ChE 204
Introduction to Transport Phenomena

Module 2

Advective Transport

2.0 How are heat, mass and momentum transported?
2.1 Advective transport of mass and integral mass balance
2.2 Advective transport of momentum and integral momentum balance

ChE 204
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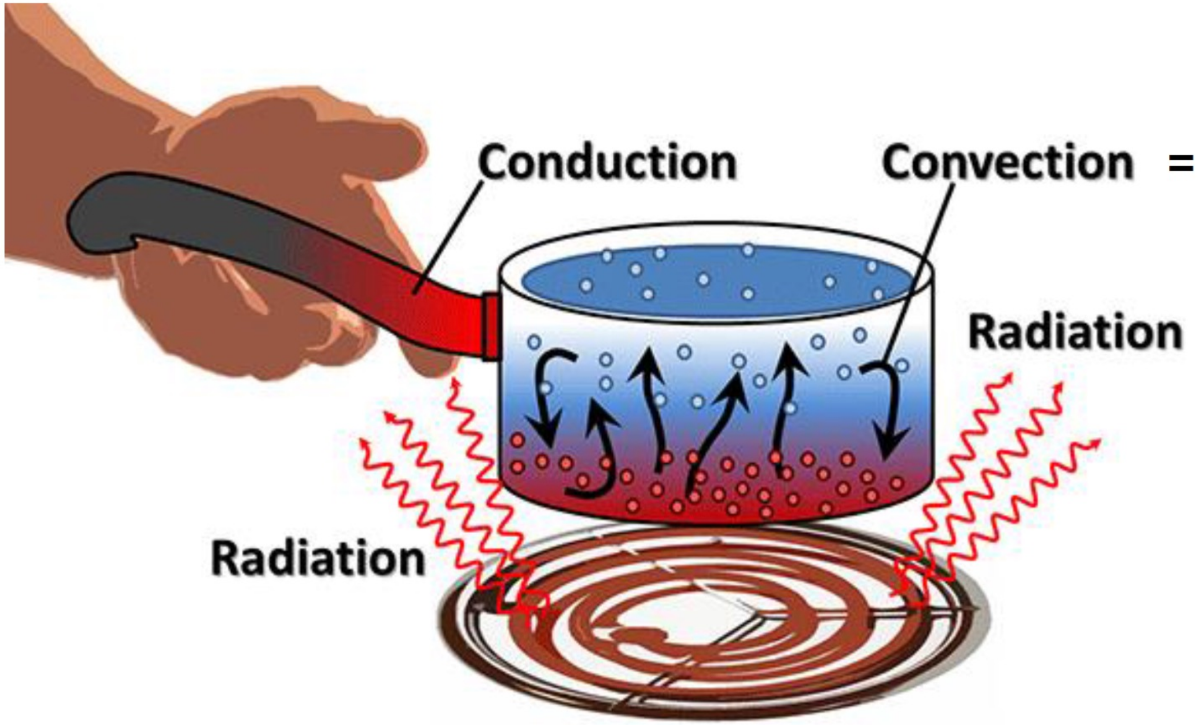
Module 2
Advective Transport

Objectives of this module:

- To apply the conservation of mass equation to balance the incoming and outgoing flow rates in a flow system
- To apply the conservation of momentum equation to calculate the forces acting in a flow system

2.0. How are heat, mass and momentum transported?



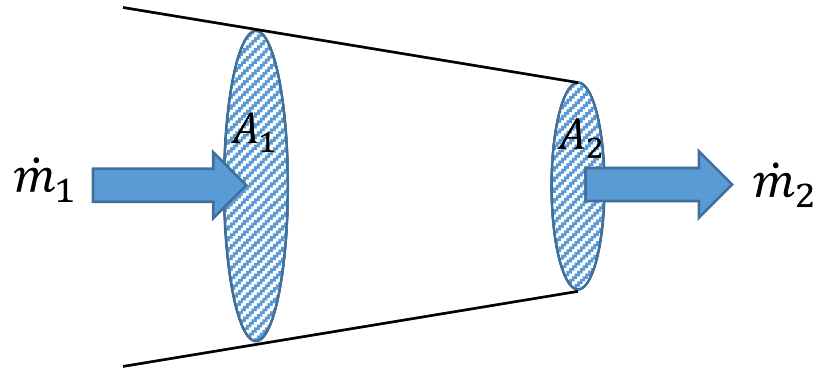
= **Advection + Diffusion**

The transfer of heat or matter due to a temperature or concentration gradient

The transfer of heat or matter by the flow of a fluid

2.1. Advective transport of mass and integral mass balance

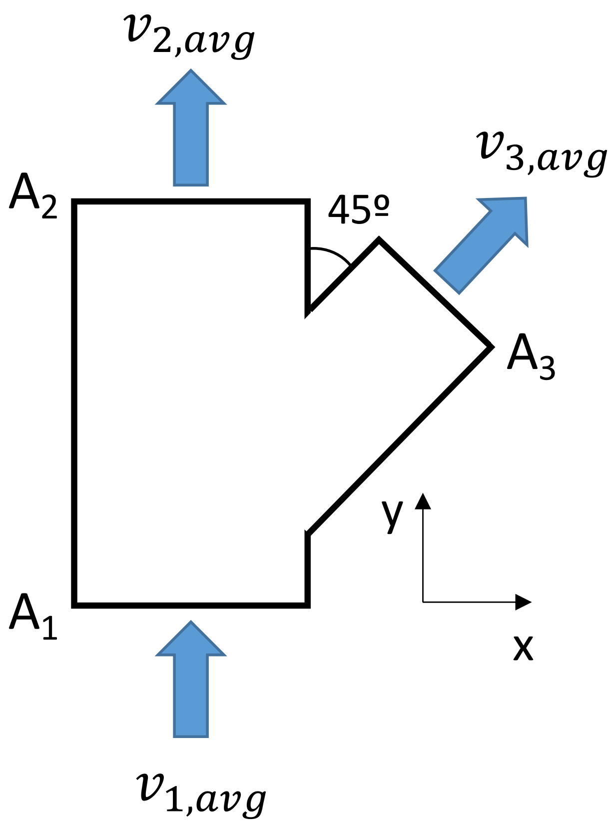
From MODULE 1: CONSERVATION OF MASS and MASS FLOW



$$Mass\ flow\ rate\ \dot{\boldsymbol{m}} = \frac{mass\ of\ fluid}{time} = \rho Q = \rho A v \qquad \left[\frac{g}{s}\right] \left[\frac{Kg}{min}\right]$$

$$\dot{m}_1 = \dot{m}_2$$

2.1. Advective transport of mass and integral mass balance



Data: water at 20ºC (assume ρ=const) flowing in the directions indicated by the arrows

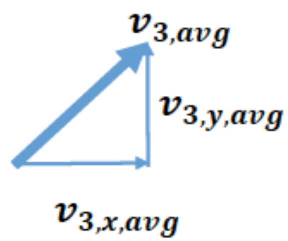
$A_1 = A_2 = 0.1$ m^2 ; $A_3 = 0.06$ m^2

$P_1 = 1.5$ bar

$\dot{m}_1 = 50$ Kg s^{-1}, $\dot{m}_2 = 30$ Kg s^{-1}

Question: what is the average velocity of the fluid moving in the x-direction?

i.e. what is $v_{3,x,avg}$?



$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

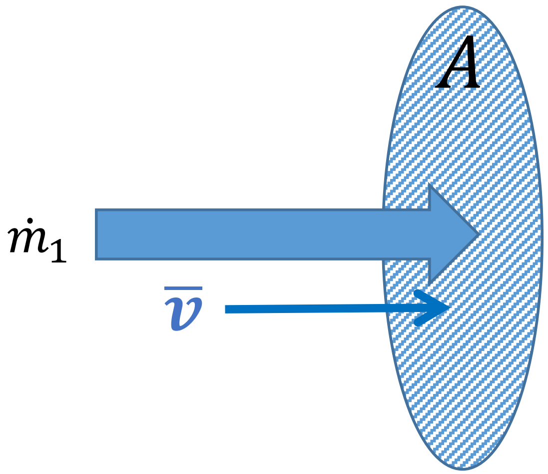
$$\dot{m}_3 = 20 \text{ Kg s}^{-1}$$

$$\dot{m}_3 = 20 \text{ Kg s}^{-1} = \rho v_{3,avg} A_3$$

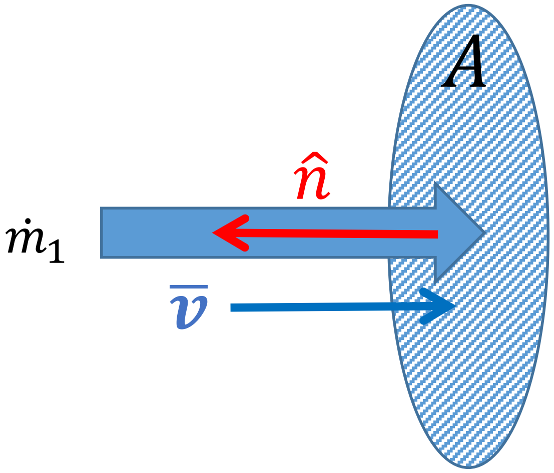
$$\boxed{\boldsymbol{v}_{3,x,avg} = \boldsymbol{v}_{3,avg} \cdot \hat{\boldsymbol{x}} = |\boldsymbol{v}_{3,avg}| \cos(90° - \theta) = \frac{\dot{m}_3}{A_3\, \rho\sqrt{2}} = 0.2364\ m\ s^{-1}}$$

2.1. Advective transport of mass and integral mass balance

The mass flow rate is a scalar product!



