Heat exchanger network design

Pinch design method

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EPFL

Conclusions of previous steps

- DTmin optimum
- Maximum heat recovery target
  - Utility heat load (hot, cold and refrigeration)
- Pinch point location
- Minimum number of heat exchangers

What is the heat exchanger network ?

Heat exchangers network synthesis

Find a heat exchangers network that satisfies:
- MER
- Minimum number of units
- Minimum investment
- Other criteria

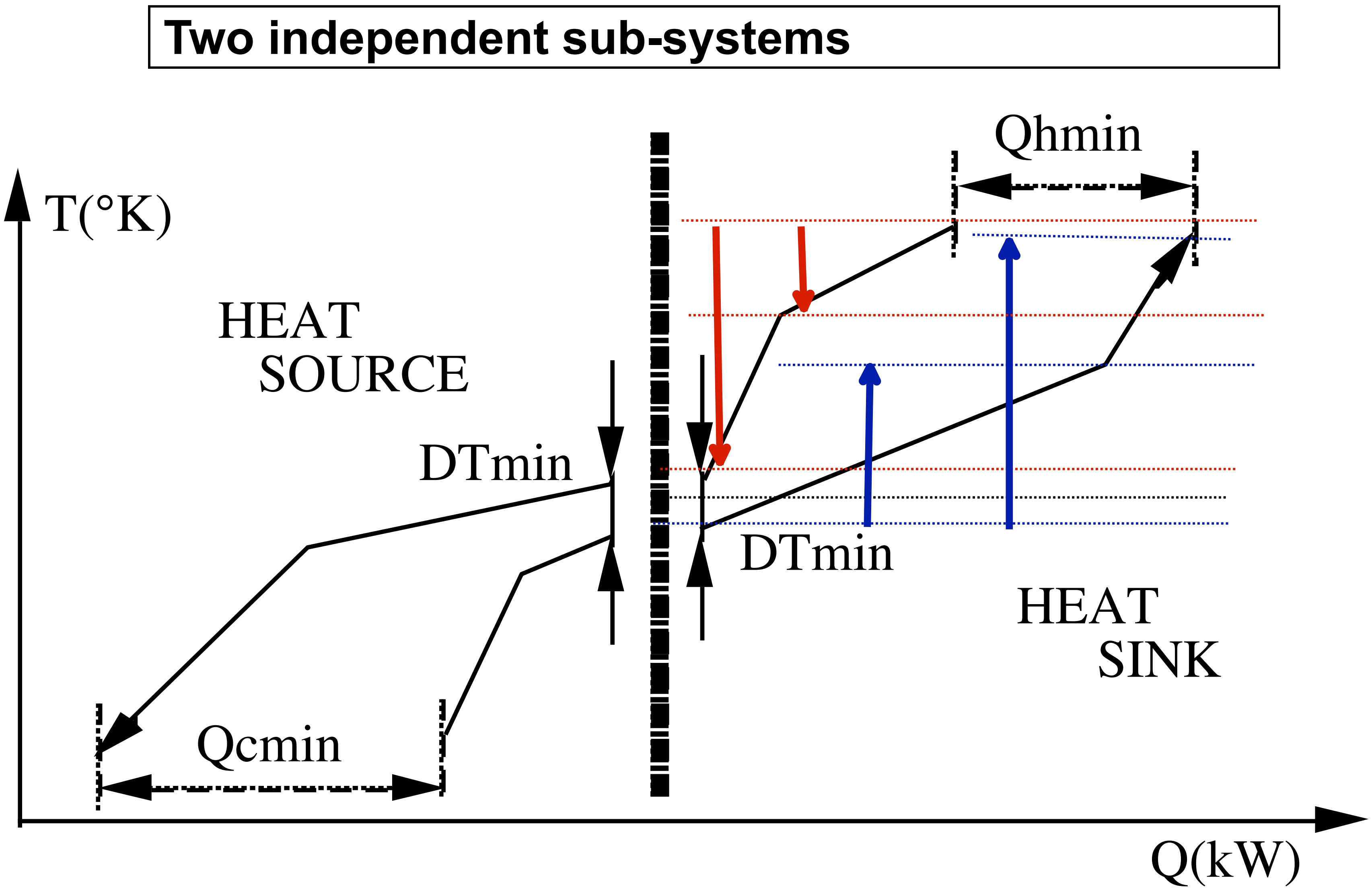
- Which hot stream with which cold stream ?

- What is the heat exchanged ?

- What is the structure : serial or //, ...

Above pinch point
Drive hot streams to the pinch point without cold utility
Below pinch point
Drive the cold streams to the pinch point without hot utility

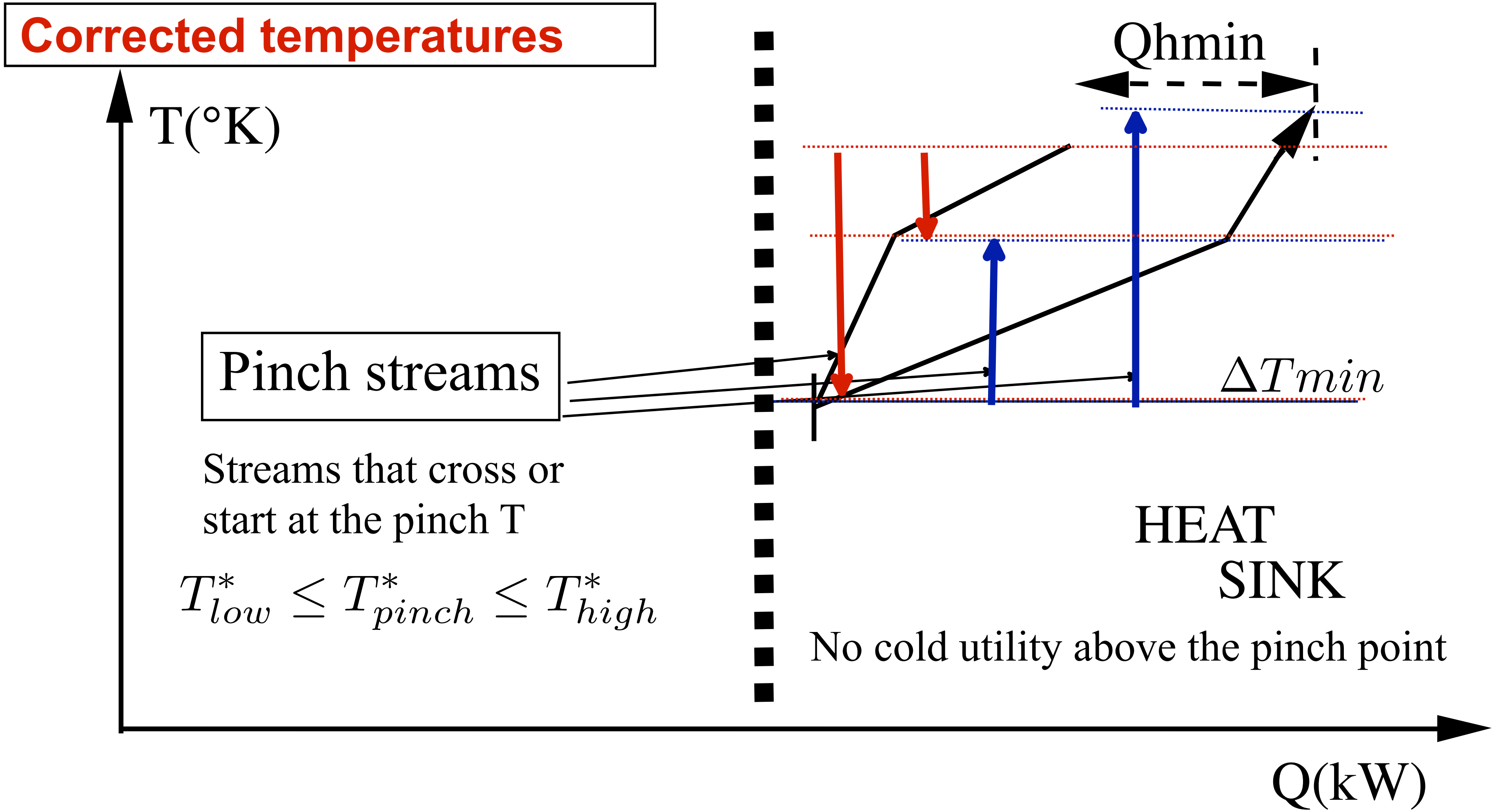
Pinch point



Two independent sub-systems
Qhmin
T(°K)
HEAT
SOURCE
DTmin
DTmin
HEAT
SINK
Qcmin
Q(kW)

Above the Pinch point

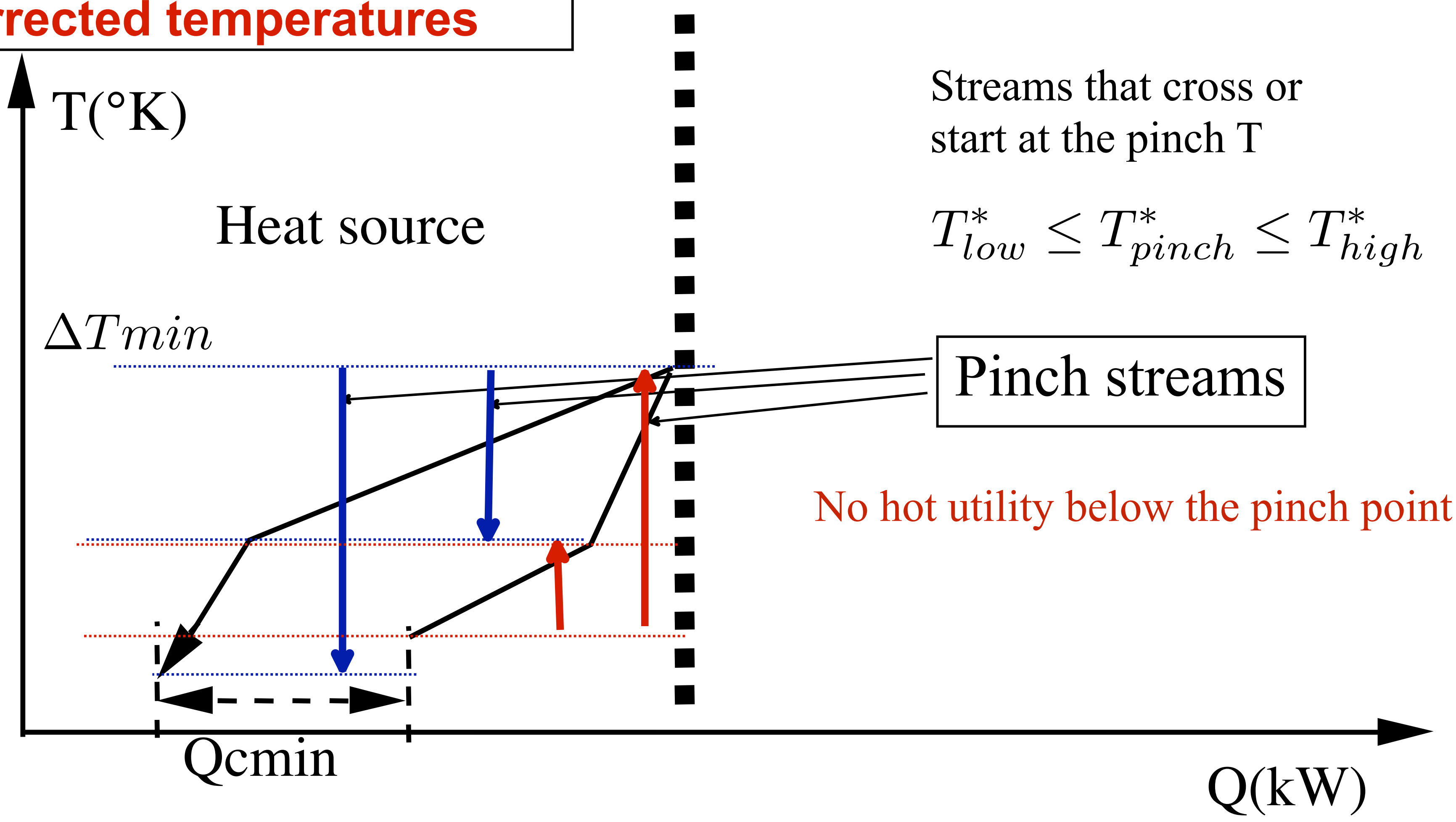
The goal is to cool down hot streams to the pinch temperature without additional utility streams



