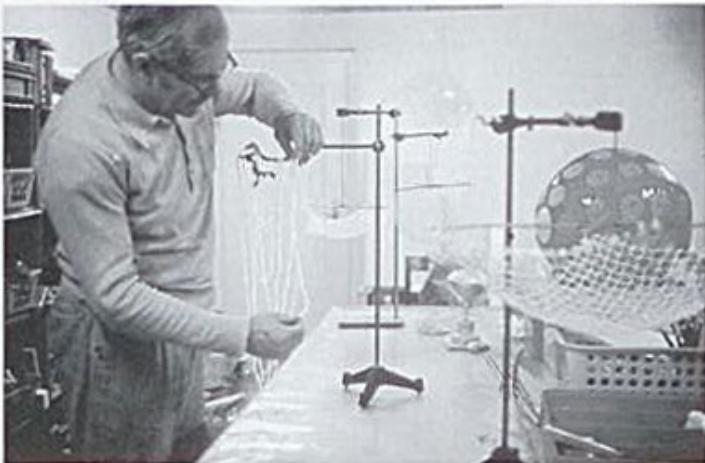
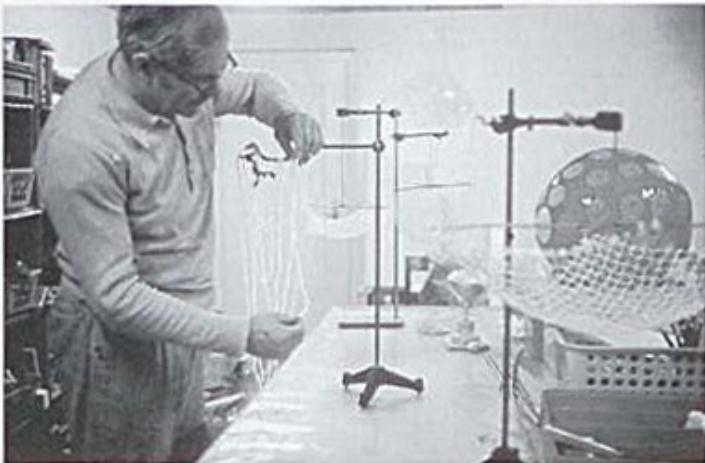
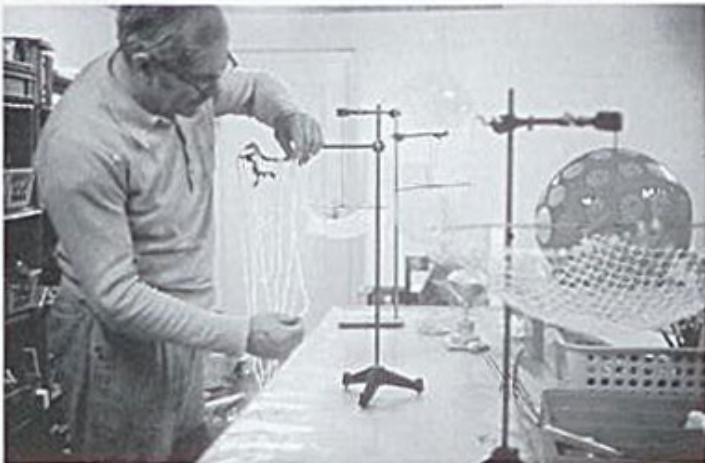
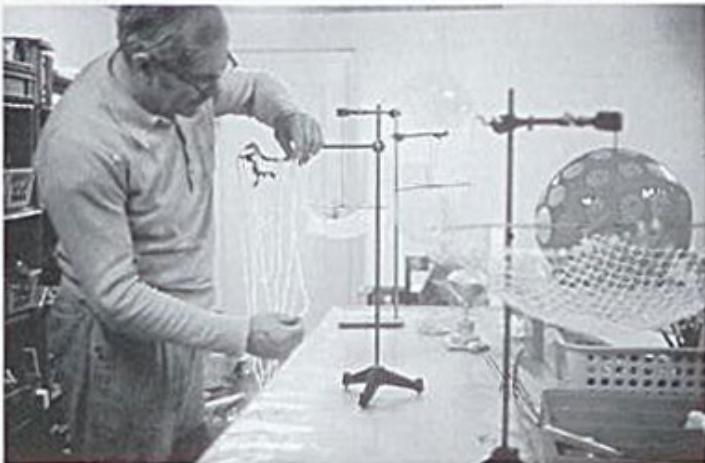
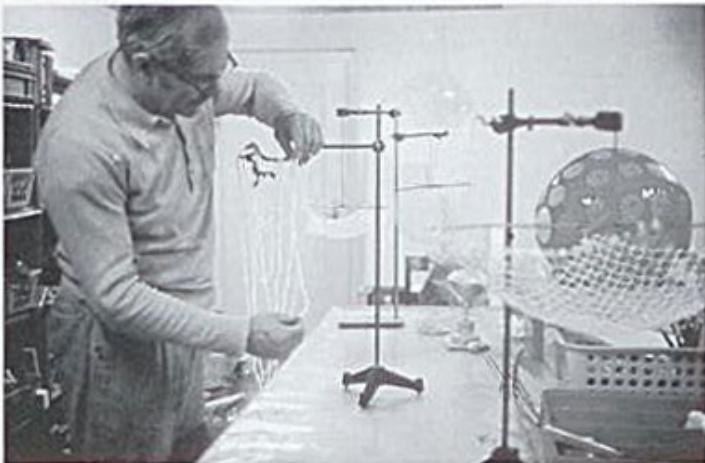
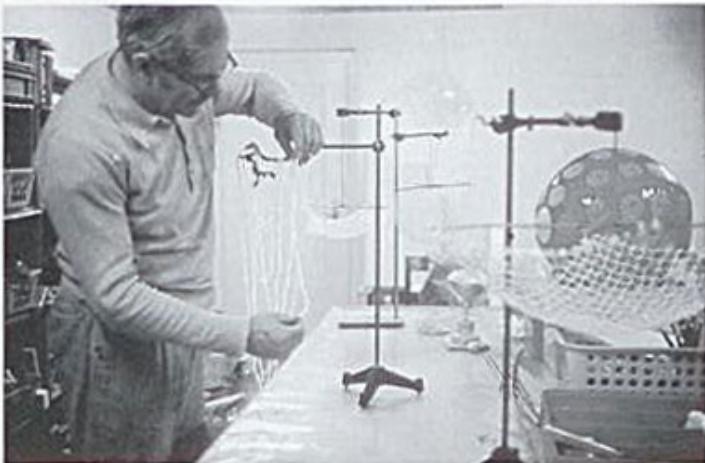
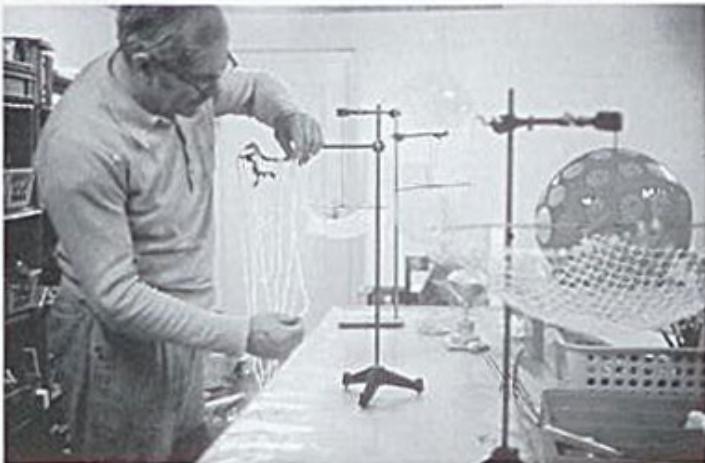
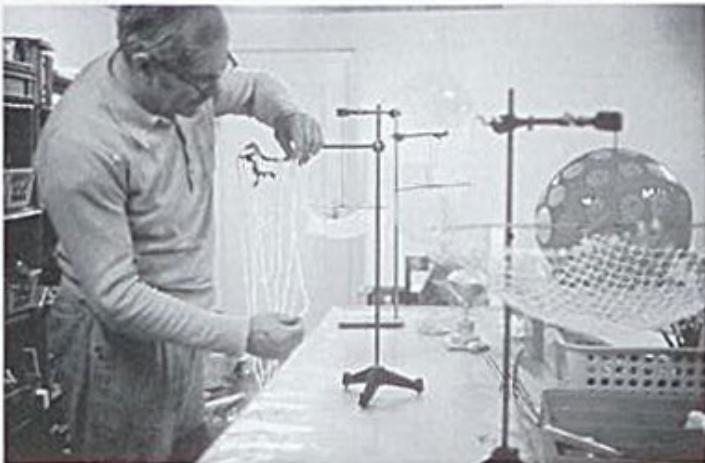
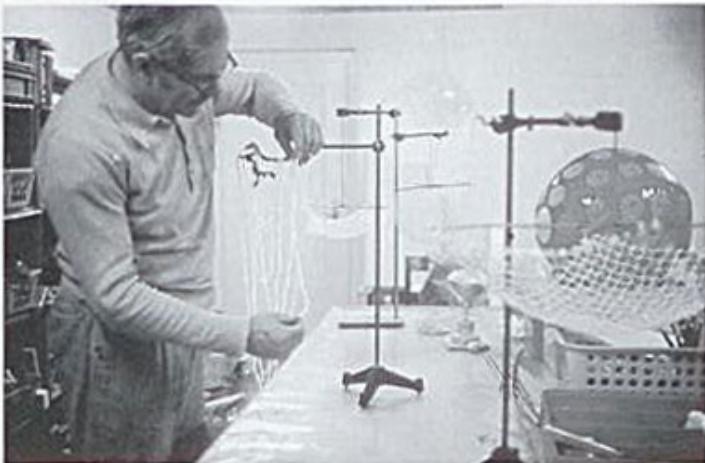
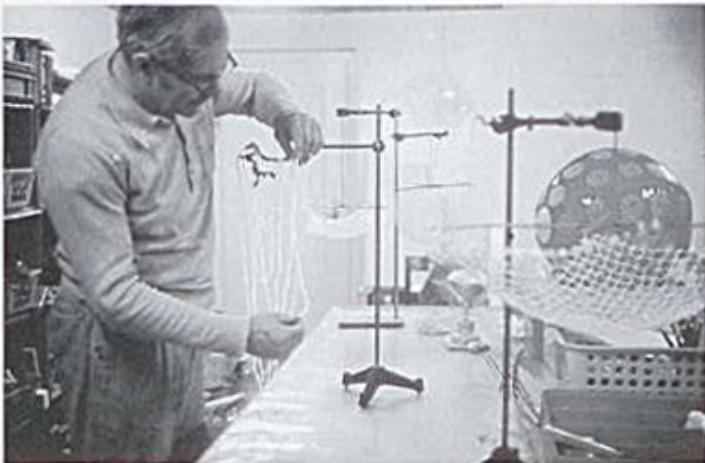
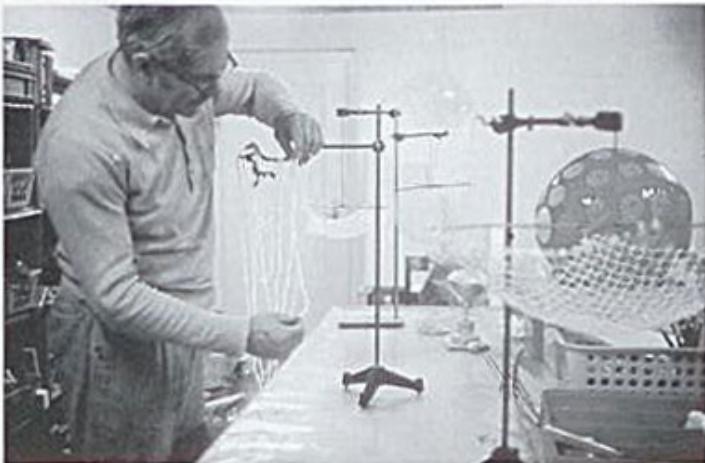
