

Homework 3

Given the following function:

$$f(x) = |\sin(100x) + 0.5|$$

0. Plot the function $f(x)$ in $[0, 0.05]$ and highlight the area under the curve.
1. Compute $\int_a^b f(x) dx$ where $a = 0$ and $b = 0.05$ by using Simpson 1/3 and Simpson 3/8 rules (write these two new functions). Use the minimum number of intervals needed by each of the two methods and discuss their estimation errors.

The estimation error is defined as the difference between the integral obtained through these methods and the value returned by `scipy.integrate.quad()`.
2. Perform the previous computation by splitting the interval into 8 regions. Choose the most appropriate method between Simpson 1/3 and Simpson 3/8 rules and discuss your choice.
3. Solve the problem discussed in Exercise 2 using Romberg integration with the Trapezoid rule (see notebook section 3 for this function or `romb.py`). Use `max_iter = 3, 4, 5` and discuss the results (explain the function of the `max_iter` parameter). Compare with the previous exercise.
4. Compute $\int_a^b f(x) dx$, where $a = 0$ and $b = 0.05$, by splitting the interval into 13 regions. Choose the most appropriate method amongst the ones you previously used and discuss your choice.

Final remarks

- You should write and execute your code in Python: Use the given template `HW3_name_surname.ipynb` (Jupyter notebook)
- Your solution should be provided as a Jupyter notebook containing all needed functions. You should use Markdown cells to answer the “theoretical” questions in the notebook.
- Homework 3 should be delivered by **Friday, 25th of April 2025, 23:59** through the Moodle page.
- One submission per person is expected.
- Solutions are corrected and graded. The grades will be posted on the Moodle page.