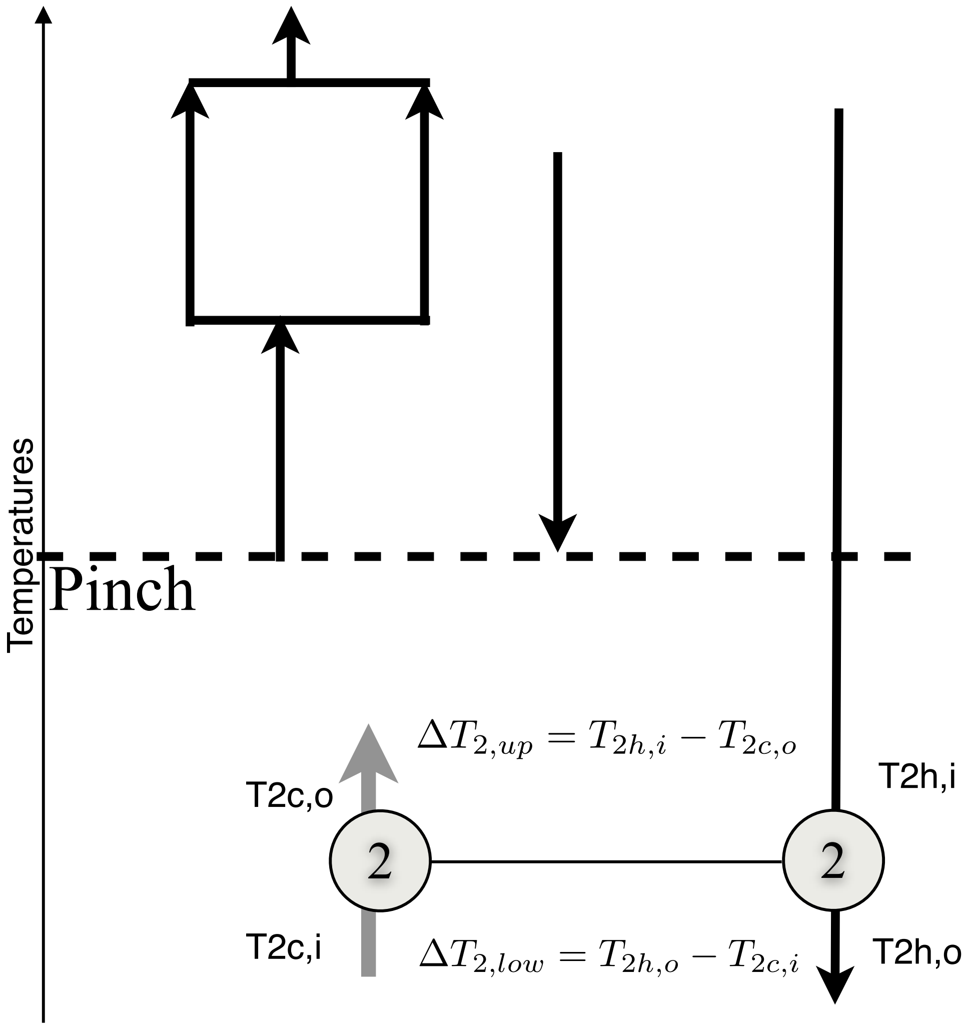
below the Pinch point

The goal is to heat up cold streams to the pinch temperature without additional hot utility

Corrected temperatures



Grid representation of HEN



$$\dot{Q}_2 = \dot{M}_{2h} cp_{2h} (T_{2h,i} - T_{2h,o})$$

$$\dot{Q}_2 = \dot{M}_{2c} cp_{2c} (T_{2c,o} - T_{2c,i})$$

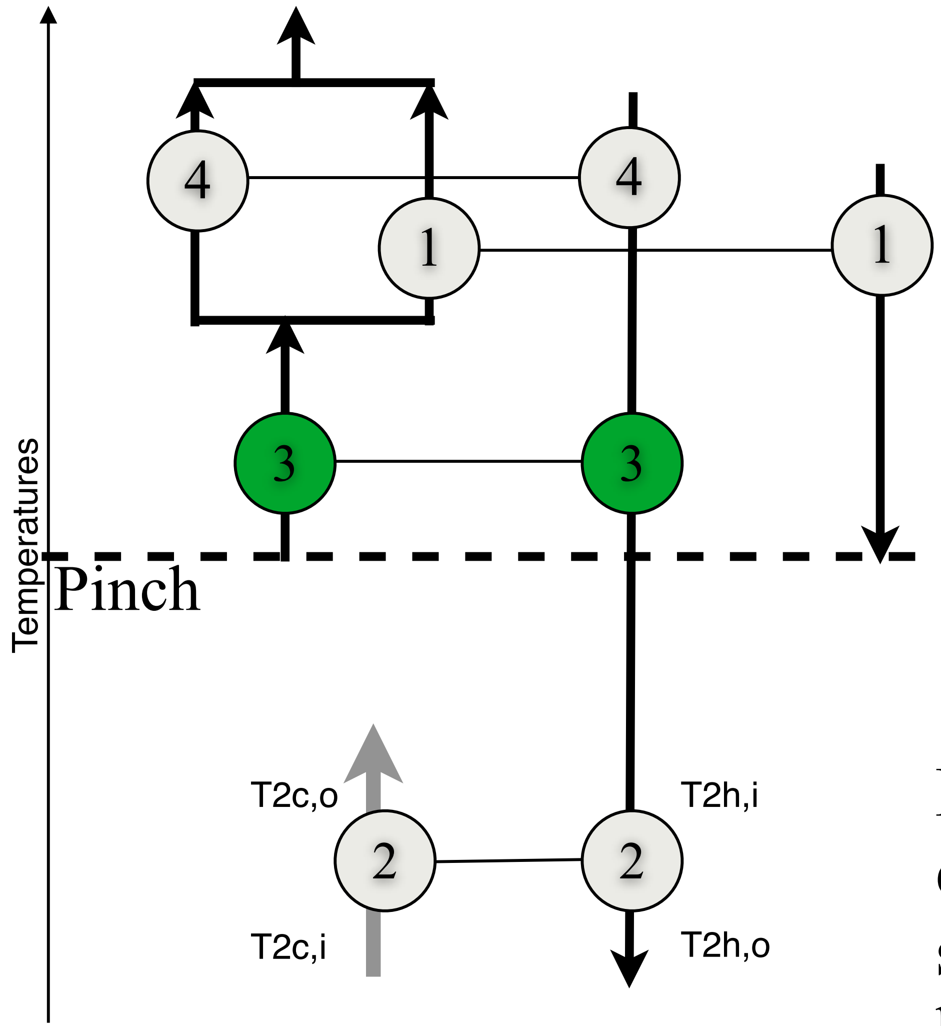
$$\Delta T_{lm,2} = \frac{\Delta T_{2,up} - \Delta T_{2,low}}{ln(\frac{\Delta T_{2,up}}{\Delta T_{2,low}})}$$

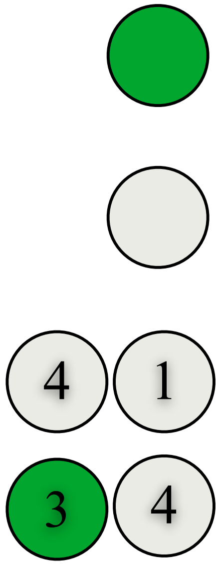
$$A_2 = \frac{\dot{Q}_2 \cdot (\frac{1}{\alpha_{2h}} + \frac{1}{\alpha_{2c}})}{\Delta T_{lm,2}}$$

Streams are vertical lines with a clear location of the pinch point (do not use real temperature scales as streams might have infinite cp when phase change occur).
Pinch streams are streams that are start from or cross the pinch
Heat exchangers are represented by horizontal lines with 2 circles identifying the connected streams. Counter current heat exchangers have therefore the hot end at the top and the cold end at the bottom, logmean temperature difference are easy to identify

Grid representation of HEN

Streams with inlet and outlet temperatures





Pinch heat exchangers

Non Pinch heat exchangers

Parallel exchangers

Sequential exchangers (hot side)

Pinch heat exchangers are heat exchanger for which one end (both streams) is at the pinch point, i.e. have $\Delta Tmin$ at one end

Feasibility rules for heat exchanger placement

- At the pinch point : i.e. for pinch streams
  - Temperature difference is known : DTmin
  - Above (or Below) the goal is known
    - above : cool down to pinch without cold utility
    - below : heat up to pinch without hot utility
- => Feasibility rules for pinch heat exchangers

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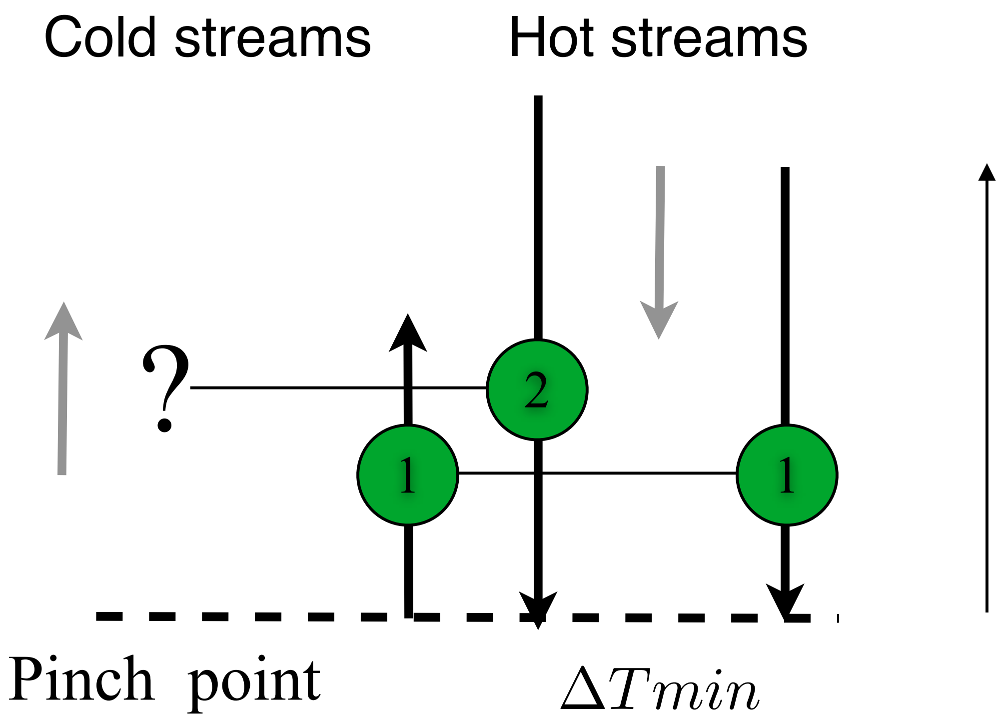
Number of streams rule

