Solution Exercise 3 – week 3

A car laps a race track in 54 seconds. The speed of the car at each 6-second interval is determined using a radar gun and is given, from the beginning of the lap, in meters per second, by the entries in the following table:

T (s)	0	6	12	18	24	30	36	42	48	54
v (m/s)	37.81	40.83	45.11	47.25	44.81	40.53	36.89	33.22	30.17	35.36

A) Determine the time points of the maximal acceleration and deceleration using the appropriate finite difference method

We will use the central difference method for all the interior points due to the lower truncation error when compared to the forward/backward difference method. The results of the differences are the following:

Time (s)	Speed (m/s)	Acceleration (m/s^2)
0	37.81	0.50
6	40.83	0.61
12	45.11	0.54
18	47.25	-0.025
24	44.81	-0.56
30	40.53	-0.66
36	36.89	-0.69
42	32.22	-0.56
48	30.17	0.26
54	35.36	0.865

From this we can see that the highest acceleration is at T=54 s, obtained using a backward difference method. The highest deceleration is at T=36s, obtained using the central difference method.

B) Taylor series analysis indicates that the error to approximate the derivative of a smooth function $f(x) = e^x$ by the forward finite difference should follow:

$$\left| f\prime(x) - \frac{f(x+h) - f(x)}{h} \right| = O(h)$$

however, after calculating the derivative of $f(x) = e^x$ at $x = 1$ in MATLAB with 16-decimal-digit accuracy for $h = 10^{-7}$ and $h = 10^{-14}$ the following was obtained:

$$\left| e^1 - \frac{e^{1+10^{-7}} - e^1}{10^{-7}} \right| \approx 1.3 \times 10^{-7}$$

and

$$\left|e^1 - \frac{e^{1+10^{-14}} - e^1}{10^{-14}}\right| \approx 0.053$$

<u>Explain the results above</u>

The answer for this lies in the fact that at $\mathrm{h} = 10^{-7}$, the truncation error still dominates over the round-off error, whereas at $\mathrm{h} = 10^{-14}$, the round-off error dominates, and we don't obtain a better result – as seen in the question, we actually obtain a worse result.