Solution exercise 2 – Week 2

Consider the following nonlinear equation:

$$g(x) = e^x - x - 2$$

a) Using the Newton-Raphson method find numerically the root of this equation with the precision better than 10^{-3}. Take as initial point $x_0=1$; the exact solution is $x^*=1.146193$.

First we derive g(x) :

$$g'(x) = e^x - 1$$

Second we use the definition of Newton Raphson to compute x1, x2,x3,... until we reach the desired accuracy:

$$x_i = x_{i-1} - \frac{e^{x_{i-1}} - x_{i-1} - 2}{e^{x_{i-1}} - 1}$$

$X_0 = 1.000$, $x_1=1.164$, $x_2 = 1.146$

b) Do three steps of Newton-Raphson iteration starting from $x_0=10$. Discuss shortly the convergence properties of the iterative scheme around this initial point.

$X_0 = 10.00$, $x_1=9.00$, $x_2 = 8.00$ & $x_3 = 7.00$

c) How many iterations would be needed to attain the accuracy of 10^{-9} if we use the bisection method on the interval (0, 5)?

$$error \leq \frac{b-a}{2^n}$$

$$2^n \leq \frac{b-a}{error}$$

$$n \leq \log_2 \frac{b-a}{error}$$

$$n \leq \log_2 \frac{5}{10^{-9}}$$

$$n \leq 32.22$$

so up to 33 iterations