Above the pinch :
Start from the pinch point and go towards increasing temperatures
The goal is to cool down hot streams to the pinch without cold utility



$$N_{pinch,c} \geq N_{pinch,h}$$

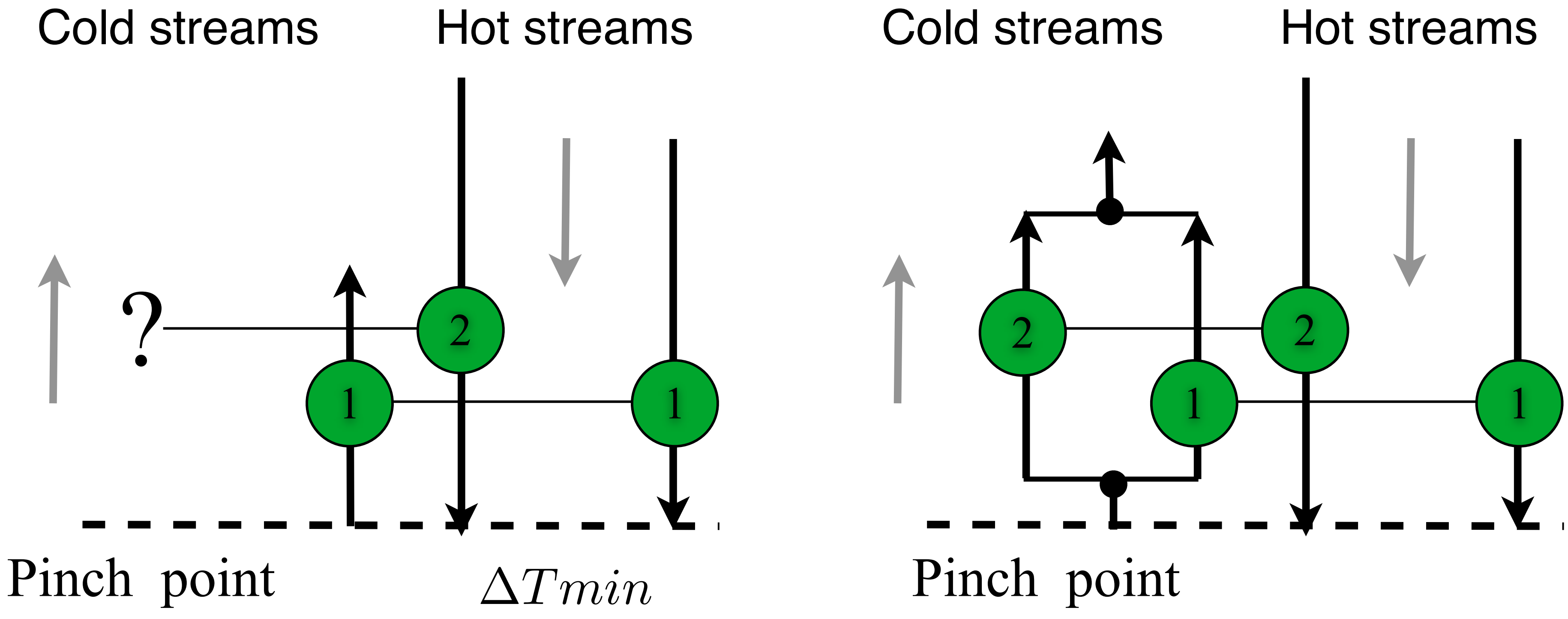
Direction of calculation : from the pinch to the highest temperatures
Note for the key streams (hot streams) we start from the target temperature , i.e. reach the pinch and go to highest temperatures, i.e. to the start temperature. the reverse for the cold streams, we start from th epinch or start temperature and go to the target (finishing with the hot utility).

Number of streams solution : splitting cold streams

Above the pinch :

Start from the pinch point and go towards increasing temperatures

The goal is to cool down hot streams to the pinch without cold utility

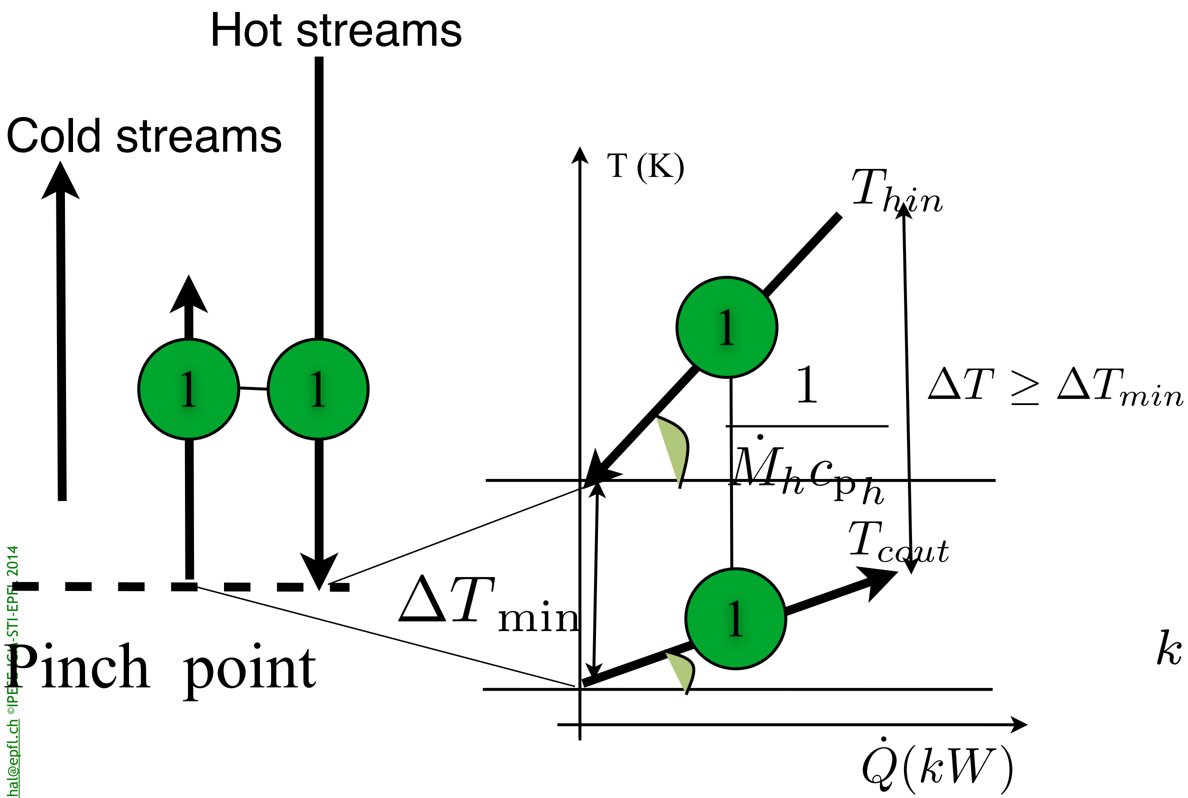


$$N_{pinch,c} \geq N_{pinch,h}$$ otherwise split 1 cold stream

At the pinch we know that the temperature difference is the ΔT_{min} , it can therefore be used as the starting point for the calculation in order to calculate the other side of the counter current heat exchanger

CP Rule : for pinch exchangers above the pinch

Above the pinch :
Start from the pinch point and go towards increasing temperatures
The goal is to cool down hot streams to the pinch without cold utility



Connexion feasibility between c and h

$$\dot{M}{c_p}_h \leq \dot{M}{c_p}_c$$

AND

$$\sum_{k=1,k\neq h}^{n_k} \dot{M}{c_p}_k \leq \sum_{j=1,j\neq c}^{n_{k-1}} \dot{M}{c_p}_j$$

At the pinch we know that the temperature difference is the ΔT_{min} , it can therefore be used as the starting point for the calculation in order to calculate the other side of the counter current heat exchanger

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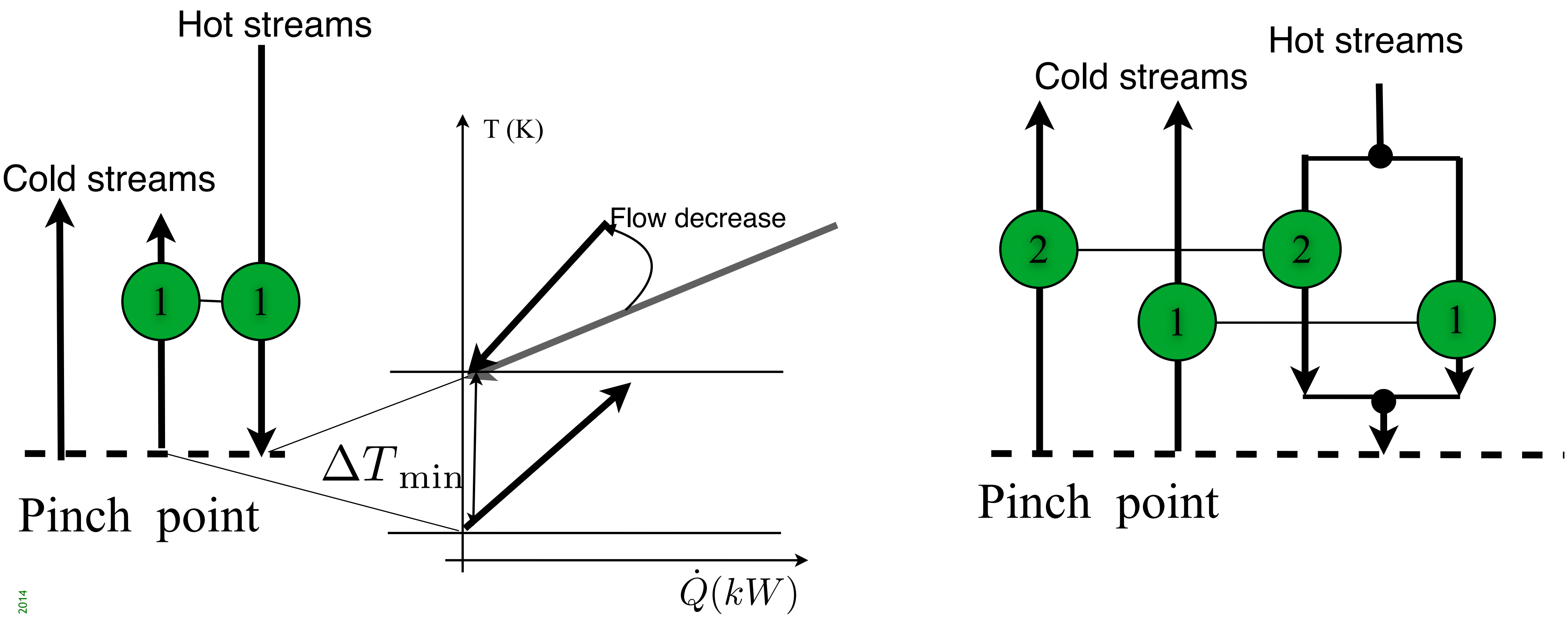
CP Rule : for pinch exchangers above the pinch

If Cp rule not satisfied split 1 hot stream



This is reducing the Mcp of hot stream in heat exchanger
CP rule is satisfied
Is Number of streams rule still valid ?