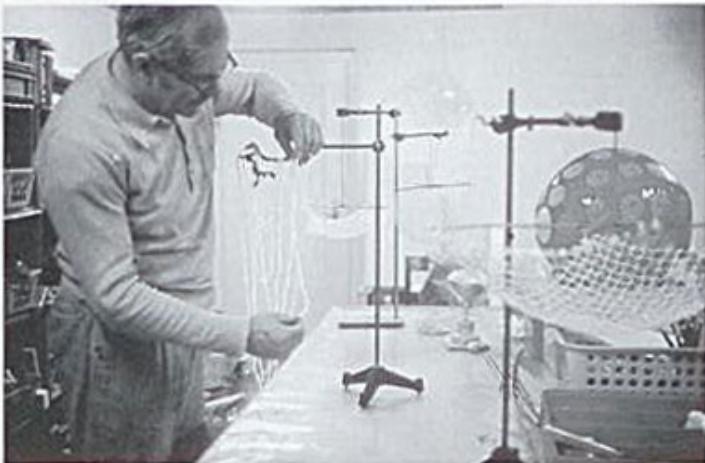
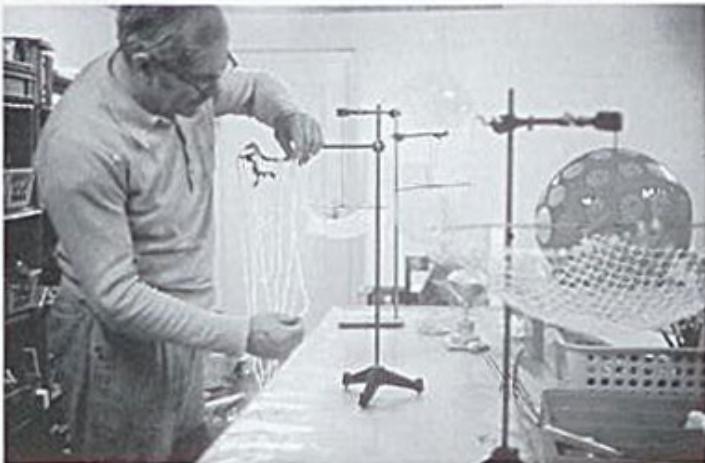
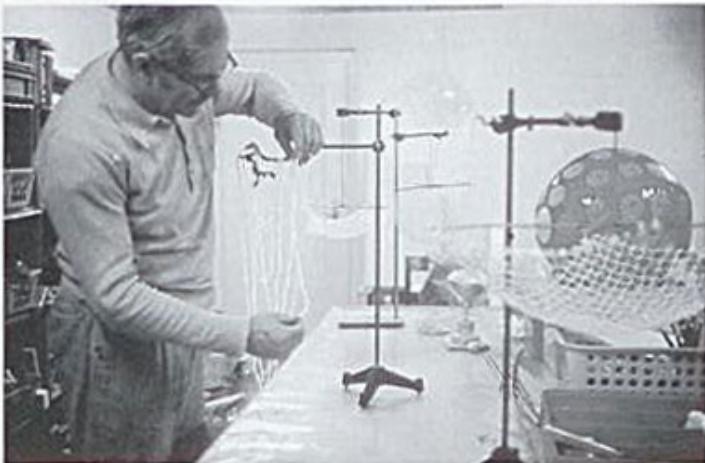
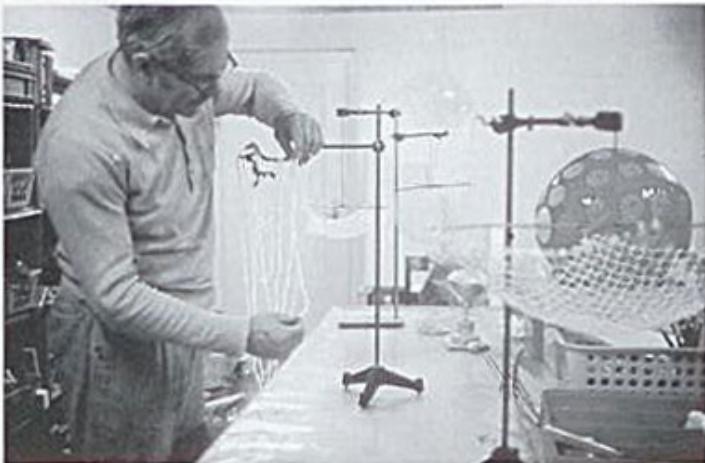
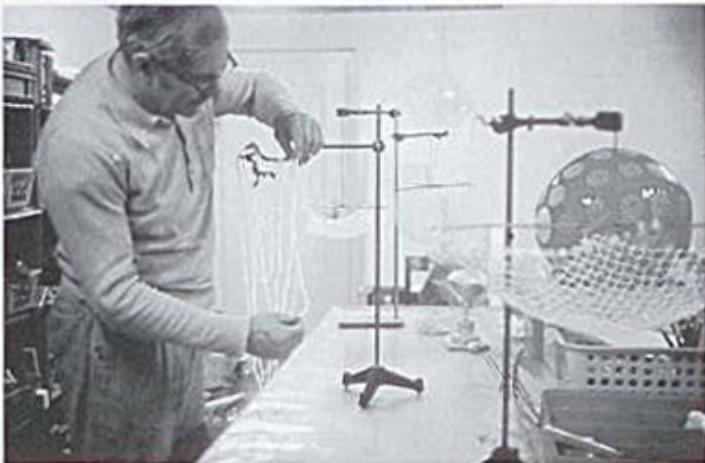
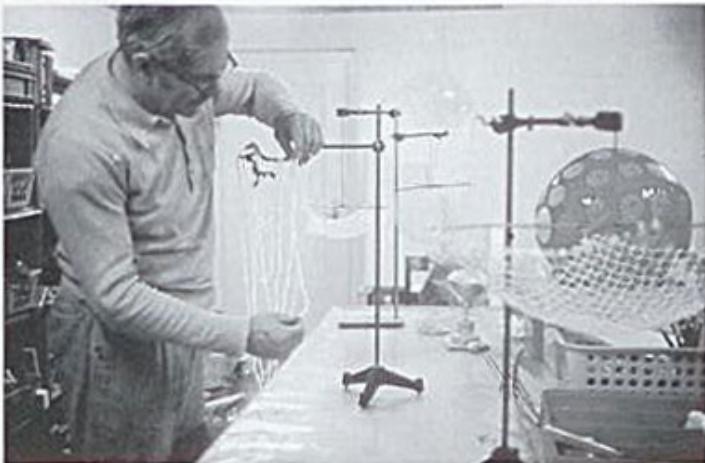
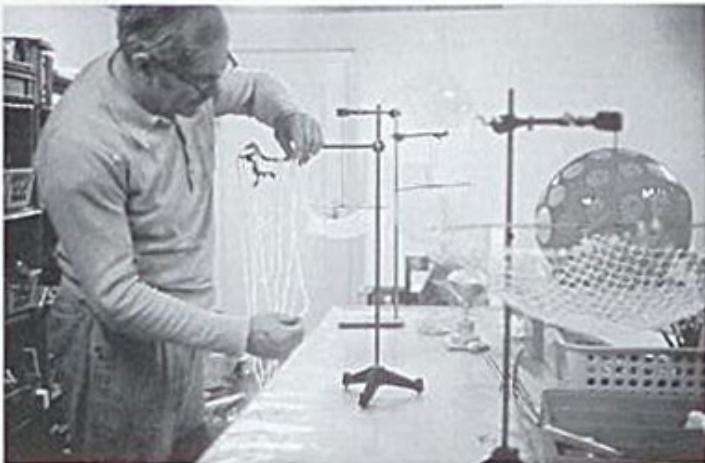
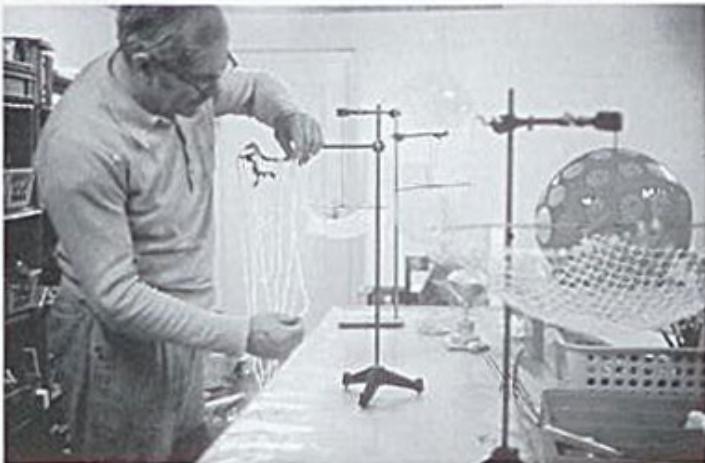
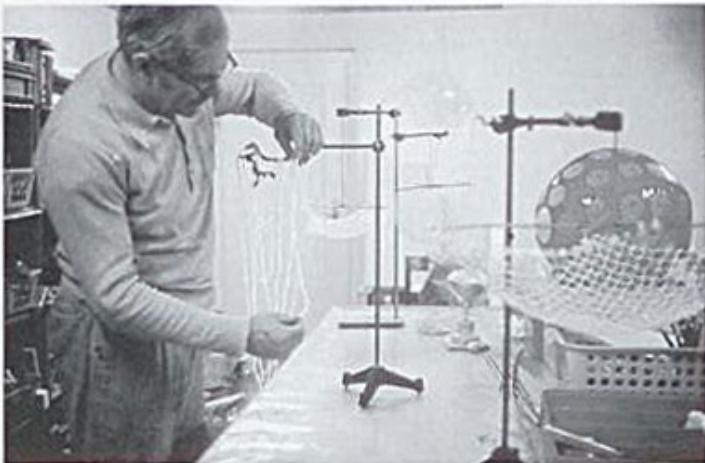
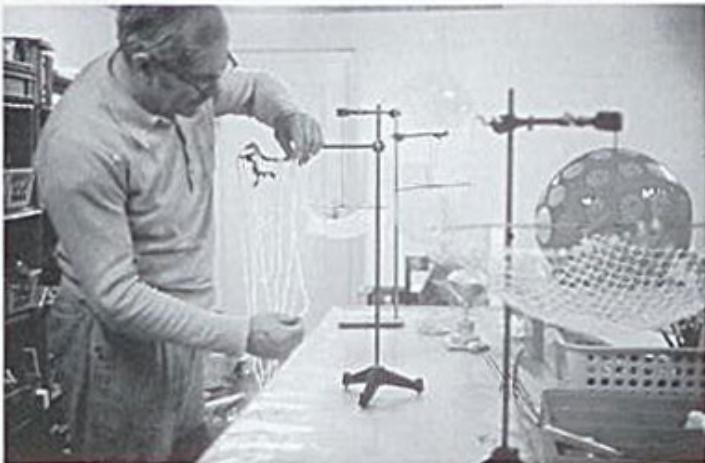
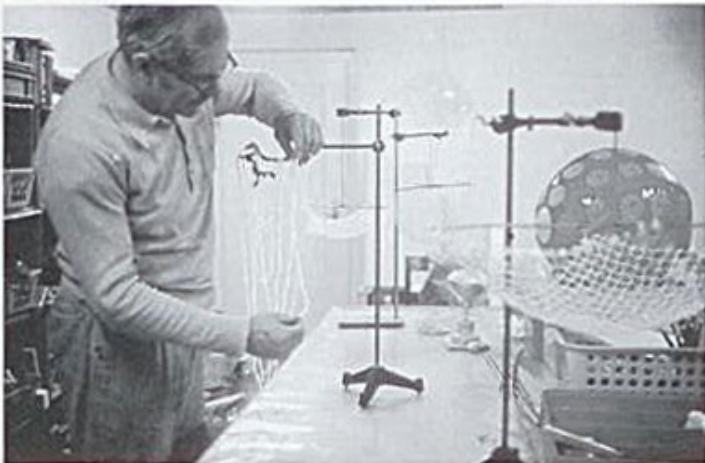