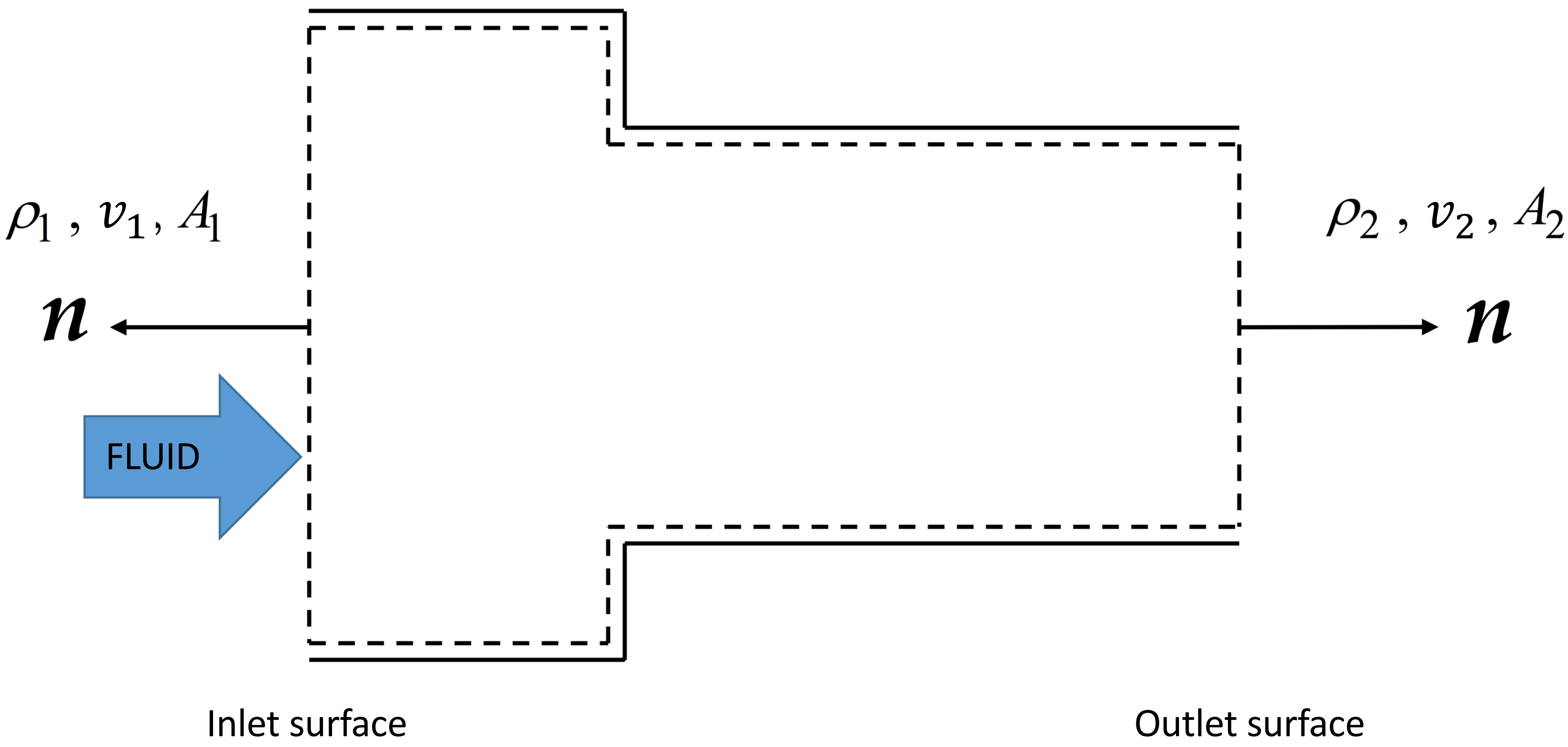
$$\dot{\boldsymbol{m}} = \rho A \cdot \bar{v}$$



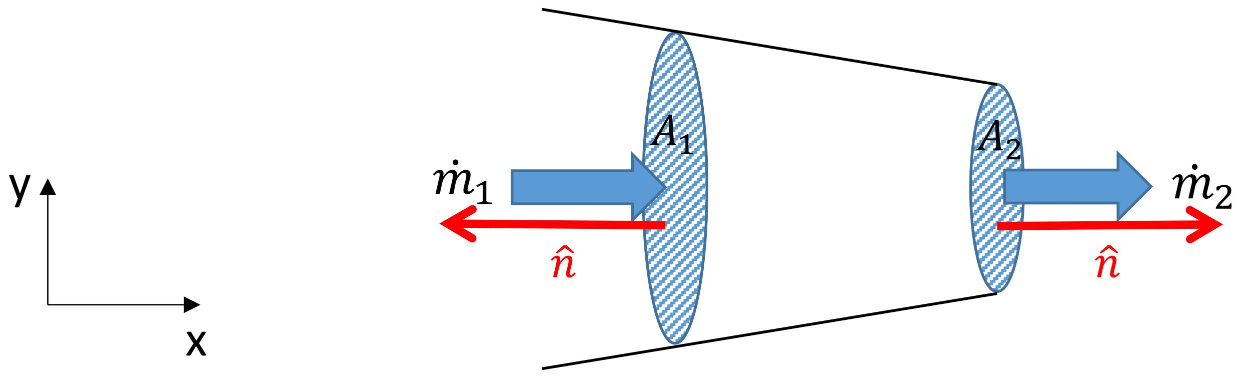
$$\dot{\boldsymbol{m}} = \rho A \cdot \bar{v} = \rho A(\hat{n} \cdot \bar{v})$$

2.1. Advective transport of mass and integral mass balance

SIGN CONVENTION



2.1. Advective transport of mass and integral mass balance



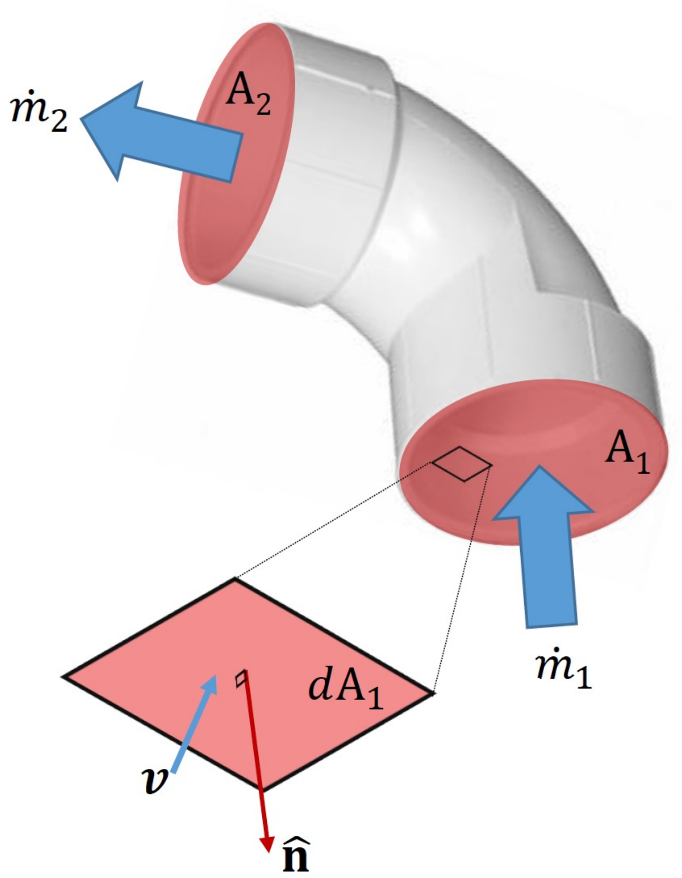
Mass flow rate inlet + mass flow rate outlet = accumulation rate (which is zero because we are at steady state)

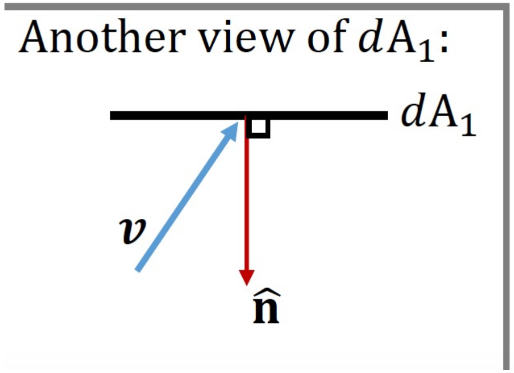
$$\dot{\boldsymbol{m}}_{\mathbf{1}} + \dot{\boldsymbol{m}}_{\mathbf{2}} = 0$$

$$\rho A_1(\hat{n} \cdot \overline{v_1}) + \rho A_2(\hat{n} \cdot \overline{v_2}) = 0$$

$$-\rho A_1 \boldsymbol{v}_{\mathbf{1}} + \rho A_2 \boldsymbol{v}_{\mathbf{2}} = 0$$

2.1. Advective transport of mass and integral mass balance





At **steady-state** the mass balance is:

$$\dot{m}_1 = \dot{m}_2$$

$\dot{m} = mass\ flow\ rate = [Kg\ s^{-1}]$

The mass transport by **advection** over the area dA

$$\dot{m} = \frac{dm}{dt} = \rho\,(v\ \cdot \hat{\mathrm{n}})dA$$

$\hat{\mathrm{n}} = unit\ vector\ normal\ to\ the\ surface = [\phi]$

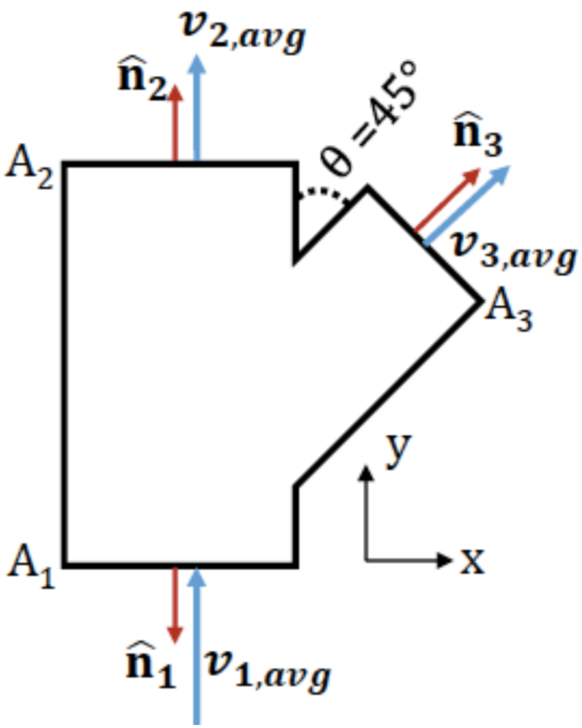
$dA = infinitesimally\ small\ surface\ area = [m^2]$

Our steady-state mass balance can be then written:

$$\int_{A_1} \rho(\boldsymbol{v}\cdot\hat{\mathbf{n}})dA_1 + \int_{A_2} \rho(\boldsymbol{v}\cdot\hat{\mathbf{n}})dA_2 = 0$$