Feasibility rules for heat exchanger placement

- **At the pinch point : i.e. for pinch streams**
  - Temperature difference is known : ΔT_{min}
  - Above (or Below) the goal is known
    - above : cool down to pinch without cold utility
    - below : heat up to pinch without hot utility
- **=> Feasibility rules for pinch heat exchangers**
  - Number of streams
  - Cp rule

"this allows to select in the list of streams the one that could be potentially connected"

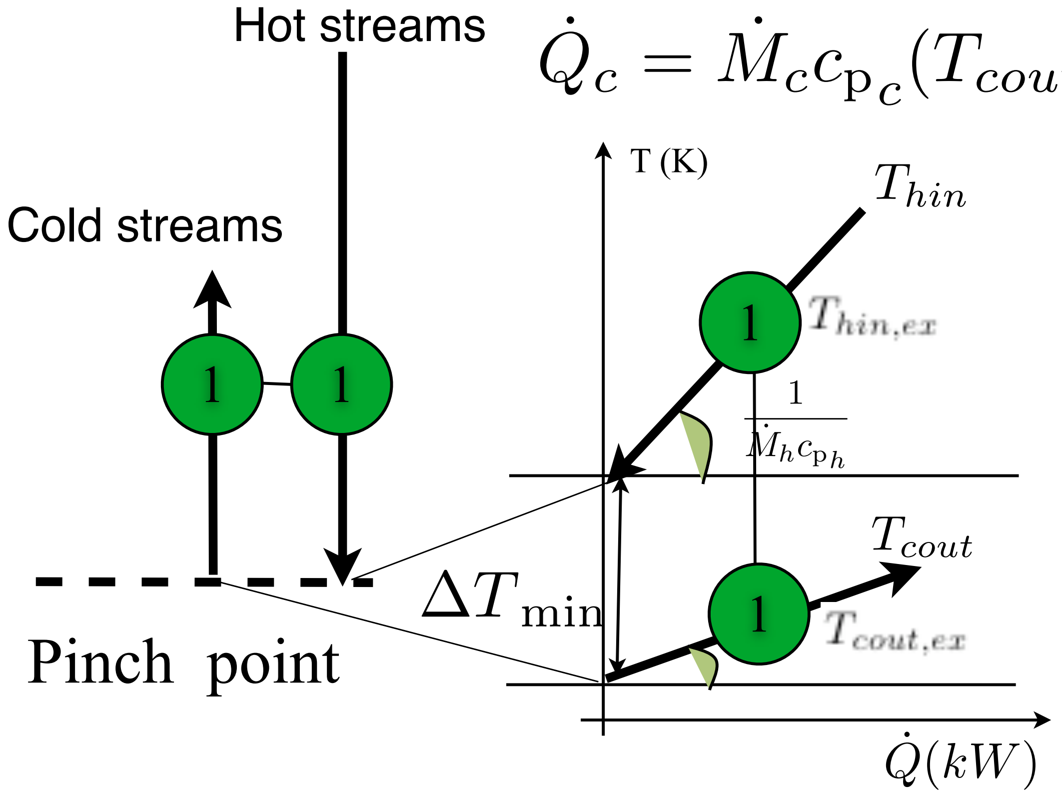
  - What is the heat load ?

Tick off rule : satisfy the heat load of one stream

In order to satisfy the minimum number of units rule

$$\dot{Q}_h = \dot{M}_h c_{p_h}(T_{hin} - (T_p^* + \Delta T_{\min}/2))$$

$$\dot{Q}_c = \dot{M}_c c_{p_c}(T_{cout} - (T_p^* - \Delta T_{\min}/2))$$



Above the pinch
Calculation starts from the pinch and goes to higher temperatures

Both side !!!

We calculate the inlet conditions of the hot stream and the outlet conditions of the cold stream

$$\dot{Q} = min(\dot{M}_c c_{p_c}(T_{cout} - (T_p^* - \Delta T_{\min}/2)), \dot{M}_h c_{p_h}(T_{hin} - (T_p^* + \Delta T_{\min}/2)))$$

$$\Rightarrow T_{hin,ex}, T_{cout,ex}$$

$$T_{hin,ex} = T_p^* + \Delta T_{\min}/2 + \frac{\dot{Q}}{\dot{M}_h c_{p_h}} \qquad T_{cout,ex} = T_p^* - \Delta T_{\min}/2 + \frac{\dot{Q}}{\dot{M}_c c_{p_c}}$$

Heuristic rules

1 - Order the streams by decreasing Cp

Goals :

Above the pinch point: cool down the hot streams without cold utilities.

Below the pinch point : heat up the cold streams without hot utilities.

Start with pinch exchangers

2 - verify feasibility rules and split if no connection found

3 - The heat load is calculated to satisfy the heat load of one of the two stream involved : “tick-off”

- work from the pinch

4 - Place the utilities at the end of the streams (control purposes)

Remaining problem analysis

Remaining problem

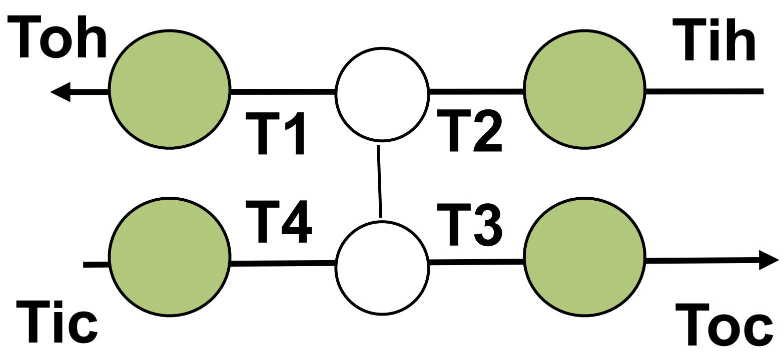
Initial problem:

Hot stream : Tih -> Toh

Cold stream: Tic -> Toc

=> MER0

Place a heat exchanger



New Hot streams:

- Tih -> T2
- T1 -> Toh

New Cold streams:

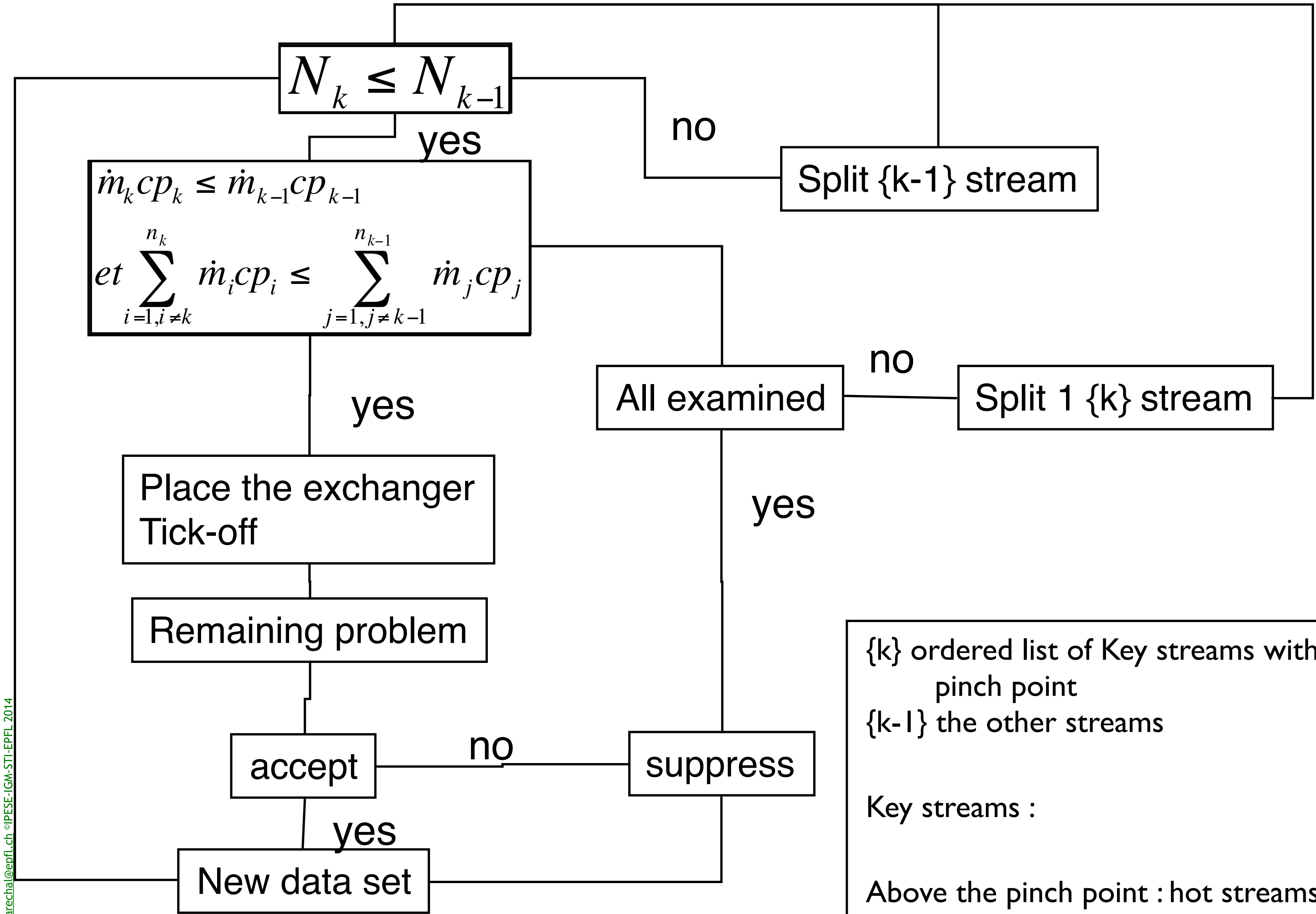
- Tic -> T4
- T3 -> Toc

=>MER1

if MER1=MER0

=> Exchanger is well placed

The synthesis of the HEN synthesis algorithm



{k} ordered list of Key streams with decreasing Cp at the pinch point

{k-1} the other streams

Key streams :