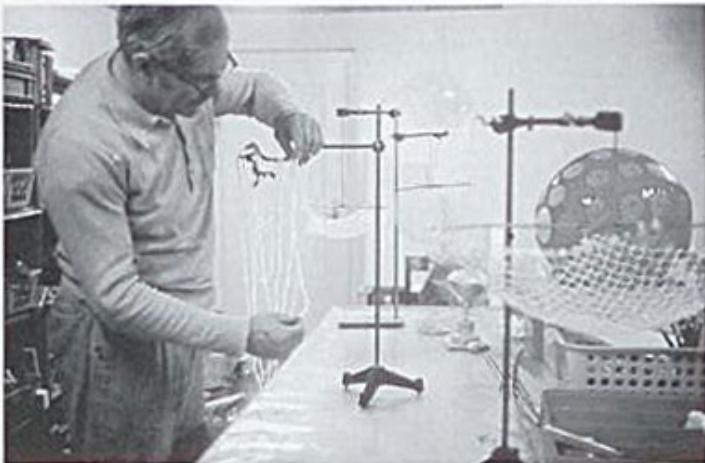
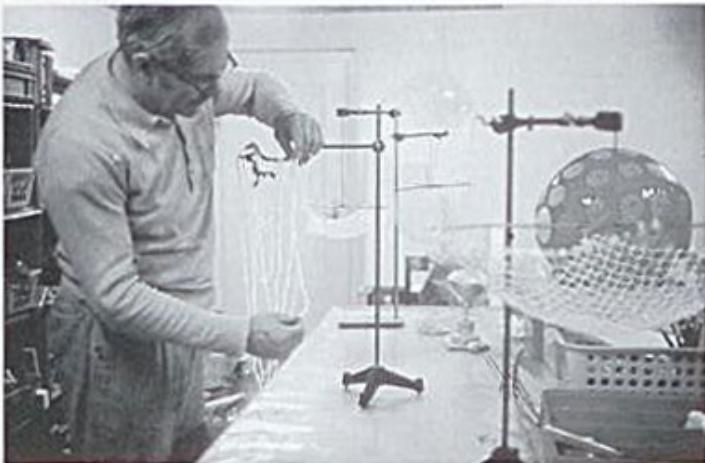
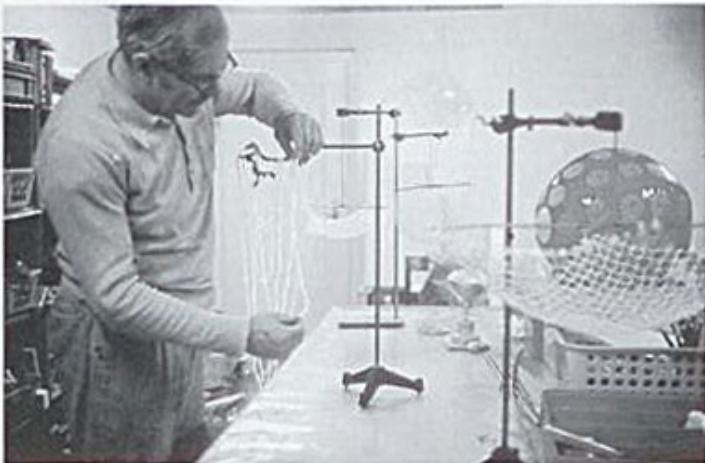
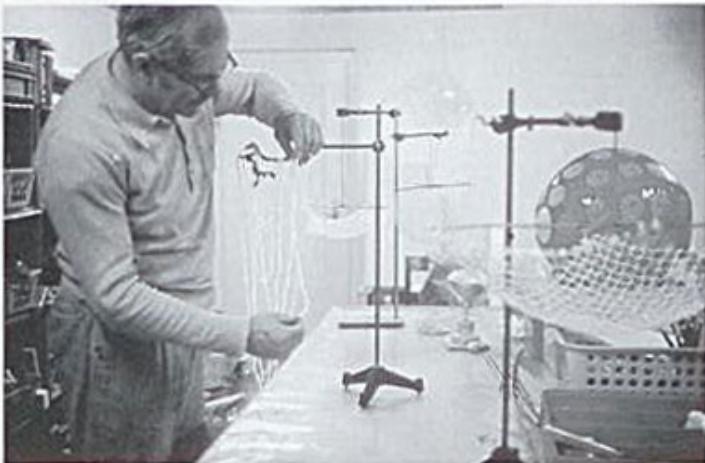
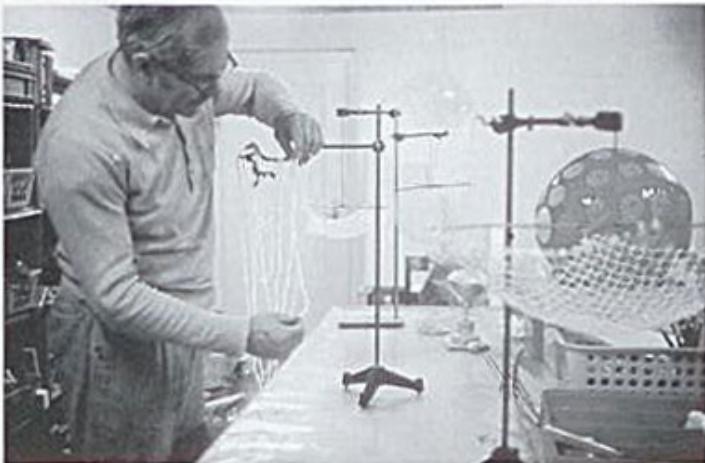
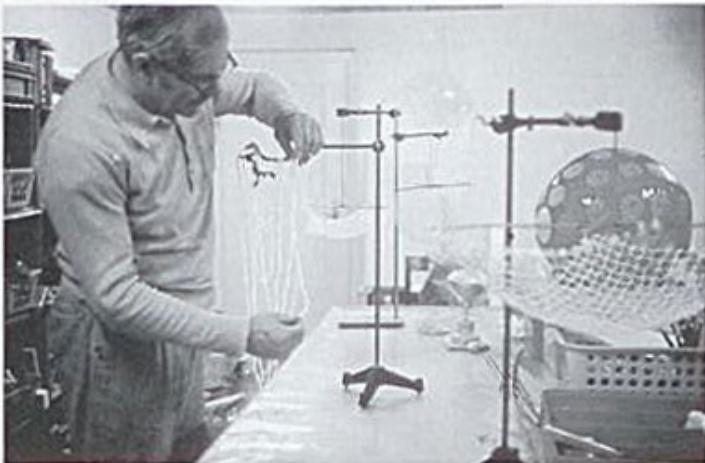
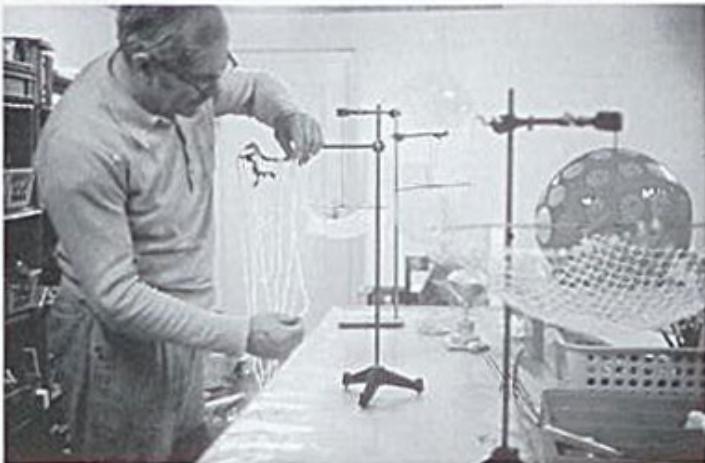
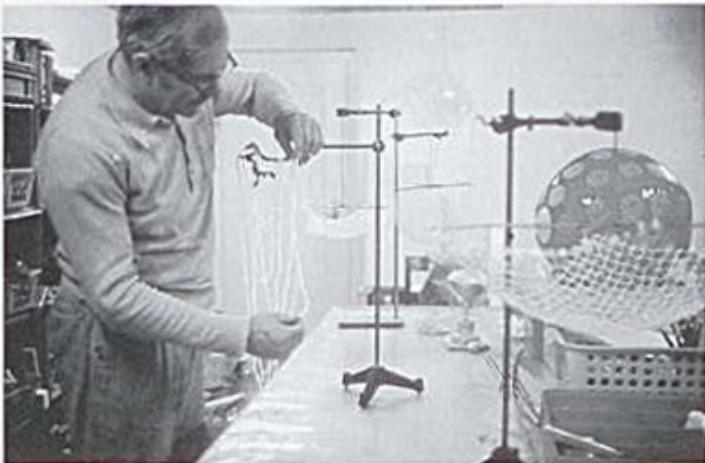
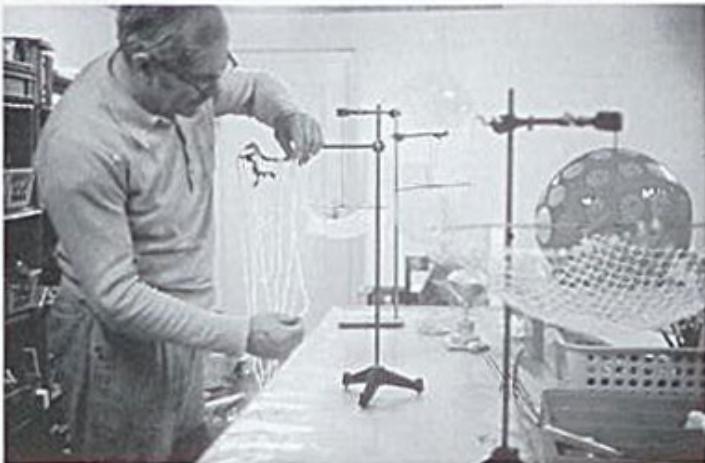
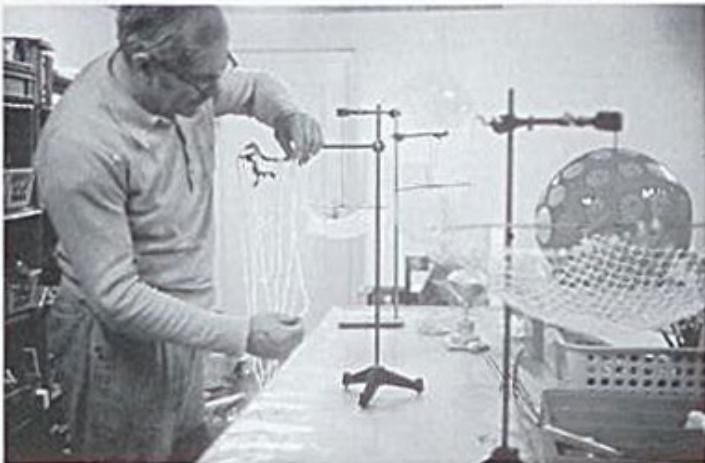
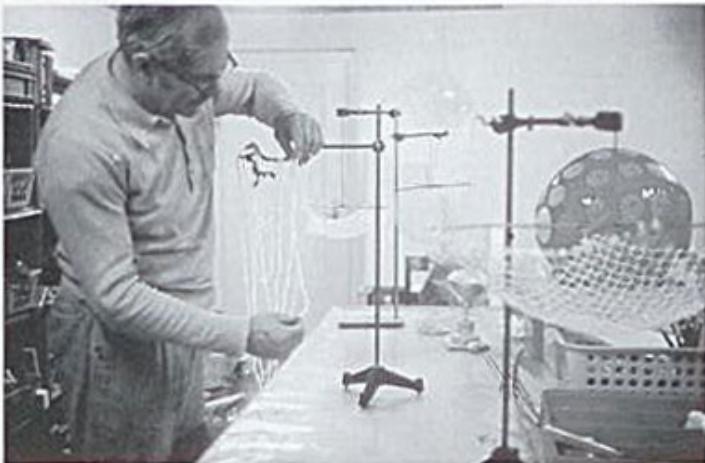