In general for N number of entrances/exits: $$\sum_{i=1}^{N}\int_{A_i} \rho(\boldsymbol{v}\cdot\hat{\mathbf{n}})dA_i = 0$$

2.1. Advective transport of mass and integral mass balance



Data: system flowing water at 20ºC (assume ρ=const)
$A_1 = A_2 = 0.1\ m^2$; $A_3 = 0.06\ m^2$
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$\dot{m}_1 = 50$ Kg s^{-1}, $\dot{m}_2 = 30$ Kg s^{-1}

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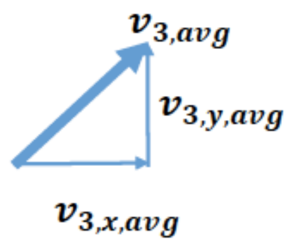


$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

$$\dot{m}_3 = 20 \text{ Kg s}^{-1}$$

$$\sum_{i=1}^{N} \int_{A_i} \rho(\boldsymbol{v} \cdot \hat{\mathbf{n}}) dA_i = 0$$

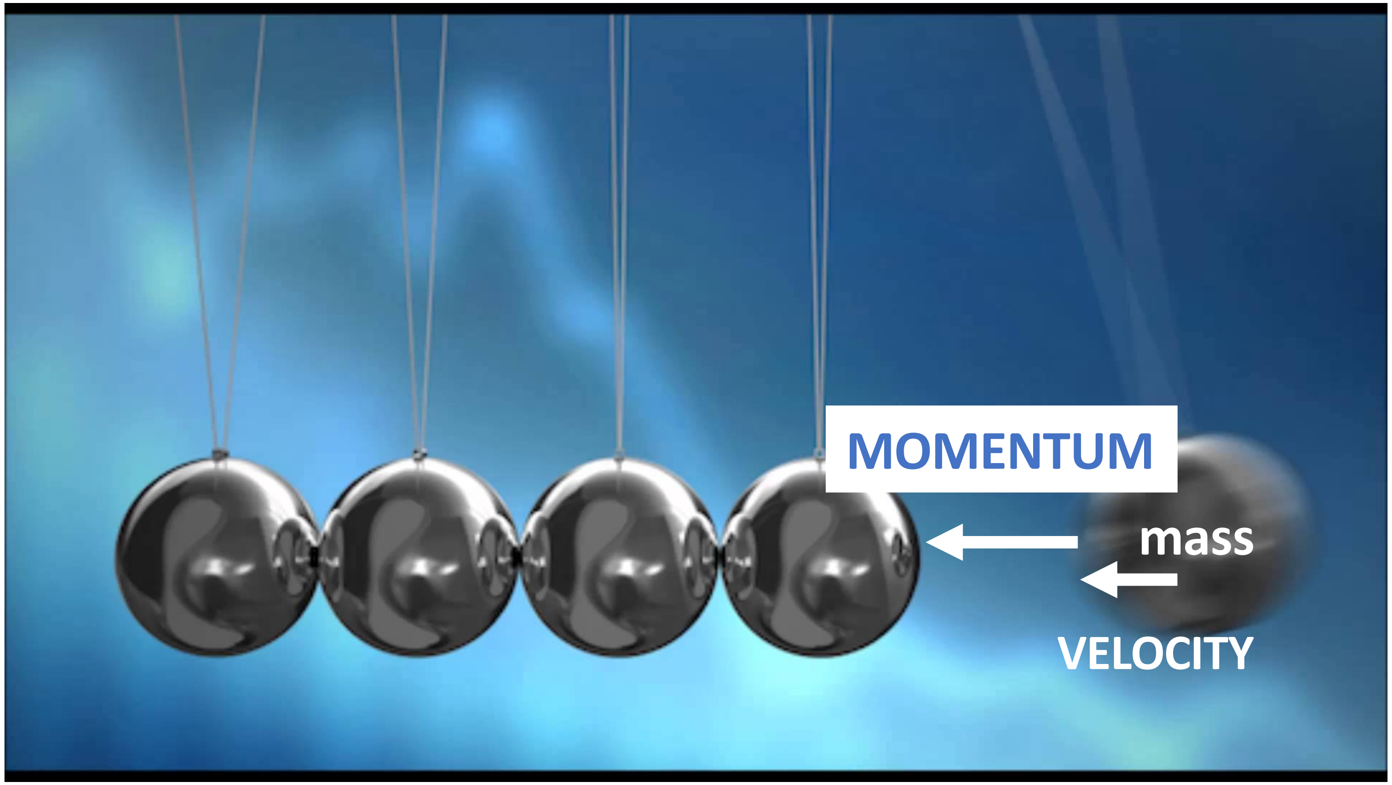
$$\Rightarrow \int_{A_1} \rho(\boldsymbol{v}_{1,avg} \cdot \hat{\mathbf{n}}_1) dA_1 + \int_{A_2} \rho(\boldsymbol{v}_{2,avg} \cdot \hat{\mathbf{n}}_2) dA_2 + \int_{A_3} \rho(\boldsymbol{v}_{3,avg} \cdot \hat{\mathbf{n}}_3) dA_3 = 0$$

$$-\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = 0$$

$$\dot{m}_3 = 20 \text{ kg s}^{-1} = A_3\, \rho\, \boldsymbol{v}_{3,avg} \Rightarrow \boldsymbol{v}_{3,x,avg} = \boldsymbol{v}_{3,avg} \cdot \hat{\boldsymbol{x}} = |\boldsymbol{v}_{3,avg}| \cos(90° - \theta) = \frac{\dot{m}_3}{A_3\, \rho \sqrt{2}}$$

2.2. Advective transport of momentum and integral momentum balance

What is momentum?



$$\vec{M} = m\vec{v}$$

2.2. Advective transport of momentum and integral momentum balance

RELATION BETWEEN MOMENTUM, MOMENTUM FLOW AND FORCES

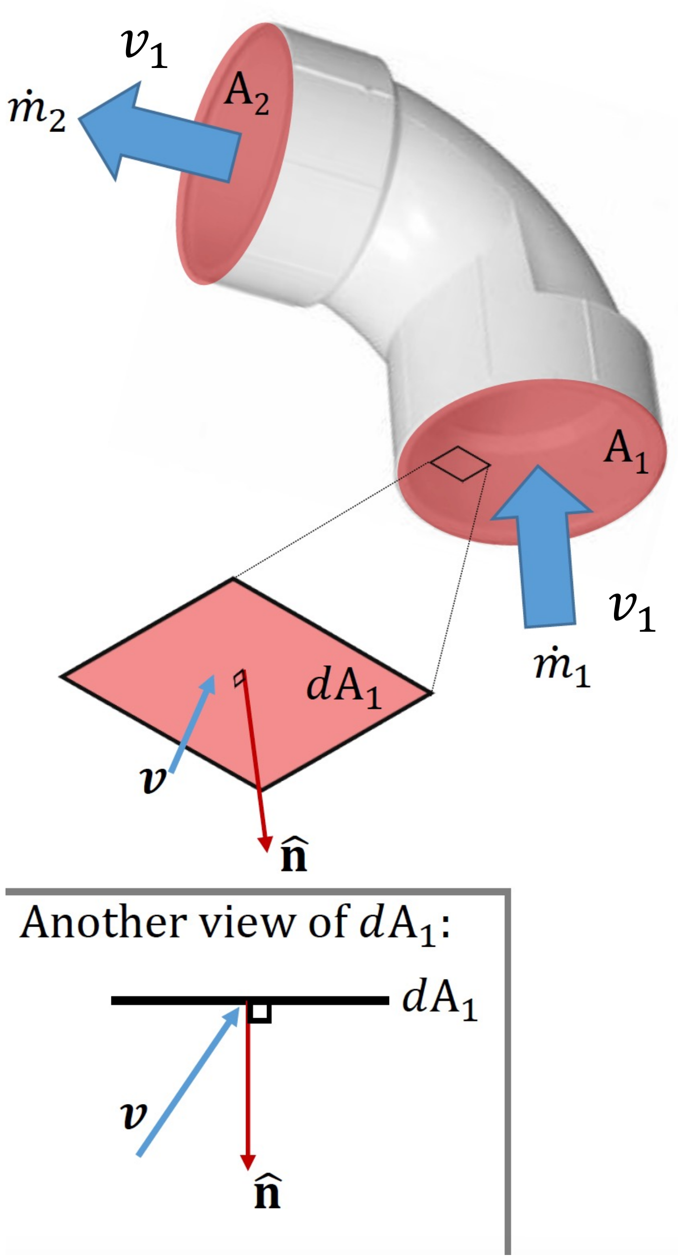
For a fluid flowing in steady state, we can write the **momentum flow rate** in the following way:

$$\dot{M} = \frac{d\vec{M}}{dt} = \frac{d(m\vec{v})}{dt} = \dot{m} \cdot \vec{v} \; = \rho v A \cdot \vec{v}$$

From classical physics (Newton’s second law):

$$\vec{\boldsymbol{F}} \; = \frac{\boldsymbol{d}\left(\overrightarrow{\boldsymbol{M}}\right)}{\boldsymbol{dt}} = \frac{\boldsymbol{d}(\boldsymbol{m}\vec{\boldsymbol{v}})}{\boldsymbol{dt}} = \dot{\boldsymbol{m}}\vec{\boldsymbol{v}} \; = \rho v \mathrm{A} \cdot \vec{\boldsymbol{v}}$$

2.2. Advective transport of momentum and integral momentum balance

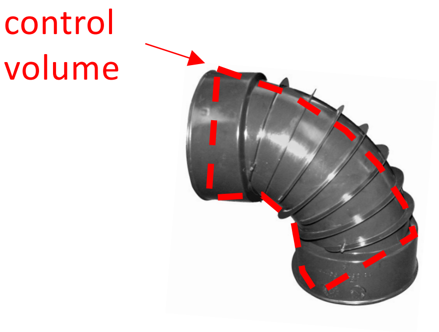


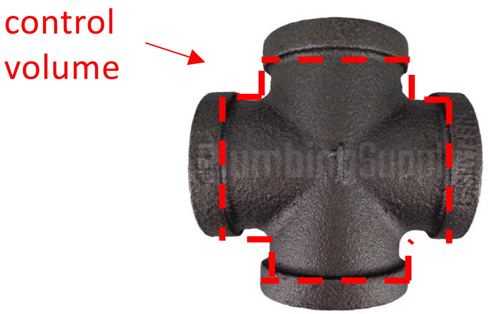
Our **steady-state** momentum balance can be finally written as:

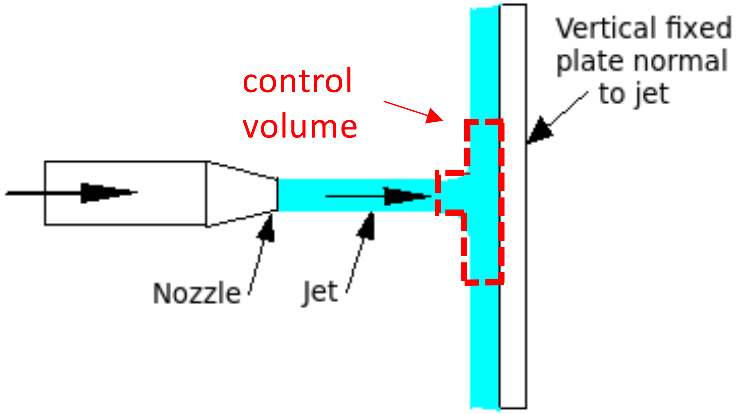
$$\vec{F} = \dot{m} \cdot \vec{v} = \rho v A \cdot \vec{v}$$

$$\boldsymbol{\Sigma}\vec{\boldsymbol{F}} = \sum_{i=1}^{N} \int_{A_i} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}) dA_i = \mathbf{0}$$

2.2. Advective transport of momentum and integral momentum balance







DEFINITION OF THE CONTROL VOLUME

Control Volume of Fluids Flow

In fluid mechanics and thermodynamics , a **control volume** is a mathematical abstraction employed in the process of creating mathematical models of physical processes. In an inertial frame of reference , it is a volume fixed in space or moving with constant velocity through which the fluid (gas or liquid) flows. The surface enclosing the control volume is referred to as the **control surface** .

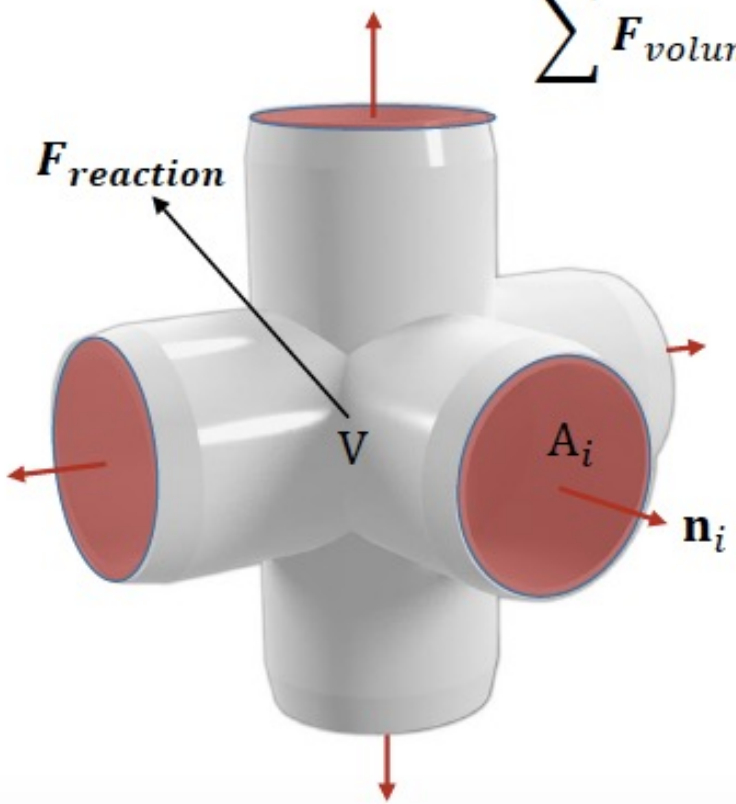
In thermodynamics, a *control volume* was defined as a fixed region in space where one studies the masses and energies crossing the boundaries of the region. This concept of a control volume is also very useful in analyzing fluid flow problems. The boundary of a control volume for fluid flow is usually taken as the physical boundary of the part through which the flow is occurring. The control volume concept is used in fluid dynamics applications, utilizing the continuity, momentum, and energy principles mentioned at the beginning of this chapter. Once the control volume and its boundary are established, the various forms of energy crossing the boundary with the fluid can be dealt with in equation form to solve the fluid problem. Since fluid flow problems usually treat a fluid crossing the boundaries of a control volume, the control volume approach is referred to as an "open" system analysis, which is similar to the concepts studied in thermodynamics. There are special cases in the nuclear field where fluid does not cross the control boundary. Such cases are studied utilizing the "closed" system approach.

Regardless of the nature of the flow, all flow situations are found to be subject to the established basic laws of nature that engineers have expressed in equation form. Conservation of mass and conservation of energy are always satisfied in fluid problems, along with Newtons laws of motion. In addition, each problem will have physical constraints, referred to mathematically as boundary conditions, that must be satisfied before a solution to the problem will be consistent with the physical results

2.2. Advective transport of momentum and integral momentum balance

For an arbitrary **control volume** with volume V and *i* entrances/exits:

$$\sum \boldsymbol{F}_{volume} + \sum \boldsymbol{F}_{surface} = \sum_{i=1}^{N} \int_{A_i} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}) dA_i$$



Main volume force:

$$\boldsymbol{F}_{gravity} = \int_V \rho \boldsymbol{g} dV$$

Main surface forces:

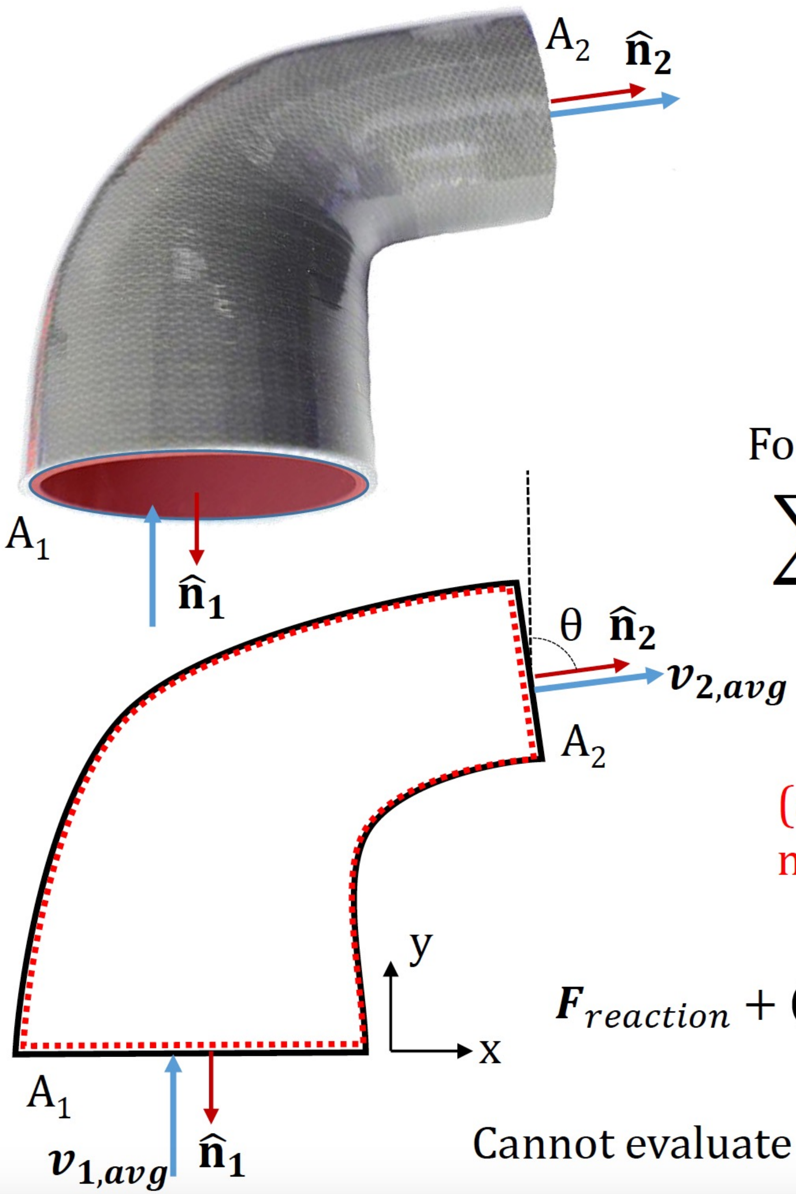
$$\boldsymbol{F}_{pressure} = -p_i A_i \hat{\mathbf{n}}_i \quad \text{(for all surfaces } i\text{)}$$

$\boldsymbol{F}_{friction}$ Due to viscosity (For all surfaces)

$\boldsymbol{F}_{reaction}$ Force by control volume on fluid

Note: in this course, unless explicitly asked, we will neglect gravity

2.2. Advective transport of momentum and integral momentum balance



Water (50 kg per second at 80°C and a pressure of 1.5 bar) enters a reducing elbow joint. Estimate the <u>force</u> that the elbow joint exerts on the fluid.