Above the pinch point : hot streams

Below the pinch point : cold streams

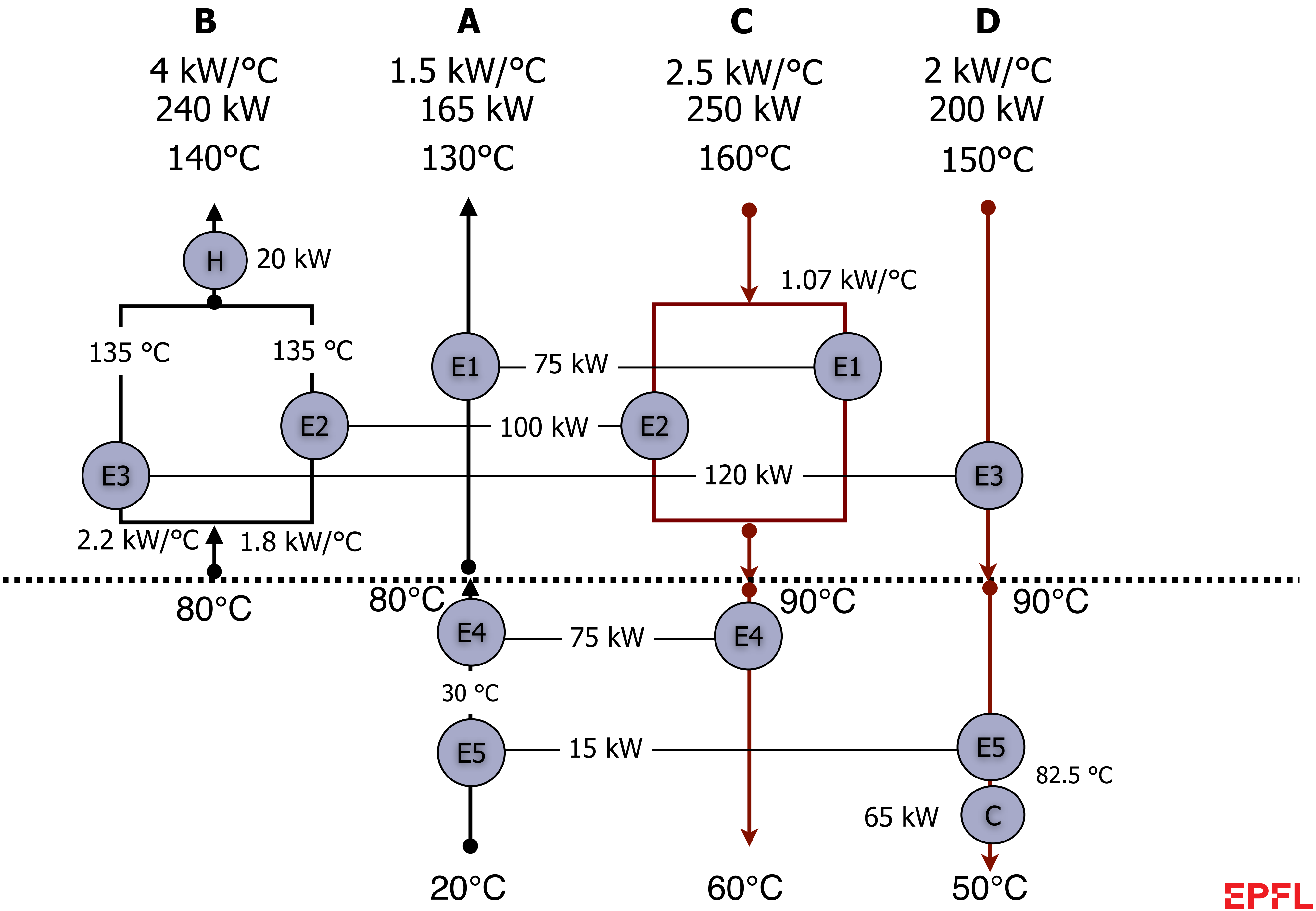
Conclusions

- Systematic method for heat exchanger network design
  - Start from the pinch
    - Goals with respect to pinch location
  - Feasibility rules to select streams
  - Split if necessary
  - Heat load by tick-off rule (minimum number of units)
  - Heat exchanger network calculation
    - from the pinch !

Is it the best solution ?

- **Reduce the number of heat exchangers**
  - Path following
  - Loop following
- **Find the best compromise between**
  - Investment
    - Less heat exchangers
    - Other DTmin value (Exchanger minimum approach temperature : EMAT for each exchanger)
  - Operating cost (more/less heat consumption)
  - Other good arguments
    - operability
    - safety
    - retrofit

Pinch design method design



Heat exchangers after Pinch design method

Operating cost = 2325 CHF/year

	T_{hot} [C]	ΔT_{hot} [C]	T_{cold} [C]	ΔT_{cold} [C]	ΔT_{lm} [C]	$\dot{Q}$ [kW]	A [m^2]	Cost [CHF]
	160		90					
E1		30		10	18.20	75.0	16.5	32945
	130		80					
	160		90					
E2		25		10	16.37	100.0	24.4	43327
	135		80					
	150		90					
E3		15		10	12.33	120.0	38.9	60055
	135		80					
	90		60					
E4		10		30	18.20	75.0	16.5	32947
	80		30					
	90		82.5					
E5		43.3		62.5	61.24	15.0	1.0	4630
	30		20					
Total						385.0	97.3	173905

Table 7: Heat Exchangers obtained with Pinch Design Method

Number of exchangers : 5+2 = 7 = 4+2-1+3-1

Fluid heat transfer coefficient U = 0.5 kW/C°/m2

Final results : comparing solutions

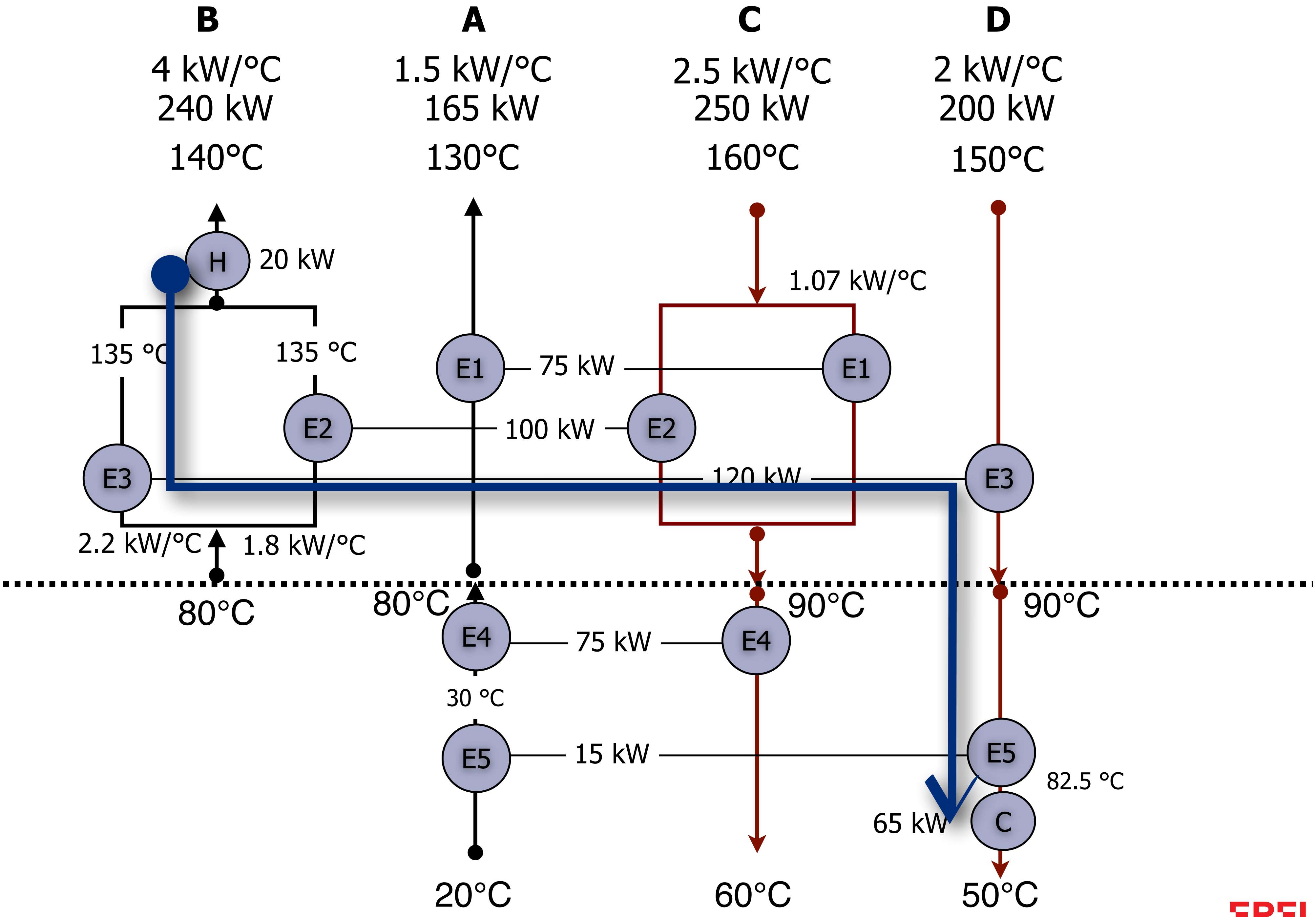
	Invest [CHF]	CAPEX [$CHF/year$]	OPEX [$CHF/year$]	Total [$CHF/year$]	Nb Ex.
0	173905	20340	2352	22692	7

Table 14: Summary of the heat exchanger network design

Path following technique

- **A path indicates the path of the heat from one hot utility to one cold utility through heat exchangers**
- **If I'm adding 1 kW of hot utility**
  - what are the heat exchangers that are affected ?
    - the next one of the path on the cold stream reduces by 1 kW, therefore on the hot side there will be the need to increase the heat load of one on the hot stream, etc… until we reach a cold utility which will increase by 1 kW.
  - in a path adding heat in the hot utility means reducing the heat exchanged in the next heat exchanger ... up to the cold utility is reached.
- **One could choose the heat load to suppress one of the heat exchangers in the path.**