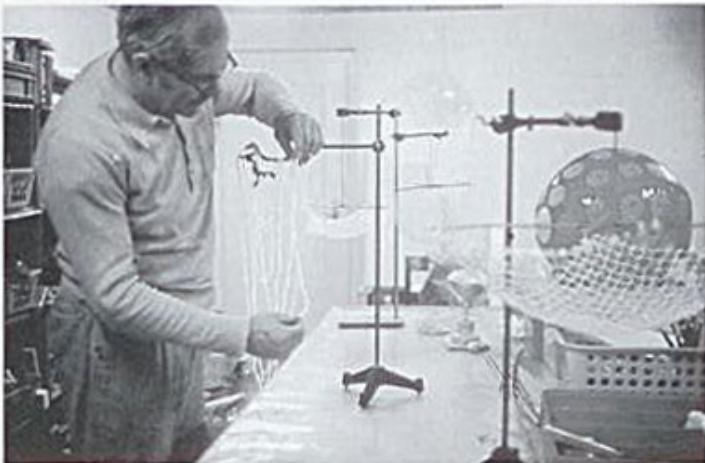
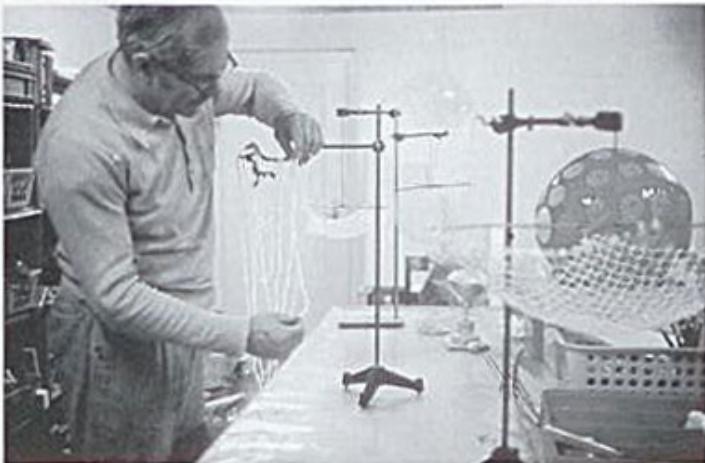
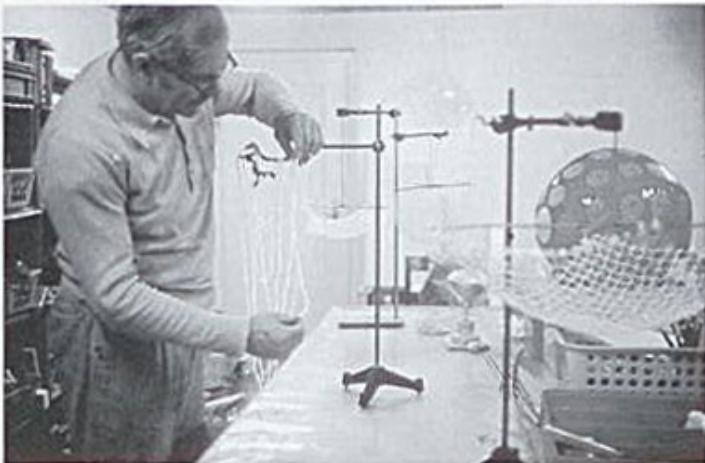
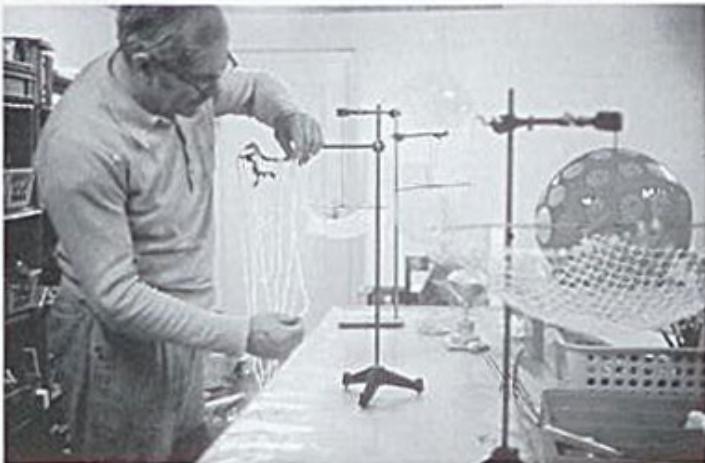
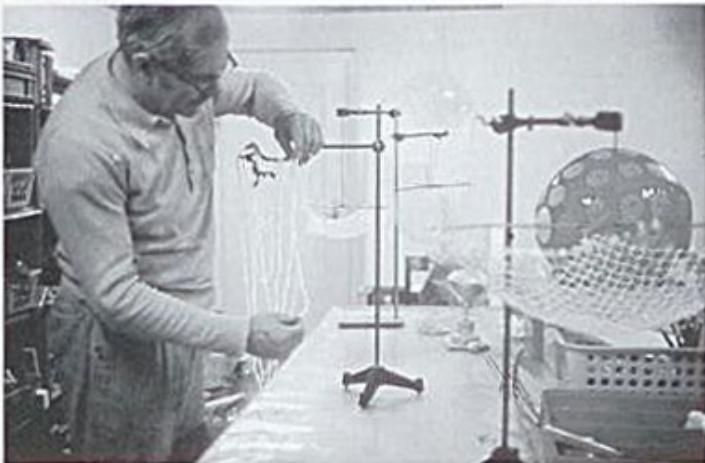
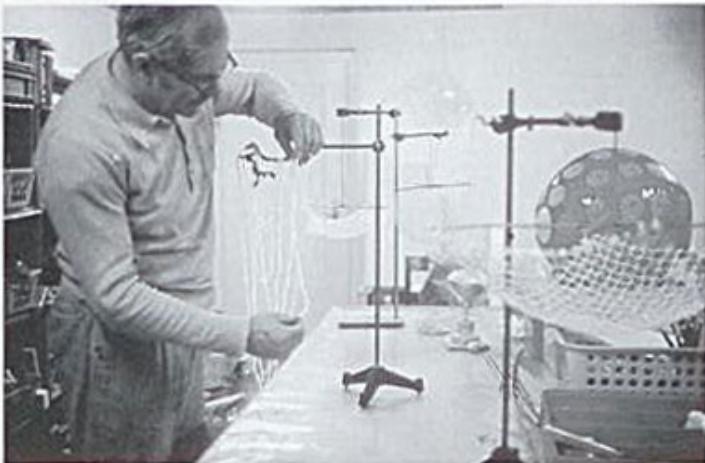
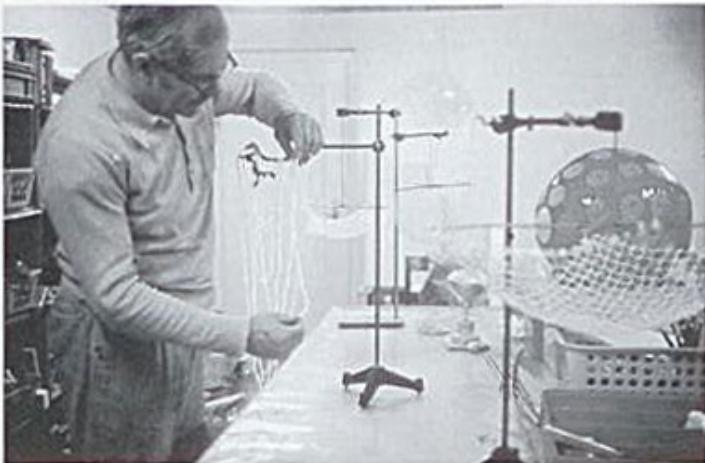
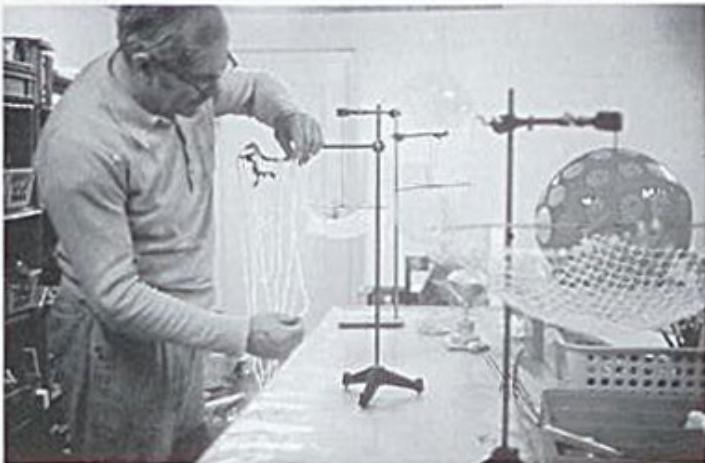
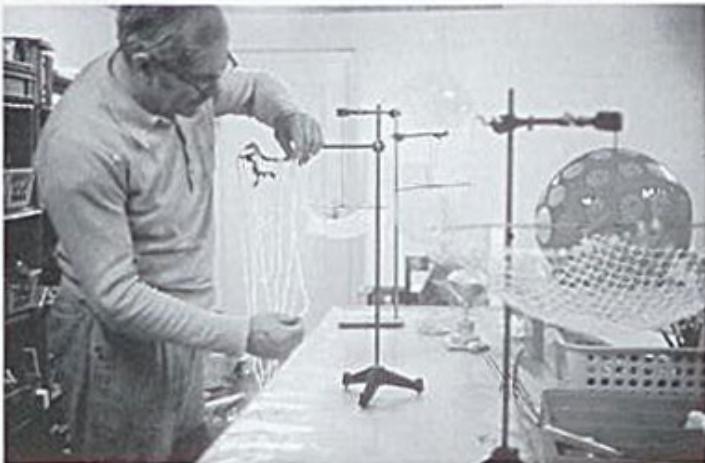
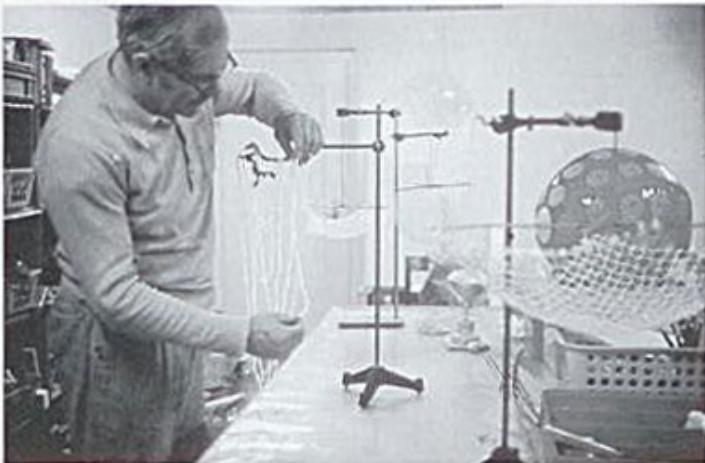
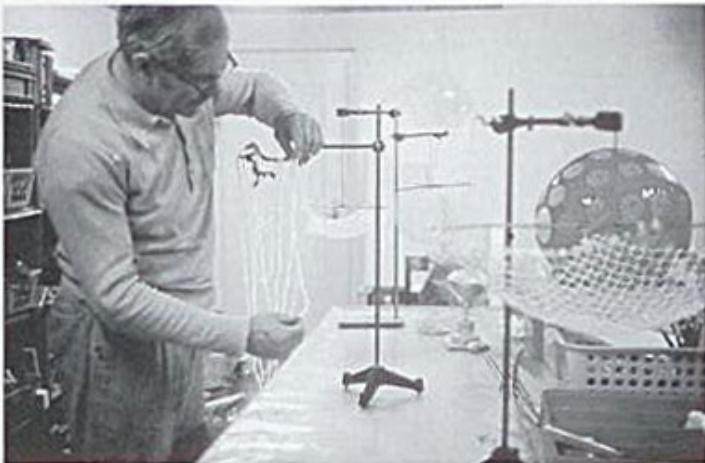