Data: $\theta = 80°$

$A_1 = 8.66\times10^{-3}\ m^2$

$A_2 = 3.70\times10^{-3}\ m^2$

$\rho_{H20,\ 80°C,\ 1.5\ bar} = 971.8\ kg\ m^{-3}$

Force/momentum balance:

$$\sum \cancel{\boldsymbol{F}_{volume}} + \sum \cancel{\boldsymbol{F}_{surface}} = \sum_{i=1}^{N} \int_{A_i} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}) dA_i$$

$\boldsymbol{F}_{gravity} = 0$

(assume gravity not important)

$\boldsymbol{F}_{pressure} = -p_i A_i \hat{\mathbf{n}}_i$

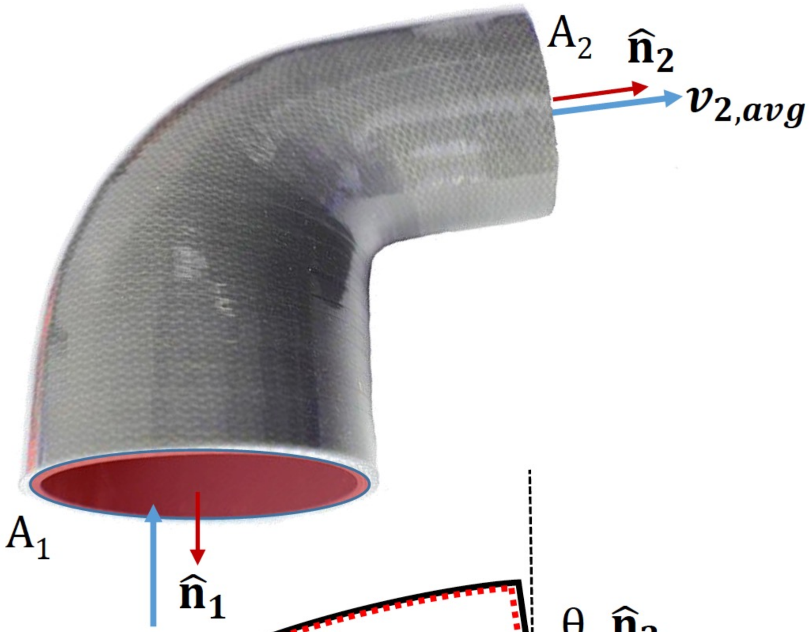
$\cancel{\boldsymbol{F}_{friction}}$ (assume negligible)

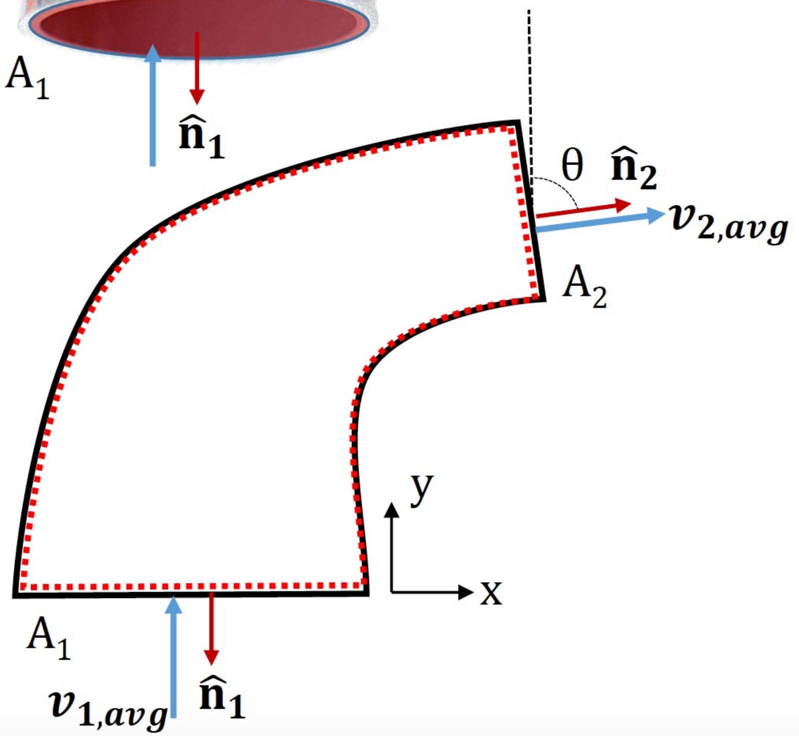
$\boldsymbol{F}_{reaction}$

$$\boldsymbol{F}_{reaction} + (-p_1 A_1 \hat{\mathbf{n}}_1) + (-p_2 A_2 \hat{\mathbf{n}}_2) = \sum_{i=1}^{N} \int_{A_i} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}) dA_i$$

Cannot evaluate this yet! We need p_2 and the velocities!

2.2. Advective transport of momentum and integral momentum balance





Water (50 kg per second at 80°C and a pressure of 1.5 bar) enters a reducing elbow joint. Estimate the force that the elbow joint exerts on the fluid.

Data: θ = 80°

$A_1 = 8.66 \times 10^{-3}\ \mathrm{m}^2$

$A_2 = 3.70 \times 10^{-3}\ \mathrm{m}^2$

$\rho_{H20,\ 80°C,\ 1.5\ bar} = 971.8\ \mathrm{kg\ m^{-3}}$

$$\text{Mass balance}: -\dot{m}_1 + \dot{m}_2 = 0$$

Calculate average velocities (assume ρ=const.) :

$$\dot{m}_1 = A_1\, \rho\, \boldsymbol{v}_{1,avg}$$

$$50\ \mathrm{kg\ s^{-1}} = 8.66 \times 10^{-3}\ \mathrm{m}^2\ \ 971.8\ \mathrm{kg\ m^{-3}}\ \boldsymbol{v}_{1,avg}$$

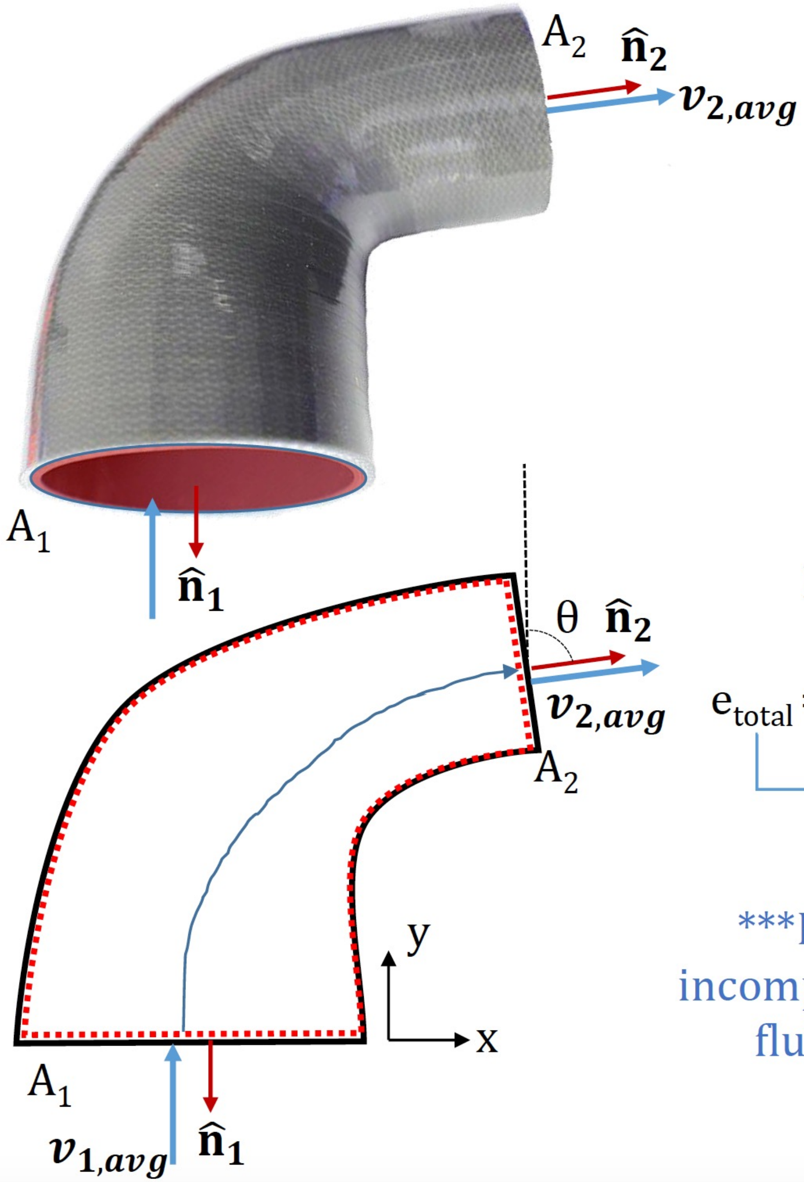
$$\boldsymbol{v}_{1,avg} = 5.9\ \mathrm{m\ s^{-1}}$$

$$\dot{m}_2 = A_2\, \rho\, \boldsymbol{v}_{2,avg}$$

$$50\mathrm{kg\ s^{-1}} = 3.70 \times 10^{-3}\ \mathrm{m}^2\ \ 971.8\ \mathrm{kg\ m^{-3}}\ \boldsymbol{v}_{2,avg}$$

$$\boldsymbol{v}_{2,avg} = 13.9\ \mathrm{m\ s^{-1}}$$

2.2. Advective transport of momentum and integral momentum balance



Water (50 kg per second at 80°C and a pressure of 1.5 bar) enters a reducing elbow joint. Estimate the <u>force</u> that the elbow joint exerts on the fluid.

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$A_1 = 8.66\times10^{-3}\ m^2$

$A_2 = 3.70\times10^{-3}\ m^2$

$\rho_{H20,\ 80°C,\ 1.5\ bar} = 971.8\ kg\ m^{-3}$

$\boldsymbol{v}_{1,avg} = 5.9\ \mathrm{m\ s^{-1}}$

$\boldsymbol{v}_{2,avg} = 13.9\ \mathrm{m\ s^{-1}}$

What about p_2?