Path following through E3



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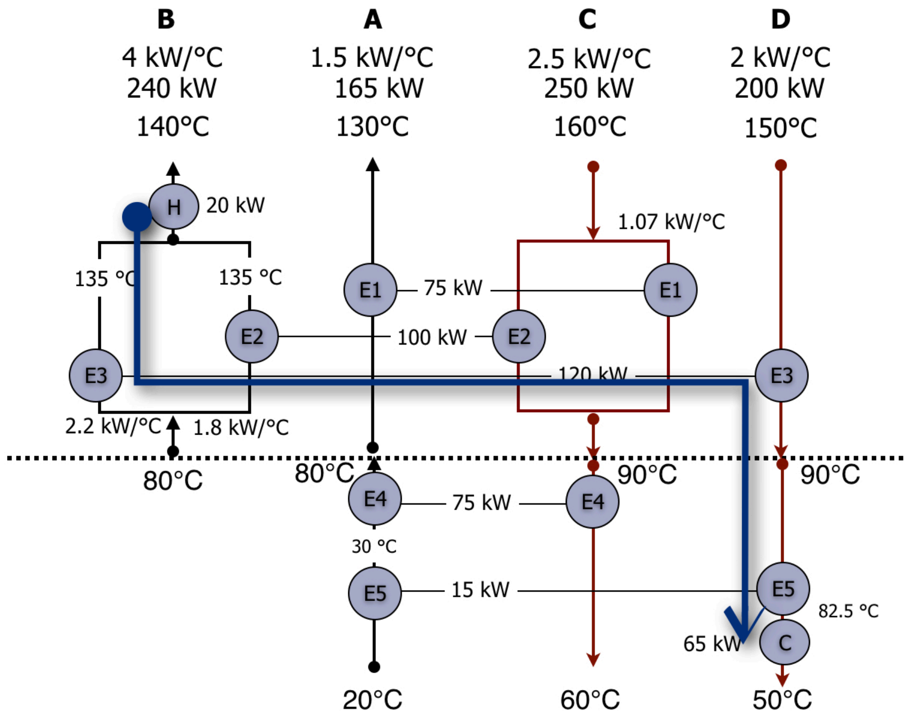
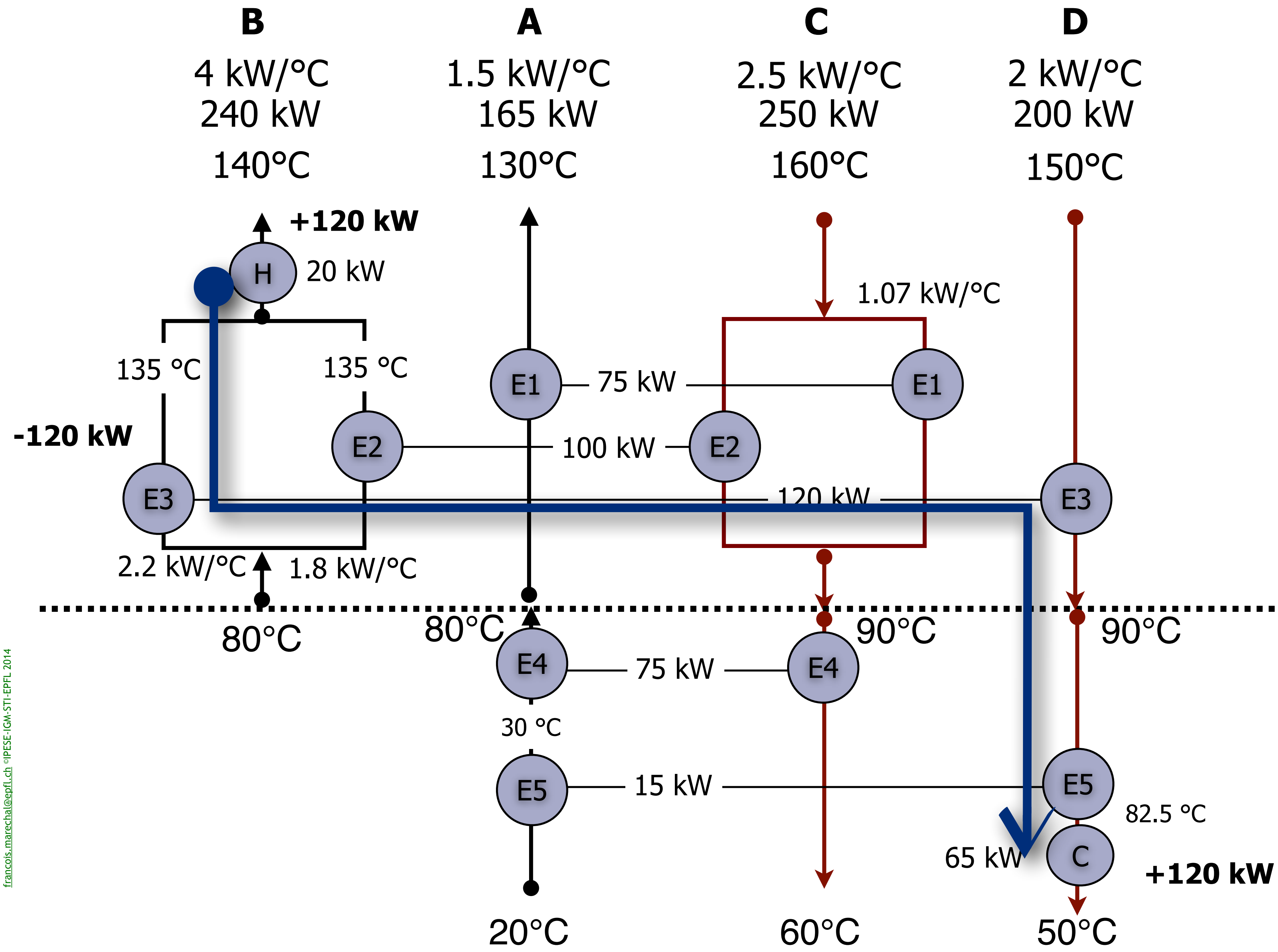
Path explanation

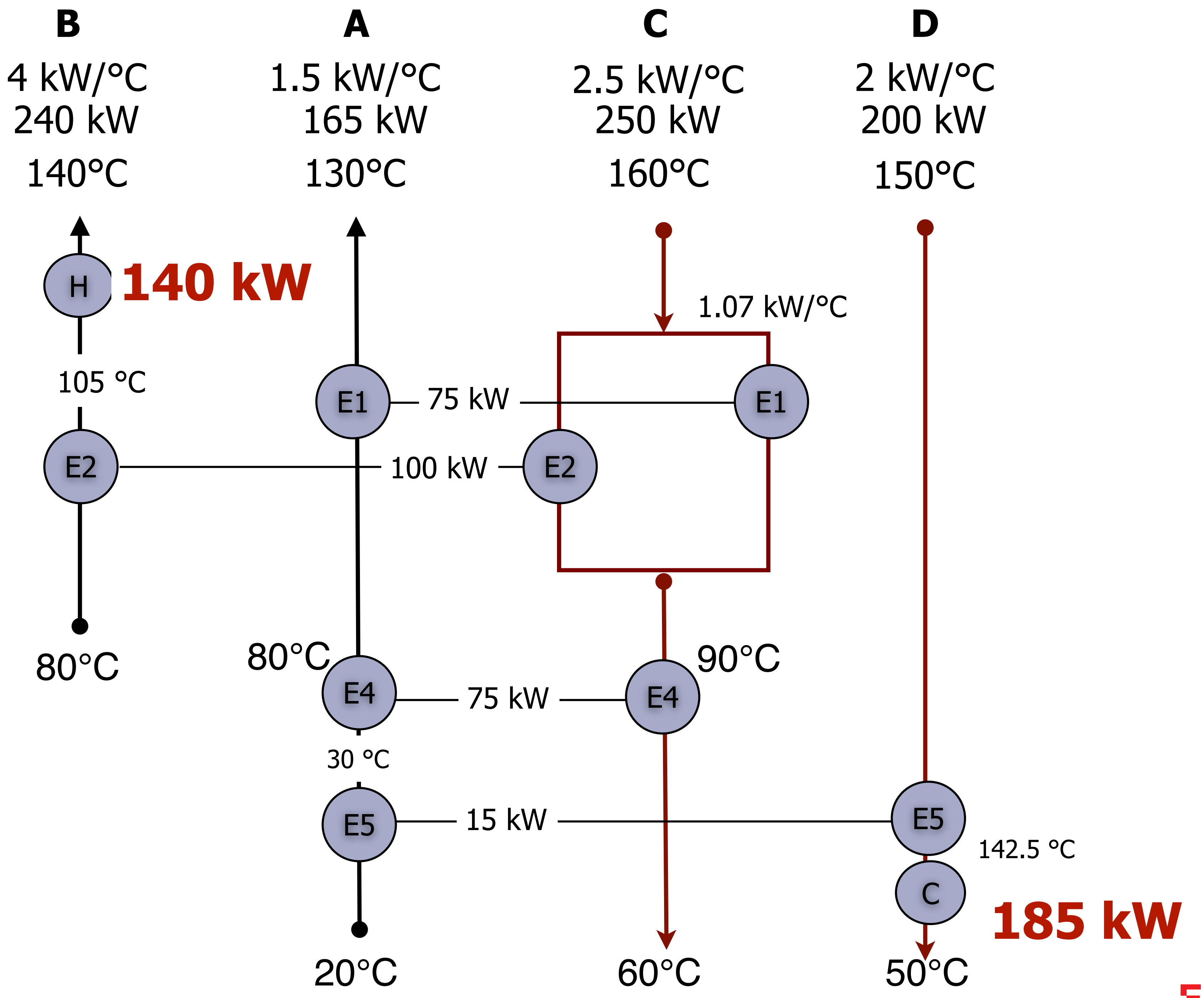


- There exist a path that goes from hot utility (H) on cold stream (B) to the cold utility (C) on hot stream (D) via the heat exchanger E3 that links the cold stream B and the hot stream D
- if the load of E3 is reduced by X kW,
- there will be less heat recovery for stream B therefore the hot utility will have to be increased by X kW to compensate the heat recovery reduction for the cold stream B,
- similarly the cold utility will have to increase by X kW to compensate from the fact that the hot stream D in exchanger E3 has not given the heat to the cold stream B and therefore has more heat that has to be evacuated by the cold utility.
- When X is equal to the heat load of the heat exchanger E3, the heat exchanger will vanish. As a consequence the investment disappears at the price of an increase of both the hot and the cold utility.
- The energy cost will have to be compared with the CAPEX reduction

Path following E3



Path following supressing E3 : Network design

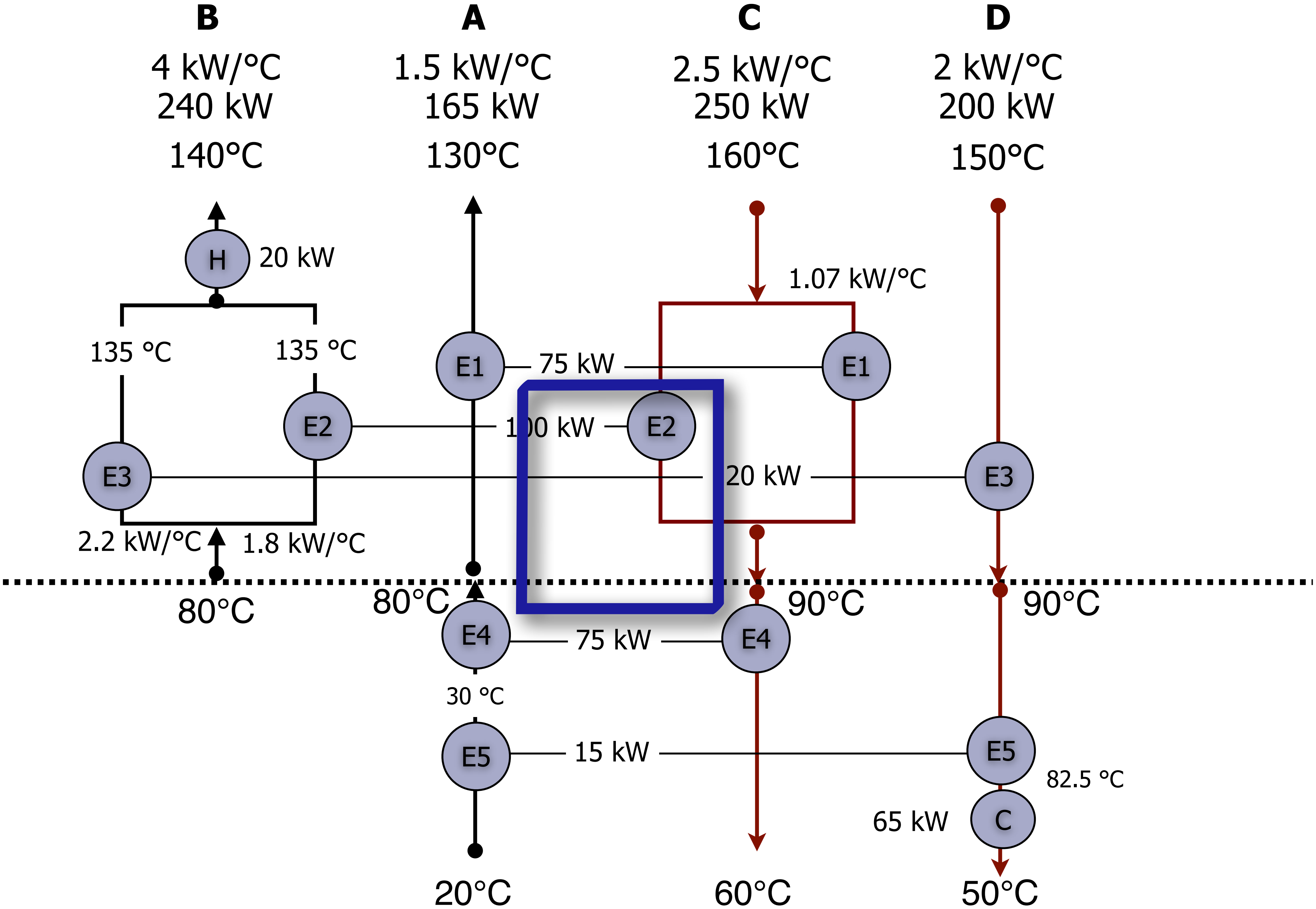


Loop following technique

- **A loop identifies a list of heat exchangers that exchange twice with the same hot and cold streams.**
- **If I'm adding 1 kW to one heat exchanger in the loop**
  – the neighbouring exchanger will reduce by 1 KW then the next one will increase until we are back to the first one ?
  – The energy balance is not changed (i.e. no increase in hot and cold utility).
  – The heat distribution between the heat exchangers in the loop is changed