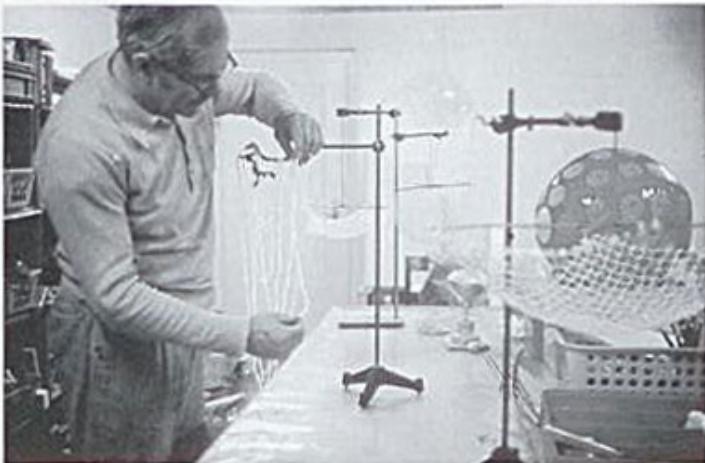
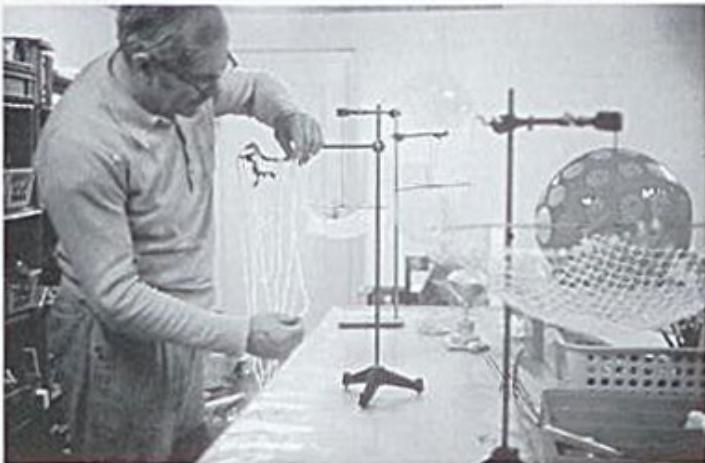
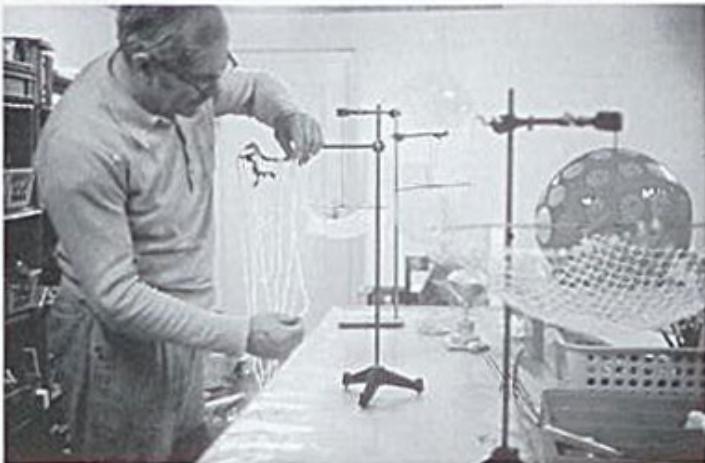
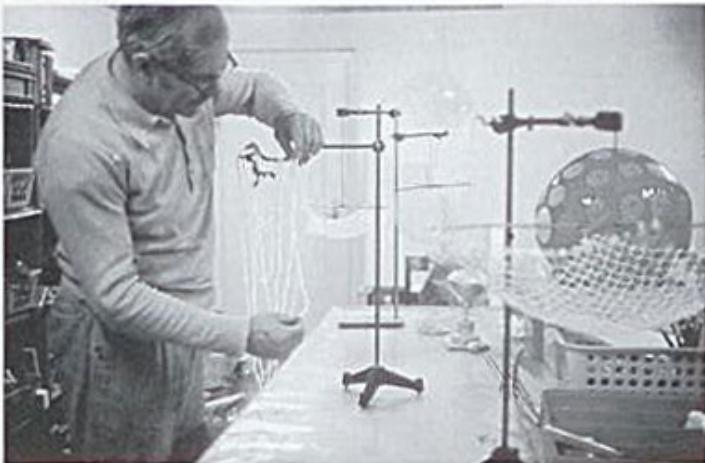
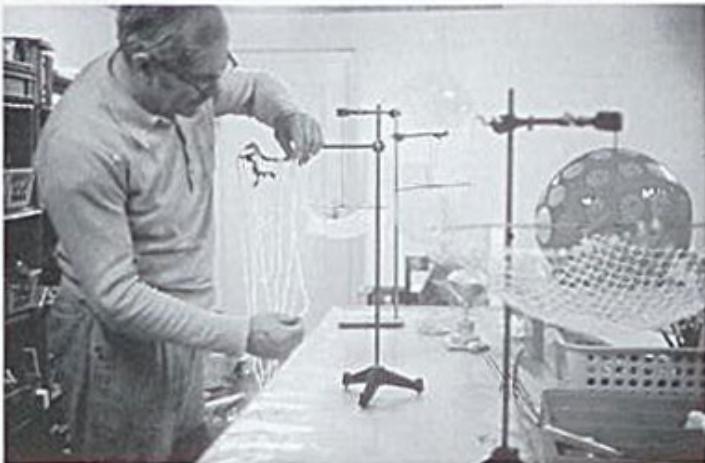
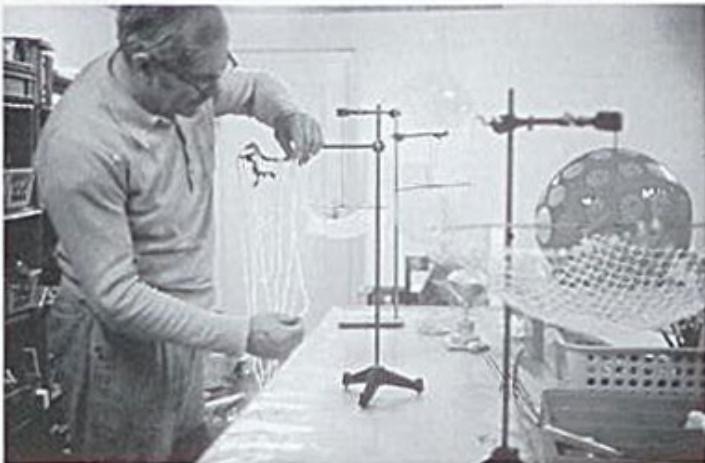
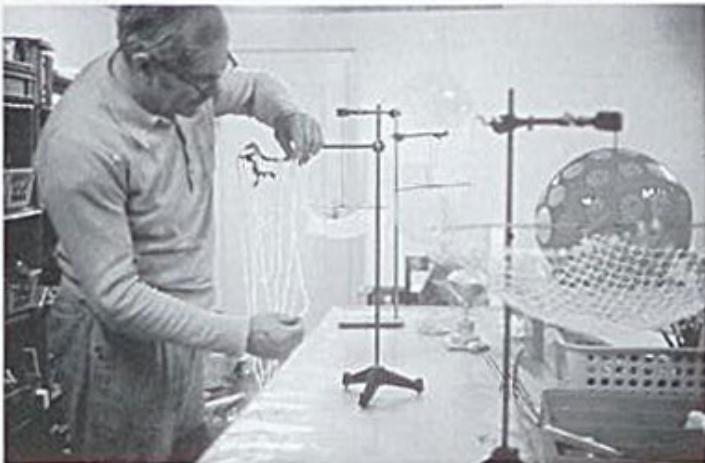
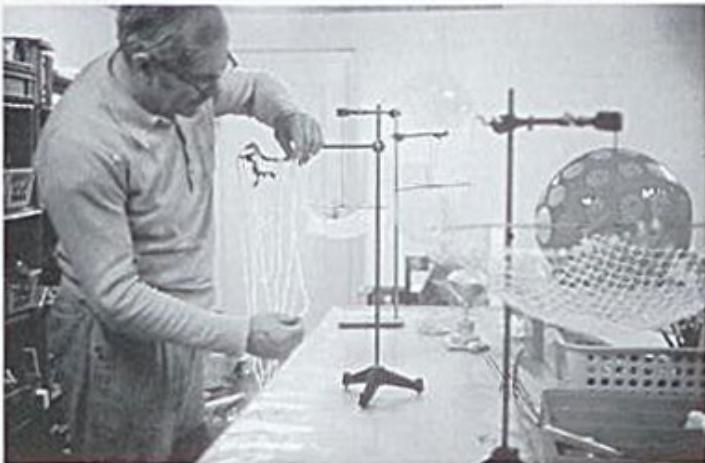
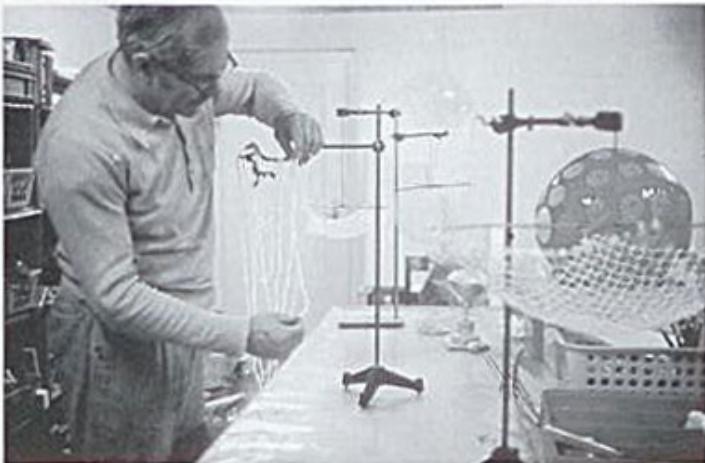
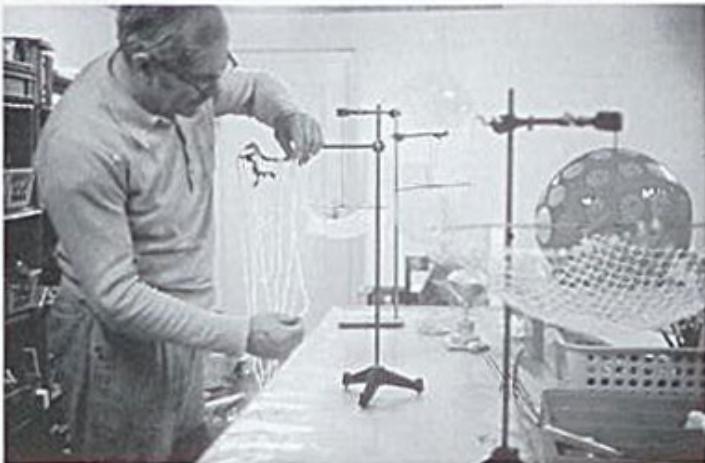
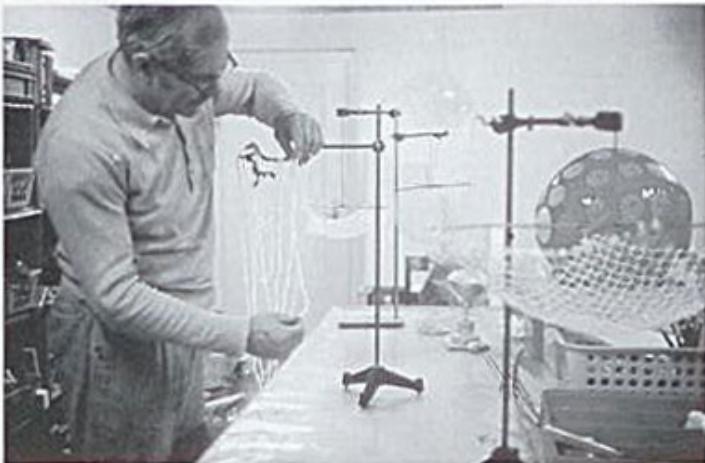