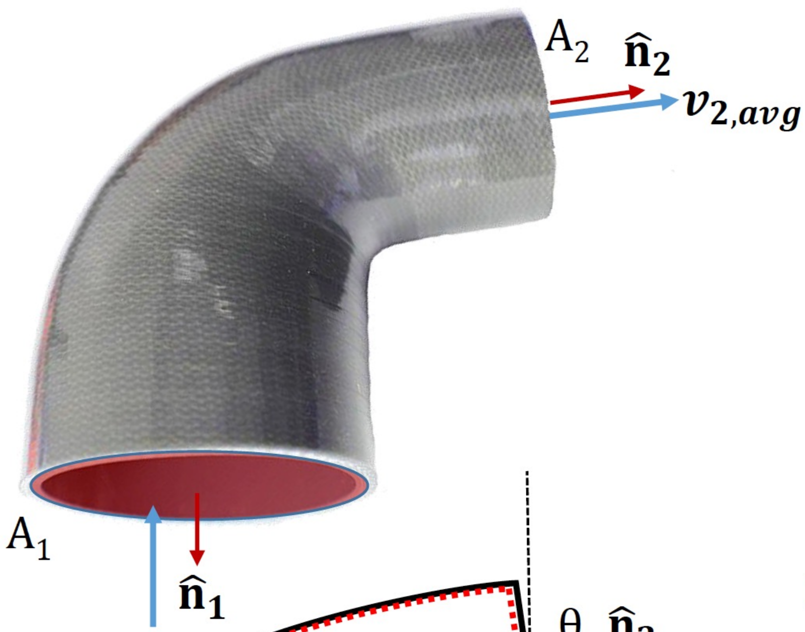
Mechanical energy balance (Bernoulli)

$e_{total} = e_{kin} + e_{pot} + e_{pres}$ = constant along a streamline

- e_{total} → Energy density of fluid = [J m^{-3}] = [Pa]
- e_{kin} → Kinetic energy density = $\frac{1}{2}\rho(|\boldsymbol{v}|)^2$
- e_{pot} → Potential energy density = $\rho \boldsymbol{g} z$
- e_{pres} → Pressure energy density = p

For an incompressible fluid

2.2. Advective transport of momentum and integral momentum balance



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Data: $\theta = 80°$

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$\rho_{H2O,\ 80°C,\ 1.5\ bar} = 971.8\ kg\ m^{-3}$

$\boldsymbol{v}_{1,avg} = 5.9\ m\ s^{-1}$

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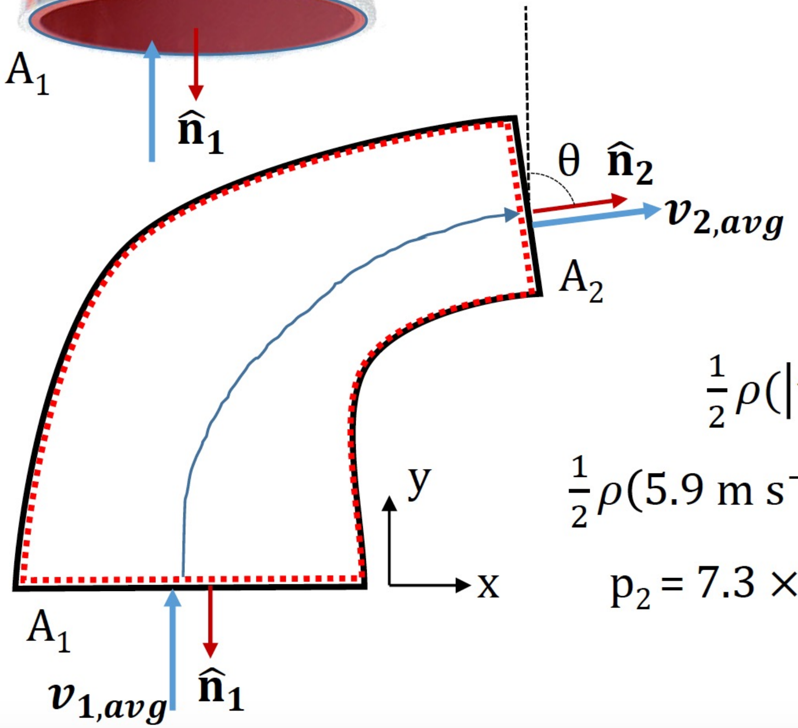
What about p_2?



Mechanical energy balance (Bernoulli)

From 1 to 2:

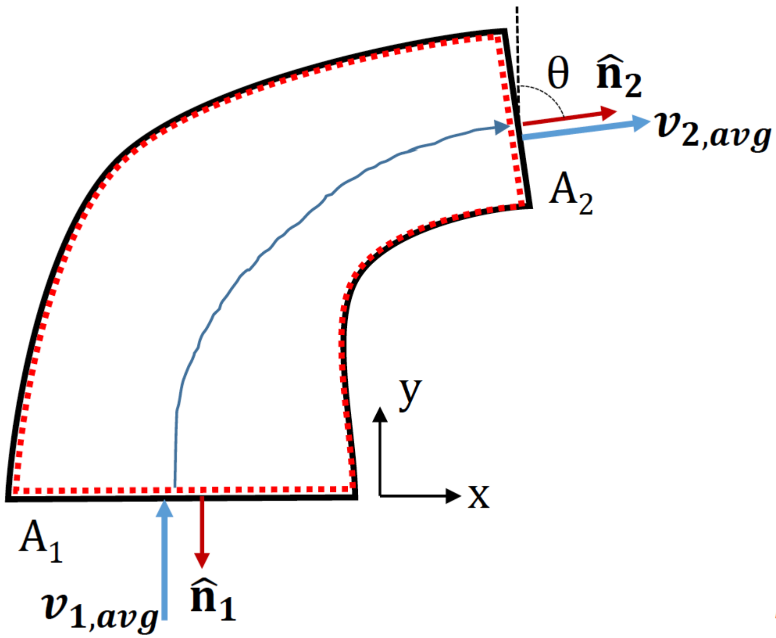
$e_{kin,1} + e_{pot,1} + e_{pres,1} = e_{kin,1} + e_{pot,1} + e_{pres,1}$

$$\frac{1}{2}\rho(|\boldsymbol{v}_{1,avg}|)^2 + \rho\boldsymbol{g}z_1 + p_1 = \frac{1}{2}\rho(|\boldsymbol{v}_{2,avg}|)^2 + \rho\boldsymbol{g}z_2 + p_2$$

$$\frac{1}{2}\rho(5.9\ m\ s^{-1})^2 + 0 + 1.5\times10^5\ Pa = \frac{1}{2}\rho(13.9\ m\ s^{-1})^2 + 0 + p_2$$

$$p_2 = 7.3\times10^4\ Pa$$

2.2. Advective transport of momentum and integral momentum balance



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$\rho_{H20,\ 80°C,\ 1.5\ bar} = 971.8$ kg m^{-3}

$\boldsymbol{v}_{1,avg} = 5.9$ m s^{-1}

$\boldsymbol{v}_{2,avg} = 13.9$ m s^{-1}

$p_2 = 7.3\times10^4$ Pa

$$\boldsymbol{F}_{reaction} + (-p_1A_1\hat{\mathbf{n}}_1) + (-p_2A_2\hat{\mathbf{n}}_2) = \int_{A_1} \cancelto{\rho(v_{1,avg})^2A_1\hat{\mathbf{n}}_1}{\rho\boldsymbol{v}_{1,avg}(\boldsymbol{v}_{1,avg}\cdot\hat{\mathbf{n}}_1)dA_1} + \int_{A_2} \cancelto{\rho(v_{2,avg})^2A_2\hat{\mathbf{n}}_2}{\rho\boldsymbol{v}_{2,avg}(\boldsymbol{v}_{2,avg}\cdot\hat{\mathbf{n}}_2)dA_2}$$

x-component:

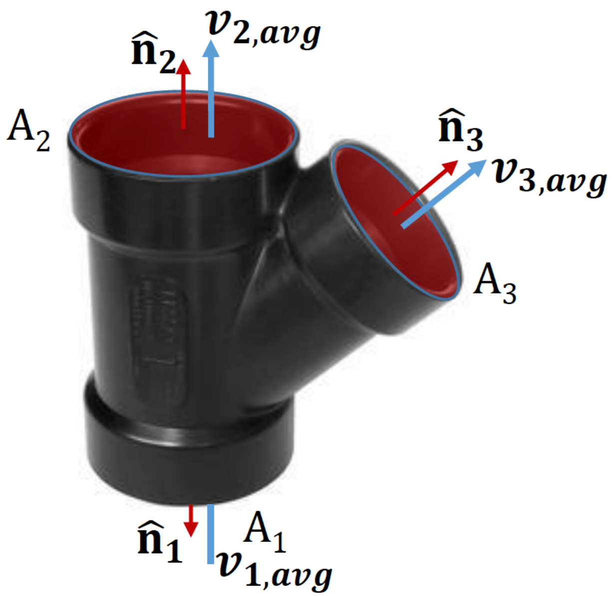
$$\boldsymbol{F}_{rxn,x} + \cancelto{0}{(-p_1A_10)} + (-p_2A_2\cos(90° - \theta)) = \cancelto{0}{\rho(v_{1,avg})^2A_10} + \rho(v_{2,avg})^2A_2\cos(90° - \theta)$$

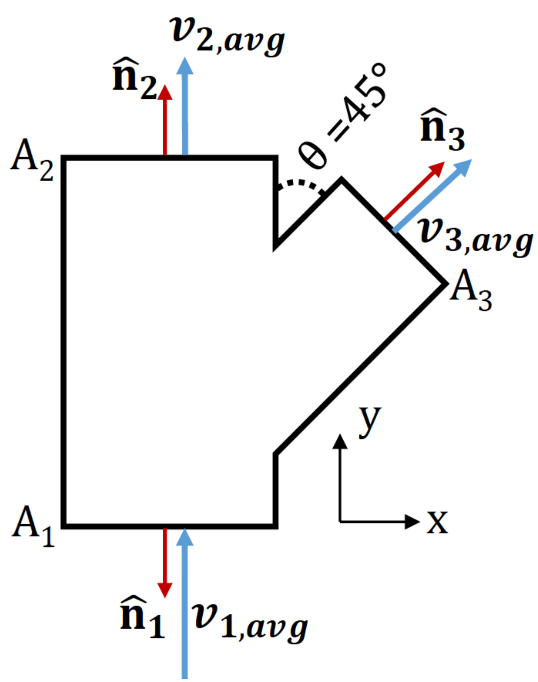
y-component:

$$\boldsymbol{F}_{rxn,y} + (-p_1A_1(-1)) + (-p_2A_2\cos(\theta)) = \rho(v_{1,avg})^2A_1(-1) + \rho(v_{2,avg})^2A_2\cos(\theta)$$