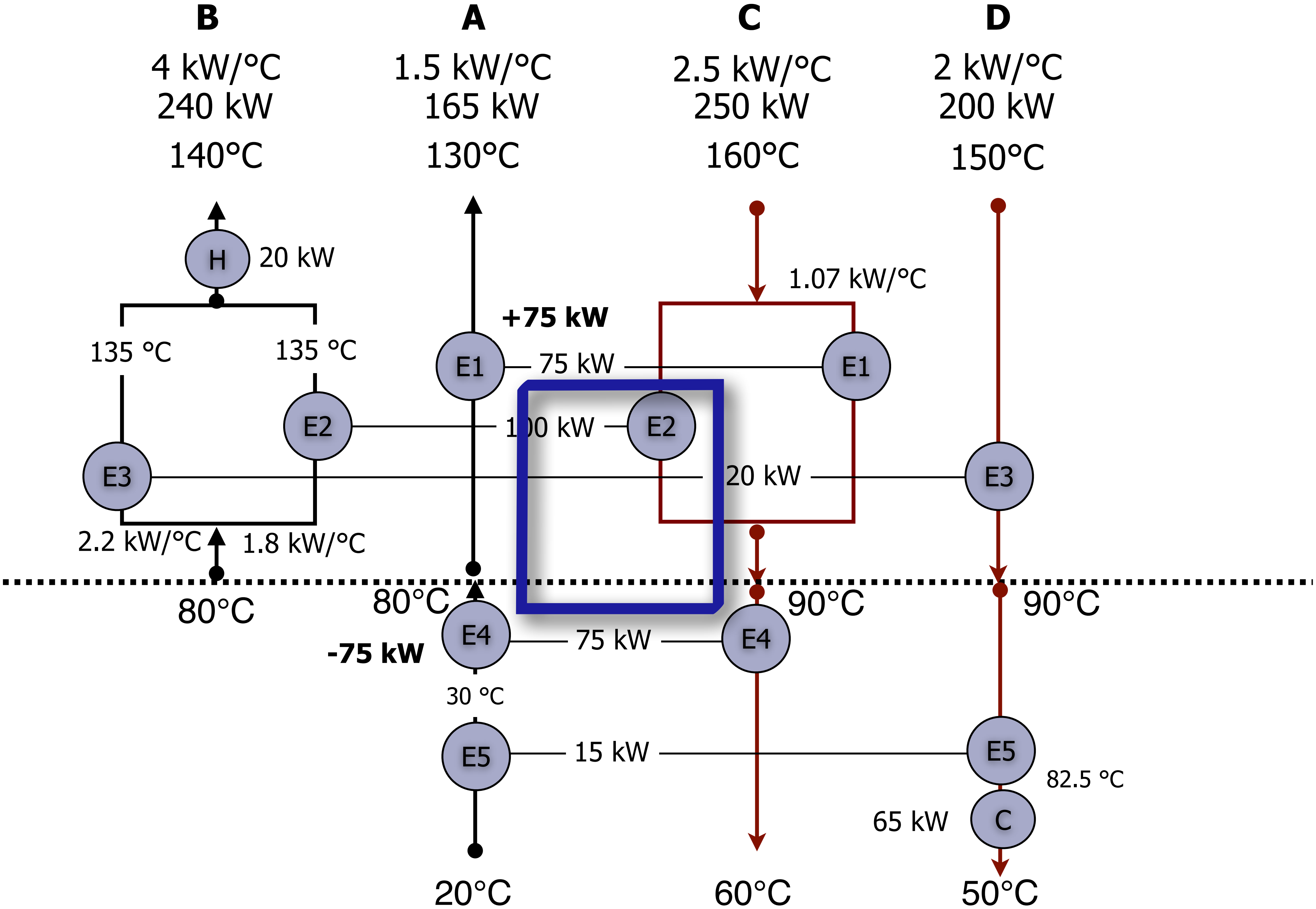
- **One could choose the heat load distribution change to suppress one of the heat exchangers in the loop.**

Loops



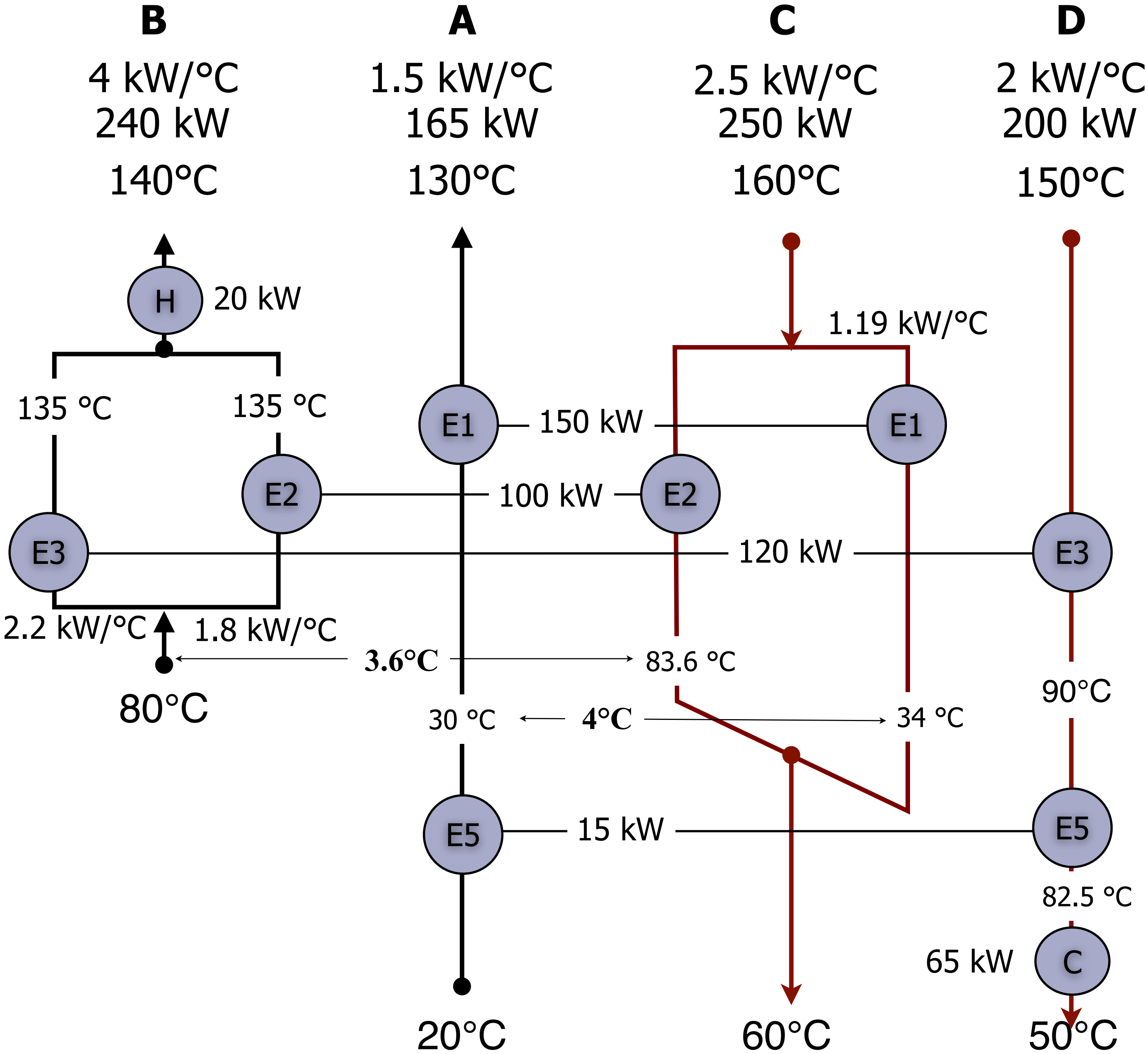
B
4 kW/°C
240 kW
140°C
A
1.5 kW/°C
165 kW
130°C
C
2.5 kW/°C
250 kW
160°C
D
2 kW/°C
200 kW
150°C
H
20 kW
135 °C
135 °C
E1
75 kW
E1
1.07 kW/°C
E2
100 kW
E2
E3
20 kW
E3
2.2 kW/°C
1.8 kW/°C
80°C
80°C
90°C
90°C
E4
75 kW
E4
30 °C
E5
15 kW
E5
82.5 °C
65 kW
C
20°C
60°C
50°C

Loops



B
4 kW/°C
240 kW
140°C
A
1.5 kW/°C
165 kW
130°C
C
2.5 kW/°C
250 kW
160°C
D
2 kW/°C
200 kW
150°C
H
20 kW
135 °C
135 °C
+75 kW
E1
75 kW
E1
1.07 kW/°C
E2
100 kW
E2
E3
20 kW
E3
2.2 kW/°C
1.8 kW/°C
80°C
80°C
90°C
90°C
-75 kW
E4
75 kW
E4
30 °C
E5
15 kW
E5
82.5 °C
65 kW
C
20°C
60°C
50°C

Loops



B
4 kW/°C
240 kW
140°C
A
1.5 kW/°C
165 kW
130°C
C
2.5 kW/°C
250 kW
160°C
D
2 kW/°C
200 kW
150°C
H
20 kW
1.19 kW/°C
135 °C
135 °C
E1
150 kW
E1
E2
100 kW
E2
E3
120 kW
E3
2.2 kW/°C
1.8 kW/°C
3.6°C
83.6 °C
80°C
30 °C
4°C
34 °C
90°C
E5
15 kW
E5
82.5 °C
65 kW
C
20°C
60°C
50°C

Recalculate the temperatures

- Operating cost : 2352 CHF/year

	T_{hot} [C]	ΔT_{hot} [C]	T_{cold} [C]	ΔT_{cold} [C]	ΔT_{lm} [C]	$\dot{Q}$ [kW]	A [m^2]	Cost [CHF]
	160		34					
E1		30		4	12.90	150.0	46.5	68043
	130		30					
	160		83.6					
E2		25		3.6	11.04	100.0	36.2	57131
	135		80					
	150		90					
E3		15		10	12.33	120.0	38.9	60055
	135		80					
	90		82.5					
E5		43.3		62.5	61.24	15.0	1.0	4630
	30		20					
Total						385.0	122.6	189859

Table 8: Heat Exchangers after loop 1 breaking to suppress E4

Final results : comparing solutions

	Invest $[CHF]$	CAPEX $[CHF/year]$	OPEX $[CHF/year]$	Total $[CHF/year]$	Nb Ex.
0	173905	20340	2352	22692	7
1	189859	22206	2352	24558	6
2	190001	22222	2352	24574	6
3	145847	17058	4118	21176	6
4	**133491**	**15613**	**4118**	**19731**	**6**
5	147924	17301	4118	21419	5
6	99723	11664	16471	28135	6

Table 14: Summary of the heat exchanger network design

Choose the best option !