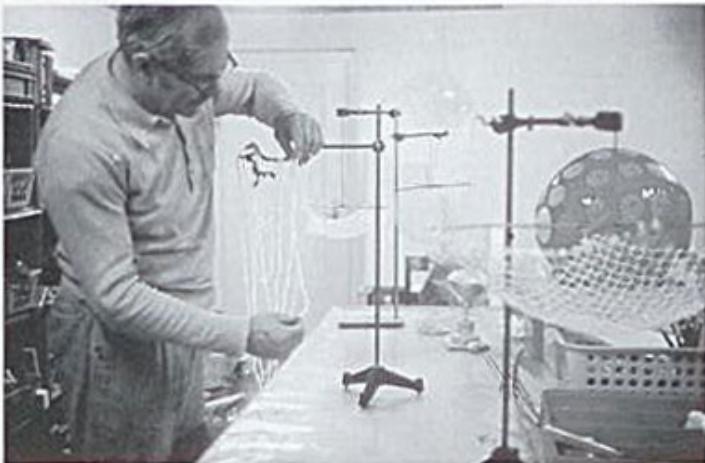
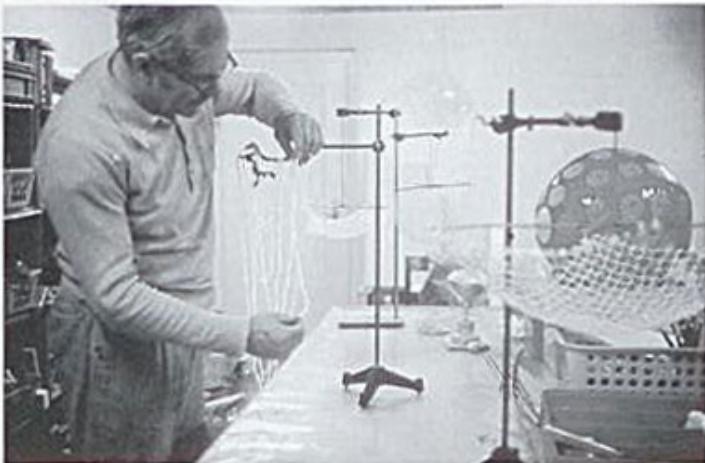
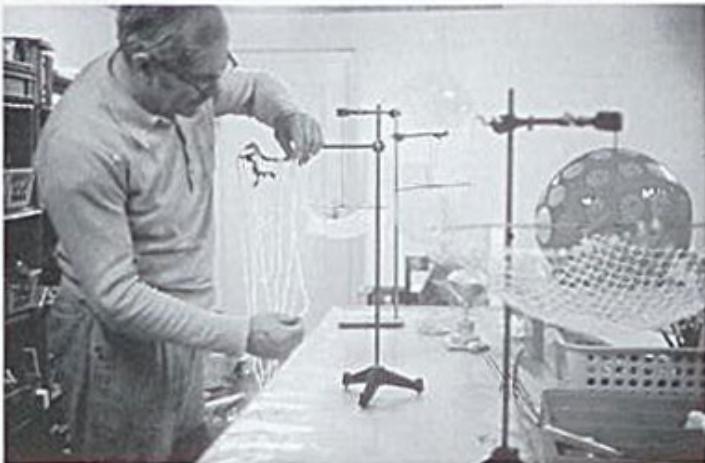
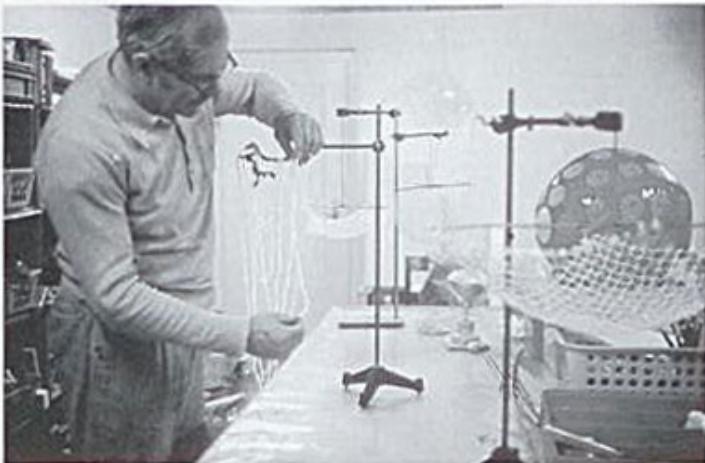
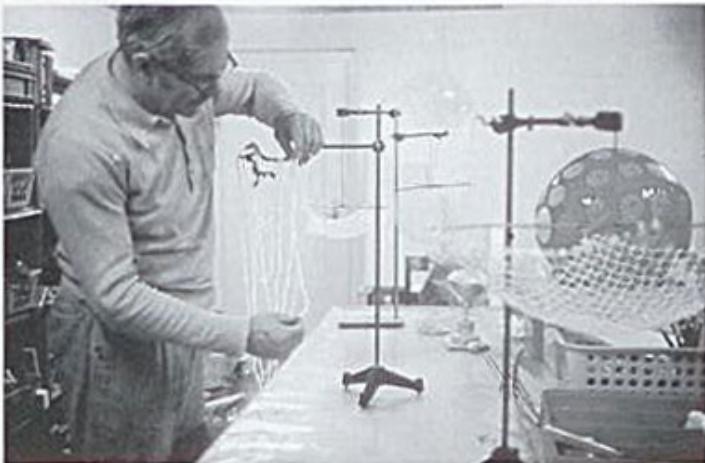
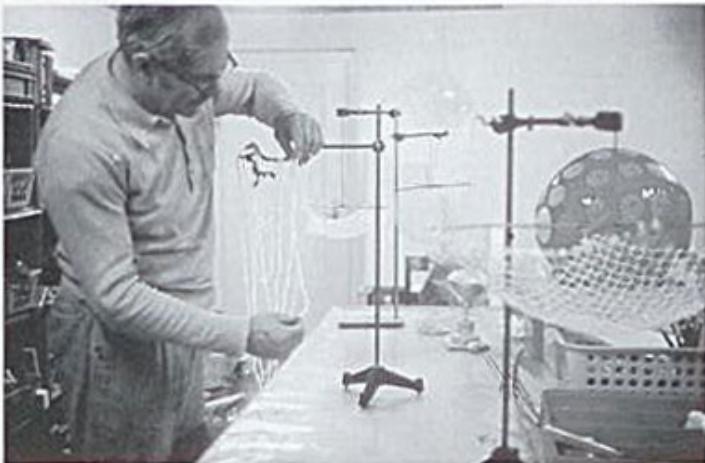
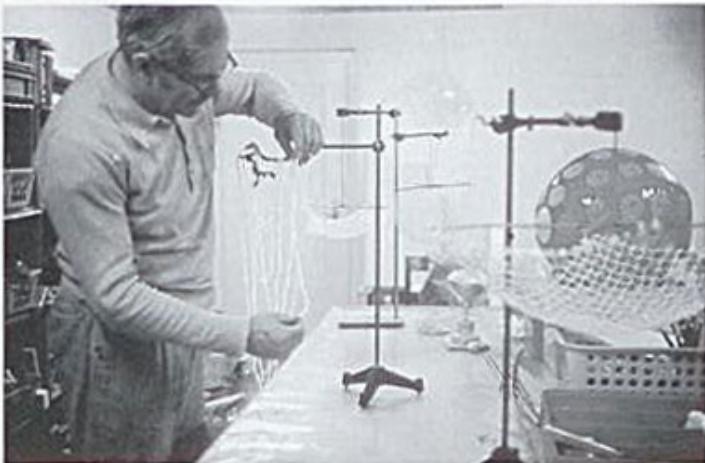
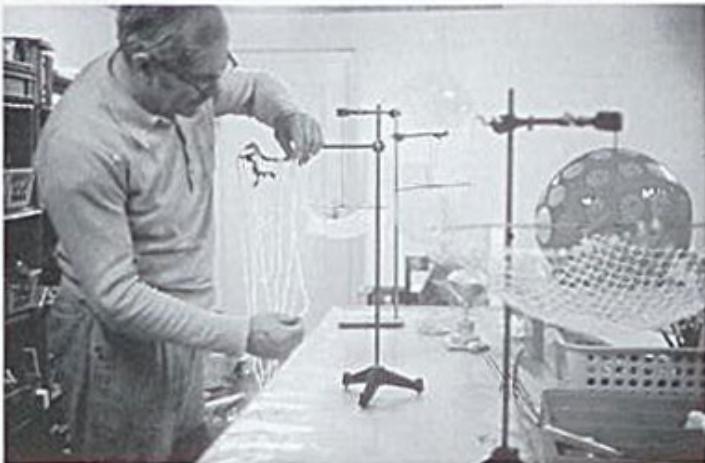
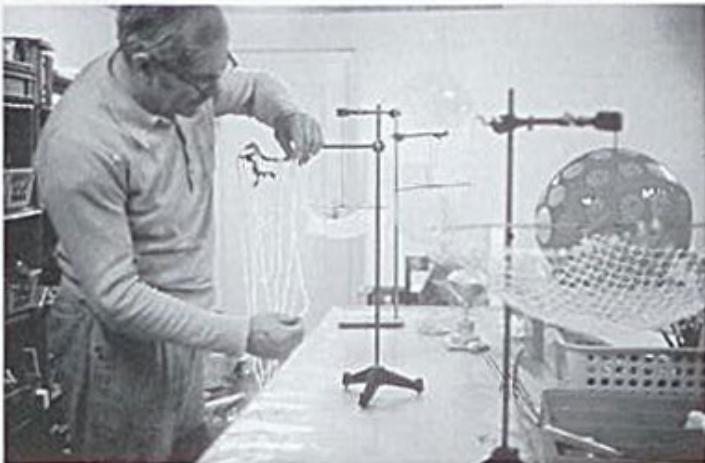
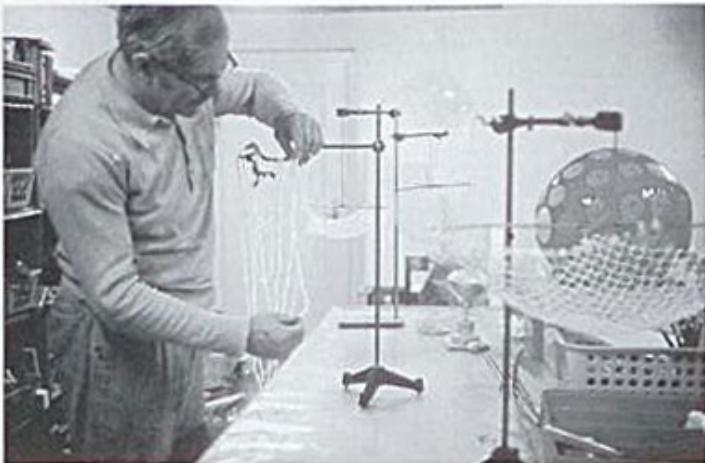
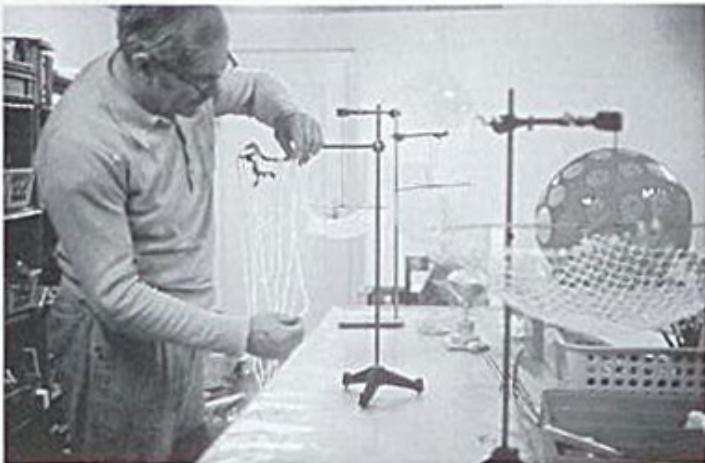