2.2. Advective transport of momentum and integral momentum balance

Exercize





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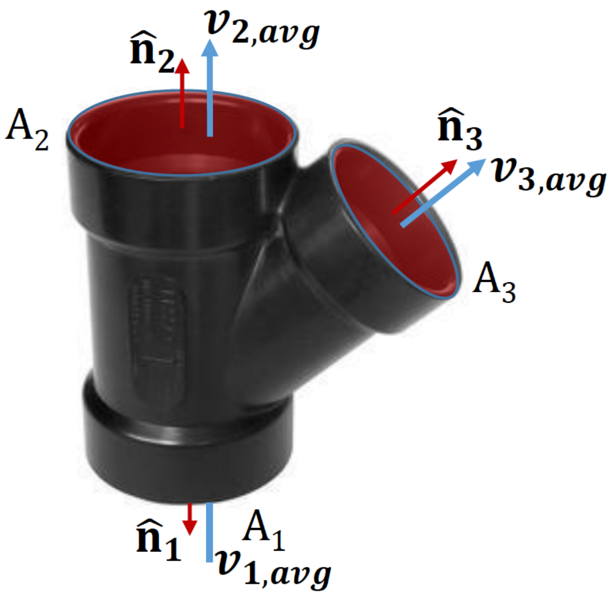
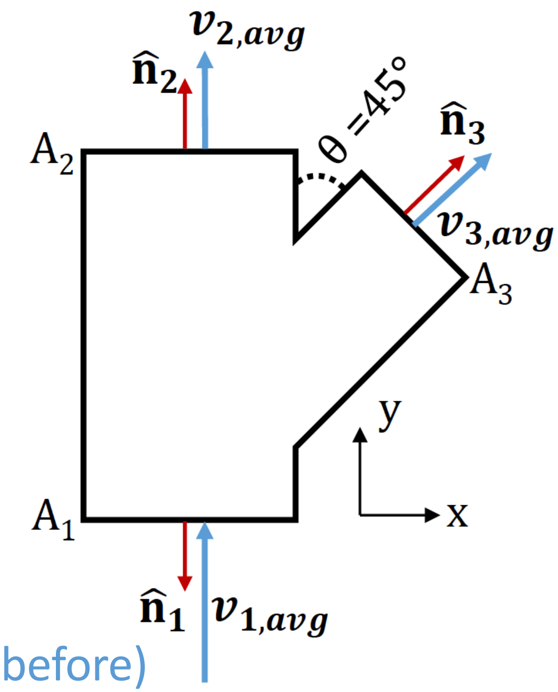
$p_1 = 1.5$ bar
$\dot{m}_1 = 50\ kg\ s^{-1}$, $\dot{m}_2 = 30\ kg\ s^{-1}$

Question: Estimate the force the pipe exerts in the x-direction?

2.2. Advective transport of momentum and integral momentum balance





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$p_1 = 1.5$ bar
$\dot{m}_1 = 50\ kg\ s^{-1}$, $\dot{m}_2 = 30\ kg\ s^{-1}$

Question: Estimate the force the pipe exerts in the x-direction?

Mass balance: (we have done this before)

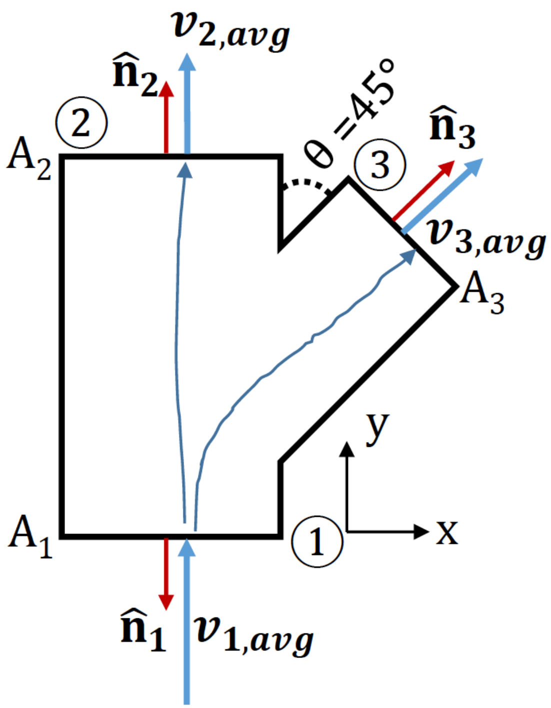
$$\dot{m}_3 = 20\ \mathrm{kg\ s^{-1}} \qquad \boldsymbol{v}_{3,x,avg} = \frac{\dot{m}_3}{A_3\, \rho \sqrt{2}} = \frac{20\ \mathrm{kg\ s^{-1}}}{0.06\ \mathrm{m^2}\ 1000 \mathrm{kg\ m^{-3}} \sqrt{2}} = 0.236\ \mathrm{m\ s^{-1}}$$

Momentum balance:

$$\sum \boldsymbol{F}_{volume} + \sum \boldsymbol{F}_{surface} = \sum_{i=1}^{N} \int_{A_i} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}) dA_i$$

$$\boldsymbol{F}_{pressure} = -p_i A_i \hat{\mathbf{n}}_i \qquad \boldsymbol{F}_{reaction} \qquad \boldsymbol{F}_{friction}$$

2.2. Advective transport of momentum and integral momentum balance



Data: system flowing water at 20°C (assume ρ =const.),
$A_1 = A_2 = 0.1$ m^2 , $A_3 = 0.06$ m^2 $\dot{m}_1 = 50$ kg s^{-1} , $\dot{m}_2 = 30$ kg s^{-1}
$p_1 = 1.5$ bar $\quad \dot{m}_3 = 20$ kg s^{-1}

Question: Estimate the force the pipe exerts in the x-direction?

What are p_2 and p_3? For pressure we always apply Bernoulli

$e_{total} = e_{kin} + e_{pot} + e_{pres}$ = constant along a streamline

- e_{total} → Energy density of fluid = [J m^{-3}] = [Pa]
- e_{kin} → Kinetic energy density = $\frac{1}{2}\rho(|\boldsymbol{v}|)^2$
- e_{pot} → Potential energy density = $\rho \boldsymbol{g} z$
- e_{pres} → Pressure energy density = p

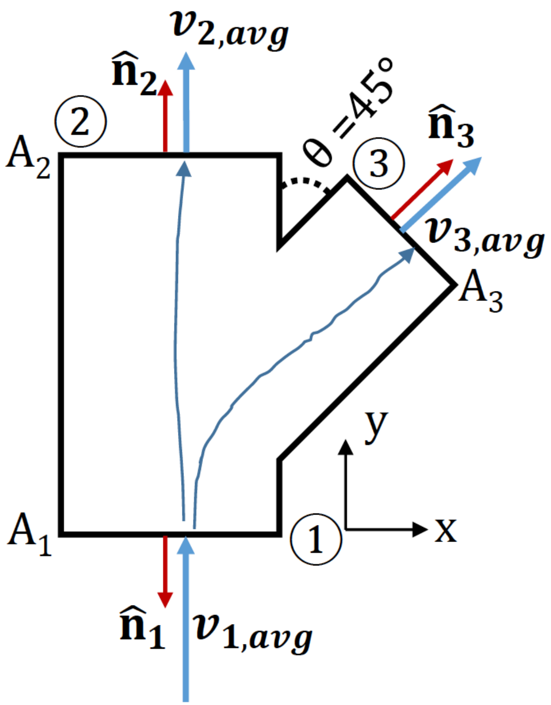
From 1 to 3:

$$e_{kin,1} + e_{pot,1} + e_{pres,1} = e_{kin,3} + e_{pot,3} + e_{pres,3}$$

$$\frac{1}{2}\rho(|\boldsymbol{v}_{1,avg}|)^2 + \rho \boldsymbol{g} z_1 + p_1 = \frac{1}{2}\rho(|\boldsymbol{v}_{3,avg}|)^2 + \rho \boldsymbol{g} z_3 + p_3$$

$$\frac{1}{2}\rho\left(\frac{\dot{m}_1}{\rho A_1}\right)^2 + 0 + 1.5 \times 10^5 \text{ Pa} = \frac{1}{2}\rho\left(\frac{\dot{m}_3}{\rho A_3}\right)^2 + 0 + p_3$$

2.2. Advective transport of momentum and integral momentum balance



From 1 to 3:

$$\frac{1}{2}\rho\left(\frac{\dot{m}_1}{\rho\, A_1}\right)^2 + 0 + 1.5\times10^5\ \text{Pa} = \frac{1}{2}\rho\left(\frac{\dot{m}_3}{\rho\, A_3}\right)^2 + 0 + \text{p}_3$$

$$\frac{1}{2}1000\ \text{kg m}^{-3}\left(\frac{50\ \text{kg s}^{-1}}{1000\ \text{kg m}^{-3}\ 0.1\ \text{m}^2}\right)^2 + 1.5\times10^5\ \text{Pa}$$

$$= \frac{1}{2}1000\ \text{kg m}^{-3}\left(\frac{20\ \text{kg s}^{-1}}{1000\ \text{kg m}^{-3}\ 0.06\ \text{m}^2}\right)^2 + \text{p}_3$$

$$125\ \text{Pa} + 1.5\times10^5\ \text{Pa} = 20\ \text{Pa} + \text{p}_3$$

$$\text{p}_3 = 1.501\times10^5\ \text{Pa}$$

From 1 to 2:

$$\frac{1}{2}\rho\left(\frac{\dot{m}_1}{\rho\, A_1}\right)^2 + 0 + 1.5\times10^5\ \text{Pa} = \frac{1}{2}\rho\left(\frac{\dot{m}_2}{\rho\, A_2}\right)^2 + 0 + \text{p}_2$$

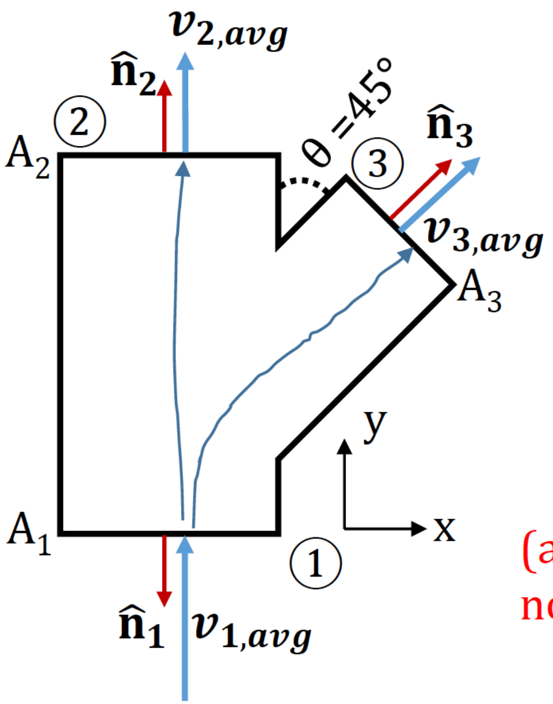
$$\frac{1}{2}1000\ \text{kg m}^{-3}\left(\frac{50\ \text{kg s}^{-1}}{1000\ \text{kg m}^{-3}\ 0.1\ \text{m}^2}\right)^2 + 1.5\times10^5\ \text{Pa}$$

$$= \frac{1}{2}1000\ \text{kg m}^{-3}\left(\frac{30\ \text{kg s}^{-1}}{1000\ \text{kg m}^{-3}\ 0.1\ \text{m}^2}\right)^2 + \text{p}_2$$

$$125\ \text{Pa} + 1.5\times10^5\ \text{Pa} = 45\ \text{Pa} + \text{p}_2$$

$$\text{p}_2 = 1.5008\times10^5\ \text{Pa}$$

2.2. Advective transport of momentum and integral momentum balance



Data: system flowing water at 20°C (assume ρ =const.),
$A_1 = A_2 = 0.1\ m^2$, $A_3 = 0.06\ m^2$ $\dot{m}_1 = 50\ kg\ s^{-1}$, $\dot{m}_2 = 30\ kg\ s^{-1}$
$p_1 = 1.5$ bar $\quad \dot{m}_3 = 20\ kg\ s^{-1} \quad p_3 = 1.501 \times 10^5$ Pa