The best option is the one that has the smallest total cost. It is possible that this solution will have ΔT smaller than the ΔTmin and a smaller number of heat exchangers. This is mainly due to the fact that minimising the number of connexions does not consider the size of the heat exchangers. Smaller heat exchangers having a higher specific cost, the higher number of units can have a higher cost than a smaller number of units even if the total area is increased. It is also important to realise that at the end of the procedure, the ΔTmin is not anymore a guiding value but is a value that exist specifically for each heat exchanger.

Calculating of the optimal HEX areas

NLP(Non Linear Programming) optimisation problem

$$\min_{\dot{M}_u, A_e} = \left(\sum_{u=1}^{N_u} c_u^+ \cdot \dot{M}_u \right) \cdot t_{op} + \frac{1}{\tau} \sum_{e=1}^{n_{htx}} \left(a_e + b_e \cdot (A_e)^{c_e} \right)$$

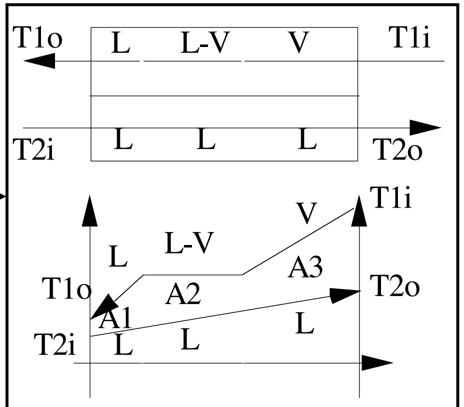
Constraints

Heat and mass balances
Rating equations
Specifications :

F(X) = 0

Bounds and limits

G(X) ≤ 0



$$A_i = \sum_{z=1}^{nz_i} A_{z,i} = \sum_{z=1}^{nz_i} \frac{Q_{iz}}{U_{iz} \Delta Tlm_{iz}}$$

X : flows (split factors, utility flows), pressure, temperature, area, heat exchanged, ...

Optimising the flows (e.g. in splitters and utility streams) and temperatures therefore changing the heat exchange areas allows to calculate the best ΔTmin value for each of the heat exchangers (EMAT). The calculation is a non linear programming problem that can be quite complex to solve, due to the interrelations between the heat exchangers and the difficulty of the infeasible heat exchanges (ΔT < 0).
Non linear programming : class of optimisation that involves non linear equality (heat balances, heat transfer) and inequality constraints (flows >0, ΔT>0).

Difficulty of the method

- **Sequential approach**
  - interdependent decisions
- **Multiple solutions**
- **Do we reach the minimum number of units ?**
- **Is the network optimal ?**

- **Reducing the number of units**
  - save the (fixed part) of investment
  - first m2 costs more
- **Optimisation is needed**
  - $EMAT_i$ optimisation

Application of mathematical programming

- **Calculating heat exchanger network design**
  - given a list of all the hot and cold streams
  - given the ΔTmin and the pinch(s) location
  - Calculate :
    - the heat exchanger network structure
    - The heat loads, temperature and flows in the network
    - The heat exchange area of the heat exchangers
    - The total cost of the heat exchanger network
    - The optimal value of ΔTmin for each HEN
- **Mathematical programming (optimisation) can help identifying the matches between the hot and the cold streams and therefore reduce the risk of taking a wrong decision in the sequential approach.**

HEN synthesis

Draw backs of the Pinch Design Method

- **multiple solutions**
- **combinatorial problem**
- **sequential**

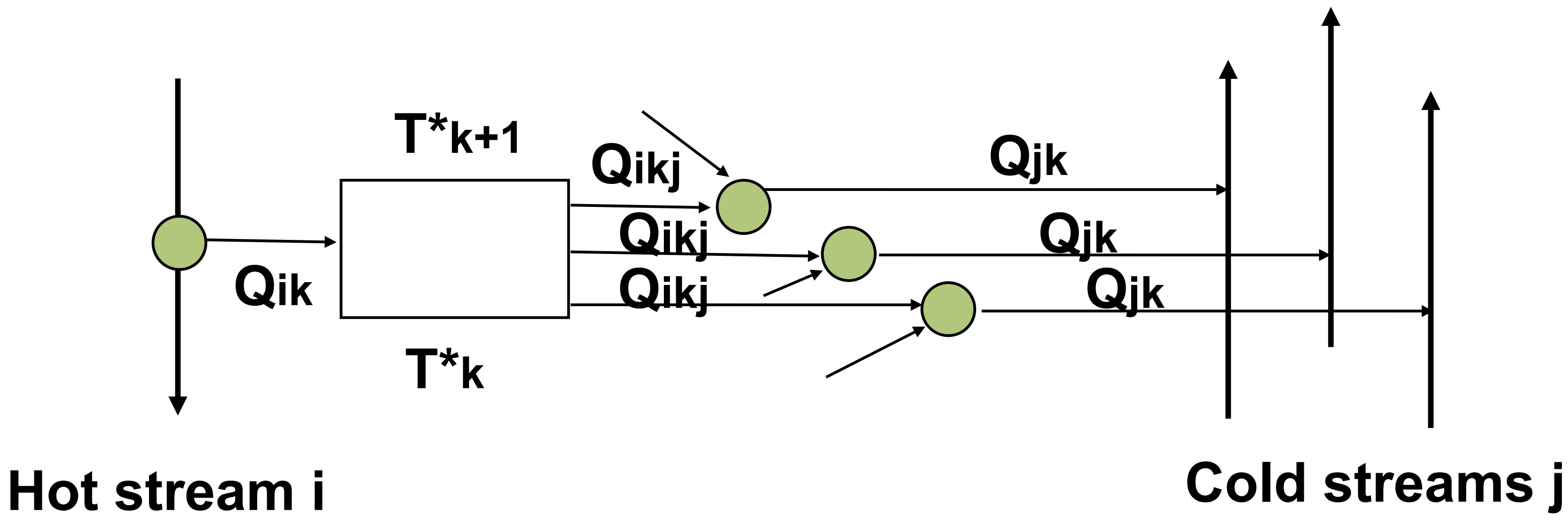
Use of mathematical programming:

Heat load distribution:

- **which streams exchange heat**
- **How much heat is exchanged**
- **minimize the number of connections**
- **satisfies DTmin and MER**

Remaining problem : find the HEN structure

Heat load distribution



Hot stream i in temperature interval k

$$\sum_{j=1}^{nc} Q_{ikj} = Q_{ik} \qquad \forall i = 1,\ldots,nh; \forall k = k_1,\ldots,k_2$$

Cold stream j in and above temperature interval k

$$\sum_{i=1}^{nh} \sum_{r=k}^{k_2} Q_{irj} - \sum_{r=k}^{k_2} Q_{jr} \le 0 \qquad \forall j = 1,\ldots,nc; \forall k = k_1,\ldots,k_2$$

connection between i et j (integer variable)

$$\sum_{r=k}^{k_2} Q_{irj} - y_{ij} Q\max_{ij} \le 0 \qquad \forall j = 1,\ldots,nc; \forall i = 1,\ldots,nh$$

Q_{ikj} the heat load of the hot stream i in temperature interval k that is sent to the cold stream j
Q_{ik} the heat load of hot stream i in temperature interval k
Q_{jk} the heat load of cold stream j in the temperature interval k

Heat load distribution

MILP formulation (Mixed Integer Linear Porgamming)

Minimize the number of connections

$$\min_{y_{ij},Q_{ikj}} \sum_{i=1}^{nh} \sum_{j=1}^{nc} y_{ij} \qquad y_{ij} \in \{0,1\}$$

$$\sum_{j=1}^{nc} Q_{ikj} = Q_{ik} \qquad \forall i = 1,\ldots,nh; \forall k = k_1,\ldots,k_2$$

$$\sum_{i=1}^{nh} \sum_{r=k}^{k_2} Q_{irj} - \sum_{r=k}^{k_2} Q_{jr} \leq 0 \qquad \forall j = 1,\ldots,nc; \forall k = k_1,\ldots,k_2$$

$$\sum_{r=k}^{k_2} Q_{irj} - y_{ij} Q\max_{ij} \leq 0 \qquad \forall j = 1,\ldots,nc; \forall i = 1,\ldots,nh$$

The problem is a mixed integer linear programming problem (MILP). This is class of optimisation problem that has only linear constraints and objective function and in which the decision variables are continuous (Q_{ikj}) or integer (y_{ij}).

Multiple solutions

- **Add heuristic rules**
  - favour the connexion with utility streams
  - favour close connexions
  - favour connexion in closer sub-systems
- **A heuristic rule is applied only if it does not penalize the minimum number of solution target**

$$\min_{y_{i,j},Q_{i,j,k}} \sum_{i=1}^{N_h} \sum_{j=1}^{N_c} W_{i,j} \cdot y_{i,j}$$

$W_{i,j}$: weighting factor of connexion i,j

There exist most of the time more than one solution that minimises the number of connections. In order to distinguish the solutions it is therefore important to give an importance to some of the solutions. This can be done by preferences or by other weighting criteria (e.g. the geographical distance that would indicate the pimping required to connect two streams).

Introduce heuristic rules in MILP programs

- **The weight of priority rule k is given by :**
  - the number of possible connexions satisfying rule k

$$P_k = \sum_{j=1}^{n_c} \sum_{i=1}^{n_h} (p_{kij})$$

  - an im

$$\min_{y_{ij} Q_{ikj}} NT = \sum_{j=1}^{n_c} \sum_{i=1}^{n_h} (\prod_{k=1}^{p_{ij}-1} (P_k + 1) \, y_{ij})$$

$$\frac{\prod_{k=1}^{r} (P_k + 1)}{\prod_{k=1}^{r-1} (P_k + 1)} = P_r + 1 > \sum_{i=1}^{n_h} \sum_{j=1}^{n_c} P_{rij}$$

When we can define a order of preferences between the solutions, the above formulation guarantees that the number of connections will be minimized (i.e. the sum of the previous preference will never exceed the value of the next preference level with one more unit)

Generating multiple solutions

- Integer cut constraint : generate the next best solution
  - assuming that we know k solutions
  - problem k + 1 is defined by adding to the previous MILP problem the integer cut constraint $Problem^{k+1} =$

$$Problem^k +$$

$$\sum_{p=1}^{n_p} \sum_{c=1}^{n_c} (2 * y_{p,c}^k - 1) * y_{p,c} \leqslant \sum_{p=1}^{n_p} \sum_{c=1}^{n_c} y_{p,c}^k$$

The integer cut contraints indicates as a constraints the list of solutions that are already known. The solution to this problem is therefore the next best solution.

Conclusion

- **HEN design algorithm**
  - sequential approach
  - thermo-economic evaluation of options
- **HEN design with mathematical programming**
  - systematic with a holistic approach
  - decides the connections with mathematical programming approach
  - HEN network still needs to be defined
    - Split streams
    - Non linear programming