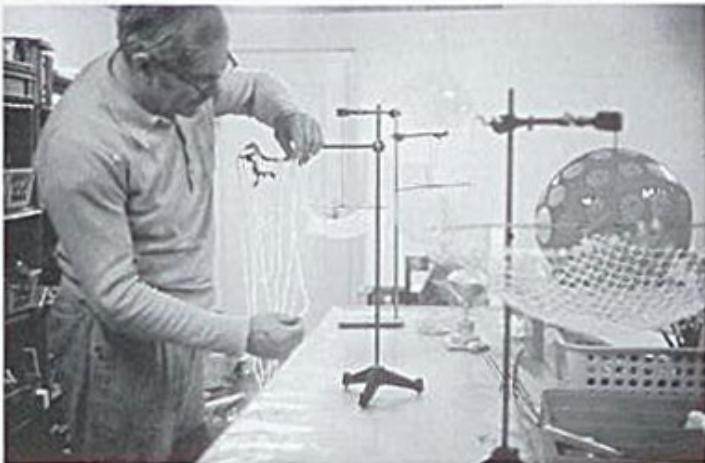
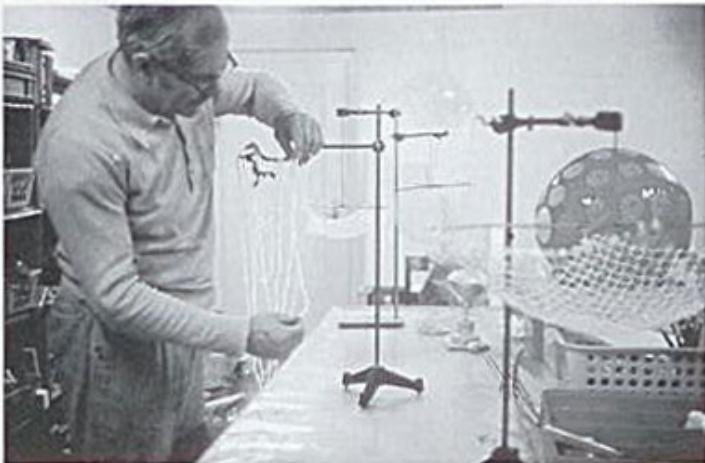
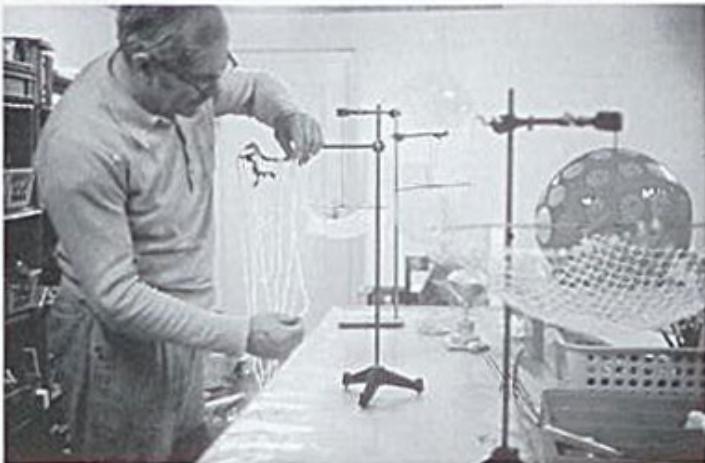
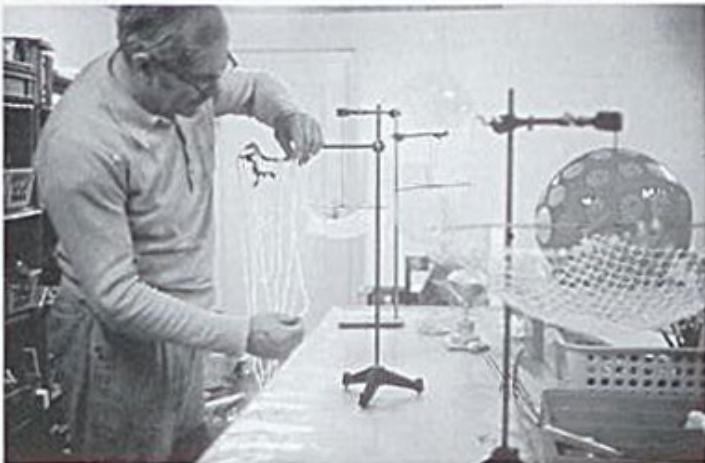
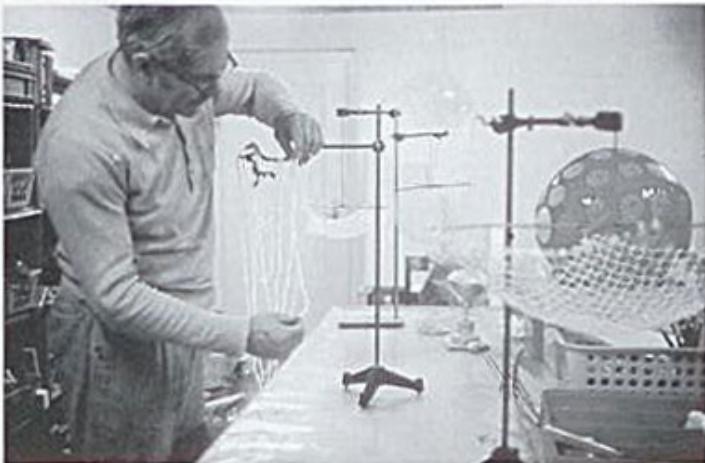
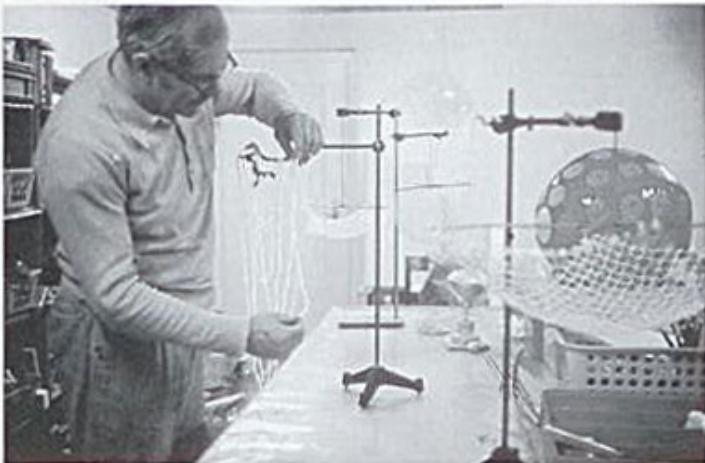
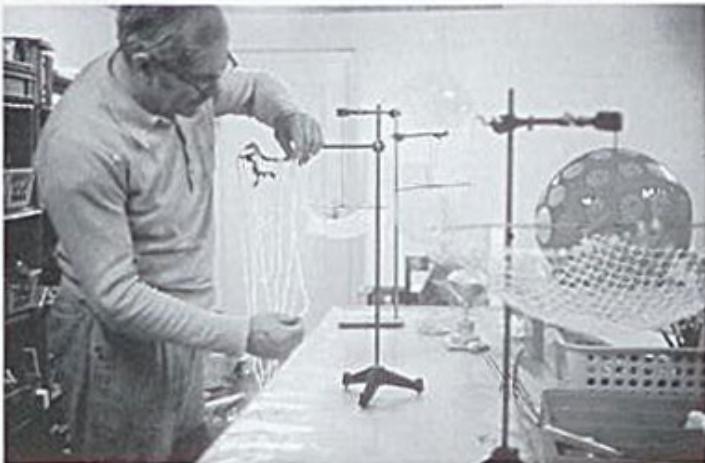
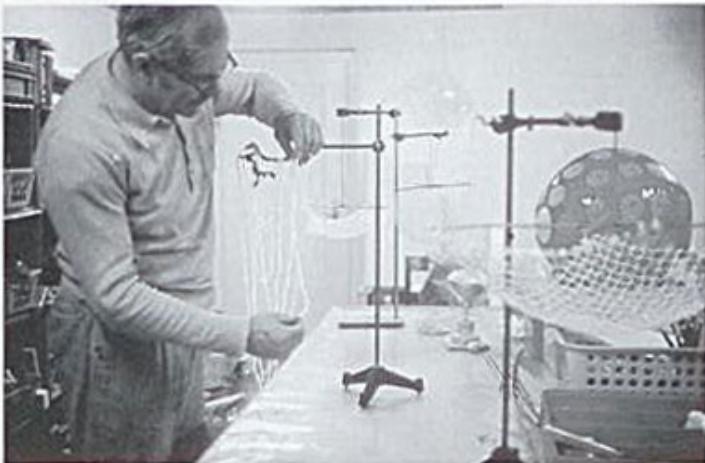
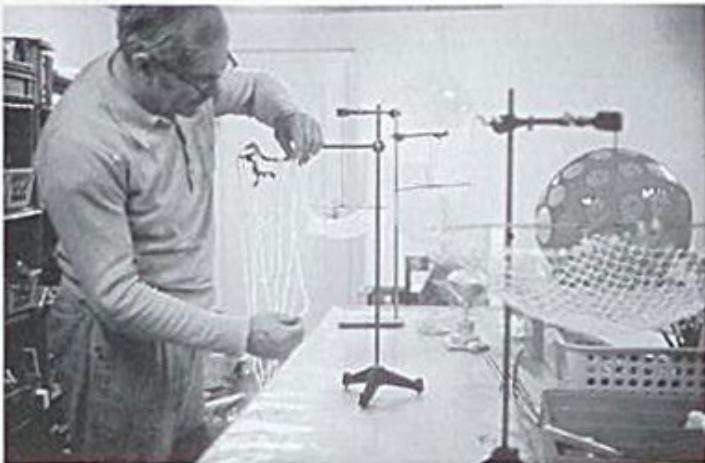
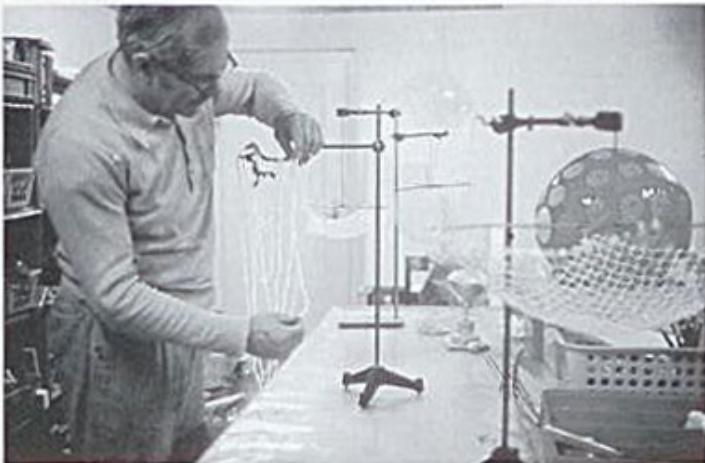
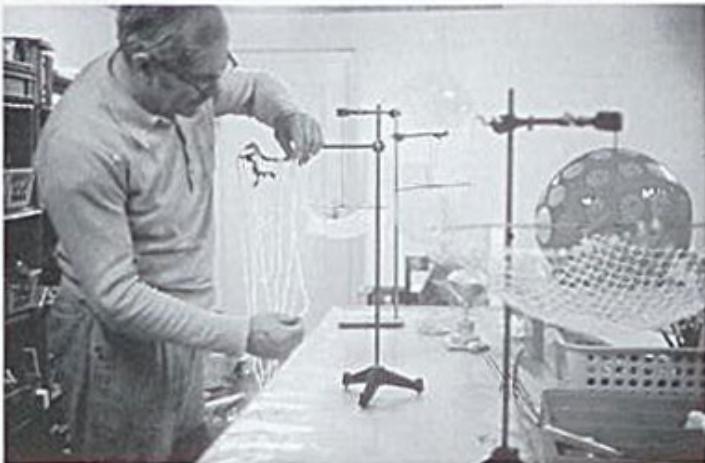
