Selected solutions to diffEQs with one independent variable: $\varphi(x)$

Most common solutions :

$$\frac{d\varphi}{dx} = \frac{f(x)}{g(\varphi)} \longrightarrow \int g(\varphi)d\varphi = \int f(x)dx + \mathbb{C}_1$$

$$\frac{d\varphi}{dx} + f(x)\varphi = g(x) \longrightarrow \varphi(x) = e^{-\int f(x)dx}\left(\int e^{\int f(x)dx} g(x)dx + \mathbb{C}_1\right)$$

$$\frac{d^2\varphi}{dx^2} + a^2\varphi = 0 \longrightarrow \varphi(x) = \mathbb{C}_1 \cos a\,x + \mathbb{C}_2 \sin a\,x$$

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{d\varphi}{dx}\right) + a^2\varphi = 0 \longrightarrow \varphi(x) = \frac{\mathbb{C}_1}{x}\cos a\,x + \frac{\mathbb{C}_2}{x}\sin a\,x$$

$$\frac{d^2\varphi}{dx^2} - a^2\varphi = 0 \longrightarrow \varphi(x) = \mathbb{C}_1 \cosh a\,x + \mathbb{C}_2 \sinh a\,x$$

or

$$\varphi(x) = \mathbb{C}_3 e^{+ax} + \mathbb{C}_4 e^{-ax}$$

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{d\varphi}{dx}\right) - a^2\varphi = 0 \longrightarrow \varphi(x) = \frac{\mathbb{C}_1}{x}\cosh a\,x + \frac{\mathbb{C}_2}{x}\sinh a\,x$$

$$x^2\frac{d^2\varphi}{dx^2} + ax\frac{d\varphi}{dx} + b\varphi = 0 \longrightarrow \varphi(x) = \mathbb{C}_1 x^{m_1} + \mathbb{C}_2 x^{m_2}$$

Where m_1 and m_2 are the roots of

$$m^2 + (a-1)m + b = 0$$

$$\frac{d^2\varphi}{dx^2} + 2x\frac{d\varphi}{dx} = 0 \longrightarrow \varphi(x) = \mathbb{C}_1\int_0^x e^{-x'^2}dx' + \mathbb{C}_2 = \mathbb{C}_1\frac{\sqrt{\pi}}{2}\operatorname{erf} x + \mathbb{C}_2$$

$$x^2\frac{d^2\varphi}{dx^2} + x\frac{d\varphi}{dx} + (x^2 - a^2)\varphi = 0 \longrightarrow \varphi(x) = \mathbb{C}_1 \mathrm{J}_a(x) + \mathbb{C}_2 \mathrm{Y}_a(x)$$

Where $\mathrm{J}_a(x)$ and $\mathrm{Y}_a(x)$ are Bessel functions