Question: Estimate the force the pipe exerts in the x-direction?

$$\sum \underbrace{\cancel{\boldsymbol{F}_{volume}}}_{0} + \sum \boldsymbol{F}_{surface} = \sum_{i=1}^{N} \int_{A_i} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}) dA_i$$

0
(assume gravity not important)

$$\boldsymbol{F}_{pressure} = -p_i A_i \hat{\mathbf{n}}_i$$

$\boldsymbol{F}_{rxn}$

$\boldsymbol{F}_{friction}$ (assume negligible)

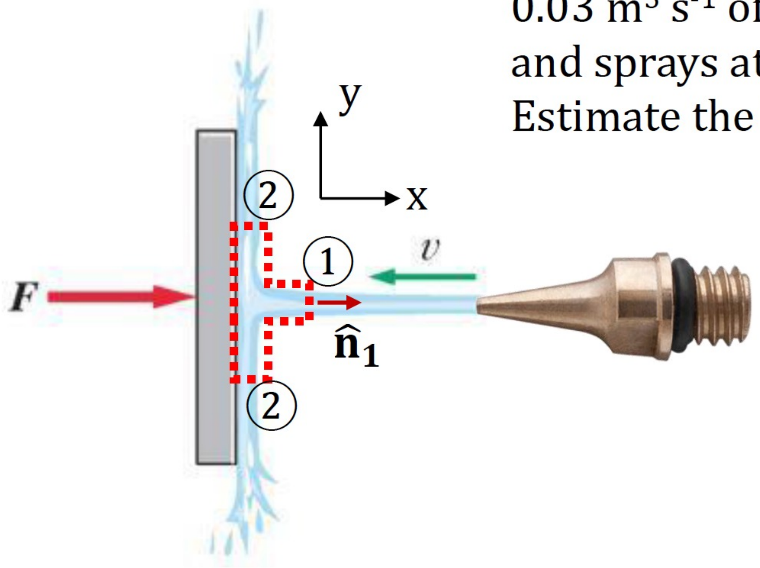
$$\boldsymbol{F}_{rxn,} + (-p_1 A_1(\hat{\mathbf{n}}_1)) + (-p_2 A_2(\hat{\mathbf{n}}_2)) + (-p_3 A_3(\hat{\mathbf{n}}_3))$$

$$= \underbrace{\int_{A_1} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}_1) dA_1}_{\rho(v_{1,avg})^2 A_1 \hat{\mathbf{n}}_1} + \underbrace{\int_{A_2} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}_2) dA_2}_{\rho(v_{2,avg})^2 A_2 \hat{\mathbf{n}}_2} + \underbrace{\int_{A_3} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}_3) dA_3}_{\rho(v_{3,avg})^2 A_3 \hat{\mathbf{n}}_3}$$

x-component: $\boldsymbol{F}_{rxn,x} + 0 + 0 + (-p_3 A_3 \cos(90° - \theta)) = \rho (v_{3,avg})^2 A_3 \cos(90° - \theta)$

$\boldsymbol{F}_{rxn,x} - 6368\ \text{N} = 4.7\ \text{N} \qquad F_{rxn,x} = 6373\ N$

2.2. Advective transport of momentum and integral momentum balance



0.03 m^3 s^{-1} of water at 20°C is shot out of a nozzle at 30.5 m s^{-1} and sprays at a flat plate perpendicular to the jet.
Estimate the force needed to hold the plate motionless.

$$\sum \cancel{\boldsymbol{F}}_{volume} + \sum \cancel{\boldsymbol{F}}_{surface} = \sum_{i=1}^{N} \int_{A_i} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}}) dA_i$$

$\boldsymbol{F}_{gravity} = 0$

(assume gravity not important)

$\boldsymbol{F}_{pressure} = -p_i A_i \hat{\mathbf{n}}_i$

$\boldsymbol{F}_{friction}$ (assume negligible)

$\boldsymbol{F}_{rxn}$

$$\boldsymbol{F}_{rxn} = \int_{A_1} \rho \boldsymbol{v}_{1,avg}(\boldsymbol{v}_{1,avg} \cdot \hat{\mathbf{n}}_1) dA_1 + \int_{A_2} \rho \boldsymbol{v}_{2,avg}(\boldsymbol{v}_{2,avg} \cdot \hat{\mathbf{n}}_2) dA_2$$

x-component: no component along x

$$\boldsymbol{F}_{rxn,x} = \int_{A_1} \rho \boldsymbol{v}_{1,avg}(\boldsymbol{v}_{1,avg} \cdot \hat{\mathbf{n}}_1) dA_1 = \rho (v_{1,avg})^2 A_1 \hat{\mathbf{n}}_1$$

$$= 1000 \text{ kg m}^{-3} (30.5 \text{ m s}^{-1})^2 (0.000983 \text{ m}^2)(1) = 914 \text{ N}$$

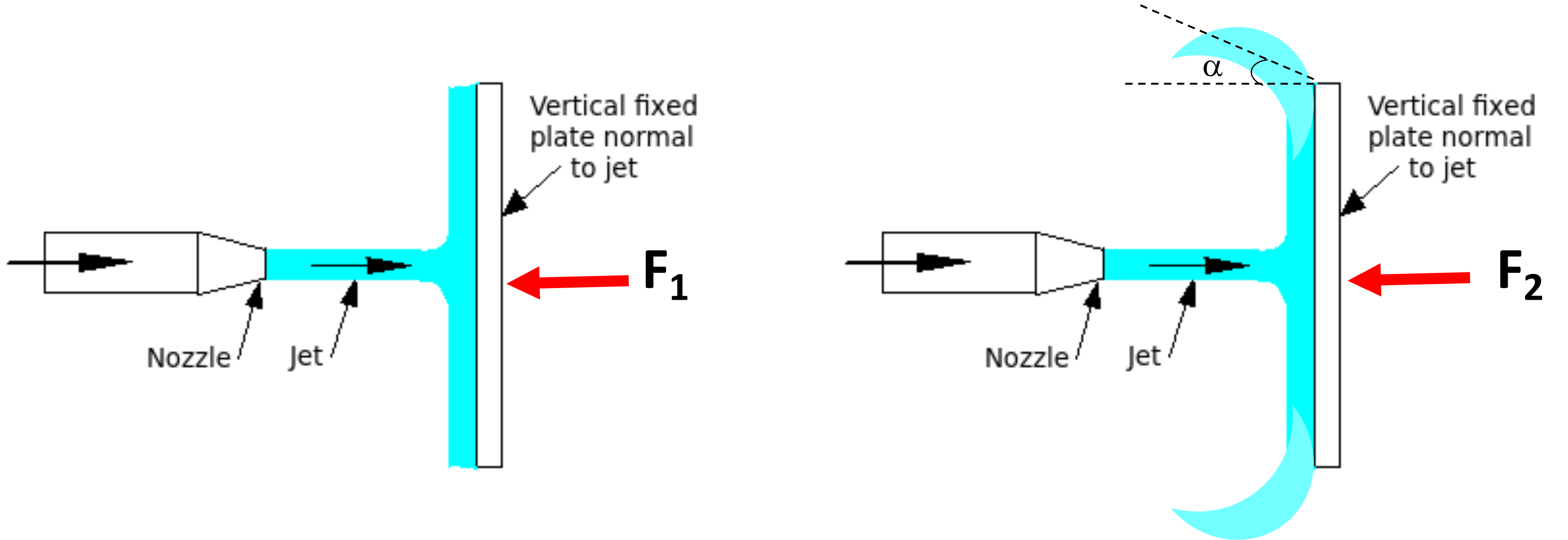
What is A_1?

$\dot{m}_1 = \rho \dot{V}_1 = A_1 \rho \boldsymbol{v}_{1,avg}$

0.03 m^3 s^{-1} = A_1 30.5 m s^{-1}

$A_1 = 0.000983$ m^2

2.2. Advective transport of momentum and integral momentum balance



What is the relations between F_1 and F_2? Which is greater?

Recap: Macroscopic Fluid Mechanics

Consider a fluid consisting of a given set of particles with a total mass *m*, total momentum *M* and total energy *E* (E = internal + kinetic + potential) flowing through a pipe in **steady-state flow**, it will obey the following basic laws:

1) **Conservation of Energy (energy balance)** $\frac{dE}{dt} = 0$

Bernoulli's Equation

$$\boxed{\frac{P_1}{\rho g} + h_1 + \frac{v_1^2}{2g} + H_p = \frac{P_2}{\rho g} + h_2 + \frac{v_2^2}{2g} + H_L}$$

2) **Conservation of Mass (mass balance)** $\frac{dm}{dt} = 0$

$$\boxed{\sum_{i=1}^{N} \int_{A_i} \rho(\boldsymbol{v} \cdot \hat{\mathbf{n}})\, dA_i = 0}$$

3) **Conservation of Momentum (momentum balance)** $\frac{dM}{dt} = 0$

$$\boxed{\Sigma \vec{F} = \sum_{i=1}^{N} \int_{A_i} \rho \boldsymbol{v}(\boldsymbol{v} \cdot \hat{\mathbf{n}})\, dA_i = 0}$$