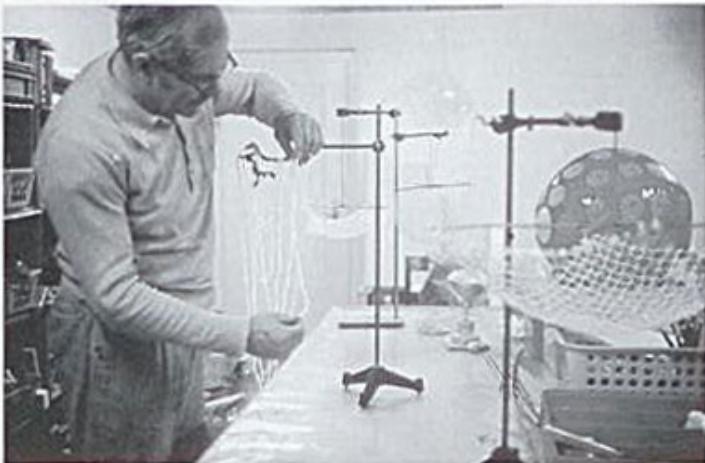
Comparaison câble-arc



$$H = \frac{ql^2}{8f}; \quad V = \frac{ql}{2}$$

$$H = \frac{ql^2}{8f}; \quad V = \frac{ql}{2}$$

	Câble	Arc
<i>Effort normal</i>	Traction	Compression
<i>Matériaux</i>	Acier Fibre de carbone Fibre naturelle	Béton (avec faible armature) Pierre naturelle Bois



Chapitres à étudier dans le TGC 1

- **Chapitre 10:** Câbles (sauf 4.5)

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